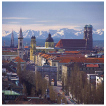
Quantum-corrected Boltzmann equations for Leptogenesis

Mathias Garny (TU Munich)

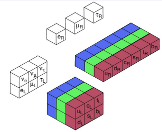
TECHNISCHE UNIVERSITÄT MÜNCHEN



PLANCK, Geneva, May 31 – June 04 2010

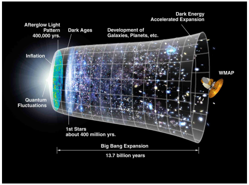
based on 1005.5385, 1002.0331, 0911.4122, 0909.1559

with A. Hohenegger, A. Kartavtsev, M. Lindner; M. M. Müller

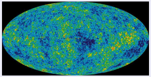


$+$?

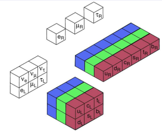
$\longleftrightarrow$



$\longleftrightarrow$

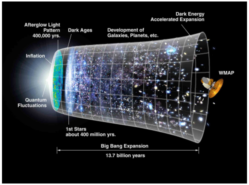


- Baryogenesis
- Dark matter freeze-out
- Inflation, Reheating
- ...

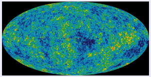


+ ?

⟷



⟷



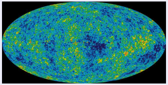
- Baryogenesis
- Dark matter freeze-out
- Inflation, Reheating
- ...

Nonequilibrium dynamics at high energy

Quantum-corrected Boltzmann equations for Leptogenesis

Baryogenesis: three Sakharov conditions

- baryon number violation
- CP violation
- deviation from thermal equilibrium

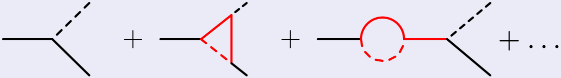


$\eta_{10} = (n_b - n_{\bar{b}})/(s \cdot 10^{-10})$

$4.7 < \eta_{10} < 6.5$ (95% CL)

Leptogenesis: decay of heavy right-handed neutrino N_i

$\mathcal{M}_{N_i \to l_\alpha h^\dagger} =$



CP violation in decay described by loop process

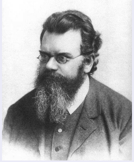
Quantum nonequilibrium effects ?

Outline

- The classical approach: Boltzmann
- Nonequilibrium quantum field theory: Kadanoff-Baym
- Corrections to Boltzmann

Semi-classical approach: Boltzmann equations

Boltzmann equation



$$p^\alpha \mathcal{D}_\alpha f_\ell(t, \mathbf{x}, \mathbf{p}) \quad = \quad \mathcal{C}_\ell^{gain}(p)(1 - f_\ell) - \mathcal{C}_\ell^{loss}(p) f_\ell$$

$$\mathcal{C}_\ell^{gain} = \int d\Pi^3_{p_N} d\Pi^3_{p_h} (2\pi)^4 \delta(p_N - p_\ell - p_h) |\mathcal{M}|^2_{N_i \to \ell_\alpha h^\dagger} \; f_{N_i}(1 + f_{h^\dagger}) + \ldots$$

$$\mathcal{C}_\ell^{loss} = \int d\Pi^3_{p_N} d\Pi^3_{p_h} (2\pi)^4 \delta(p_N - p_\ell - p_h) |\mathcal{M}|^2_{\ell_\alpha h^\dagger \to N_i} \; (1 - f_{N_i}) f_{h^\dagger} + \ldots$$

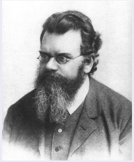
$|\mathcal{M}|^2$: microscopic interactions, off-shell processes

$f_\psi(t, \mathbf{x}, \mathbf{p})$: macroscopic propagation of on-shell particles

