BEYOND MFV IN FAMILY SYMMETRY THEORIES OF FERMION MASSES

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Based on existing ideas but, hopefully, contributing to a systematic comparison of various approaches

The success of the SM in the FCNC and CP violating sectors relies on:

- absence of tree-level effects

- GIM mechanism (unitarity of the quark mixing matrix)

- flavour global symmetry

  $$SU(3)^5 = SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R} \otimes SU(3)_{L_L} \otimes SU(3)_{E_R}$$

  broken only by Yukawa matrices

- pattern of quark masses and mixing, taken from experiment

$$\mathcal{L}_{eff}^{\Delta S=2} \sim \frac{1}{M_W^2}\frac{g^4}{(4\pi)^2}\{(U_{ts}^*U_{td})^2 + $$

$$+ (U_{cs}^*U_{cd})^2\frac{m_c^2}{M_W^2} + \ldots\}(\bar{s}_L\gamma_\mu d_l)(\bar{s}\gamma^\mu d_L)$$

$$\sim \frac{1}{M_W^2}\frac{g^4}{(4\pi)^2}10^{-5}(\bar{s}_L\gamma_\mu d_L)(\bar{s}_L\gamma^\mu d_L)$$

suppression scale

loop factor

Go beyond SM:

- **to explain the pattern of quark masses, mixing**
  – physics with the characteristic scale M_f

- **to ease the hierachy problem**
  –physics with the scale M_h around 1 TeV

It may be that

$M_f >> M_h$ (e.g. supersymmetric models)

or

$M_f \sim M_h$ (e.g. RS models of flavour)

Physics BSM may have new sources of FCNC and CP violation

Precision of the FCNC and CP violation data
leaves little room for new effects from physics BSM.

With generic (anarchical) flavour structure, for

$$\frac{C}{\Lambda^2}\bar{Q}_i Q_j \bar{Q}_k Q_l \sim \mathcal{L}_{SM}^{\text{eff}}$$

one gets

$$\Lambda \sim 10^{\frac{5}{2}}\frac{1}{\alpha_W}M_W \sim 600\text{TeV}, \qquad C \sim 1$$

$$\Lambda \sim 10^{\frac{5}{2}}\frac{\alpha_s}{\alpha_W}M_W \sim 100\text{TeV}, \qquad C \sim \alpha_s^2$$

Thus, if the scale of new physics is around 1 TeV,
to solve the hierarchy problem, its flavour structure
must be strongly constrained.

Minimal Flavour Violation and spurion technique

(D'Ambrosio, Giudice, Isidori, Strumia, 2002):

Hypothesis:

in BSM physics the flavour symmetry is broken by Yukawa interactions (as in the SM)

$$\mathcal{L} = \bar{Q}_L Y_D D_R H + \bar{Q}_L Y_U U_R H_C + \bar{L}_L Y_E E_R H + h.c.$$

Treating Yukawa couplings as spurion fields transforming as

$$Y_U \sim (3, \bar{3}, 1)_{SU(3)^3_q}, \ Y_D \sim (3, 1, \bar{3})_{SU(3)^3_q}$$

where

$$SU(3)^3_q = SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R}$$

and similarly for leptons,

the Lagrangian has $SU(3)^5$ invariant form

MFV gives then the higher dimension operators as products of $SU(3)^3_q$ invariant two-fermion operators

$$\bar{Q}_L Y_u Y_u^\dagger Q_L, \quad \bar{D}_R Y_D^\dagger Y_u Y_u^\dagger Q_L$$

$$\lambda_{FC} = (Y_u Y_u^\dagger)_{ij} \approx \lambda_t^2 U_{3i}^* U_{3j}$$

The operators with external up quarks are strongly supperessed due to the smallness of the down quark Yukawa coupling

Experimental bounds on the effective suppression scale for dim 6 operators, e.g.

$$\frac{1}{\Lambda_{eff}^2}(\bar{Q}_L Y_u Y_u^\dagger Q_L)^2$$

(D'Ambrosio et al.)

