Vacuum Stability Constraints.

in Models with Extended Scalar Sectors

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[Hollik, Weiglein, JW; 1812.04644]

[Ferreira, Mühlleitner, Santos, Weiglein, JW; 1905.xxxxx]

Vacuum Stability

The EW vacuum is a local minimum of the scalar potential. The universe has been in that state at least since BBN (so for $\sim T_H$).

- if it is the global minimum $\Rightarrow$ absolute stability
- if there are deeper minima
  - $\rightarrow$ lifetime of the EW vacuum $> T_H \Rightarrow$ long-lived metastability
  - $\rightarrow$ lifetime of the EW vacuum $< T_H \Rightarrow$ short-lived instability

Both absolute and metastability are fine with observations, but a short-lived EW vacuum is not.

This can be used to constrain models of particle physics.

Vacuum Stability in Scalar Potentials

The most general renormalizable scalar potential at tree-level is

$$V(\phi_a) = \lambda_{abcd}\phi_a\phi_b\phi_c\phi_d + A_{abc}\phi_a\phi_b\phi_c + m^2_{ab}\phi_a\phi_b + t_a\phi_a + c\,.$$

Expand around EW vacuum $\vec{\phi} \to \vec{v} + \vec{\varphi}$:

$$V(\varphi^a) = \lambda(\vec{v})_{abcd}\varphi_a\varphi_b\varphi_c\varphi_d + A(\vec{v})_{abc}\varphi_a\varphi_b\varphi_c + m^2(\vec{v})_{ab}\varphi_a\varphi_b\,.$$

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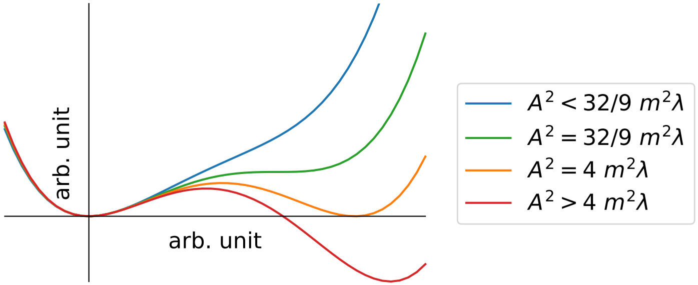
Introduce polar coordinates $\vec{\varphi} \to \varphi\hat{\varphi}$:

$$V(\varphi) = \lambda(\hat{\varphi})\varphi^4 - A(\hat{\varphi})\varphi^3 + m^2(\hat{\varphi})\varphi^2\,.$$

- $\lambda > 0$ for physical potentials (bounded from below)
- $A > 0$ by choice ($\varphi \leftrightarrow -\varphi$)
- $m^2 > 0$ if the EW-vacuum is a local minimum

Stability of Fieldspace Directions



- at most one additional minimum for each $\hat{\varphi}$
- the additional minimum is deeper if $A(\hat{\varphi})^2 > 4m^2(\hat{\varphi})\lambda(\hat{\varphi})$

We use polynomial homotopy continuation to find all stationary points and identify deep minima from there.

Lifetime and Vacuum Decay

The vacuum tunneling decay width is given by

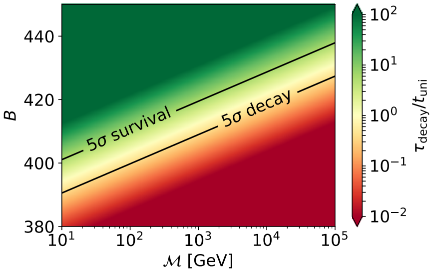
$$\frac{\Gamma}{V} = K e^{-B}$$

The bounce action B

> analytic solution in straight path approximation
[hep-ph/9302321, Adams]

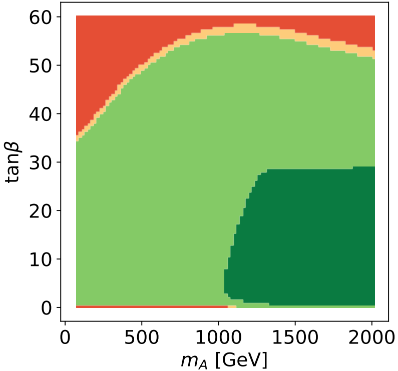
> associated uncertainty of $\mathcal{O}(10\%)$
[Masoumi, Olum, Wachter; 1702.00356]

The prefactor $K \sim \mathcal{M}^4$



$\Leftrightarrow B \in [390, 440] \sim 10\%$

Vacuum Stability of MSSM benchmark scenarios



[Hollik, Weiglein, JW; 1812.04644]

- stable, EW vacuum is the global minimum
- long-lived
- $390 < B < 440$
- short-lived