Semi-classical approach: Boltzmann equations

Boltzmann equation

$$p^\alpha \mathcal{D}_\alpha f_\ell(t, \mathbf{x}, \mathbf{p}) \quad = \quad \mathcal{C}_\ell^{gain}(p)(1 - f_\ell) - \mathcal{C}_\ell^{loss}(p) f_\ell$$



$$\mathcal{C}_\ell^{gain} = \int d\Pi^3_{p_N} d\Pi^3_{p_h} (2\pi)^4 \delta(p_N - p_\ell - p_h) |\mathcal{M}|^2_{N_i \to \ell_\alpha h^\dagger} \; f_{N_i}(1 + f_{h^\dagger}) + \ldots$$

$$\mathcal{C}_\ell^{loss} = \int d\Pi^3_{p_N} d\Pi^3_{p_h} (2\pi)^4 \delta(p_N - p_\ell - p_h) |\mathcal{M}|^2_{\ell_\alpha h^\dagger \to N_i} \; (1 - f_{N_i}) f_{h^\dagger} + \ldots$$

$|\mathcal{M}|^2$: microscopic interactions, off-shell processes

$f_\psi(t, \mathbf{x}, \mathbf{p})$: macroscopic propagation of on-shell particles

$$\Delta x_{interaction} \ll \lambda_{mfp}, \quad \lambda_{de-Broglie} \ll \lambda_{mfp}$$

$$1/M \ll 1/\Gamma, \quad 1/T \ll 1/y^2 T$$

Corrections within Boltzmann picture: bottom-up

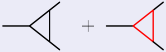
Bose-enhancement, Pauli-Blocking; kinetic (non-)equilibrium

- quantum statistical factors $1 \pm f_k$
- non-integrated Boltzmann equations

Hannsestad, Basbøll 06; Garayoa, Pastor, Pinto, Rius, Vives 09; Hahn-Woernle, Plümacher, Wong 09

Medium corrections via thermal QFT

- medium correction to decay rates



$$\epsilon_i = \frac{\Gamma(N_i \to l h^\dagger) - \Gamma(N_i \to l^c h)}{\Gamma(N_i \to l h^\dagger) + \Gamma(N_i \to l^c h)} = \epsilon_i^{vac} + \delta\epsilon_i^{th}(T, \ldots)$$

- thermal masses

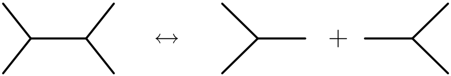
Covi, Rius, Roulet, Vissani 98; Giudice, Notari, Raidal, Riotto, Stumia 04; Kiessig, Thoma, Plümacher 10...

Flavour effects

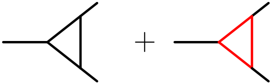
Nardi, Nir, Roulet, Racker 06; Adaba, Davidson, Ibarra, Josse-Micheaux, Losada, Riotto 06; Blanchet, diBari 06...

Limitations of the bottom-up Boltzmann approach

- Double Counting Problem(s) for real intermediate states [RIS]

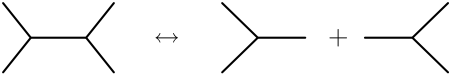


- Ambiguities of thermal QFT applied to non-equilibrium processes

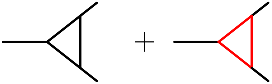


Limitations of the bottom-up Boltzmann approach

- Double Counting Problem(s) for real intermediate states [RIS]



- Ambiguities of thermal QFT applied to non-equilibrium processes



- Higher gradient terms, memory effects

$$\frac{dY_{B-L}}{dz} = \underbrace{D_i(Y_{N_i} - Y_{N_i}^{eq})}_{\mathcal{S}_0} - WY_{B-L} + \mathcal{S}_1 + \ldots$$

- Spectral function $\neq$ quasi-particles (resonant case), ...

