Mass and Spin Measurement with M_{T2} and MAOS Momentum

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Outline

1. Motivation: New Physics Events with Missing Energy
2. M_{T2} and MAOS (M_{T2}-Assisted-On-Shell) Momentum
3. Application to Mass and Spin Measurement
4. Conclusion

Motivations for new physics at the TeV scale:

- Hierarchy Problem

$$\delta m_H^2 \sim \frac{g^2}{8\pi^2}\Lambda_{\rm SM}^2 \sim M_Z^2 \quad \Longrightarrow \quad \Lambda_{\rm SM} \sim 1 \text{ TeV}$$

- Dark Matter

Thermal WIMP with $\Omega_{\rm DM}h^2 \sim \frac{0.1}{g^4}\left(\frac{m_{\rm DM}}{1\text{ TeV}}\right)^2 \sim 0.1$

$\Longrightarrow \quad m_{\rm DM} \sim 1 \text{ TeV}$

Many new physics models solving the hierarchy problem while providing a DM candidate involve a Z_2-symmetry under which the new particles are Z_2-odd, while the SM particles are Z_2-even:

SUSY with R-parity, UED with KK-parity,
Little Higgs with T-parity, ...

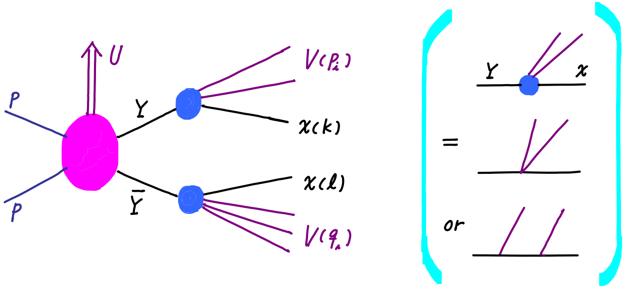
* At colliders, new particles are produced always in pairs.
* Lightest new particle is stable, so a good candidate for WIMP DM.

LHC Signal

Pair-produced new particles $(Y + \bar{Y})$ decaying into visible SM particles (V) plus invisible WIMPs (χ):

$$pp \rightarrow U + Y + \bar{Y} \rightarrow U + \sum_i V(p_i) + \chi(k) + \sum_j V(q_j) + \chi(l)$$

Multi-jets $\left(+ \text{Leptons} \right) + \not{p}_T$ Events:



$U \equiv$ Upstream momenta carried by the visible SM particles not from the decay of $Y + \bar{Y}$, e.g. initial state radiation (ISR).

To identify the underlying theory for these new physics events, it is crucial to determine the mass and spin of the involved new particles.

If we don't want to rely on specific-model-dependent matrix elements, we need to use a kinematic method for mass and spin measurement.

However kinematic method suffers from

* Initial parton momenta in the beam-direction are unknown.
* Each event involves two missing WIMPs.

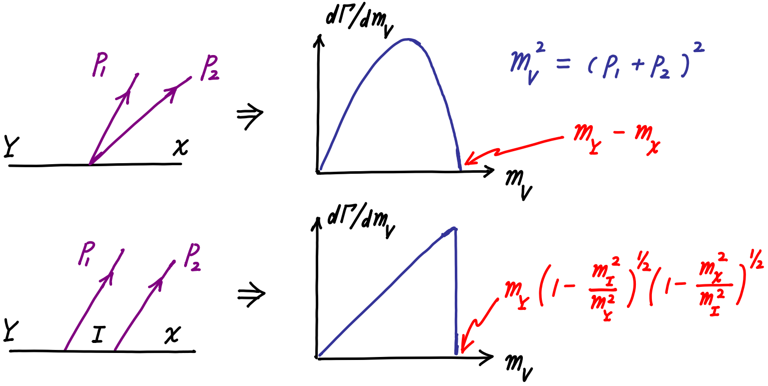
There are several approaches proposed to overcome these difficulties:

M_{T2} and MAOS momentum provide one of those ways to determine the mass and/or spin of new particles in missing energy events at LHC, which can work even when long decay chain is not available.

