SUSY Breaking in $\mathcal{N} = 2$ QFT

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Overview

- Motivation for studying $\mathcal{N} = 2$ SUSY breaking
- A general tool: the supercurrent multiplet
- Review of the Ferrara-Zumino (FZ) multiplet
- Generalization to $\mathcal{N} = 2$ for theories with (spontaneously broken) $SU(2)_R$
- RG flow of anomaly multiplet and SUSY breaking

$\mathcal{N} = 2$ **SUSY breaking motivation I**

- Better behaved SUSY breaking theories (see post SW literature) where we can work at strong coupling?

- Simple dynamical models? $\mathcal{N} = 2$ ISS, metastable theories?

- Could be relevant for describing the hidden sector where SUSY is broken (see D-brane constructions, etc.)

- Interesting IR signatures?

$\mathcal{N} = 2$ SUSY breaking historical interlude

• First example [P. Fayet, Nucl. Phys. B113 (1976)]

$$\begin{aligned}\mathcal{L} \ = \ & \int d^2\theta(\frac{1}{8}\mathrm{Tr}W^2 + \frac{1}{4}W'^2 + \frac{1}{\sqrt{2}}(2g\tilde{\Phi}_a N^a{}_b\Phi^b - g'N'\tilde{\Phi}_a\Phi^a) + \frac{f}{\sqrt{2}}N') \\ & +h.c. + \int d^4\theta(\bar{\Phi}_a e^{2gV^a{}_b - g'V'\delta^a{}_b}\Phi^b + \tilde{\Phi}_a e^{-2gV^a{}_b + g'V'\delta^a{}_b}\bar{\tilde{\Phi}}^b \\ & +\bar{N}_i e^{2gV^i{}_j}N^j + \bar{N}'N') \qquad (1)\end{aligned}$$

• f is put in by hand and breaks SUSY and $SU(2)_R$.

• This model was studied in the asymptotically free regime classically. It has some quantum peculiarities.

• Similar example but with more matter studied in the strongly coupled (weak SUSY breaking) regime by [Arai et. al., 0708.0668]

• Other known examples are variations on this model... Magnetic FI terms with non-trivial prepotential and partial breaking after SW [I. Antoniadis, H. Partouche, and T. Taylor, 9512006]

$\mathcal{N} = 2$ SUSY breaking motivation II

• Dynamical models?

• $\mathcal{N} = 2$ constraining, as hinted at by the few SUSY breaking examples that have been engineered. Why?

• We can give some broad answers to these questions by grouping theories by their symmetries and studying the corresponding supercurrent multiplets.

• We will study theories with $SU(2)_R$ at most spontaneously broken in addition to $\mathcal{N} = 2$.

• **Claim:** Such theories will not have SUSY breaking vacua as long as the IR is a weakly coupled soup of goldstinos and Goldstone bosons.

• Our reasoning will not rest on SW-type solutions (which are only valid for small SUSY breaking anyway since they ignore higher-derivative corrections).

Intro to the $\mathcal{N}=1$ FZ supercurrent multiplet

• Know from SUSY algebra that supercurrent, S^μ_α, and stress tensor, $T^{\mu\nu}$, should be grouped together with R-current, j^μ_R, i.e.

$$\{\bar{Q}_{\dot\alpha}, S^\mu_\alpha\} \sim \sigma_{\nu\alpha\dot\alpha}T^{\mu\nu}, \quad [Q_\alpha, j^\mu_R] \sim S^\mu_\alpha, \tag{2}$$

up to Schwinger terms.

• This notion was made more precise by Ferrara and Zumino (FZ). For a superconformal theory

$$\bar{D}^{\dot\alpha}\mathcal{J}_{\alpha\dot\alpha} = 0 \tag{3}$$

• Solving this equation, they found

$$\mathcal{J}_\mu = j^R_\mu + \theta^\alpha S_{\mu\alpha} + \bar\theta_{\dot\alpha}\bar{S}^{\dot\alpha}_\mu + (\theta\sigma^\nu\bar\theta)(T_{\mu\nu} - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho}j^{\sigma]}) + ... \tag{4}$$

- 8 bosonic and 8 fermionic components.

- Here j^R_μ is the current for the symmetry that assigns charge $+2/3$ to chiral superfields, Φ.

Intro to the $\mathcal{N}=1$ FZ supercurrent multiplet (cont...)