Goal: QFT description
quantify corrections to Boltzmann

Going beyond the standard Boltzmann picture

Statistical propagator	$D_F^{ij}(x,y)$	$=$	$\langle \Phi^i(x)\bar{\Phi}^j(y) + \bar{\Phi}^j(y)\Phi^i(x)\rangle/2$	$=$	$[D_>^{ij} + D_<^{ij}]/2$
Spectral function	$D_\rho^{ij}(x,y)$	$=$	$i\langle \Phi^i(x)\bar{\Phi}^j(y) - \bar{\Phi}^j(y)\Phi^i(x)\rangle$	$=$	$i[D_>^{ij} - D_<^{ij}]$

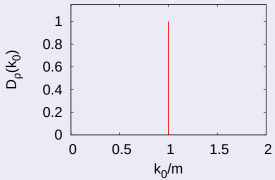
Going beyond the standard Boltzmann picture

Statistical propagator $D_F^{ij}(x,y) = \langle \Phi^i(x)\bar{\Phi}^j(y) + \bar{\Phi}^j(y)\Phi^i(x)\rangle/2 = [D_>^{ij} + D_<^{ij}]/2$

Spectral function $D_\rho^{ij}(x,y) = i\langle \Phi^i(x)\bar{\Phi}^j(y) - \bar{\Phi}^j(y)\Phi^i(x)\rangle = i[D_>^{ij} - D_<^{ij}]$

Boltzmann limit

- on-shell quasi-stable particles



$$D_\rho^{ij}(k) \sim \delta^{ij}\delta(k^2 - m_i^2)$$

- equilibrium-like KMS relation

$$D_F^{ij}(t,k) = \left(f_k^i(t) + \frac{1}{2}\right) D_\rho^{ij}(k)$$

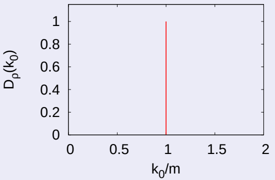
Going beyond the standard Boltzmann picture

Statistical propagator $D_F^{ij}(x,y) = \langle \Phi^i(x)\bar{\Phi}^j(y) + \bar{\Phi}^j(y)\Phi^i(x)\rangle/2 = [D_>^{ij} + D_<^{ij}]/2$

Spectral function $D_\rho^{ij}(x,y) = i\langle \Phi^i(x)\bar{\Phi}^j(y) - \bar{\Phi}^j(y)\Phi^i(x)\rangle = i[D_>^{ij} - D_<^{ij}]$

Boltzmann limit

- on-shell quasi-stable particles



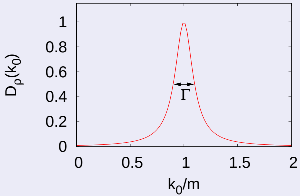
$$D_\rho^{ij}(k) \sim \delta^{ij}\delta(k^2 - m_i^2)$$

- equilibrium-like KMS relation

$$D_F^{ij}(t,k) = \left(f_k^i(t) + \frac{1}{2}\right) D_\rho^{ij}(k)$$

Propagation beyond Boltzmann

- spectrum with (thermal) width



$$D_\rho^{ij}(t,k) \sim \frac{\delta^{ij} 2k_0 \Gamma_i(t)}{(k^2 - m_{th,i}^2(t))^2 + k_0^2 \Gamma_i(t)^2} + \ldots$$

- off-shell excitations

$$D_F^{ij}(t,k) = \begin{pmatrix} D_F^{11} & D_F^{12} \\ D_F^{21} & D_F^{22} \end{pmatrix}$$

Kadanoff-Baym equations

$$\left(\Box_x + m_i^2(x)\right) D_F^{ij}(x,y) = \int_0^{y^0} d^4z\, \Sigma_F^{ik}(x,z) D_\rho^{kj}(z,y) - \int_0^{x^0} d^4z\, \Sigma_\rho^{ik}(x,z) D_F^{kj}(z,y)$$

$$\left(\Box_x + m_i^2(x)\right) D_\rho^{ij}(x,y) = \int_{x_0}^{y^0} d^4z\, \Sigma_\rho^{ik}(x,z) D_\rho^{kj}(z,y)$$

- Statistical propagator encodes time-evolution of the state
- Spectral function includes off-shell effects self-consistently
- Memory integrals

Buchmüller, Fredenhagen 00; Lindner, Müller 05,07; DeSimone, Riotto 07;
Anisimov, Buchmüller, Drewes, Mendizabal 08,10;
MG, Hohenegger, Kartavtsev, Lindner 09,10; Gagnon 09; Beneke, Garbrecht, Herranen, Schwaller 10