$M_h^{125}(\tilde{\tau})$-scenario [Bahl, et.al.; 1808.07542]

$$X_t = A_t - \frac{\mu}{\tan\beta} = 2.8\,\text{TeV}\,, \quad A_b = A_t\,, \quad \mu = 1\,\text{TeV}\,,$$

$$m_{Q_3,U_3,D_3} = 1.5\,\text{TeV}\,, \quad m_{L_3,E_3} = 350\,\text{GeV}\,, \quad A_\tau = 800\,\text{GeV}$$

The N2HDM

Extension of the SM by a second scalar doublet Φ_2 and a real scalar singlet S (*i.e.* 2HDM + real singlet):

$$V = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + h.c.\right)$$
$$+ \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + h.c.\right]$$
$$+ \frac{m_S^2}{2}S^2 + \frac{\lambda_6}{8}S^4 + \frac{\lambda_7}{2}|\Phi_1|^2S^2 + \frac{\lambda_8}{2}|\Phi_2|^2S^2\,.$$

> 2HDM: no $\not{C}\!\!P$ or $\not{e}$ vacua can coexist with the EW vacuum

[Barosso, Ferreira, Santos; hep-ph/0507224]

> this is no longer true in the N2HDM

[Mühlleitner, Sampaio, Santos, JW; 1612.01309]

Vacuum Structure of the N2HDM

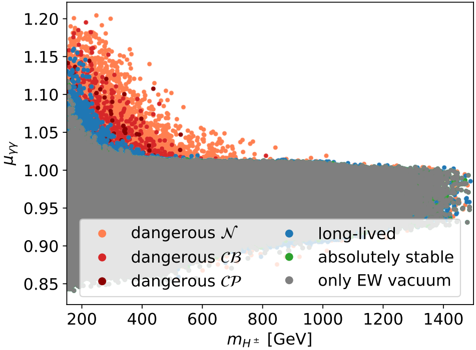
Most general N2HDM vacuum configuration:

$$\Phi_1 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \Phi_2 = \begin{pmatrix} v_{\text{CB}} \\ v_2 + \mathrm{i} v_{\text{CP}} \end{pmatrix} \quad S = v_s$$

N2HDM-like	2HDM-like
$\mathcal{N}s = (v_1, v_2, v_s)$	$\mathcal{N} = (v_1, v_2)$
$\mathcal{CB}s = (v_1, v_2, v_s, v_{\text{CB}})$	$\mathcal{CB} = (v_1, v_2, v_{\text{CB}})$
$\mathcal{CP}s = (v_1, v_2, v_s, v_{\text{CP}})$	$\mathcal{CP} = (v_1, v_2, v_{\text{CP}})$
$S = (v_s)$	

An $\mathcal{N}s$ EW vacuum can coexist with possibly deeper $\mathcal{N}$, $\mathcal{CB}$, $\mathcal{CP}$, S, and $\mathcal{N}s$ minima.

Vacuum Stability constraints in the N2HDM



$$\mu_{\gamma\gamma} = \frac{\sigma_{pp\to h_{125}\to\gamma\gamma}}{\sigma_{pp\to h_{\mathrm{SM}}\to\gamma\gamma}}$$

> all points fulfill all other constraints

[Ferreira, Mühlleitner, Santos, Weiglein, JW; 1905.xxxxx]

Vacuum stability excludes enhancements of $\mu_{\gamma\gamma}$ unless $m_{H^\pm}$ is very small.

Conclusions

Vacuum stability provides important constraints on the parameter space of models with large scalar sectors and can provide complementary constraints to experimental searches.

- Vacuum stability constraints can put direct constraints on collider observables.
- Tree-level, straight-path tunnelling is a good enough approximation to derive stability constraints.

We aim to provide a tool that provides efficient and reliable bounds from vacuum stability in any renormalizable model.

Upcoming paper: [Ferreira, Mühlleitner, Santos, Weiglein, JW; 1905.xxxxx]
The code will be published later this year.

Thanks for your attention.

The Scalar Sector of the MSSM

In SUSY theories every SM fermion gains a scalar superpartner.

In the MSSM the scalar potential including only the real, neutral Higgs and real $\widetilde{t}_r,\ \widetilde{t}_l$ fields reads

$$V((h_u)^{0,r},(h_d)^{0,r},(\widetilde{t}_r)^r,(\widetilde{t}_l)^r) =$$

$$\frac{g_1^2}{288}\left(3(h_u^2-h_d^2)+\widetilde{t}_l^{\,2}-4\widetilde{t}_r^{\,2}\right)^2+\frac{g_2^2}{32}\left(h_d^2-h_u^2+\widetilde{t}_l^{\,2}\right)^2+\frac{g_3^2}{24}\left(\widetilde{t}_l^{\,2}-\widetilde{t}_r^{\,2}\right)^2$$

$$+\frac{y_t^2}{4}\left(h_u^2(\widetilde{t}_l^{\,2}+\widetilde{t}_r^{\,2})+\widetilde{t}_l^{\,2}\widetilde{t}_r^{\,2}\right)+\frac{y_t}{\sqrt{2}}(A_t h_u-\mu h_d)\widetilde{t}_l\widetilde{t}_r$$

$$+\frac{g_1^2+g_2^2}{16}\left(v_0^2 c_{2\beta}(h_u^2-h_d^2)\right)+\frac{m_A^2}{2}\left(c_\beta h_u-s_\beta h_d\right)^2+\frac{m_{Q_3}^2}{2}\widetilde{t}_l^{\,2}+\frac{m_{U_3}^2}{2}\widetilde{t}_r^{\,2}$$

Tree-level — the 1-loop effective potential has numerical and theoretical issues.

