Emergent Gauge Fields in Holographic Superconductors

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based on

O. Domènech, M. Montull, A. Pomarol, A. S. and P. J. Silva:
arXiv:1005.1776

Effective theories of superconductors

A superconductor is a material in which $U(1)_{\rm em}$ is spontaneously broken.

dynamical fields: $a_\mu \equiv (a_0, a_i)\,,\ \Phi_{\rm cl}$

for time-independent configurations and without electric fields

$$\textit{free energy} = F = \int d^{d-1}x\, \mathcal{L}_{\rm eff}\left(\mathcal{F}_{ij}^2, |D_i\Phi_{\rm cl}|^2, |\Phi_{\rm cl}|, ...\right)$$

For small enough fields we expect a Ginzburg-Landau (GL) free energy:

$$F_{\rm GL} = \int d^{d-1}x \left\{ \frac{1}{4e_0^2}\mathcal{F}_{ij}^2 + |D_i\Phi_{\rm GL}|^2 + V_{\rm GL}(|\Phi_{\rm GL}|)\right\}$$

$$\Phi_{\rm GL} = \textit{constant} \times \Phi_{\rm cl}\ , \quad V_{\rm GL} \equiv -\frac{1}{2\xi_{\rm GL}^2}|\Phi_{\rm GL}|^2 + b_{\rm GL}|\Phi_{\rm GL}|^4$$

non-dynamical $a_i \leftrightarrow$ superfluid limit

Comparing superconductors with superfluids

to illustrate the important role of the dynamical a_i in superconductors

$\rightarrow$ focus on vortices: $a_\phi = a_\phi(r)$, $\Phi_{\rm cl} = e^{in\phi}\psi_{\rm cl}(r)$, $n = integer$

(r, ϕ) are the polar coordinates restricted to $0 \leq r \leq R$, $0 \leq \phi < 2\pi$.

	superfluids	**superconductors**
field behavior	$\psi_{\rm cl} \overset{B=0}{\underset{\text{large } r}{\simeq}} \psi_\infty \left(1 - n^2 \frac{\xi^2}{r^2}\right)$	$\psi_{\rm cl} \overset{\text{large } r}{\simeq} \psi_\infty + \frac{\psi_1}{\sqrt{r}} e^{-r/\xi'}$ $a_\phi \overset{\text{large } r}{\simeq} n + a_1 \sqrt{r} e^{-r/\lambda'}$
vortex energy	$F_n - F_0 \overset{\text{large } R}{\sim} n^2 \ln \frac{R}{\xi} - \frac{n}{2} B R^2$	finite as $R \rightarrow \infty$
1st critical field	$H_{c1} \overset{\text{large } R}{\simeq} \frac{2}{R^2} \ln \frac{R}{\xi}$	generically $\neq 0$ as $R \rightarrow \infty$
2nd critical field	$H_{c2} = \frac{1}{2\xi^2_{\rm GL}}$	$H_{c2} = \frac{1}{2\xi^2_{\rm GL}}$

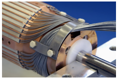
Motivations for holographic superconductors

- To understand how and when the spontaneous symmetry breaking of $U(1)_{\mathrm{em}}$ occurs one needs a microscopic theory.
- BCS theory **(Bardeen, Cooper, Schrieffer, 1957)** describes "conventional superconductors" only.
- There are also "unconventional superconductors".

e.g. some high-temperature superconductors (HTSC) which, unlike BCS theory, seem to involve strong coupling.

important applications;
e.g. HTSC current leads

for the LHC magnets



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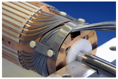


→ apply the AdS/CFT correspondence

The holographic model (Hartnoll, Herzog, Horowitz, 2008; Horowitz, Roberts, 2008)

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + dx_1^2 + \ldots + dx_{d-1}^2\right] + \frac{L^2}{z^2 f(z)}dz^2\,, \quad f(z) = 1 - \left(\frac{z}{z_h}\right)^d$$

