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Asymptotic safety of gravity and the Higgs boson mass

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What if gravity is asymptotically safe?

Asymptotic safety = existence of non-Gaussian UV fixed point for gravity. Conjecture + ϵ-expansion argument - Weinberg '79

$$S_G = -\frac{1}{16\pi G_0}\int d^D x\sqrt{g}R$$

$$G(\mu) = \mu^{D-2}G_0, \quad \mu\frac{d}{d\mu}G(\mu) = (D-2)G(\mu) - bG^2(\mu) \,.$$

Fixed point:

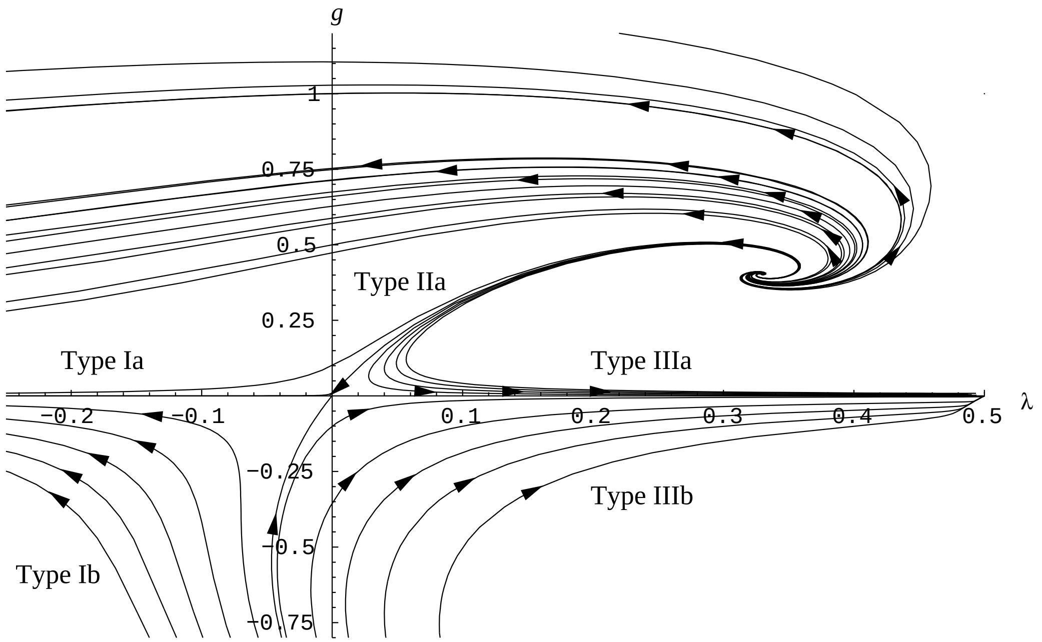
$$G^* = \frac{D-2}{b}, \quad G_0^*(\mu) = \frac{G^*}{\mu^{D-2}} \to 0 \text{ if } \mu \to \infty$$

Computations give $b > 0$. Gastmans et al '77, Christensen and Duff '77, Kawaiand and Ninomiya '90, Percacci '06,...

Functional RG analysis - Reuter '96, Percacci et al, Niedermaier '09, ...

$$S_{\mathrm{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, \{-R + 2\Lambda\} \ ,$$

$$k\,\partial_k \Gamma_k = \frac{1}{2}\, \mathrm{STr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k\,\partial_k \mathcal{R}_k \right] \ .$$



Reuter and Saueressig '02

Possible consequence: Electroweak theory + Gravity is a final theory

Experimental evidence for physics beyond the SM

- i. Neutrino masses and oscillations
- ii. Dark matter
- iii. Baryon asymmetry of the Universe
- iv. Inflation

require only a modest extension of the SM (νMSM) by 3 singlet right-handed fermions (needed for i-iii) with masses in keV - GeV area, and non-minimal coupling of the Standard Model Higgs field to Ricci scalar (needed for iv).

To be true: all the couplings of the SM must be asymptotically safe or asymptotically free

Problem for:

- U(1) gauge coupling g_1, $\mu\frac{dg_1}{d\mu} = \beta_1^{\mathrm{SM}} = \frac{41}{96\pi^2} g_1^3$
- Scalar self-coupling λ, $\mu\frac{d\lambda}{d\mu} = \beta_\lambda^{\mathrm{SM}} =$

$$= \frac{1}{16\pi^2}\left[(24\lambda + 12h^2 - 9(g_2^2 + \frac{1}{3}g_1^2))\lambda - 6h^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2\right]$$

- Fermion Yukawa couplings, t-quark in particular h, $\mu\frac{dh}{d\mu} = \beta_h^{\mathrm{SM}} =$

$$= \frac{h}{16\pi^2}\left[\frac{9}{2}h^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2\right]$$

