Non-perturbative effects and D-brane instanton resummation in string theory

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Motivation & introduction

Non-perturbative effects in string theory are relevant in a number of phenomenological model building issues

- Kahler moduli stabilization in KKLT models
- Generation of perturbatively forbidden couplings
  
  Neutrino masses, Yukawas, mu term, ...

Focus on type II (or orientifolds):
D-brane instantons

Euclidean D-branes wrapped on CY cycles and localized on 4d Minkowski
D-brane instanton effects

[Becker’s, Strominger; Witten; Harvey, Moore; ...]

Violate certain perturbatively exact U(1) global symmetries

Ex: Take one complex structure modulus in IIA CY orientifold

\[ T = t + i a = \int_C \text{Re} \Omega + i \int_C C_3 \]

PQ symmetry \( a \to a + \lambda \) violated by D2-brane instanton \( \sim e^{-T} \)

\[ \Rightarrow \text{Stabilization of moduli perturbatively protected by PQ} \]

[Kachru, Kallosh, Linde, Trivedi]

Need to sum over all such instantons!

Only very partial results...

e.g. classify contributing instantons  [Denef, Douglas, Florea+Grassi, Kachru]
Field theory operators

With 4d gauge D6-branes, gauging of PQ by U(1) in U(N)
From D6-branes on C’

\[
\int_{C’ \times M_4} C_5 \wedge \text{tr } F \rightarrow \int_{M_4} B_2 \wedge \text{tr } F \rightarrow \int d^4 x (\partial_\mu a + A_\mu)^2
\]

⇒ Instanton on C violates U(1)_C’ by n=I_{CC’} units
⇒ Fermion zero modes at I_{CC’} intersections

⇒ Role in generating perturbatively forbidden couplings

\[
e^{-T} \Phi_1 \ldots \Phi_n
\]

[Blumenhagen, Cvetic, Weigand; Ibanez, AU; Florea, Kachru, McGreevy, Saulina]

Need to sum over all such instantons!
Essentially no results...
Particles and instantons

A whole lot is known about the spectrum of BPS particles in CY compactifications

[Denef, Moore; ...]

Sometimes even full list at any point in moduli space

e.g. conifold

[Jafferis, Moore]

toric singus with no 4-cycles

[Aganagic, Ooguri, Vafa, Yamazaki]

Relate BPS D-brane particles to BPS D-brane instantons via circle compactification to 3d and T-duality

[Ooguri, Vafa ’96]

One-loop diagram of BPS particles ⇔ BPS D-brane instanton

Momentum of BPS particles ⇔ Multiwrapping of instanton

Bound states of BPS particles ⇔ Multi-instanton processes
D1/D(-1) instantons in type IIB on CY  

- Consider 4d D2/D0 particles in IIA on CY
  Compactification to 3d + T-duality gives D1/D(-1)-instantons on IIB on same CY
  e.g. corrections to kinetic term of Kahler moduli

- Multiplicities of D2/D0 can be counted by lift to M-theory
  in terms of Gopakumar-Vafa invariants $n_g,[C]$
  count 5d BPS particles from M2-branes on 2-cycles C

All at once: One-loop of 5d particles on $T^2$

$$Z_{M2} = \frac{1}{4\pi} \sum_k n_{k_a}^{(0)} \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m + \tau n|^{3/2}} (1 + 2\pi |m + \tau n| k_a t^a) e^{-S_{k_1,k_2}}$$

$$Z_{D(-1)\text{-inst}} = \frac{1}{4\pi} \sum_{k_a} n_{k_a} \sum_{m\neq 0,n\in\mathbb{Z}} \frac{|z + q|^{1/2}}{|p|^{3/2}} \left[ 1 + \sum_{k=1}^{\infty} \frac{\Gamma(3/2 + k)}{k! \Gamma(3/2 - k)} \left( 4\pi |p\tau_2| |z + q| \right)^{-k} \right] e^{-S_{(p,q)}}$$

Reproduces [Robles-Llana, Rocek, Saueressig, Theis, Vandoren, ‘06]
D(-1) instantons in type IIB with O7/D7

Understand better the role of O-planes and gauge D-branes
- Instantons localized on top of O-plane
- Instantons leading to gauge operators

Consider a simple 8d example
type IIB on $T^2$ with O7-planes and D7’s
D(-1)-instantons yield $\text{tr} F^4$, $(\text{tr} F^2)^2$, $R^4$
etc corrections