[Andreassen, Farhi, Frost, Schwartz; 1604.06090]

Considered Field Sets in the MSSM

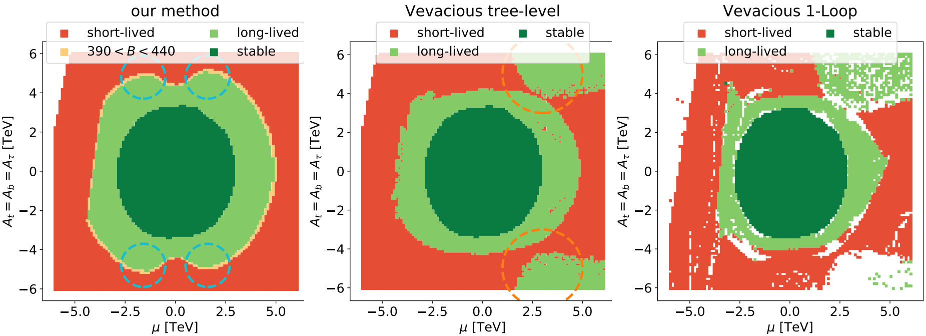
We look for stationary points using these three sets of fields:

$$\left\{\mathrm{Re}(h_u^0),\, \mathrm{Re}(h_d^0),\, \mathrm{Re}(\tilde{t}_L),\, \mathrm{Re}(\tilde{t}_R),\, \mathrm{Re}(\tilde{b}_L),\, \mathrm{Re}(\tilde{b}_R)\right\}$$

$$\left\{\mathrm{Re}(h_u^0),\, \mathrm{Re}(h_d^0),\, \mathrm{Re}(\tilde{t}_L),\, \mathrm{Re}(\tilde{t}_R),\, \mathrm{Re}(\tilde{\tau}_L),\, \mathrm{Re}(\tilde{\tau}_R)\right\}$$

$$\left\{\mathrm{Re}(h_u^0),\, \mathrm{Re}(h_d^0),\, \mathrm{Re}(\tilde{b}_L),\, \mathrm{Re}(\tilde{b}_R),\, \mathrm{Re}(\tilde{\tau}_L),\, \mathrm{Re}(\tilde{\tau}_R)\right\}$$

Impact of the 1-loop Effective Potential



Finding Deep Directions in Fieldspace

1. Solve $\vec{\nabla}_\phi V = 0$ to find all stationary points using polynomial homotopy continuation (PHC).
2. Compare the potential values at each stationary point to the value at the EW vacuum.
3. Get $\lambda(\hat{\varphi})$, $A(\hat{\varphi})$ and $m^2(\hat{\varphi})$ for $\hat{\varphi}$ pointing towards each deeper stationary point.

PHC is in theory guaranteed to find all isolated solutions.

> All gauge redundancies need to be removed from the scalar potential.

Increased reliability if the system ir rescaled to reduce coefficient variability.

Perturbative Expansion of the Bounce

$$V(\phi) = \lambda\phi^4 \quad \text{with } \lambda < 0$$

Analytic bounce solution

$$\phi_c(\rho) = \sqrt{-\frac{2}{\lambda}}\frac{R}{R^2+\rho^2} \;\Rightarrow\; B = -\frac{2\pi}{3\lambda}$$

The 1-loop effective action up to two derivatives is

$$S_{\text{eff}} = \underbrace{\lambda\phi^4}_{\text{LO}} + \underbrace{\frac{9\lambda^2}{4\pi^2}\phi^2\left(\ln\frac{12\lambda\phi^2}{\mu^2}-\frac{3}{2}\right)}_{\text{1L eff. potential}} + \underbrace{\frac{(\partial_\mu\phi)^2}{2}\left(1+\frac{\lambda}{4\pi^2}\right)}_{\text{1L } p^2} + \underbrace{\mathcal{O}(\partial^4)}_{\text{1L } p^4}$$

Leading to

$$B = -\frac{2\pi}{3\lambda} + 3\ln\frac{R\mu}{2\sqrt{6}} + \frac{19}{4} + \frac{1}{3} + \mathcal{O}(1)$$

[Andreassen, Farhi, Frost, Schwartz; 1604.06090]