Lattice spectroscopy of SU(2) Adjoint Higgs model

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Der Wissenschaftsfonds.

Natural Sciences

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- FMS example: SU(N) + fundamental Higgs

$$O_{0^+}(x) = (\phi^\dagger \phi)(x)$$

- Fix the gauge to non-vanishing vev: $\phi(x) = \frac{v}{\sqrt{2}} n + h(x)$
- Expand the correlator

$$\langle O^\dagger_{0^+}(x) O_{0^+}(y) \rangle = const. + 4v^2 \langle h(x)^\dagger h(y) \rangle + O(h^4)\,.$$

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- Poles of bound states are at same position as elementary fields. [Maas,Mufti-1412.6440(hep-lat)]
- FMS mechanism can be applied to fermions (no lattice results so far).
- Exist examples with no correspondence. [Maas,Sondenheimer,Toerek-1709.07477(hep-ph)]

SU(2) Gauge theory coupled with an adjoint Scalar

- The Lagrangian of the theory:

$$\mathcal{L} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + \text{tr}\left[(D_\mu \Phi)^\dagger (D^\mu \Phi) \right] - V(\Phi)\,.$$

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- $\Phi(x) = \Phi^a(x)T^a = \Phi^a(x)\sigma^a/2$ is the scalar field in the adjoint representation.
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- Transformation of the field: $\Phi(x) \to U(x)\Phi(x)U(x)^\dagger$.
- Potential spanned by invariant Casimirs of the gauge group, E.g:

$$V = -\mu^2 \,\mathrm{tr}\, \Phi^2 + \frac{\lambda}{2} (\mathrm{tr}\, \Phi^2)^2 \,.$$

- Center symmetry Z_2^U, Custodial symmetry Z_2^Φ.

Brout-Englert-Higgs Effect

- (We look for potentials that allow the BEH effect.)
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$$(M_A^2)^{ab} = -2(gw)^2 \, \mathrm{tr}\Big([T^a, \Phi_0][T^b, \Phi_0]\Big)\,.$$

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- To see whether a gauge boson acquires a mass we must check if the generator associated commutes with Φ_0.
- The relevant breaking pattern lead to a potential with a minimum. In this case $SU(2)\to U(1)$.

Gauge invariant operators for the SU(2) adjoint Higgs

Construct operators which expand to a single gauge field in leading order:

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$$\begin{aligned} O^{\mu}_{1^-} &= -\, w^2 \operatorname{tr}\left[\Phi_0 A^{\mu}_{\perp}\right](x) + \mathcal{O}(A^2, \phi) \\ &= -\, w^2 \operatorname{tr}\left[\Phi_0 (\delta^{\mu}_{\nu} - \partial^{\mu}\partial_{\nu}/\partial^2) A^{\nu}\right](x) + \mathcal{O}(A^2, \phi)\,. \end{aligned}$$

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Scalar channel:

- $O_{0^+}(x) = \operatorname{tr}\left[\Phi^2\right](x)$.
- $H(x) = \Phi^a_0 \phi^a(x)$.
- FMS: $O_{0^+}(x) = \frac{w^2}{2} + wH(x) + \frac{1}{2}\phi^a(x)\phi^a(x)$.

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FMS mechanism:

- A mass for the scalar ground state m_H^2.
- One massless gauge boson (it comes from the first order expansion).
- A massive scattering state with mass $2m_A^2$ (it comes from an expansion to higher orders).
  [Maas,Sondenheimer,Toerek-1709.07477(hep-ph)]

Operator for lattice spectroscopy

- Massless bound state is predicted by FMS.

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- Lattice operator:
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- We give the operator a non-zero momentum via

$$B^j(\vec{p},t) = \frac{1}{\sqrt{V_{\vec{x}}}}\,\mathrm{Re}\sum_{\vec{x}} B^j(\vec{x},t)e^{i\vec{p}\cdot\vec{x}}$$

- We chose as momentum the smallest one in the z direction

$$\vec{p}_z = \left(0, 0, \frac{2\pi}{N_z}\right)$$

Transverse and Longitudinal Correlator

We split the correlator in the transverse and the longitudinal part

$$C_\perp(t) = \frac{1}{N_t}\sum_{t'=0}^{L_t-1}\sum_{j=1}^{2}\langle B^j(\vec{p}_z, t')B^j(\vec{p}_z, t+t')\rangle$$

$$C_\parallel(t) = \frac{1}{N_t}\sum_{t'=0}^{L_t-1}\langle B^3(\vec{p}_z, t')B^3(\vec{p}_z, t+t')\rangle$$

