

# Bottom and strange Yukawa couplings at the NNLO in the MSSM

Margherita Ghezzi



ALPS 2019, Obergurgl, 26/04/2019

# Introduction

## The MSSM Higgs sector

- 2 Higgs doublets, 5 physical Higgs bosons
- 2 input parameters at LO:  $M_A$  and  $\text{tg}\beta \equiv \frac{v_2}{v_1}$
- couplings of the Higgs bosons:

$\Phi$		$g_u^\Phi$	$g_d^\Phi$	$g_V^\Phi$
SM	$H$	1	1	1
MSSM	$h$	$\cos \alpha / \sin \beta$	$-\sin \alpha / \cos \beta$	$\sin(\beta - \alpha)$
	$H$	$\sin \alpha / \sin \beta$	$\cos \alpha / \cos \beta$	$\cos(\beta - \alpha)$
	$A$	$1/\text{tg}\beta$	$\text{tg}\beta$	0

- For large  $\text{tg}\beta$  the down-type Yukawa couplings are enhanced
- Higgs decay to  $b\bar{b}$  and Higgs bremsstrahlung from b quarks are dominant

# Introduction

## Yukawa couplings in the large $\tan\beta$ case

- Down-type Yukawa couplings are enhanced
- LR mixing terms in the squark mass matrices can be enhanced
- Soft SUSY breaking:  $-\lambda_u A_u \phi_2 \tilde{u}_R^\dagger \tilde{Q}_L - \lambda_d A_d \phi_1 \tilde{d}_R^\dagger \tilde{Q}_L$

$$\mathcal{M}_{\tilde{f}}^2 = \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{RL}^2 & M_{RR}^2 \end{pmatrix} = \begin{pmatrix} M_{\tilde{f}_L}^2 + m_f^2 & m_f(A_f - \mu r_f) \\ m_f(A_f - \mu r_f) & M_{\tilde{f}_R}^2 + m_f^2 \end{pmatrix}$$

with  $r_d = 1/r_u = \tan\beta = \frac{v_2}{v_1}$

# Effective Lagrangian

$$\begin{aligned}
 \mathcal{L}_{eff} &= - \sum_{q=b,s} \lambda_q \bar{q}_R [(1 + \Delta_{q,1}) \phi_1^0 + \Delta_{q,2} \phi_2^{0*}] q_L + h.c. \\
 &= - \sum_{q=b,s} m_q \bar{q} \left[ 1 + i\gamma_5 \frac{G^0}{v} \right] q - \frac{m_q/v}{1 + \Delta_q} \bar{q} \left[ g_q^h \left( 1 - \frac{\Delta_q}{\text{tg}\alpha \text{tg}\beta} \right) h \right. \\
 &\quad \left. + g_q^H \left( 1 + \Delta_q \frac{\text{tg}\alpha}{\text{tg}\beta} \right) H - g_q^A \left( 1 - \frac{\Delta_q}{\text{tg}^2\beta} \right) i\gamma_5 A \right] q
 \end{aligned}$$

1-loop contributions for the bottom:

$$\begin{aligned}
 \Delta_{b,1} &= -\frac{C_F}{2} \frac{\alpha_s(\mu_R)}{\pi} m_{\tilde{g}} A_b I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) \\
 \Delta_{b,2} &= \Delta_{b,2}^{QCD} + \Delta_{b,2}^{elw} \\
 \Delta_{b,2}^{QCD} &= \frac{C_F}{2} \frac{\alpha_s(\mu_R)}{\pi} m_{\tilde{g}} \mu I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) \\
 \Delta_{b,2}^{elw} &= \frac{\lambda_t^2(\mu_R)}{(4\pi)^2} A_t \mu I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, \mu^2)
 \end{aligned}$$

$$\Delta_q = \frac{\Delta_{q,2} \text{tg}\beta}{1 + \Delta_{q,1}}$$

$$I(a, b, c) = \frac{ab \log \frac{a}{b} + bc \log \frac{b}{c} + ca \log \frac{c}{a}}{(a-b)(b-c)(a-c)}$$

Carena, Garcia, Nierste and Wagner, Nucl.Phys. B577 (2000) 88  
 Guasch, Häfliger and Spira, Phys.Rev. D68 (2003) 115001