Flavour violating dimension six operator	$\Lambda^{\text{eff}}_{MFV}$ (in TeV) $-$	$+$
$\frac{1}{2}(\bar{Q}_L \lambda_{FC} \gamma_\mu Q_L)^2$	6.4	5.0
$H^\dagger (\bar{D}_R \lambda_d \lambda_{FC} \sigma^{\mu\nu} Q_L) F_{\mu\nu}$	9.3	12.4
$H^\dagger (\bar{D}_R \lambda_d \lambda_{FC} \sigma^{\mu\nu} T^a q_L) G^a_{\mu\nu}$	2.6	3.5
$(\bar{Q}_L \lambda_{FC} \gamma^\mu Q_L)(\bar{L}_L \gamma_\mu L_L)$	3.1	2.7

MFV: no tension with the scale O(1 TeV) needed for solving the hierarchy problem provided new physics contributes only at loop level

$$\Lambda^{\rm phys}_{\rm MFV} = \alpha\, \Lambda^{\rm eff}_{\rm MFV}$$

(e.g. MSSM with MFV)

However: MFV does not address the origin of the Yukawa couplings

Theories of fermion masses and mixing may (indeed do) violate the MFV hypothesis

(Feldmann, Mannel 2006, Feng, Lester, Nir, Shadmi 2007, Hiller, Hochberg, Nir 2010, Altmanhofer et al.. 2010)

How well they do for FCNC ? (Quark sector only)

Our laboratory: Froggatt-Nielsen-like models

Seminal papers: Leurer, Nir, Seiberg

Our approach: use the SM effective field theory and spurion technique

(extending D'Ambrosio et al.; see also Feldmann, Mannel 2006)

New elements of such an analysis:

- **horizontal symmetries must be imposed on the higher dim operators**

- **the familon fields can be used in their construction**

Simple example

U(1) family symmetry, spontaneously broken by a single familon Θ, with U(1) charge +1

Fermion charges (all $\geq$ 0):

left-handed doublets $\quad Q_L^i : \quad q_i$

left-handed singlets $\quad U_i^C,\, D_i^C : \quad u_i,\, d_i$

Higgs field $\quad q_H = 0$

Several models studied

Yukawa matrix

$$\bar{Q}_L Y_U U_R H_c = \bar{Q}_L^i [a_i^j (\frac{\theta}{M})^{q_i + u_j}] U_R^j H_c$$

a_i^j ≡ 3 x 3 matrix of O(1) coefficients

$$\epsilon \equiv \frac{\theta}{M} \sim \text{Cabibbo angle}$$

Spurion analysis:

assign $(3, \bar{3}, 1)$ under $SU(3)_q^3$

to $[a_i^j (\frac{\theta}{M})^{q_i + u_j}]$

or,

regard 3 x 3 matrix of a_i^j as a spurion field transforming as

$$(3,\ \bar{3},\ 1) \quad \text{under} \quad SU(3)_q^3$$

and the factors

$$\Phi_L^i = (\frac{\theta}{M})^{q_i} \quad \text{and} \quad \Phi_u^{\dagger i} = (\frac{\theta}{M})^{u_i}$$

as U(1) spurions which are singlets under the flavour group

Notation: $$a_i^j \Phi_L^i \phi_{U j}^{\dagger} \equiv \Phi_L a^{LU} \Phi_U^{\dagger}$$

Effective quark bilinear operators – building blocks for flavour violating dim 6 operators*

$$\bar{Q}_L \Phi_L a^{LD} \Phi_D^\dagger D_R, \quad \bar{Q}_L \Phi_L a^{LU} \Phi_U^\dagger U_R, \quad \bar{Q}_L \Phi_L a^{LL} \Phi_L^\dagger Q_L,$$

$$\bar{U}_R \Phi_U a^{UU} \Phi_U^\dagger U_R, \quad \bar{D}_R \Phi_D a^{DD} \Phi_D^\dagger D_R$$

* Beyond MFV, there are dim 6 operators that do not factorise into quark bilinears invariant under flavour group

The matrices a^{LL}, a^{UU}, a^{DD} transform

as (8, 1, 1), (1, 8, 1) and (1, 1, 8)

Notation

$$\bar{Q}_L \Phi_L a^{LL} \Phi_L^\dagger Q_L \equiv \bar{Q}_L X^Q_{LL} Q_L, \text{ etc}$$