Landau pole behaviour

Gravity contribution to RG running

Let x_j is a SM coupling. Gravity contribution to RG:

$$\mu\frac{dx_j}{d\mu} = \beta_j^{\mathrm{SM}} + \beta_j^{grav} \ .$$

On dimensional grounds

$$\beta_j^{grav} = \frac{a_j}{8\pi}\frac{\mu^2}{M_P^2(\mu)}x_j \ .$$

where

$$M_P^2(\mu) = M_P^2 + 2\xi_0\mu^2 \ ,$$

with $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV, $\xi_0 \approx 0.024$
from a numerical solution of FRGE

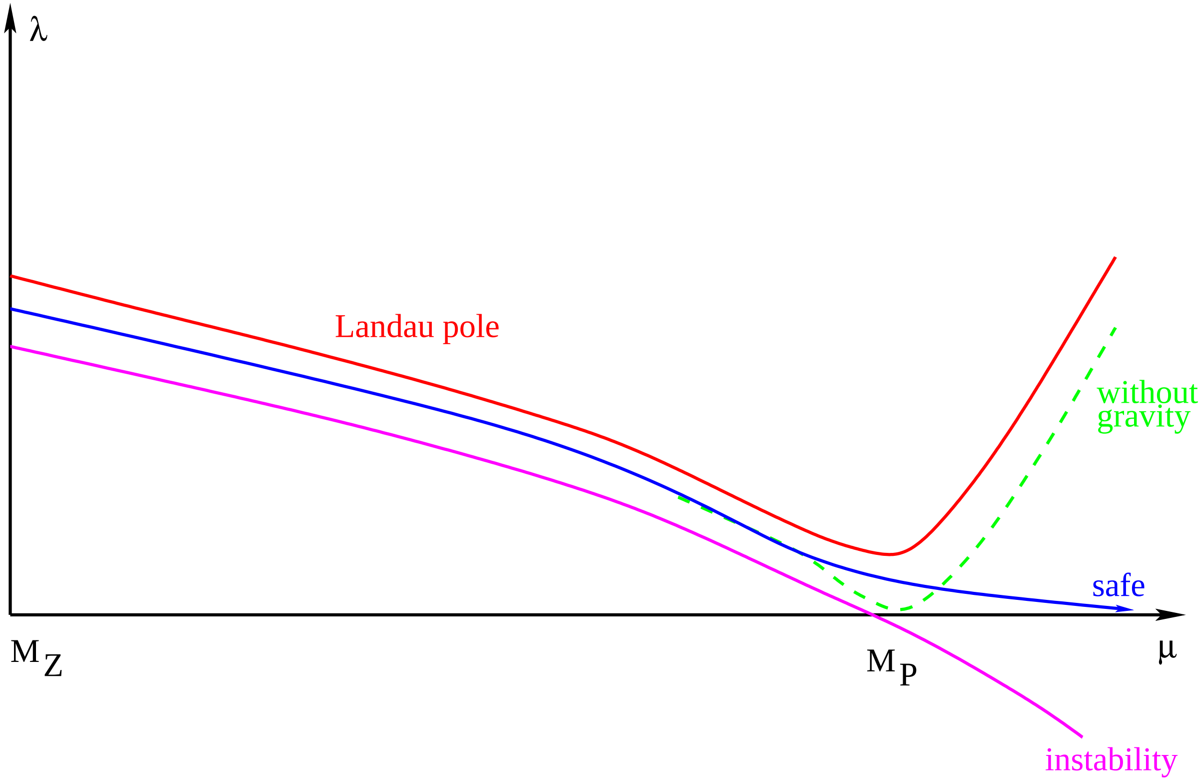
Computations of a_j are ambiguous and controversial

Robinson and Wilczek '05, Pietrykowski '06, Toms '07&'08, Ebert, Plefka and Rodigast '07, Narain and Percacci '09, Daum, Harst and Reuter '09, Zanusso et al '09, ...

- Most works get for gauge couplings a universal value $a_1 = a_2 = a_3 < 0$: U(1) gauge coupling get asymptotically free in asymptotically safe gravity
- $a_\lambda \simeq 2.6 > 0$ according to Percacci and Narain '03 for scalar theory coupled to gravity
- $a_h >< 0$?? The case $a_h > 0$ is not phenomenologically acceptable - only massless fermions are admitted

Suppose that indeed $a_1 < 0,\ a_h < 0,\ a_\lambda > 0$. Then the Higgs mass is predicted with theoretical uncertainty $\simeq \pm 2$ GeV