• When the superconformal symmetry is broken, we must add an appropriate representation of SUSY to the RHS of the conservation equation, i.e., an anomaly. FZ chose

$$\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_{\alpha}X \tag{5}$$

where X is chiral, i.e., $\bar{D}_{\dot{\alpha}}X = 0$.

• The solution to this equation is

$$\begin{aligned}\mathcal{J}_{\mu} \;=\; & j^{R}_{\mu} + \theta^{\alpha}(S_{\mu\alpha} + \frac{1}{3}(\sigma_{\mu}\bar{\sigma}^{\rho}S_{\rho})_{\alpha}) + \bar{\theta}_{\dot{\alpha}}(\bar{S}^{\dot{\alpha}}_{\mu} + \frac{1}{3}\epsilon^{\dot{\alpha}\dot{\beta}}(\bar{S}_{\rho}\bar{\sigma}^{\rho}\sigma_{\mu})_{\dot{\beta}}) \\ & +(\theta\sigma^{\nu}\bar{\theta})(T_{\mu\nu} - \frac{2}{3}\eta_{\mu\nu}T - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho}j^{\sigma]}) + \frac{i}{2}\theta^{2}\partial_{\mu}x - \frac{i}{2}\theta^{2}\partial_{\mu}\bar{x}\end{aligned} \tag{6}$$

and

$$X = x + \sqrt{2}\theta^\alpha(\frac{\sqrt{2}}{3}\sigma^\mu_{\alpha\dot{\alpha}}\bar{S}^{\dot{\alpha}}_\mu) + \theta^2(\frac{2}{3}T + i\partial_\mu j^\mu) \qquad (7)$$

• 12 bosonic and 12 fermionic components in the multiplet.

• Solutions not unique: $(\mathcal{J}_\mu + i\partial_\mu(Y - \bar{Y}), X - \frac{1}{2}\bar{D}^2\bar{Y})$. These solutions are related by improvement terms for the (conserved) component currents.

• The existence of this multiplet is subject to several obstructions, but it is well-defined for a large class of theories [Z. Komargodski and N. Seiberg, 0904.1159, 1002.2228].

• A simple $\mathcal{N} = 2$ generalization of the FZ multiplet will be valid for the theories we consider.

• There are other possible embeddings of the supercurrent multiplet, e.g., when the theory has an R-symmetry can define $\bar{D}^\alpha R_{\alpha\dot{\alpha}} = \chi_\alpha$. If the theory also has an FZ multiplet, and one can solve $X = -\frac{1}{2}\bar{D}^2 U$, then one can write $R_{\alpha\dot{\alpha}} = \mathcal{J}_{\alpha\dot{\alpha}} + [D_\alpha, \bar{D}_{\dot{\alpha}}]U$. Also other options `Z. Komargodski and N. Seiberg 1002.2228, S. Kuzenko 1002.4932`.

The FZ supercurrent multiplet in the UV

- Example

$$S = \int d^4\theta K(\Phi, \bar{\Phi}) + (\int d^2\theta W(\Phi) + h.c.) \tag{8}$$

- Have

$$\mathcal{J}_{\alpha\dot{\alpha}} = 2D_\alpha\Phi \cdot \bar{D}_{\dot{\alpha}}\bar{\Phi} - \frac{2}{3}[D_\alpha, \bar{D}_{\dot{\alpha}}]K \tag{9}$$

and

$$X = 4W - \frac{1}{3}\bar{D}^2 K \tag{10}$$

- Note if $K = \Phi\bar{\Phi}$ and $W = \Phi^3$, then $X = 0$ and we have a (classically) conformal theory.

The FZ supercurrent multiplet along the RG flow

• Supercharge commutators are well-defined along the RG flow even if SUSY is spontaneously broken.

• Therefore, the X operator satisfies chiral commutation relations along the full RG flow:

$$[\xi Q, x] \sim \xi\psi, \quad [\bar{\xi}\bar{Q}, x] = 0$$

$$[\xi Q, \psi_\alpha] \sim \xi_\alpha F, \quad [\bar{\xi}\bar{Q}, \psi_\alpha] \sim \bar{\xi}\bar{\sigma}^\mu \partial_\mu x \tag{11}$$

$$[\xi Q, F] = 0, \quad [\bar{\xi}\bar{Q}, F] \sim \partial_\mu \psi \sigma^\mu \bar{\xi} \tag{12}$$

and we have a chiral superfield even in the IR.