# Resummation

The corrections  $\Delta_{b,s}$  induce a modification of the relation between the mass  $m_b$  ( $m_s$ ) and the Yukawa coupling  $\lambda_b$  ( $\lambda_s$ ):

$$m_q = \frac{\lambda_q v_1}{\sqrt{2}} [1 + \Delta_{q,1} + \Delta_{q,2} \operatorname{tg}\beta] \quad (q = b, s)$$

Resummed couplings:

$$\mathcal{L}_{\text{eff}} = - \sum_{q=b,s} \frac{m_q}{v} \bar{q} [ \tilde{g}_q^h h + \tilde{g}_q^H H - \tilde{g}_q^A i\gamma_5 A ] q$$

$$\begin{aligned} \tilde{g}_q^h &= \frac{g_q^h}{1 + \Delta_q} \left[ 1 - \frac{\Delta_q}{\operatorname{tg}\alpha \operatorname{tg}\beta} \right] \\ \tilde{g}_q^H &= \frac{g_q^H}{1 + \Delta_q} \left[ 1 + \Delta_q \frac{\operatorname{tg}\alpha}{\operatorname{tg}\beta} \right] \\ \tilde{g}_q^A &= \frac{g_q^A}{1 + \Delta_q} \left[ 1 - \frac{\Delta_q}{\operatorname{tg}\beta^2} \right] \end{aligned} \quad \Delta_q = \frac{\Delta_{q,2} \operatorname{tg}\beta}{1 + \Delta_{q,1}}$$

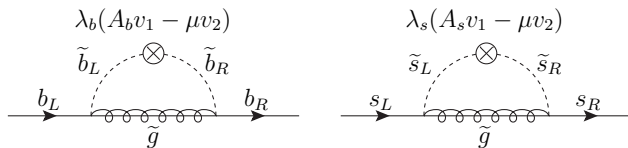
Carena, Garcia, Nierste and Wagner, Nucl.Phys. B577 (2000) 88  
 Guasch, Häfliger and Spira, Phys.Rev. D68 (2003) 115001

# Low Energy theorem

- Calculate the self-energies at vanishing momentum
- Substitute:

$$v_1 \rightarrow \sqrt{2}\phi_1^0$$

$$v_2 \rightarrow \sqrt{2}\phi_2^{0*}$$



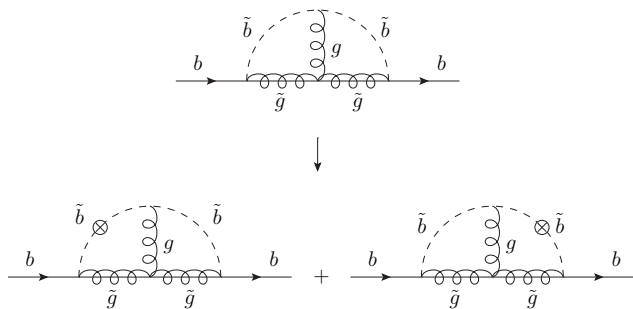
$$m_q = \frac{\lambda_q}{\sqrt{2}} v_1 + \Sigma_S(m_q^2)$$

$$\Sigma_S(m_q^2) = \frac{\lambda_b}{\sqrt{2}} v_1 [\Delta_{q,1} + \Delta_{q,2} \text{tg}\beta]$$

Ellis, Gaillard and Nanopoulos, Nucl.Phys. B106 (1976) 292

Shifman, Vainshtein, Voloshin and Zakharov, Sov.J.Nucl.Phys.30 (1979) 711

# Bottom Yukawa coupling at NNLO



Extract leading terms for large  $\tan\beta$ :

$$m_b = \frac{\lambda_b}{\sqrt{2}} v_1 + \Sigma_S(m_b)$$

$$\Sigma_S(m_b) = \frac{\lambda_b}{\sqrt{2}} v_1 (\Delta_{b,1} + \Delta_{b,2} \tan\beta)$$

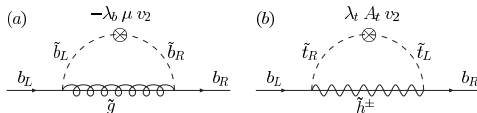
Mihaila and Reisser, JHEP 1008 (2010) 021