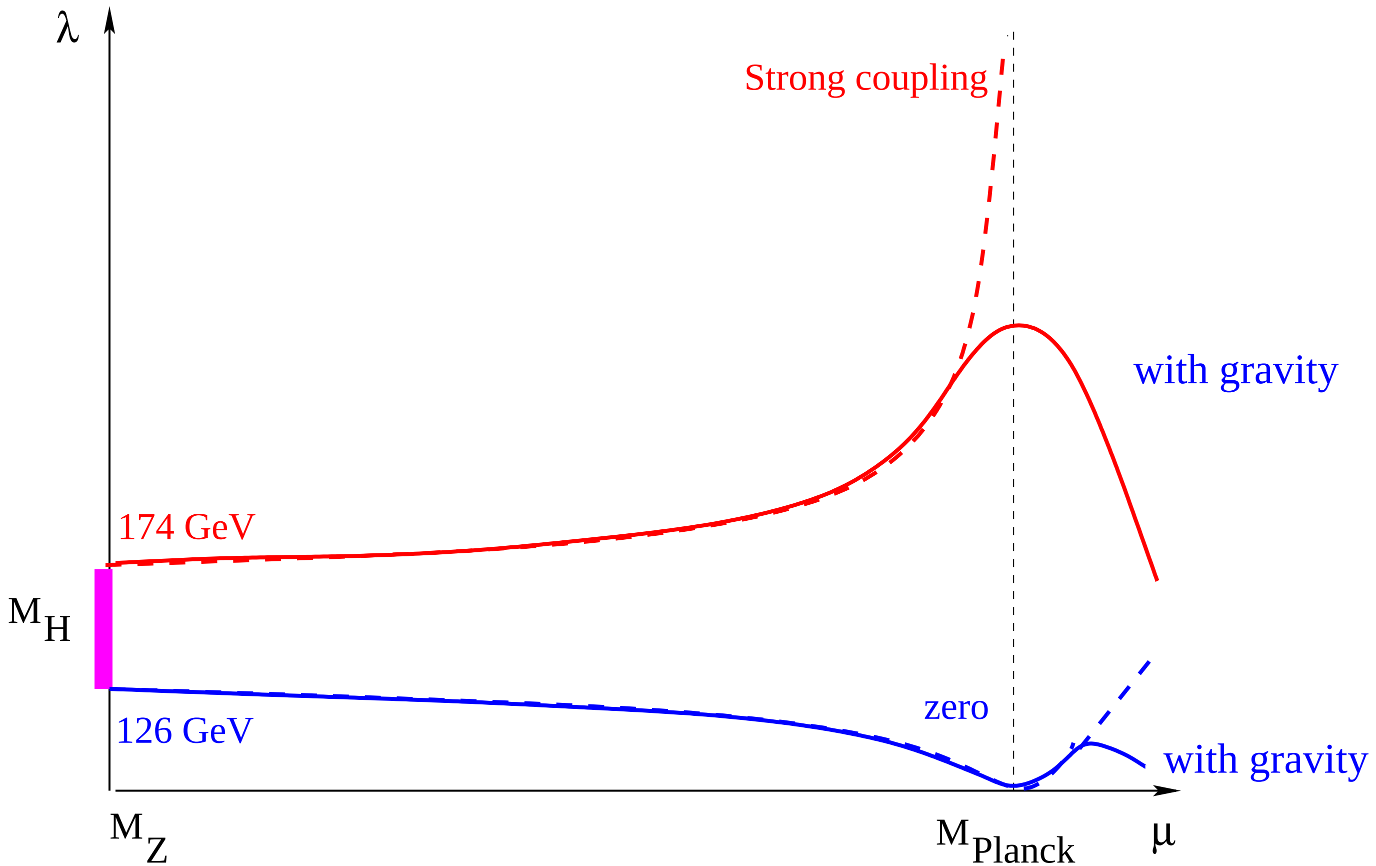
$$m_{\rm H} = [126.3 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5]\ {\rm GeV}\ ,$$



Possible understanding of the amazing fact that $\lambda(M_P) = 0$ and $\beta_\lambda^{\rm SM}(M_P) = 0$ simultaneously at the Planck scale.

Suppose that $a_1 < 0,\ a_h < 0,\ a_\lambda < 0$. Then the Higgs mass is predicted with theoretical uncertainty $\simeq 50$ GeV

$$126\ \text{GeV} < m_H < 174\ \text{GeV}$$



Conclusions

If gravity is asymptotically safe then the possible outcome of the LHC experiments is:

- Higgs and nothing else
- $m_H \simeq \mathbf{126}$ GeV (for central values of m_t and α_s) if, as some computations show, $a_\lambda > 0$
- $\mathbf{126\ GeV} < m_{\rm H} < \mathbf{174\ GeV}$ if $a_\lambda < 0$
- Waiting time $\sim$ 6 years (?)