SUSY breaking and the FZ supercurrent multiplet in the deep IR

• When SUSY is broken, the supercurrent flows to its spin half component, i.e., the goldstino

$$S^{\mu}_{\alpha} \sim f \sigma^{\mu}_{\alpha\dot{\alpha}} \bar{G}^{\dot{\alpha}} \tag{13}$$

• Therefore, one can identify a universal goldstino superfield [Z. Komargodski and N. Seiberg, 0907.2441]

$$X = x + \sqrt{2}\theta^{\alpha} G_{\alpha} + \theta^2 F \tag{14}$$

• In general, SUSY partner of goldstino is not massless. Then, at leading order, the corresponding operator can create a two goldstino state. The norrmalization of the operator is fixed by the SUSY algebra, and one finds[Z. Komargodski and N. Seiberg, 0907.2441]

$$X_{IR} = \frac{G^2}{2F} + \sqrt{2}\theta^\alpha G_\alpha + \theta^2 F \tag{15}$$

- With

$$X_{IR}^2 = 0 \tag{16}$$

X and the low energy effective action

• At low energies, can construct an Akulov-Volkov Lagrangian

$$\mathcal{L} = \int d^4\theta X_{IR}\bar{X}_{IR} + (\int d^2\theta f X_{IR} + h.c.) + ... \qquad (17)$$

• One consistency condition on the above action is that the divergence of the FZ multiplet of the above theory matches the RG evolved FZ multiplet of the original theory, i.e.

$$\bar{D}^{\dot{\alpha}}\mathcal{J}_{IR\alpha\dot{\alpha}} = f D_\alpha X_{IR} \qquad (18)$$

• This must be the case for the theory to be a consistent low energy version of the original theory.

Summary of $\mathcal{N}=1$

- We start in the UV with some theory, say massive $\mathcal{N}=1$ SQCD a la ISS that has an FZ multiplet

$$\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} \sim D_\alpha(cW^2) + ... \tag{19}$$

- And go to the IR where we find a consistent low energy action of the type above

$$\mathcal{L} = \int d^4\theta X_{IR}\bar{X}_{IR} + (\int d^2\theta f X_{IR} + h.c.) + ... \tag{20}$$

- With possible additional particles (goldstone bosons, etc.).

- **Question:** Is there an analog in $\mathcal{N}=2$ QFT?

Generalizing to $\mathcal{N} = 2$: SCFT warmup

• The supercurrent structure of $\mathcal{N} = 2$ QFT is significantly more complicated. At the superconformal level, we package the $\mathcal{N} = 2$ supercurrent in a dimension two field that satisfies [`M. F. Sohnius, P. Lett. B 81, 1979`]

$$D^{\langle ij \rangle} \mathcal{J} = 0 \tag{21}$$

where i, j are $SU(2)_R$ indices and $D^{\langle ij \rangle}$ is the $SU(2)_R$ spin one differential operator.

• This constraint immediately implies the following interesting spectrum of component operators

$$
\begin{array}{ccc}
 & SU(2)_R & \text{Dim} \\
 & & \\
J & \mathbf{1} & 2 \\
J_\alpha^{\langle i\rangle} & \mathbf{2} & 5/2 \\
J_{\alpha\beta} & \mathbf{1} & 3 \\
J^{\langle i\rangle}_{\langle j\rangle\mu} & \mathbf{3} & 3 \\
J_\mu & \mathbf{1} & 3 \\
J_{\mu\alpha}^{\langle i\rangle} & \mathbf{2} & 7/2 \\
T_{\mu\nu} & \mathbf{1} & 4
\end{array}
\qquad (22)
$$

• Standard conservation, symmetrization, and trace identities satisfied.

• Unlike the $\mathcal{N}=1$ superconformal case, the $\mathcal{N}=2$ case comes with more degrees of freedom than just the supercurrents, $U(2)_R$ currents, and $T^{\mu\nu}$. Have 24 bosonic and 24 fermionic components.