Nonequilibrium Quantum Field Theory

toy model $\mathcal{L} \supset -y_i \tilde{N}_i \tilde{\ell} h^\dagger - y_i^* \tilde{N}_i \tilde{\ell}^\dagger h$

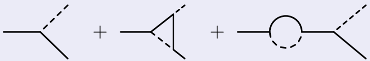
Boltzmann (bottom-up)

$$n_{B-L} = -\int \frac{d^3p}{(2\pi)^3} \left[f_\ell - f_{\bar{\ell}} \right]$$

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3 n_{B-L}\right) =$$

$$-\int \frac{d^3p}{(2\pi)^3 2E_p} \Big[\mathcal{C}_\ell^{gain}(1 - f_\ell) - \mathcal{C}_\ell^{loss} f_\ell$$

$$-\mathcal{C}_{\bar{\ell}}^{gain}(1 - f_{\bar{\ell}}) + \mathcal{C}_{\bar{\ell}}^{loss} f_{\bar{\ell}} \Big]$$



Nonequilibrium Quantum Field Theory

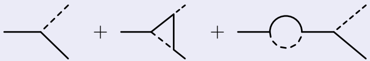
toy model $\mathcal{L} \supset -y_i \tilde{N}_i \tilde{\ell} h^\dagger - y_i^* \tilde{N}_i \tilde{\ell}^\dagger h$

Boltzmann (bottom-up)

$$n_{B-L} = -\int \frac{d^3p}{(2\pi)^3} \left[f_\ell - f_{\bar{\ell}} \right]$$

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3 n_{B-L}\right) =$$

$$-\int \frac{d^3p}{(2\pi)^3 2E_p} \left[\mathcal{C}_\ell^{gain}(1-f_\ell) - \mathcal{C}_\ell^{loss} f_\ell - \mathcal{C}_{\bar{\ell}}^{gain}(1-f_{\bar{\ell}}) + \mathcal{C}_{\bar{\ell}}^{loss} f_{\bar{\ell}} \right]$$



Non-equilibrium QFT

$$\begin{aligned} j_\mu(x) &= 2i \left\langle \left[\mathcal{D}_\mu \tilde{\ell}(x) \right] \tilde{\ell}^\dagger(x) - \tilde{\ell}(x)\, \mathcal{D}_\mu \tilde{\ell}^\dagger(x) \right\rangle \\ &= (n_{B-L}, \vec{j}_{B-L}) \end{aligned}$$

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3 n_{B-L}\right) = \mathcal{D}^\mu j_\mu = \qquad \text{[in FRW]}$$

$$\int_{t_{\rm init}}^{t} dt' \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\Big\{ \Sigma_<(t,t')D_>(t',t) - \Sigma_>(t,t')D_<(t',t) - \bar{\Sigma}_<(t,t')\bar{D}_>(t',t) + \bar{\Sigma}_>(t,t')\bar{D}_<(t',t) \Big\}$$

Non-equilibrium QFT

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3 n_{B-L}\right) = \mathcal{D}^\mu j_\mu = \int_{t_{\rm init}}^{t} dt' \int \frac{d^3p}{(2\pi)^3} \,\mathrm{Tr}\Big\{ \Sigma_<(t,t')D_>(t',t) - \Sigma_>(t,t')D_<(t',t) - \bar{\Sigma}_<(t,t')\bar{D}_>(t',t) + \bar{\Sigma}_>(t,t')\bar{D}_<(t',t) \Big\}$$

Caveat:

interactions switched on at $t = t_{init}$ $\Leftrightarrow$ Gaussian initial state

Kadanoff, Baym 62

Gauss: $\mathcal{D}^\mu j_\mu(x)|_{t=t_{init}} = 0 \quad \Leftrightarrow n_{B-L}(t) - n_{B-L}^{init} \sim (t - t_{init})^2 + \ldots$