Kinematic Methods for Mass Measurement

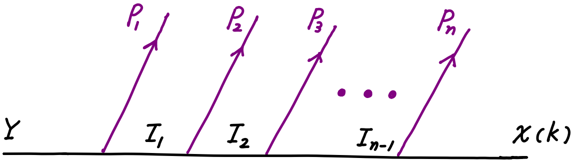
Endpoint Method Hinchliffe et. al.; Allanach et. al.; Gjelsten et. al.;...

Endpoint value of the invariant mass distribution of visible (SM) decay products depend on the involved new particle masses.



Long decay chain can be particularly useful as it can provide enough number of endpoint values to determine all of the involved new particle masses.

n-step cascade decay:



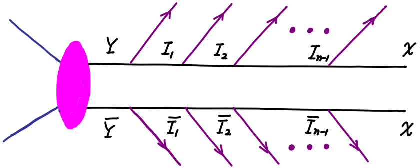
* Number of measurable invariant mass distributions: $2^n - (n+1)$
* Number of unknown new particle masses: $n+1$.

$\Longrightarrow$ For $n \geq 3$, there can be enough number of independent endpoint values to determine (in principle) all of the involved new particle masses. (cf: discrete ambiguities)

Mass Relation Method Nojiri, Polesello, Tovey; Cheng et. al.; ...

Reconstruct the missing particle momenta with on-shell and other available constraints.

Symmetric pair of n-step cascade decays:



* Number of unknowns for N-events: $8N + (n+1)$

(2N-missing momenta + $(n+1)$-unknown masses)

* Number of constraints: $[2(n+1)+2]\,N$

(mass relations + $\not{p}_T$ constraints)

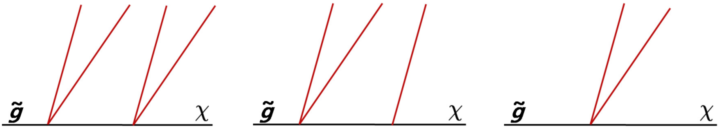
$\Longrightarrow$ For $n \geq 3$ and $N \geq (n+1)/2(n-2)$, on-shell mass relations and $\not{p}_T$ constraints can provide more constraints than those necessary for reconstructing the missing momenta. (cf: discrete ambiguities)

Long decay chain can be particularly useful for the endpoint and mass relation methods.

On the other hand, there are many new physics models which do not provide such a long decay chain:

*** SUSY models with relatively heavy sfermions:**
Focus point scenario, Yukawa-unified SUSY GUT, D-term mediation, Loop-split SUSY, ...

Dominant decay modes:

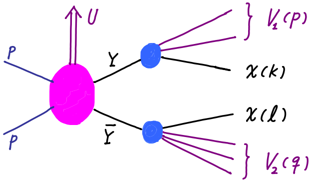


- Enough number of constraints for mass relation method are not available.
- Endpoint point method can provide only a partial information on the new particle spectrum, e.g. mass differences.

MAOS (M_{T2}-Assisted-On-Shell) Reconstruction Cho, KC, Kim, Park

Even when enough number of constraints are not available, one can attempt to (approximately) reconstruct the missing particle momenta just using the available minimal constraints:

$$pp \;\rightarrow\; U + Y + \bar{Y} \;\rightarrow\; U + \sum V(p_i) + \chi(k) + \sum V(q_j) + \chi(l)$$



Introduce trial masses m_Y and m_χ $\left(\text{true masses: } m_Y^{\text{true}} \text{ and } m_\chi^{\text{true}}\right)$ of mother particle and WIMP, and impose the minimal constraints:

$$k^2 = l^2 = m_\chi^2, \quad (k+P)^2 = (l+Q)^2 = m_Y^2, \quad \mathbf{k}_T + \mathbf{l}_T = \not{\mathbf{p}}_T$$

$$\left(P = \sum p_i, \quad Q = \sum q_j \right)$$

6 constraints for 8 unknowns (k^μ, l^μ), so two-parameter set of solutions for each event, which can be parameterized by $\mathbf{k}_T$.