$\mathcal{O} \leftrightarrow \Psi$

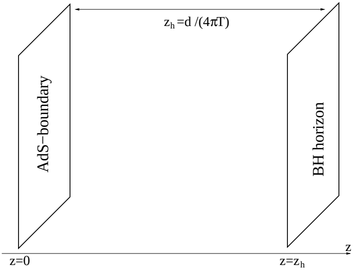
$\Psi|_{z=0} = s =$ source of $\mathcal{O}$

$\hat{J}_\mu \leftrightarrow A_M$

$A_\mu|_{z=0} = a_\mu =$ source of $\hat{J}_\mu$

$$S = \frac{1}{g^2}\int d^{d+1}x\,\sqrt{-G}\left(-\frac{1}{4}\mathcal{F}_{MN}^2 - |D_M\Psi|^2\right)$$

$$J_\mu = \langle \hat{J}_\mu \rangle \propto z^{3-d}\mathcal{F}_{z\mu}|_{z=0}\,, \quad \Phi_{\rm cl} = \langle \mathcal{O} \rangle \propto z^{1-d} D_z\Psi^*|_{z=0}$$



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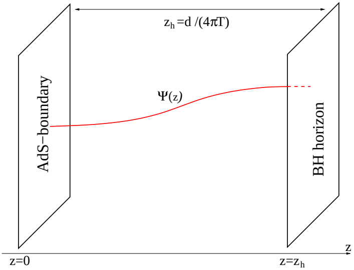
Superconducting phase

no x^μ-dependence (homogeneous solutions) and $A_i = 0$

$\mu = A_0|_{z=0}$

$T < T_c = 0.03(0.05)\mu$ for $d = 3(4)$



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Non homogeneous solutions with $A_i \neq 0$ have also been found.

(Albash, Johnson, 2008; Nakano, Wen, 2008; Maeda, Okamura, 2008; Hartnoll, Herzog, Horowitz, 2008; Montull, Pomarol, Silva, 2009; Keranen, Keski-Vakkuri, Nowling, Yogendran, 2009; Wang, Wu, Yang, 2010)

However, that (Dirichlet) boundary condition corresponds to a superfluid.

$\rightarrow$ non-dynamical a_i!

- impose a **dynamical equation for a_μ**

$$J^\mu + \frac{1}{e_b^2}\partial_\nu \mathcal{F}^{\nu\mu} + J^\mu_{ext} = 0$$

Here, for generality, we have added a kinetic term for a_μ and a background external current J^μ_{ext}.

- Then we must add to S the following term

$$\int d^d x \left[-\frac{1}{4e_b^2}\mathcal{F}^2_{\mu\nu} + A_\mu J^\mu_{ext} \right]_{z=0} .$$

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- by using $J_\mu = \frac{L^{d-3}}{g^2} z^{3-d} \mathcal{F}_{z\mu}|_{z=0}$

$$\frac{L^{d-3}}{g^2} z^{3-d} \mathcal{F}_z{}^\mu \Big|_{z=0} + \frac{1}{e_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0} + J^\mu_{ext} = 0$$

This is an AdS-boundary condition of the Neumann type.

Dynamical a_μ in holography

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$d = 3 + 1$ case

J_μ is logarithmically divergent:

$$\frac{1}{z} \partial_z A_\mu \Big|_{z=0} = -\partial^\nu \mathcal{F}_{\nu\mu} \ln z \Big|_{z=0} + \ldots$$

We can absorb the divergence in $\frac{1}{e_b^2} \partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0}$
to define a renormalized electric charge e_0
in the normal phase ($\Phi_{\rm cl} = 0$):

$$\frac{1}{e_0^2} = \frac{1}{e_b^2} - \frac{L}{g^2} \ln z|_{z=0} + \textit{finite terms}$$

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$d = 2+1$ case

no divergence $\Rightarrow$
we can take $e_b \to \infty$
so $\frac{1}{e_b^2} \partial_\nu \mathcal{F}^{\nu\mu} \Big|_{z=0} \to 0$

In this case a_μ *does not* break conformal invariance and can be considered as an emerging phenomenon:
its kinetic term is induced by the dynamics.
(see also Witten, 2003)