Non-equilibrium QFT

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3 n_{B-L}\right) = \mathcal{D}^\mu j_\mu = \int_{t_{\rm init}}^{t} dt' \int \frac{d^3p}{(2\pi)^3} \,\mathrm{Tr}\Big\{ \Sigma_<(t,t')D_>(t',t) - \Sigma_>(t,t')D_<(t',t) - \bar{\Sigma}_<(t,t')\bar{D}_>(t',t) + \bar{\Sigma}_>(t,t')\bar{D}_<(t',t) \Big\}$$

Caveat:

interactions switched on at $t = t_{init} \Leftrightarrow$ Gaussian initial state

Kadanoff, Baym 62

Gauss: $\mathcal{D}^\mu j_\mu(x)|_{t=t_{init}} = 0 \quad \Leftrightarrow n_{B-L}(t) - n_{B-L}^{init} \sim (t - t_{init})^2 + \ldots$

Solution(s):

- Non-Gaussian initial state *Danielewicz 83, ..., Borsanyi, Reinosa 08; MG, Müller 09*
- Pre-evolution, $t_{init} > t_{Gauss} (\rightarrow -\infty)$ *Kadanoff, Baym, Schwinger, Keldysh, ...*

Non-Gauss: $\mathcal{D}^\mu j_\mu(x)|_{t=t_{init}} \neq 0 \quad \Leftrightarrow n_{B-L}(t) - n_{B-L}^{init} \sim t - t_{init} + \ldots$

Top-down approach

Kadanoff-Baym equations

$\downarrow$

Wigner transformation in $s = x - y$ [+ pre-evolution]

$$D(X,k) = \int d^4s\ e^{iks}\ D(x,y), \quad t_{Gauss} \to -\infty$$

$\downarrow$

Gradient expansion in $X = (x+y)/2$ [- memory effects $\frac{t_{micro}}{t_{macro}} \sim \frac{\Gamma}{M}, \frac{H}{M}$]

$$\int d^4z\ \Sigma(x,z)D(z,y) \to \Sigma(X,k)D(X,k) + \frac{i}{2}\left\{\frac{\partial\Sigma}{\partial X}\frac{\partial D}{\partial k} - \frac{\partial\Sigma}{\partial k}\frac{\partial D}{\partial X}\right\} + \mathcal{O}(\partial_X^2)$$

$\downarrow$

Quasi-particle approximation [(-) off-shell effects $\sim \frac{\Gamma}{M}, \frac{\Gamma_\ell}{m_{th,\ell}}, \frac{\Gamma_h}{m_{th,h}}$]

$$D_\rho(X,k) = \frac{2k_0\Gamma}{(k^2-m_{th}^2)^2+k_0^2\Gamma^2} \to 2\pi\text{sign}(k^0)\delta(k^2-m^2)$$

$\downarrow$

Boltzmann equations

Top-down approach

Kadanoff-Baym equations

$\downarrow$

Wigner transformation in $s = x - y$ [+ pre-evolution]

$$D(X,k) = \int d^4s \; e^{iks} \; D(x,y), \quad t_{Gauss} \to -\infty$$

$\downarrow$

Gradient expansion in $X = (x+y)/2$ [- memory effects $\frac{t_{micro}}{t_{macro}} \sim \frac{\Gamma}{M}, \frac{H}{M}$]

$$\int d^4z \; \Sigma(x,z)D(z,y) \to \Sigma(X,k)D(X,k) + \frac{i}{2}\left\{\frac{\partial\Sigma}{\partial X}\frac{\partial D}{\partial k} - \frac{\partial\Sigma}{\partial k}\frac{\partial D}{\partial X}\right\} + \mathcal{O}(\partial_X^2)$$

$\downarrow$

Quasi-particle approximation [(-) off-shell effects $\sim \frac{\Gamma}{M}, \frac{\Gamma_\ell}{m_{th,\ell}}, \frac{\Gamma_h}{m_{th,h}}$]

$$D_\rho(X,k) = \frac{2k_0\Gamma}{(k^2 - m_{th}^2)^2 + k_0^2\Gamma^2} \to 2\pi \text{sign}(k^0)\delta(k^2 - m^2)$$