Solution of the minimal constraints:

$$k_z^{\text{maos}} = \frac{1}{P^2+\mathbf{P}_T^2}\left(AP_z \pm P_0\sqrt{A^2-(P^2+\mathbf{P}_T^2)(m_\chi^2+\mathbf{k}_T^2)}\right)$$
$$l_z^{\text{maos}} = \frac{1}{Q^2+\mathbf{Q}_T^2}\left(BQ_z \pm Q_0\sqrt{B^2-(Q^2+\mathbf{Q}_T^2)(m_\chi^2+\mathbf{l}_T^2)}\right)$$

$$P^\mu = \sum p_i^\mu = (P_0, \mathbf{P}_T, P_z), \quad Q^\mu = \sum q_j^\mu = (Q_0, \mathbf{Q}_T, Q_z)$$
$$A = \frac{1}{2}(m_Y^2 - m_\chi^2 - P^2) + \mathbf{P}_T\cdot\mathbf{k}_T, \quad B = \frac{1}{2}(m_Y^2 - m_\chi^2 - Q^2) + \mathbf{Q}_T\cdot\mathbf{l}_T$$

Real-valued solution exists iff

$$A^2 \geq (P^2+\mathbf{P}_T^2)(m_\chi^2+\mathbf{k}_T^2), \quad B^2 \geq (Q^2+\mathbf{Q}_T^2)(m_\chi^2+\mathbf{l}_T^2)$$
$$\Longleftrightarrow \quad m_Y \geq \max\Big(M_T(P,\mathbf{k}_T,m_\chi), M_T(Q,\mathbf{l}_T,m_\chi)\Big)$$

$$M_T^2(P,\mathbf{k}_T,m_\chi) \equiv \text{Transverse mass of } \; Y \to V(P) + \chi(k)$$
$$= P^2 + m_\chi^2 + 2\sqrt{P^2+\mathbf{P}_T^2}\sqrt{m_\chi^2+\mathbf{k}_T^2} - 2\mathbf{P}_T\cdot\mathbf{k}_T.$$

For given trial (m_Y, m_χ), to have real-valued solution for the largest possible number of events, choose (event-by-event)

$$\mathbf{k}_T = \mathbf{k}_T^{\text{maos}}, \quad \mathbf{l}_T = \mathbf{l}_T^{\text{maos}} \qquad (\mathbf{k}_T^{\text{maos}} + \mathbf{l}_T^{\text{maos}} = \not{\mathbf{p}}_T)$$

$$\begin{aligned} & \max\Big(M_T(P, \mathbf{k}_T^{\text{maos}}, m_\chi), M_T(Q, \mathbf{l}_T^{\text{maos}}, m_\chi)\Big) \\ = \quad & \min_{\mathbf{k}_T + \mathbf{l}_T = \not{\mathbf{p}}_T} \Big[\max\Big(M_T(P, \mathbf{k}_T, m_\chi), M_T(Q, \mathbf{l}_T, m_\chi)\Big) \Big] \\ \equiv \quad & M_{T2}(P, Q, \not{\mathbf{p}}_T; m_\chi) \end{aligned}$$

Lester and Summers

- MAOS momenta are real-valued for all events if

$$m_Y \geq M_{T2}^{\max}(m_\chi) \equiv \max_{\{\text{events}\}} \Big[M_{T2}(\text{event}; m_\chi) \Big]$$

- $M_{T2}^{\max}(m_\chi = m_\chi^{\text{true}}) = m_Y^{\text{true}}$ in the zero width limit.