Its suppression **by powers of ε** is fixed

by fermionic U(1) charges

Suppression: $\epsilon^{|p|}$; charges p given below

	M I	$U(1)^2$	$N-A$	MFV
$X^Q_{LLij} = \Phi_{Li} \otimes \Phi^\dagger_{Lj}$				
(12)	1	$(3,-1) \sim 5$	3	5
(13)	3	$(3,0) \sim 3$	3	3
(23)	2	$(0,1) \sim 2$	2	2
$X^D_{RRij} = \Phi_{Di} \otimes \Phi^\dagger_{Dj}$				
(12)	1	$(-5,3) \sim 11$	3	$5(\lambda_d \lambda_s)$
(13)	1	$(-1,1) \sim 3$	3	$3(\lambda_d \lambda_b)$
(23)	0	$(4,-2) \sim 8$	2	$2(\lambda_s \lambda_b)$
$X^D_{LRij} = \Phi_{Li} \otimes \Phi^\dagger_{Dj}$				
(12)	3	$(7,-1) \sim 9$	3	$5(\lambda_s)$
(13)	3	$(3,1) \sim 5$	3	$3(\lambda_b)$
(23)	2	$(0,2) \sim 4$	2	$2(\lambda_b)$

For comparison with MFV, one chooses the electroweak basis with mass diagonal down quarks

$$X^Q_{LL} = S_d^\dagger \Phi_L a^{LL} \Phi_L^\dagger S_d$$

In Abelian models, for flavour anarchical matrices of O(1) coefficients, e.g. if $a^{LL}_{11} \neq a^{LL}_{22}$, the rotation results in a universal suppression in (1,2) sector only by ε, **except for U(1)^2 model** where the Cabibbo angle comes from the up sector

MFV vs family symmetry

$$\bar{Q}_L X^D_{LR} D_R : \;\; \bar{Q}_L Y_u Y_u^\dagger Y_d D_R \;\; \text{vs} \;\; \bar{Q}_L \underbrace{\Phi_L a^{LD} \Phi_D^\dagger}_{O(Y_d)} D_R$$

$$\bar{Q}_L X^Q_{LL} Q_L : \;\; \bar{Q}_L Y_u Y_u^\dagger Q_L \;\; \text{vs} \;\; \bar{Q}_L \underbrace{\Phi_L a^{LL} \Phi_L^\dagger}_{"O(Y_u)"} Q_L$$

Consequences:

- generically, less suppression than for MFV (but model dependent and different in different family sectors)
- in MFV only $\bar{Q}_L X^Q_{LL} Q_L$ and $\bar{Q}_L X^D_{LR} D_R$ are important;
- with family symmetry, also

$$\bar{D}_R X^D_{RR} D_R,\ \bar{U}_R X^U_{RR} U_R \text{ and } \bar{Q}_L X^U_{LR} U_R$$

can be equally important

Flavor violating dimension six operator	Λ/Λ_{MFV} Ex. 1	$U(1)^2$	$N-A$
$\frac{1}{2}(\bar{q}_L X^Q_{LL} Q_L)^2$	ϵ^{-4}	1	ϵ^{-2}
$H^\dagger(\bar{D}_R X^{D\dagger}_{LR} \sigma^{\mu\nu} Q_L) F_{\mu\nu}$	$x\epsilon^{-2}$	$x\epsilon$	$x\epsilon^{-2}$
$H^\dagger(\bar{D}_R X^{D\dagger}_{LR} \sigma^{\mu\nu} T^a Q_L) G^a_{\mu\nu}$	$x\epsilon^{-2}$	$x\epsilon$	$x\epsilon^{-2}$
$(\bar{Q}_L X^Q_{LL} \gamma^\mu Q_L)(\bar{L}_L \gamma_\mu L_L)$	ϵ^{-2}	1	ϵ^{-1}

$$x = \sqrt{\frac{m_t}{m_b}}$$

- **Higher bounds on Λ_{eff}**

  than in MFV, particularly in (d,s) sector,

  except for $U(1)^2$ model

  (models with the Cabibbo angle coming from

  the up sector are favoured in this context)

$$\Lambda_{\rm eff} \gtrsim (25 - 100)\,\Lambda^{\rm eff}_{MFV}$$

Further suppression mechanism needed for lower scales. What about SUSY ?