$\downarrow$

Quantum-corrected Boltzmann equations

Quantum-corrected Boltzmann equations

- No double counting (no explicit RIS necessary)
- Includes medium corrections self-consistently
  (resolves ambiguities of thQFT ⇒ adv./ret. 3-point fctn.)
- Off-shell effects via spectral width can be included

$$\frac{dY_{B-L}}{dz} \propto \mathcal{D}^\mu j_\mu = 2\,|y_1|^2 \int d\Pi_p d\Pi_k d\Pi_q\, \Theta(p_0)(2\pi)^4 \delta(k-p-q)$$
$$\times D_\rho^{\tilde{N}_1}(k) D_\rho^{\tilde{\ell}}(p) D_\rho^h(q)$$
$$\times \epsilon(k,T) \left[f_{\tilde{N}_1}(k) - f_{\tilde{N}_1}^{eq}(k) \right]$$
$$\times \left([1 + f_{\tilde{\ell}}^{eq}(p)][1 + f_h^{eq}(q)] - f_{\tilde{\ell}}^{eq}(p) f_h^{eq}(q) \right)$$

$$\epsilon(k,T) = \epsilon^{vac} \times \int \frac{d\Omega}{4\pi} \left[1 + f_{\tilde{\ell}}(E_{\tilde{\ell}}) + f_h(E_h) \right]$$

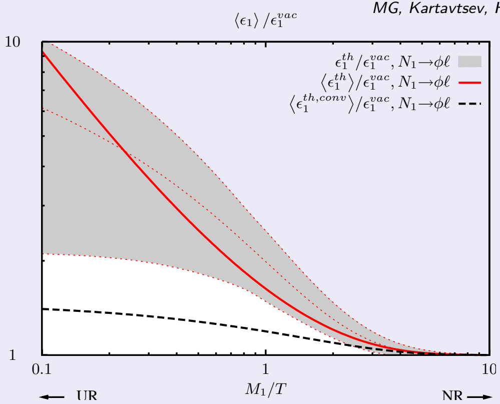
MG, Kartavtsev, Hohenegger, Lindner; Beneke, Garbrecht, Herranen, Schwaller; Buchmüller, Fredenhagen

Quantum-corrected Boltzmann equations

Medium correction to CP-violating parameter: SM

MG, Kartavtsev, Hohenegger, Lindner

$$\frac{\epsilon(k,T)}{\epsilon^{vac}}$$

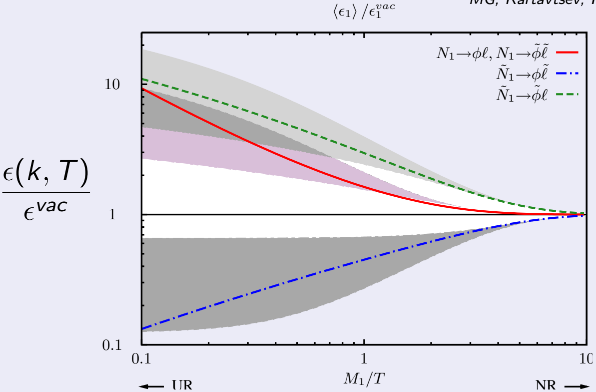


$\epsilon(k,T) = \epsilon^{vac} \times \int \frac{d\Omega}{4\pi}\left[1 - f_\ell(E_\ell) + f_h(E_h)\right]$ *Bose enhancement*

$$\epsilon(k,T) = \epsilon^{vac} \times \int \frac{d\Omega}{4\pi}\left[1 - f_\ell(E_\ell) + f_h(E_h) - 2 f_\ell(E_\ell) f_h(E_h)\right] \sim \epsilon^{vac}$$