Reformulation in terms of $\mathcal{N}=1$ superspace

• We can redefine the supercurrent as follows

$$\hat{\mathcal{J}} \equiv \mathcal{J}|, \quad \mathcal{J}_\alpha \equiv (D_\alpha^{\langle 2\rangle}\mathcal{J})|, \quad \mathcal{J}_{\alpha\dot{\alpha}} = \left(-\frac{1}{3}\left[D_\alpha^{\langle 1\rangle}, \bar{D}_{\langle 1\rangle\dot{\alpha}}\right] + \left[D_\alpha^{\langle 2\rangle}, \bar{D}_{\langle 2\rangle\dot{\alpha}}\right]\right)\mathcal{J}| \tag{23}$$

• We can then rewrite the above $\mathcal{N}=2$ conservation equation in $\mathcal{N}=1$ superspace as follows

$$\bar{D}^2\hat{\mathcal{J}} = D^2\hat{\mathcal{J}} = 0, \quad D^\alpha\mathcal{J}_\alpha = 0, \quad \bar{D}^2\mathcal{J}_\alpha = 0, \quad \bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = 0, \tag{24}$$

• Which we can solve as follows

$$\hat{\mathcal{J}} \;=\; J + \theta^\alpha J_\alpha^{\langle 1\rangle} + \bar{\theta}_{\dot{\alpha}}\bar{J}^{\dot{\alpha}}_{\langle 1\rangle} + \theta\sigma^\mu\bar{\theta}(-\frac{1}{2}J_\mu + J^{\langle 1\rangle}_{\langle 1\rangle\mu}) + \mathcal{O}(\theta^2\bar{\theta}, \bar{\theta}^2\theta)$$

$$
\begin{aligned}
\mathcal{J}_\alpha &= J_\alpha^{\langle 2\rangle} + \theta^\beta J_{\beta\alpha} + \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha} J_{\langle 1\rangle\mu}^{\langle 2\rangle} + \theta\sigma^\mu\bar\theta(-J_{\mu\alpha}^{\langle 2\rangle} + \frac{2}{3} i\sigma_{\mu\nu\alpha}{}^{\beta}\partial^\nu J_\beta^{\langle 2\rangle}) \\
&+ \mathcal{O}(\theta^2\bar\theta, \bar\theta^2\theta) \\
\mathcal{J}_\mu &= \frac{1}{3}J_\mu + \frac{4}{3}J_{\langle 1\rangle\mu}^{\langle 1\rangle} + \theta^\alpha J_{\mu\alpha}^{\langle 1\rangle} + \bar\theta_{\dot\alpha} J_{\langle 1\rangle\mu}^{\dot\alpha} + \theta\sigma^\nu\bar\theta\left(2T_{\nu\mu} - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho} j^{\sigma]}\right) \\
&+ \mathcal{O}(\theta^2\bar\theta, \bar\theta^2\theta) \qquad (25)
\end{aligned}
$$

Adding the $\mathcal{N} = 2$ anomaly

- We choose to consider theories with an $\mathcal{N} = 2$ linear anomaly

$$D^{\langle ij \rangle} \mathcal{J} = 3\mathcal{L}^{\langle ij \rangle} \tag{26}$$

This is a field satisfying

$$(\mathcal{L}^{\langle ij \rangle})^\dagger = \epsilon_{\langle ik \rangle} \epsilon_{\langle jl \rangle} \mathcal{L}^{\langle kl \rangle}, \quad \mathcal{L}^{\langle ij \rangle} = \mathcal{L}^{\langle ji \rangle}, \quad D_\alpha^{(\langle i \rangle} \mathcal{L}^{\langle jk \rangle)} = \bar{D}_{\dot{\alpha}}^{(\langle i \rangle} \mathcal{L}^{\langle jk \rangle)} = 0 \tag{27}$$

• The corresponding spectrum of operators is

$$
\begin{array}{ccc}
 & SU(2)_R & \text{Dim} \\
 & & \\
L^{\langle ij \rangle} & \mathbf{3} & 3 \\
L_\alpha^{\langle i \rangle} & \mathbf{2} & 7/2 \\
L & \mathbf{1} & 4 \\
L_\mu & \mathbf{1} & 4
\end{array}
\qquad (28)
$$

• So we see that theories described by these anomaly multiplets will have among other conserved currents a conserved $SU(2)_R$ and a conserved central charge current.

• Therefore these multiplets do not describe breaking by explicit field independent FI terms. But, theories where such terms are field dependent are covered.