Vortex solutions in holographic superconductors

Vortex ansatz: $\Psi = \psi(z,r)e^{in\phi}$, $\quad A_0 = A_0(z,r)$, $\quad A_\phi = A_\phi(z,r)$

AdS-boundary conditions: $s = 0$, $\quad \mu = constant$,
$\frac{L^{d-3}}{g^2} z^{3-d} \partial_z A_\phi \Big|_{z=0} + \frac{1}{e_b^2} r\partial_r \left(\frac{1}{r}\partial_r A_\phi\right)\Big|_{z=0} = 0\,,$ (for $J^\mu_{ext} = 0$)

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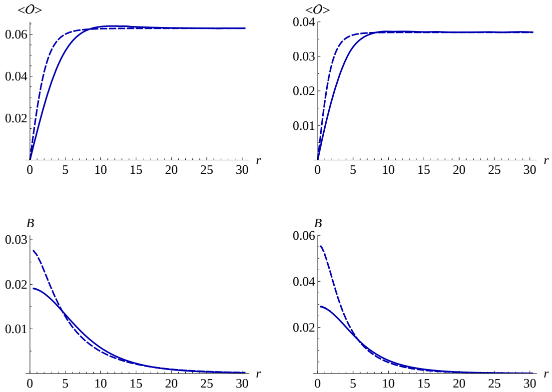
Figures

The modulus of $\langle \mathcal{O} \rangle$ (up to a factor L^{d-3}/g^2) and B versus r from our holographic model for $n = 1$ and $d = 2+1$ (solid lines on the left) and $d = 3+1$ (solid lines on the right).
The dashed lines are the corresponding profiles in the GL theory.

In units of $\mu = 1$



Determination of GL parameters:

- $\xi^2_{\rm GL} = \frac{1}{2H_{c2}}$,
- the matching at large r gives b_{GL} and e_0 in the GL free energy.

We observed $a_\phi \simeq n + a_1\sqrt{r}e^{-r/\lambda'}$, for large r.

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Figures

H_{c1} and H_{c2} versus T for $d = 2+1$ (left) and $d = 3+1$ (right).

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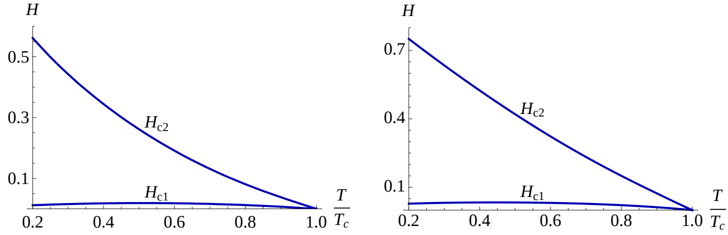

$H_{c1} < H_{c2}$ for every T, so **the holographic superconductors are of Type II**.

Interestingly, HTSC are also of Type II.

Conclusions

Summary of the main points

- *We have discussed how to introduce a dynamical gauge field in holographic superconductors.*
- *For* $d = 2 + 1$, a_μ *can be considered as an emergent phenomenon, while, for* $d = 3 + 1$, *it is external to the CFT.*
- *We have presented vortex solutions in the presence of a dynamical* a_μ.
- *The holographic superconductors are of Type II.*

to know more see arXiv:1005.1776

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Outlook

- *applications to other situations (different from vortices):*
  e.g. electromagnetic fields near the surface of a finite size superconductor or in the Josephson effect
- *extensions to p-wave and d-wave holographic superconductors*
- *extensions to non-relativistic scale invariant theories*

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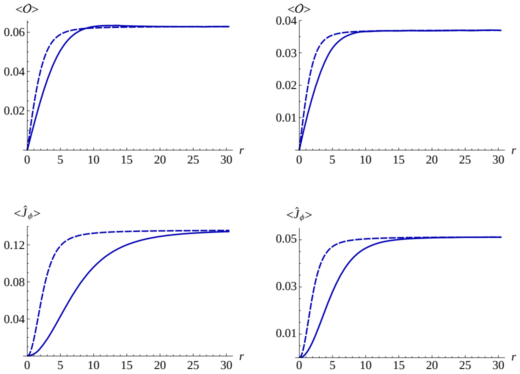

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