Family symmetry and supersymmetry

Bounds on M_{SUSY} may depend on the theory of flavour violation at M and on the mechanism of supersymmetry breaking

- one extreme case – universal soft terms & $A \sim Y$ (MFV)
- another extreme case: pattern of soft terms determined solely by the broken horizontal symmetries

Integrating out supersymmetric degrees of freedom one gets

$$\Lambda_{\mathsf{eff}} = M_{\mathsf{SUSY}}/\alpha$$

Some details:

Leading flavour changing operators

above M_{SUSY} and below M_f:

- $$M^2_{\mathrm{SUSY}}\tilde{Q}^\dagger_{Li}\tilde{Q}_{Lj}(\frac{\theta}{M})^{q_i-q_j}$$

  and similarly for RR up and down squarks

- for U(1): D-term contribution to

  the diagonal mass splitting

$$\tilde{m}_i^2-\tilde{m}_j^2=g(q_i-q_j)<D>$$

$$<D>\sim M^2_{SUSY}$$

- A terms

$$A^q_{ij} H \tilde{q}^*_{Li} \tilde{q}_{Rj}$$

The SUSY operators subsequently generate the SM dim 6 operators at one-loop through gaugino interactions with mass scale M_{SUSY}

In the electroweak basis

with the mass diagonal down quarks

one defines mass insertions

(Gabbiani, Gabrielli, Masiero, Silvestrini, 1996):

$$\delta_{ijMN} = \frac{\Delta\tilde{m}^2_{ijMN}}{\tilde{m}^2_{av}}, \qquad M, N = L, R$$

In family symmetry models, δ's are predicted as $O(\varepsilon^p)$, e.g.

$$m^2_{\tilde{q}ij}\tilde{q}_i^\dagger\tilde{q}_j\epsilon^{|q_i-q_j|}$$

$$\rightarrow \ (\delta^{\tilde{q}}_{ij})_{LL} = \epsilon^{|q_i-q_j|}$$

and can be compared with experimental bounds on them

$$
\begin{aligned}
L_{eff} &= \frac{\alpha_s^2}{216\tilde{m}_{qij}^2}((\delta_{12\,LL}^d)^2(\bar{d}_L\gamma_\mu s_L\bar{d}_L\gamma_\mu s_L) \times f(x) \\
&+ (\delta_{12\,RR}^d)^2(\bar{d}_R\gamma_\mu s_R\bar{d}_R\gamma_\mu s_R) \times f'(x) \\
&+ (\delta_{12\,LL}^d)(\delta_{12\,RR}^d)(\bar{d}_R s_L\bar{d}_L s_R) \times f''(x) + \ldots + \text{h.c.})
\end{aligned}
$$

q	ij	$(\delta_{ij}^q)_{MM}$	$\langle\delta_{ij}^q\rangle$
d	12	$0.01 \sim \epsilon^2$	$0.0007 \sim \epsilon^4$
d	13	$0.07 \sim \epsilon$	$0.025 \sim \epsilon^2$
d	23	$0.21 \sim \epsilon$	$0.07 \sim \epsilon$
u	12	$0.035 \sim \epsilon^2$	$0.003 \sim \epsilon^3$

Experimental bounds

Conclusions for supersymmetric family symmetry models:

they can remain consistent with the bounds on FCNC and CP violation for superpartner physical masses $\lesssim$O(1 TeV) but some models may require strong flavour blind renormalisation effects on the squark masses. This requires $m_{1/2}/m_0 \gtrsim 7$

e.g. $m_{1/2} = 300\text{GeV}, \ m_0 \sim 50\text{GeV}$

$$m_{\tilde{g}} = 900\text{GeV}, \ m_{\tilde{q}} = 800\text{GeV}$$

SUMMARY

- Family symmetry theories of fermion masses violate MFV
- FCNC & CP violation are typically less suppressed than in MFV; but not always...
- Compatibility with a solution to little hierarchy problem not easy but possible in SUSY models
- Effects that could discriminate between various models are expected in (13) & (23) sectors, in RR operators and with the external up quarks

Effective Lagrangian below the scales M_h and M_f (integrating out the new physics degrees)

L_{SM} + $SU(2) \times U(1)$ invariant higher dim operators

e.g. dim 6 four fermion operators contributing to FCNC

$$\mathcal{L}^{\text{eff}} = \mathcal{L}_{SM}^{\text{eff}} + \frac{C}{\Lambda^2}(\bar{Q}_i Q_j \bar{Q}_l Q_k) + \ldots$$

$$C \sim O(1)(\text{tree level}), \quad C \sim \alpha^2(1\ \text{loop})$$

Dominant contribution from operators suppressed by the lower scale, likely M_h?

Can FCNC be suppressed strongly enough just by $\Lambda \sim M_h$?