$\Longrightarrow$ With M_{T2}, one can select an one-parameter set of trial masses most favored for the reconstruction of missing particle momenta, which includes $(m_\chi^{\text{true}}, m_Y^{\text{true}})$: (See also Cheng and Han)

$$(m_\chi, m_Y) = (m_\chi, M_{T2}^{\max}(m_\chi))$$

- $M_{T2}^{\max}(m_\chi)$ by itself contains a kinematic information to determine both $m_\chi^{\rm true}$ and $m_Y^{\rm true}$:

$$\text{Kink at } m_\chi = m_\chi^{\rm true} \quad \Longrightarrow \quad M_{T2}^{\max}(m_\chi = m_\chi^{\rm true}) = m_Y^{\rm true}.$$

- For the endpoint events with $M_{T2}(m_\chi = m_\chi^{\rm true}) = m_Y^{\rm true}$, the reconstructed MAOS momenta correspond to the true missing particle momenta:

$$k_\mu^{\rm maos} = k_\mu^{\rm true} \;\; \text{for endpoint events}$$

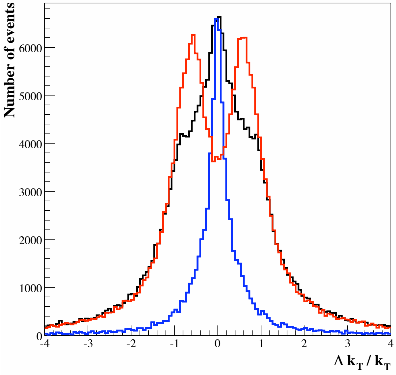
$\Longrightarrow$ MAOS momentum provides a systematic approximation to the true missing particle momentum as one can reduce $\Delta\mathbf{k}/\mathbf{k} \equiv (k_\mu^{\rm maos} - k_\mu^{\rm true})/k_\mu^{\rm true}$ with an M_{T2}-cut selecting near-endpoint events.

$$\frac{\Delta \mathbf{k_T}}{\mathbf{k_T}} \equiv \frac{\tilde{\mathbf{k}}_\mathbf{T} - \mathbf{k}_\mathbf{T}^{\text{true}}}{\mathbf{k}_\mathbf{T}^{\text{true}}}$$ distribution for $\tilde{q}\tilde{q}^* \to q\chi\bar{q}\chi$:

$\tilde{\mathbf{k}}_\mathbf{T} = \frac{1}{2}\not{\mathbf{p}}_\mathbf{T} \quad (\tilde{\mathbf{k}}_\mathbf{T} + \tilde{\mathbf{l}}_\mathbf{T} = \not{\mathbf{p}}_\mathbf{T})$

$\tilde{\mathbf{k}}_\mathbf{T} = \mathbf{k}_\mathbf{T}^{\text{maos}}$ for full events,

$\tilde{\mathbf{k}}_\mathbf{T} = \mathbf{k}_\mathbf{T}^{\text{maos}}$ for the top 10 % of near endpoint events



$\mathbf{M_{T2}}$-kink Cho, KC, Kim, Park

$$pp \;\rightarrow\; U + Y + \bar{Y} \;\rightarrow\; U + \sum V(p_i) + \chi(k) + \sum V(q_j) + \chi(l)$$

$$M_{T2}(P, Q, \not{\mathbf{p}}_T; m_\chi) = \min_{\mathbf{k}_T + \mathbf{l}_T = \not{\mathbf{p}}_T} \left[\max\Big(M_T(P, \mathbf{k}_T, m_\chi), M_T(Q, \mathbf{l}_T, m_\chi) \Big) \right]$$

$$\Big(P = \sum p_i, \quad Q = \sum q_j\Big)$$

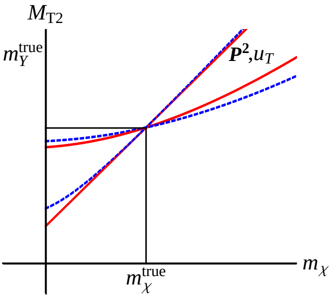
To understand the origin of kink, consider symmetric endpoint events:

$$P^2 = Q^2, \quad \mathbf{P}_T = \mathbf{Q}_T, \quad M_{T2}(m_\chi = m_\chi^{\text{true}}) = m_Y^{\text{true}}.$$