Become 4d N=2 gauge functions upon compactification on K3
Using heterotic/type IIB orientifold duality, can be computed from heterotic as threshold corrections

$$\frac{t_8 \text{Tr} F^4}{4} \log \left| \frac{\eta(4T_h)}{\eta(2T_h)} \right|^4 + \frac{t_8 (\text{Tr} F^2)^2}{16} \log \left( \text{Im} T_h \text{Im} U_h \frac{\eta(2T_h)^8 |\eta(U_h)|^4}{|\eta(4T_h)|^4} \right)$$

$$+ 2 t_8 \text{Pf} F \log \left| \frac{\eta(T_h + 1/2)}{\eta(T_h)} \right|^4$$

[Lerche, Stieberger; Gutperle; Kiritsis, Obers, Pioline;
Gava, Narain, Sarmadi]
The instanton computation

- In type IIB can be directly computed in instanton language
- Complicated integration over instanton moduli space using “localization”
- Heavy computation, explicit only order by order in instanton number; extrapolation
- Result is

\[ t_8 \text{Tr} F^4 \left\{ \frac{\pi i T_h}{12} - \frac{1}{2} \sum_{k=1}^{\infty} (d_k q_h^{4k} - d_k q_h^{2k}) \right\} + \text{c.c.} \]

\[ + t_8 (\text{Tr} F^2)^2 \left\{ \frac{1}{16} \log (\text{Im} T_h \text{Im} U_h |\eta(U_h)|^4) + \frac{1}{8} \left( \sum_{k=1}^{\infty} (d_k q_h^{4k} - 2d_k q_h^{2k}) + \text{c.c.} \right) \right\} \]

\[ + 8 t_8 \text{Pf} F \left( \sum_{k=1}^{\infty} d_{2k-1} q_h^{2k-1} + \text{c.c.} \right) \]

with \( q = \exp (2\pi iT) \) the instanton factor

- Full agreement with heterotic result

[Billo, Ferro, Frau, Gallot, Lerda, Pesando]
One-loop of D0’s

The whole D(-1) instanton sum can be obtained from a one-loop diagram of BPS particles in T-dual type I’ 9d IIA on $S^1$ and two O8 with 16 D8’s each i.e. $SO(16)^2$ with Wilson line breaking $SO(16)$ to the 8d $SO(8)^2$

- Bound states of BPS particles stuck on top of O-plane
- BPS particles charged under 4d group

The $F^4$ term from a one-loop of particle in representation $R$:

$$\Delta = \sum_m \int d^8p \int \frac{dt}{t} e^{-t(p^2 + \frac{m}{R^2} + n^2)} t^4 \text{tr}_RF^4 = \sum_w \frac{1}{w} q^n w e^{2\pi i w A} \text{tr}_RF^4$$

BPS particles:
- Bound state of $2n$ D0’s, in 120 of $SO(16)$
- Bound state of $2n+1$ D0’s, in 128 of $SO(16)$

Full result computable, agreement with IIB and heterotic results
Interpretation in Horava-Witten theory

Lift IIA with two O8’s to Horava-Witten

Each 10d boundary on $S^1$ with WL breaking $E_8$ to $SO(16)$
- Momentum $2n$ is WL invariant and gives 120
- Momentum $2n+1$ is WL odd and gives 128

Each 10d boundary on $T^2$ with WLs breaking $E_8$ to $SO(8)^2$

One-loop diagram of $E_8$ gauge bosons with WL effects

Clearly, same computation & result
Manifold interpretation of modular functions of $\Gamma(2)$
Modular transformations of $T^2$ preserving the WLs
Multi-instantons vs. Polyinstantons

Mapping to BPS particle loops allows a nice interpretations of contributions involving multiple instantons

- **Multi-instantons**: [Garcia-Etxebarria, AU]
  Correspond to diagrams with one BPS bound states of particles

- **Poly-instantons**: [Blumenhagen, Schmidt-Sommerfeld]
  Correspond to diagrams with multiparticle states
  Higher loop diagram

![Diagram](image)
Superpotentials?

Compactification on K3 leads to gauge kinetic functions in 4d N=2

Similar techniques for gauge functions in 4d N=1 using orbifolds
(corrections lie in N=2 subsectors, as in heterotic)

- What about genuine computations of superpotentials?
Not there yet...

But tantalizing hints:

Open and unoriented versions of Gopakumar-Vafa invariants

Expected to describe BPS multiplicities of particles in theories with 4 susys

Looking for a clever duality to 4d instantons...
Final comments

Non-perturbative effects in string theory are in rich interface

Duality
Moduli stabilization and susy breaking
Phenomenological applications

Basic physics understood, new physics in resummation

Duality

Modular properties

Started exploiting interplay of particles and instantons

Successful with 8 susys, or subsectors of 4d N=1
Hoping to find lessons for 4d N=1 superpotentials