Noth and Spira, JHEP 1106 (2011) 084

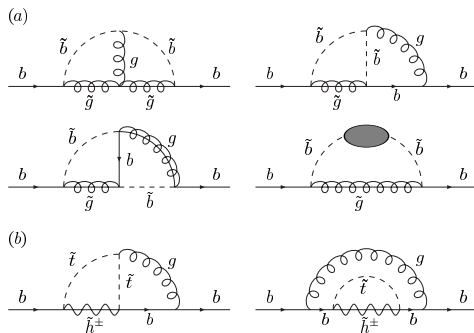
Crivellin and Greub, Phys.Rev.D 87 (2013) 015013

# Top-induced corrections

NLO



NNLO





## Calculation: technical details

- **dimensional regularization** in  $n = 4 - 2\epsilon$  dimensions
- reduction to **master integrals**:

$$A_0(m) = \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - m^2}$$

$$T_{134}(m_1, m_3, m_4) = \int \frac{d^n k}{(2\pi)^n} \frac{d^n q}{(2\pi)^n} \frac{1}{(k^2 - m_1^2)[(k - q)^2 - m_3^2](q^2 - m_4^2)}$$

- dimensional regularization violates SUSY  $\Rightarrow$  **anomalous counter terms**

$$\hat{g}_s = g_s \left[ 1 + \left( \frac{C_A}{6} - \frac{C_F}{8} \right) \frac{\alpha_s}{\pi} \right]$$

$$\lambda_{Hbb} = \lambda_{\tilde{H}\tilde{b}\tilde{b}} \left( 1 + \frac{C_F}{4} \frac{\alpha_s}{\pi} \right) = \lambda_{\tilde{H}\tilde{b}\tilde{b}} \left( 1 + \frac{3C_F}{8} \frac{\alpha_s}{\pi} \right)$$

Martin and Vaughn, Phys.Lett. B318 (1993) 331-337

# Renormalization

- 1-loop corrections are finite
- 2-loop corrections are UV-divergent
- Counter terms:

$$m_{\tilde{q}_i}^{02} = m_{\tilde{q}_i}^2 + \delta m_{\tilde{q}_i}^2$$

$$m_{\tilde{g}}^0 = m_{\tilde{g}} + \delta m_{\tilde{g}}$$

$$\lambda_t^0 = \lambda_t(\mu_R) + \delta \lambda_t$$

$$\alpha_s^0 = \alpha_s(\mu_R) + \delta \alpha_s$$

$$A_t^0 = A_t + \delta A_t$$

- $m_{\tilde{b}_i}, m_{\tilde{t}_i}, m_{\tilde{g}}$ : on-shell
- $A_t$ : on-shell
- $\alpha_s, \lambda_t$ :  $\overline{\text{MS}}$  with 5 flavours

- Anomalous counter terms (restoring SUSY)

# Bottom Yukawa coupling at NNLO

- After renormalization the SUSY-QCD corrections to the  $\Delta_{b,1}$  terms are the same as for the  $\Delta_{b,2}$ :

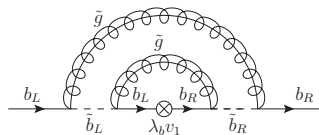
$$\Delta_{b,2}^{QCD} = \mu \Delta^{NLO} [1 + \delta_1]$$

$$\Delta_b = \frac{\mu \operatorname{tg}\beta \Delta^{NLO} [1 + \delta_1] + \Delta_{b,2}^{elw} \operatorname{tg}\beta [1 + \delta_2]}{1 - A_b^0 \Delta^{NLO} [1 + \delta_1]}$$

- Renormalization of  $A_b^0$ :

$$A_b^0 = A_b(\mu_R^2) + \delta A_b$$

$$\delta A_b = C_F \frac{\alpha_s}{\pi} \Gamma(1 + \epsilon) \left( \frac{4\pi\mu^2}{\mu_R^2} \right)^\epsilon \frac{m_{\tilde{g}}}{\epsilon} \neq \mathcal{O}(A_b)$$



Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

# Strange Yukawa coupling at NNLO

- Same calculation as for the  $b$ , as both  $b$  and  $s$  are considered **massless**
- absence of sizeable SUSY–electroweak contributions, since  $\lambda_c = 0$
- resummed terms:
  - $\mathcal{O}[(\alpha_s \mu \operatorname{tg}\beta)^n]$ ,  $\mathcal{O}[\alpha_s^{n+1} (\mu \operatorname{tg}\beta)^n]$
  - $\mathcal{O}[(\alpha_s A_s)^n]$ ,  $\mathcal{O}[\alpha_s^{n+1} A_s^n]$

$$\Delta_s = \frac{\Delta_{s,2} \operatorname{tg}\beta [1 + \delta_s]}{1 + \Delta_{s,1} [1 + \delta_s]}$$

Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

# Resummation

$$\Delta_q = \frac{\Delta_{q,2} \operatorname{tg}\beta}{1 + \Delta_{q,1}}$$

- Resummation of **tg $\beta$ -enhanced** contributions:
  - $\mathcal{O}((\alpha_s \mu \operatorname{tg}\beta)^n)$ @NLO
  - $\mathcal{O}(\alpha_s^{n+1} (\mu \operatorname{tg}\beta)^n)$ @NNLO
- Resummation of **top-induced tg $\beta$ -enhanced** contributions:
  - $\mathcal{O}[(\lambda_t^2 A_t \operatorname{tg}\beta)^n]$ @NLO
  - $\mathcal{O}[\alpha_s (\lambda_t^2 A_t \operatorname{tg}\beta)^n]$ @NNLO
- Resummation of  **$A_{b,s}$ -induced** terms (can be important when  $A_q \simeq \mu \operatorname{tg}\beta$ ):
  - $\mathcal{O}((\alpha_s A_{b,s})^n)$ @NLO
  - $\mathcal{O}(\alpha_s^{n+1} (A_{b,s})^n)$ @NNLO

# Partial decay rates

Neutral Higgs bosons decay to  $b\bar{b}$

$$\Gamma[\Phi \rightarrow b\bar{b}] = \frac{3G_F M_\Phi}{4\sqrt{2}\pi} \bar{m}_b^2(M_\Phi) \left[ 1 + \delta_{\text{QCD}} + \delta_t^\Phi \right] \tilde{g}_b^\Phi \left[ \tilde{g}_b^\Phi + \delta_{\text{SQCD}}^{\text{rem}} \right] \quad \Phi = h, H, A$$

Charged Higgs decay to  $c\bar{s}$  (and production...)

$$\Gamma[H^\pm \rightarrow c\bar{s}] = \frac{3G_F M_{H^\pm}}{4\sqrt{2}\pi} |V_{cs}|^2 \left[ \bar{m}_c^2(M_{H^\pm})(g_c^A)^2 + \bar{m}_s^2(M_{H^\pm})(\tilde{g}_s^A)^2 \right] (1 + \delta_{\text{QCD}})$$

- $\delta_{\text{QCD}}$ : QCD corrections, N<sup>4</sup>LO
- $\delta_t$ : top-induced correction, NNLO
- $\tilde{g}_{b,s}^\Phi$ : resummed Yukawa couplings
- $\delta_{\text{SQCD}}^{\text{rem}}$ : remainder from the  $\tilde{g}_b^\Phi$  resummation

# Benchmark scenarios

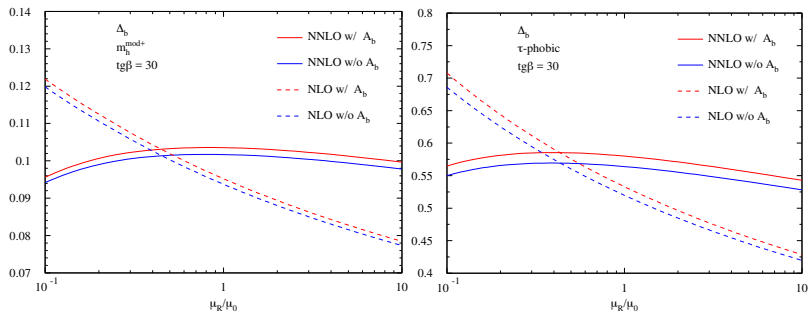
$m_h^{mod+}$  scenario:

$$\begin{aligned} \tan\beta = 30, \quad M_{\tilde{Q}} = 1 \text{ TeV}, \quad M_{\tilde{\ell}_3} = 1 \text{ TeV}, \quad M_{\tilde{g}} = 1.5 \text{ TeV}, \\ M_2 = 200 \text{ GeV}, \quad A_b = A_\tau = A_t = 1.607 \text{ TeV}, \quad \mu = 200 \text{ GeV}. \end{aligned}$$