$\Longrightarrow$ Two parameter, (E_T, u_T), set of endpoint events whose M_{T2}-curves have different slopes at $m_\chi = m_\chi^{\text{true}}$:

$$M_{T2}^2(m_\chi) = -\frac{u_T^2}{4} + \left[E_T + \sqrt{\left(\sqrt{(m_Y^{\text{true}})^2 + u_T^2/4} - E_T\right)^2 + m_\chi^2 - (m_\chi^{\text{true}})^2} \right]^2$$

$$\Big(E_T = \sqrt{P^2 + \mathbf{P}_T^2}, \quad u_T = \text{UTM (upstream transverse momentum)}\Big)$$



$$M_{T2}^{\rm max}(m_\chi) = \max_{\{\text{all events}\}} \left[M_{T2}(\text{event}; m_\chi) \right] \text{ has a kink at } m_\chi = m_\chi^{\rm true}.$$

Kink can be sharp enough to be visible if

• the invariant mass-square P^2 can have a wide range of value, which would be the case when $\sum V(p_i)$ is a multi-particle state.

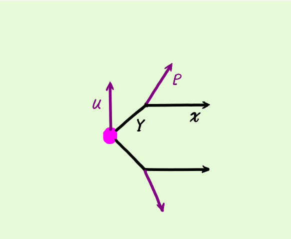
Cho, KC, Kim, Park

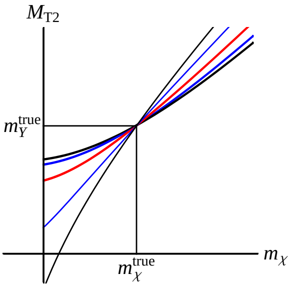
• same for the UTM u_T, which would the case when Y is produced with a large ISR or produced by the decay of heavier particle.

Gripaios; Barr, Gripaios, Lester

UTM Kink: $m_Y^{\text{true}} : m_\chi^{\text{true}} = 2 : 1$

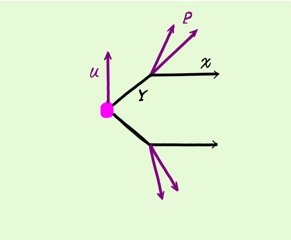
$P^2 = 0, \quad |u_T| = 0,\ m_Y^{\text{true}},\ 2m_Y^{\text{true}}$

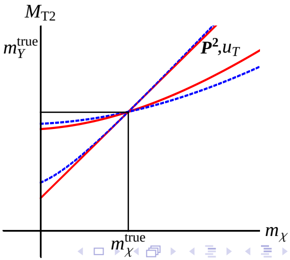




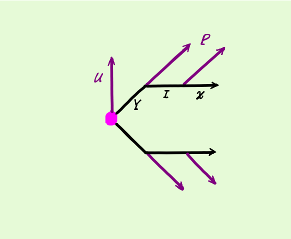
Invariant Mass Kink: $m_Y^{\text{true}} : m_\chi^{\text{true}} = 6 : 1$

$0 \leq P^2 \leq (m_Y^{\text{true}} - m_\chi^{\text{true}})^2, \quad 0 \leq |u_T| \leq m_Y$

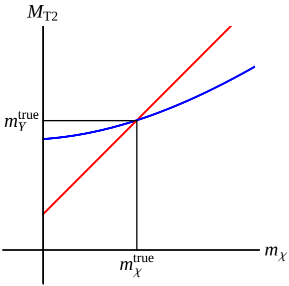




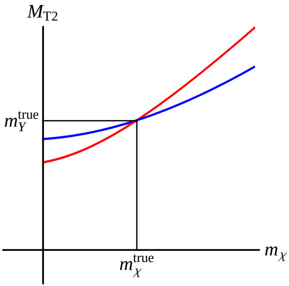
Invariant Mass Kink for Cascade Decay:



$$0 \leq P^2 \leq ((m_Y^{\text{true}})^2 - (m_I^{\text{true}})^2)(1 - (m_\chi^{\text{true}})^2/(m_I^{\text{true}})^2), \quad u_T = 0$$



$$m_I^{\text{true}} \simeq \sqrt{m_Y^{\text{true}} m_\chi^{\text{true}}}$$



$$m_Y^{\text{true}} : m_I^{\text{true}} : m_\chi^{\text{true}} = 6 : 4 : 1$$

Application to Mass and Spin Measurement

Real application should suffer from various uncertainties:

Jet momentum resolution, Combinatorics, Backgrounds, ...