Medium correction to CP-violating parameter: MSSM

MG, Kartavtsev, Hohenegger, Lindner



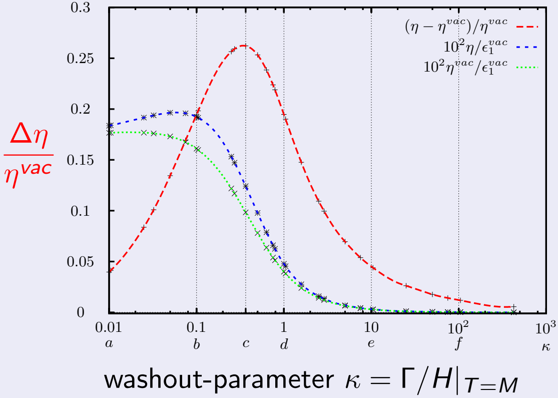
$$\tilde{N}_1 \to \tilde{h}\ell \qquad \epsilon/\epsilon^{vac} \sim 1 + f_{\tilde{\ell}} + f_h$$

$$N_1 \to h\ell, \tilde{h}\tilde{\ell} \qquad \epsilon/\epsilon^{vac} \sim 1 - f_\ell + f_h$$

$$\tilde{N}_1 \to h\tilde{\ell} \qquad \epsilon/\epsilon^{vac} \sim 1 - f_\ell - f_{\tilde{h}}$$

Medium correction to CP-violating parameter

MG, Kartavtsev, Hohenegger, Lindner



bosonic toy-model

Gradient corrections

Kadanoff-Baym equations

↓

Wigner transformation in $s = x - y$ [+ pre-evolution]

$$D(X,k) = \int d^4s\, e^{iks}\, D(x,y), \quad t_{Gauss} \to -\infty$$

↓

Gradient expansion in $X = (x+y)/2$ [- memory effects $\frac{t_{micro}}{t_{macro}} \sim \frac{\Gamma}{M}, \frac{H}{M}$]

$$\int d^4z\, \Sigma(x,z)D(z,y) \to \Sigma(X,k)D(X,k) + \frac{i}{2}\left\{\frac{\partial\Sigma}{\partial X}\frac{\partial D}{\partial k} - \frac{\partial\Sigma}{\partial k}\frac{\partial D}{\partial X}\right\} + \mathcal{O}(\partial_X^2)$$

Prokopec, Schmidt, Weinstock, Kainulainen, Konstandin, ...

↓

Quasi-particle approximation [(-) off-shell effects $\sim \frac{\Gamma}{M}, \frac{\Gamma_\ell}{m_{th,\ell}}, \frac{\Gamma_h}{m_{th,h}}$]

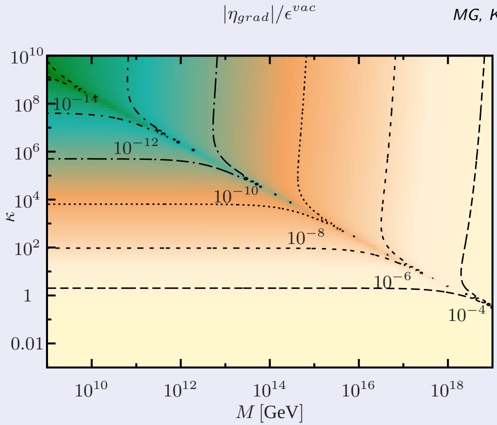
$$D_\rho(X,k) = \frac{2k_0\Gamma}{(k^2 - m_{th}^2)^2 + k_0^2\Gamma^2} \to 2\pi \mathrm{sign}(k^0)\delta(k^2 - m^2)$$

↓

Quantum-corrected Boltzmann equations

Additional source term

MG, Kartavtsev, Hohenegger



$$\frac{dY_{B-L}}{dz} = (\epsilon^{vac} + \delta\epsilon^{th} + \delta\epsilon^{grad}) D_1 (Y_{N_1} - Y_{N_1}^{eq}) - W Y_{B-L}$$ zero-order gradient

$$+ c\,\epsilon^{vac} \frac{\dot{T}}{2\pi T^2} + \ldots$$ first-order gradient

Conclusion

Check conceptual issues

- No double counting, justify/generalize RIS

Quantify corrections to Boltzmann equation

- Medium effects: larger than believed, weak washout
- Gradient corrections: additional source term, strong washout

Conclusion

Check conceptual issues

- No double counting, justify/generalize RIS

Quantify corrections to Boltzmann equation

- Medium effects: larger than believed, weak washout
- Gradient corrections: additional source term, strong washout

thank you!