Lattice spectroscopy of SU(2) Adjoint Higgs model

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- FMS example: SU(N) + fundamental Higgs

\[ O_{0+}(x) = (\phi^\dagger \phi)(x) \]

- Fix the gauge to non-vanishing vev: $\phi(x) = \frac{v}{\sqrt{2}} n + h(x)$
- Expand the correlator

\[ \langle O_{0+}^\dagger(x) O_{0+}(y) \rangle = \text{const.} + 4v^2 \langle h(x)^\dagger h(y) \rangle + O(h^4). \]
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SU(2) Gauge theory coupled with an adjoint Scalar

- The Lagrangian of the theory:

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \text{tr} \left[ (D_\mu \Phi)^\dagger (D^\mu \Phi) \right] - V(\Phi). \]
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- \( \Phi(x) = \Phi^a(x) T^a = \Phi^a(x) \sigma^a / 2 \) is the scalar field in the adjoint representation.

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- Potential spanned by invariant Casimirs of the gauge group, e.g:

\[ V = -\mu^2 \text{tr} \Phi^2 + \frac{\lambda}{2} (\text{tr} \Phi^2)^2. \]

- Center symmetry \( Z_2^U \), Custodial symmetry \( Z_2^\Phi \).
(We look for potentials that allow the BEH effect.)

Split scalar field in vev and fluctuations:

$$\Phi(x) = \langle \Phi \rangle + \phi(x) \equiv w\Phi_0 + \phi(x).$$
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- To see whether a gauge boson acquires a mass we must check if the generator associated commutes with \( \Phi_0 \).
- The relevant breaking pattern lead to a potential with a minimum. In this case \( SU(2) \rightarrow U(1) \).
Gauge invariant operators for the SU(2) adjoint Higgs

Construct operators which expand to a single gauge field in leading order:

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- FMS:

\[
O_{1-}^\mu = -w^2 \text{tr} [\Phi_0 A_{\perp}^\mu] (x) + \mathcal{O}(A^2, \phi) \\
= -w^2 \text{tr} [\Phi_0 (\delta_{\nu}^\mu - \partial^\mu \partial_\nu / \partial^2) A^\nu] (x) + \mathcal{O}(A^2, \phi).
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- $H(x) = \Phi_0^a \phi^a(x)$.
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FMS mechanism:
- A mass for the scalar ground state $m_H^2$.
- One massless gauge boson (it comes from the first order expansion).
- A massive scattering state with mass $2m_A^2$ (it comes from an expansion to higher orders).

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Operator for lattice spectroscopy

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- Lattice operator:
  \[ B^i(x) = \frac{1}{\sqrt{2 \text{Tr}(\Phi^2)}} \text{Im} \text{Tr} \left( \Phi(\vec{x}, t) U^{jk}(\vec{x}, t) \right) \]

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Lattice operator:

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We give the operator a non-zero momentum via

\[ B^j(\vec{p}, t) = \frac{1}{\sqrt{V}} \text{Re} \sum_{\vec{x}} B^j(\vec{x}, t)e^{i\vec{p}\cdot\vec{x}} \]

We chose as momentum the smallest one in the z direction

\[ \vec{p}_z = \left(0, 0, \frac{2\pi}{N_z}\right) \]
Transverse and Longitudinal Correlator

We split the correlator in the transverse and the longitudinal part

\[ C_\perp(t) = \frac{1}{N_t} \sum_{t'=0}^{L_t-1} \sum_{j=1}^{2} \langle B^j(\vec{p}_z, t') B^j(\vec{p}_z, t + t') \rangle \]

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We expect the correlators to behave as

\[
C(t) \propto \exp(-Et)
\]
B Longitudinal Correlator at smearing level 0 ($\beta = 4, \kappa = 0.750000, \lambda = 1$)
B Transverse Correlator at smearing level 0 ($\beta = 4, \kappa = 0.750000, \lambda = 1$)
Massless state investigation

For a massless state, we expect

$$E(\vec{P}_z) = |\vec{P}_z| = \frac{2\pi}{16} = \frac{\pi}{8}$$

We use the quantity

$$E_{\text{eff}}(t + 0.5) = \log \left( \frac{C_\perp(t)}{C_\perp(t + 1)} \right)$$

We plot also the expected value, with the corrected cosh behaviour.
Preliminary spectroscopy results

Effective energy at smearing level $0(\beta = 4, \kappa = 0.750000, \lambda = 1)$
Conclusions

We have found very good hints of a massless vector state present in the theory.

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Outlook:

- Extend our analysis to BSM models with larger gauge groups guided by FMS mechanism.
- Make analytic predictions coming from gauge invariant perturbation theory and confront them with the phenomenology. Review: [Maas-1712.04721(hep-ph)]
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