Some Remarks:

- To reduce the smearing of endpoint by jet momentum uncertainties,

$$M_{\text{vis}}^2 = P^2 = \left(\sum p_i\right)^2 \longrightarrow M_{T\text{vis}}^2 = 2\sum_{i>j}\left(|\mathbf{p}_{Ti}||\mathbf{p}_{Tj}| - \mathbf{p}_{Ti}\cdot\mathbf{p}_{Tj}\right)$$

- To deal with the combinatorics of $\{P, Q, u\}$, one can use

$$M_{TGen} = \min_{\{\text{combinatorics}\}} M_{T2}(P, Q, u)$$ Barr, Lester, Stephens

- M_{T2} and MAOS momentum can be useful also for some SM process:

$$t\bar{t} \to bW^+\bar{b}W^- \to b\ell^+\nu\bar{b}\ell\bar{\nu}, \quad h \to W^+W^- \to \ell^+\nu\ell\bar{\nu}$$

- Generalization to more general event topology:
  "algebraic singularity method" Parallel talk by Ian Woo Kim

M_{T2}-Kink for Mass Measurement

Example 1: Gluino-Chargino M_{T2} for Focus Point Scenario (SPS2):

$$m_{\tilde{g}} \simeq 780, \quad m_{\chi_1^\pm} \simeq m_{\chi_2^0} \simeq 230, \quad m_{\chi_1^0} \simeq 120 \quad \left(m_{\tilde{q},\tilde{\ell}} \sim 1500\right)$$

Event generation by PYTHIA6.4 at $\sqrt{s} = 14$ TeV, $\int \mathcal{L} = 100$ fb^{-1}, and detector simulation with PGS4

Select the events with

* $n(\text{hard jet}) \geq 4, \quad n(\text{isolated } \ell) = 2 \ \ (\ell = e^\pm \text{ or } \mu^\pm)$
* $|\mathbf{p}_T|(j_1, j_2, j_3, j_4) > 100,\ 50,\ 50,\ 50$ GeV
* $|\not{\mathbf{p}}_T| > \max(100, 0.2M_{\text{eff}}) \quad \left(M_{\text{eff}} = |\not{\mathbf{p}}_T| + \sum|\mathbf{p}_T|(j_i) + \sum|\mathbf{p}_T|(\ell)\right)$
* $|m_{\ell\bar{\ell}} - m_Z| > 25, \ldots$

Assume these events arise from

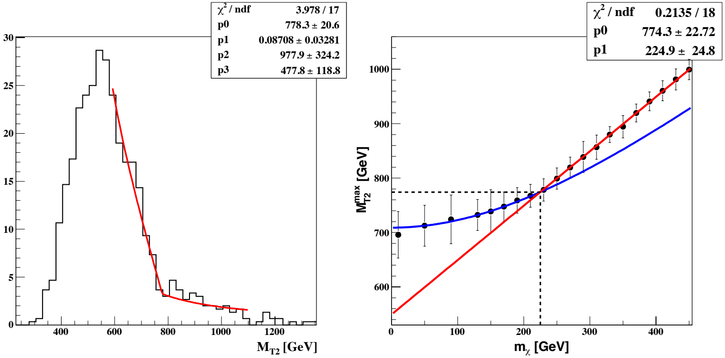
$$\tilde{g}\tilde{g} \to qq\chi_1^\pm qq\chi_1^\pm \quad (\chi_1^\pm \to \chi_1^0 W^\pm \to \chi_1^0 \ell^\pm \nu)$$

and pretend leptons to be invisible, which means $\chi_1^\pm$ are regarded as missing particles.