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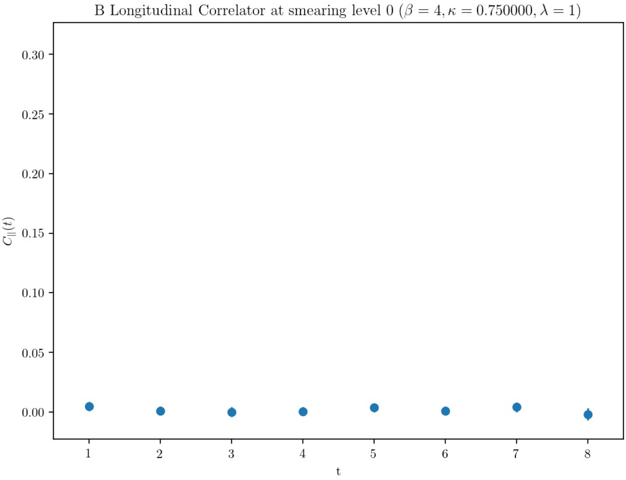
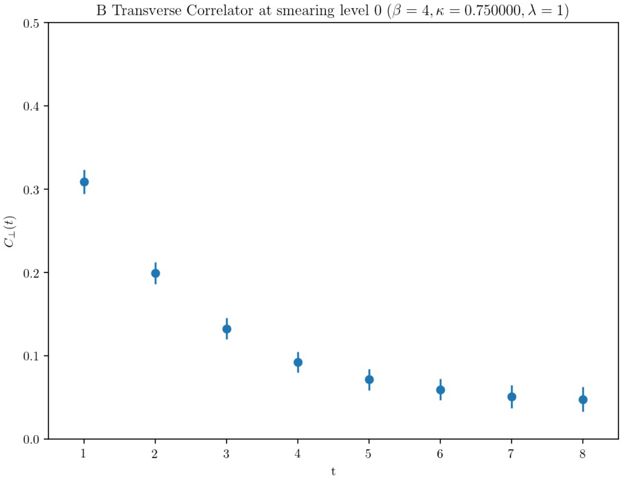
We expect the correlators to behave as

$$C(t) \propto \exp(-Et)$$


B Longitudinal Correlator at smearing level 0 ($\beta = 4, \kappa = 0.750000, \lambda = 1$)
$C_{\parallel}(t)$
0.30
0.25
0.20
0.15
0.10
0.05
0.00
1
2
3
4
5
6
7
8
t


B Transverse Correlator at smearing level 0 ($\beta = 4, \kappa = 0.750000, \lambda = 1$)
$C_\perp(t)$
0.0
0.1
0.2
0.3
0.4
0.5
1
2
3
4
5
6
7
8
t

Massless state investigation

For a massless state, we expect

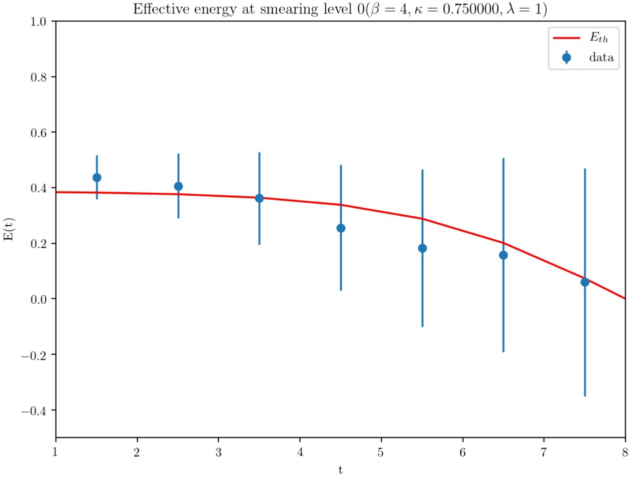
$$E(\vec{P_z}) = |\vec{P_z}| = \frac{2\pi}{16} = \frac{\pi}{8}$$

We use the quantity

$$E_{eff}(t+0.5) = \log\left(\frac{C_\perp(t)}{C_\perp(t+1)}\right)$$

We plot also the expected value, with the corrected cosh behaviour.

Preliminary spectroscopy results



Conclusions

We have found very good hints of a massless vector state present in the theory.

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- Extend our analysis to BSM models with larger gauge groups guided by FMS mechanism.
- Make analytic predictions coming from gauge invariant perturbation theory and confront them with the phenomenology. Review: [Maas-1712.04721(hep-ph)]

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