$\tau$ -phobic scenario:

$$\begin{aligned} \tan\beta = 30, \quad M_{\tilde{Q}} = 1.5 \text{ TeV}, \quad M_{\tilde{\ell}_3} = 500 \text{ GeV}, \quad M_{\tilde{g}} = 1.5 \text{ TeV}, \\ M_2 = 200 \text{ GeV}, \quad A_b = A_t = A_\tau = 4.417 \text{ TeV}, \quad \mu = 2 \text{ TeV}. \end{aligned}$$

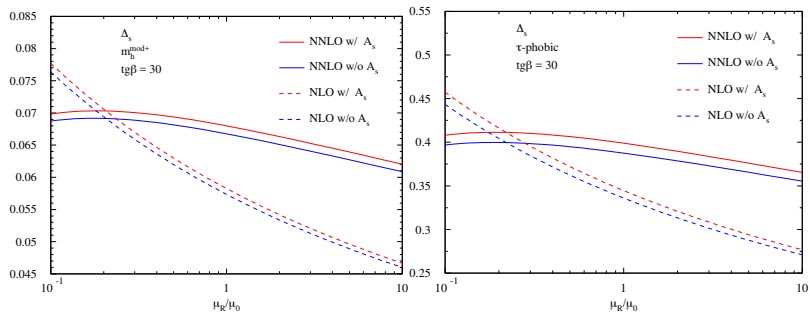
# Scale dependence of $\Delta_b$



Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

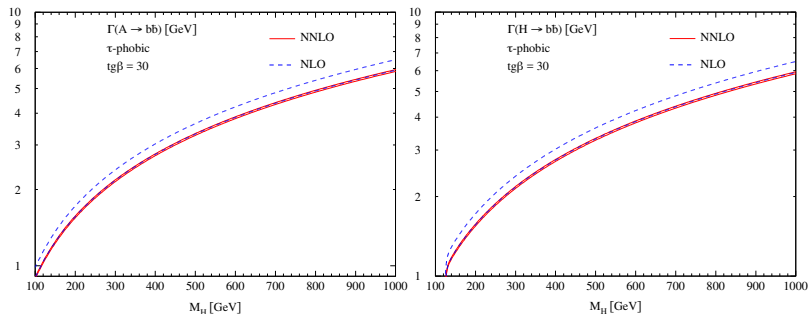


# Scale dependence of $\Delta_s$



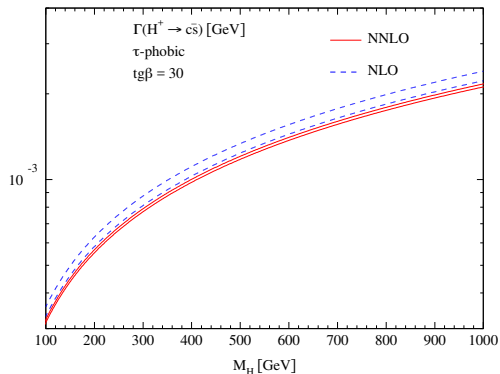
Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

# Partial decay widths of the neutral heavy Higgses



Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

# Partial decay width of the charged Higgs



Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

# Summary

- In **large- $tg\beta$**  MSSM scenarios the down-type Yukawa couplings to the heavy Higgs bosons receive large corrections.
- These  $tg\beta$ -enhanced corrections can be resummed in **effective Yukawa couplings**.
- They can be extracted from the calculation of self-energies with contributions from the off-diagonal terms of the squark mass matrices (**LET**).
- As the off-diagonal terms of the squark mass matrices are proportional to  $A_q - \mu tg\beta$ , the resummation of  **$tg\beta$ -enhanced** corrections and  **$A_q$ -induced** corrections go hand in hand.
- The calculation has been performed up to **NNLO**.
- The scale dependence was  $\mathcal{O}(10\%)$  for the NLO predictions and is currently  **$\mathcal{O}(1\%)$**  for the **NNLO**.