To deal with jet combinatorics, use

$$M_{TGen} = \min_{\{\text{jet combnatorics}\}} \left[M_{T2}(P, Q, u) \right]$$

M_{T2}-kink analysis gives rise to



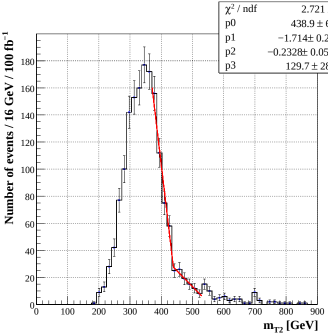
$$m_{\tilde{g}} = 774 \pm 23, \quad m_{\chi_1^\pm} = 225 \pm 25 \text{ GeV}.$$

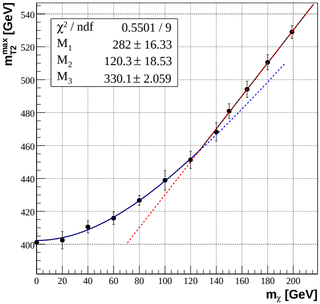
Example 2: Gluino-Neutralino M_{T2} for Yukawa-unified SUSY GUT:

Altmannshofer, Guadagnoli, Raby, Straub

$$m_{\tilde{g}} \simeq 470, \quad m_{\chi_1^\pm} \simeq m_{\chi_2^0} \simeq 118, \quad m_{\chi_1^0} \simeq 59 \quad \left(m_{\tilde{q},\tilde{\ell}} \sim \text{few TeV}\right)$$

Similar analysis for $\tilde{g}\tilde{g} \to b\bar{b}\chi_2^0 b\bar{b}\chi_2^0 \quad (\chi_2^0 \to \chi_1^0\gamma)$ and pretend photons to be invisible, which means χ_2^0 are regarded as missing particles. Parallel talk by Diego Guadagnoli





$$\Longrightarrow \quad m_{\tilde{g}} = 456 \pm 15, \quad m_{\chi_2^0} = 126 \pm 16.$$

MAOS Momentum and Spin Measurement

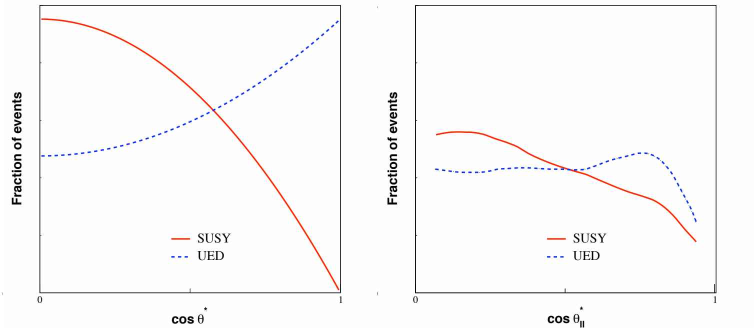
Example 1: Drell-Yan pair production of **slepton** or **KK-lepton** for SUSY SPS1a point and its UED equivalent:

$$\frac{d\Gamma}{d\cos\theta_Y} \text{ and } \frac{d\Gamma}{d\cos\theta_\ell} \quad \text{of} \quad q\bar{q} \to Z^0/\gamma \to Y\bar{Y} \to \ell\chi\bar{\ell}\chi$$

$Y =$ slepton or KK-lepton, $\chi =$ LSP or KK-photon,

$\cos\theta_Y = \hat{p}_Y \cdot \hat{p}_{\text{beam}}$ in the CM frame of $Y\bar{Y}$,

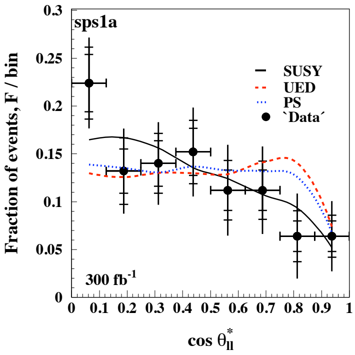
$\cos\theta_\ell = \hat{p}_\ell \cdot \hat{p}_{\text{beam}}$ in the CR(rapidity) frame of $\ell\bar{\ell}$

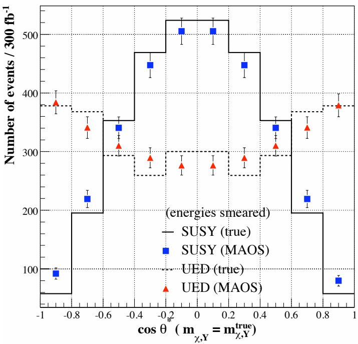


With MAOS momentum, the mother particle production angle ($\cos\theta_Y$) can be reconstructed:

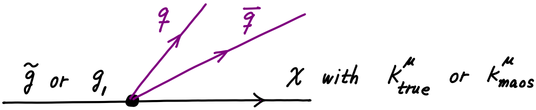
$$\frac{d\Gamma}{d\cos\theta_\ell} \text{ (Barr)} \quad \text{vs} \quad \frac{d\Gamma}{d\cos\theta_Y^{\rm maos}} \text{ (Cho, KC, Kim, Park)}$$

under appropriate event cut ($\ni$ the M_{T2}-cut selecting the top 30 %): $\big(\theta_Y^{\rm maos}$ for $m_\chi = 0, m_Y = M_{T2}^{\max}(0)$ shows a similar behavior, so the knowledge of masses is not essential.$\big)$



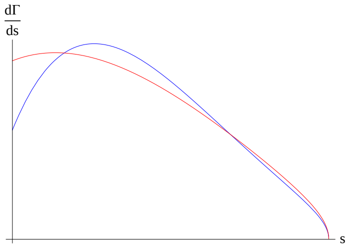


Example 2: **Gluino** or **KK-gluon** 3-body decay for SPS2 point and its UED equivalent:

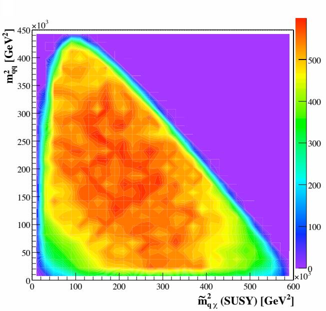


$$\mathbf{s} = \left(p_q + p_{\bar q}\right)^2, \quad \mathbf{t}_{\rm maos} = \left(p_q \ (\text{or } p_{\bar q}) + k^{\rm maos}\right)^2$$

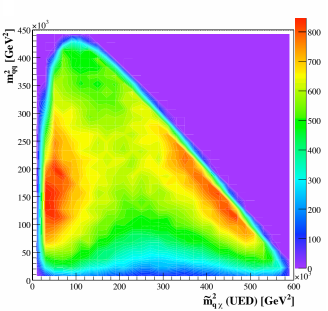
Without $k^{\rm maos}_{\mu}$, one may consider the $\mathbf{s}$-distribution $d\Gamma/d\mathbf{s}$ to distinguish **gluino** from **KK-gluon**: Csaki, Heinonen, Perelstein



With $k_\mu^{\rm maos}$, one can use the **s-t**$_{\rm maos}$ distribution $\left(d\Gamma/ds\,d\mathbf{t}_{\rm maos}\right)$:



gluino 3-body decay



KK-gluon 3-body decay

Including various uncertainties (jet momentum, combinatorics, SUSY backgrounds), the **s**-distribution $\left(d\Gamma/ds\right)$ for SPS2 can not distinguish SUSY from UED even with $\int \mathcal{L} = 300\,\mathrm{fb}^{-1}$.

On the other hand, the **s-t**$_{\rm maos}$ distribution $\left(d\Gamma/ds\,d\mathbf{t}_{\rm maos}\right)$ can clearly discriminate SUSY from UED. Cho, KC, Kim, Park

Conclusion

- M_{T2} and MAOS momentum are the collider variables which can be useful for the mass and/or spin measurement of new particles in missing energy events at LHC.

  * M_{T2}-kink might be able to determine new particle masses even when a long cascade decay is not available.

  * MAOS momentum provides a systematic approximation to the missing particle momentum, so might be useful for the determination of new particle properties, e.g. mass and spin.

- M_{T2} and MAOS momentum can be useful also for some SM process with two missing neutrinos:

  * $t\bar{t} \to bW^+\bar{b}W^- \to b\ell^+\nu\bar{b}\ell\bar{\nu}$
  * $h \to W^+W^- \to \ell^+\nu\ell\bar{\nu}$