The $\mathcal{N}=2$ anomaly in $\mathcal{N}=1$ superspace

- In $\mathcal{N}=1$ language these constraints amount to

$$\bar{D}_{\dot{\alpha}}X = 0, \quad D^2L = \bar{D}^2L = 0 \tag{29}$$

- The component conservation equations then become

$$\bar{D}^2\hat{\mathcal{J}} = 3X, \quad D^\alpha\mathcal{J}_\alpha = -3iL, \quad \bar{D}^2\mathcal{J}_\alpha = 0, \quad \bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X \tag{30}$$

- We can again solve these equations

$$\begin{aligned}\mathcal{J}_\alpha \;=\; & J_\alpha^{\langle 2\rangle} + \theta^\beta\left(J_{\beta\alpha} - \frac{3}{2}i\epsilon_{\beta\alpha}\ell\right) + \sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}J^{\langle 2\rangle}_{\langle 1\rangle\mu} + \theta\sigma^\mu\bar{\theta}(-J^{\langle 2\rangle}_{\mu\alpha} \\ - \; & \frac{1}{2}(\sigma_\mu\bar{\sigma}^\rho J^{\langle 2\rangle}_\rho)_\alpha + \frac{2}{3}i\sigma_{\mu\nu\alpha}{}^{\beta}\partial^\nu J^{\langle 2\rangle}_\beta) + 2\theta^2\sigma^\mu_{\alpha\dot{\alpha}}\bar{J}^{\dot{\alpha}}_{\langle 2\rangle\mu}\end{aligned} \tag{31}$$

$$
\begin{aligned}
& + \quad \theta^2\bar{\theta}_{\dot{\alpha}}\left[\frac{3}{2}\sigma^{\mu\dot{\alpha}}_{\alpha}\left(-\frac{1}{2}\partial_\mu\ell + i\ell_\mu\right) + \frac{i}{2}\sigma^{\mu\dot{\alpha}}_{\beta}\partial_\mu J_{\alpha}{}^{\beta}\right] + \mathcal{O}(\bar{\theta}^2\theta) \\
\mathcal{J}_\mu \quad & = \quad j^{\mathcal{N}=1}_\mu + \theta^\alpha\left(J^{\langle 1\rangle}_{\mu\alpha} + \frac{1}{3}(\sigma_\mu\bar{\sigma}^\rho J^{\langle 1\rangle}_\rho)_\alpha\right) \\
& + \quad \bar{\theta}_{\dot{\alpha}}\left(\bar{J}^{\dot{\alpha}}_{\langle 1\rangle\mu} + \frac{1}{3}\epsilon^{\dot{\alpha}\dot{\beta}}(\bar{J}_{\langle 1\rangle\rho}\bar{\sigma}^\rho\sigma_\mu)_{\dot{\beta}}\right) \\
& + \quad \theta\sigma^\nu\bar{\theta}\left(2T_{\nu\mu} - \frac{2}{3}\eta_{\mu\nu}T - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho}j^{\sigma]}_{\mathcal{N}=1}\right) + \frac{i}{2}\theta^2\partial_\mu\bar{x} - \frac{i}{2}\bar{\theta}^2\partial_\mu x \\
& + \quad \mathcal{O}(\theta^2\bar{\theta}, \bar{\theta}^2\theta)
\end{aligned}
$$

- And

$$
\begin{aligned}
X &= x + \theta^\alpha\left(\tfrac{2}{3}\sigma^\mu_{\alpha\dot{\alpha}}\bar{J}^{\dot{\alpha}}_{\langle 1\rangle\mu}\right) + \theta^2\left(\tfrac{2}{3}T + i\partial_\mu j^\mu_{\mathcal{N}=1}\right) + \mathcal{O}(\theta\bar{\theta}), \\
L &= \ell - \tfrac{i}{2}\theta^\alpha\left(\tfrac{2}{3}\sigma^\mu_{\alpha\dot{\alpha}}\bar{J}^{\dot{\alpha}}_{\langle 2\rangle\mu}\right) + \tfrac{i}{2}\bar{\theta}_{\dot{\alpha}}\left(\tfrac{2}{3}\sigma^{\mu\dot{\alpha}}_{\alpha}J^{\langle 2\rangle\alpha}_{\mu}\right) + \theta\sigma^\mu\bar{\theta}\ell_\mu \\
&\quad + \mathcal{O}(\theta^2\bar{\theta}, \bar{\theta}^2\theta). \qquad (32)
\end{aligned}
$$

• Improvement terms

$$\mathcal{J} - \frac{3}{2}(W + \bar{W}), \quad \mathcal{L}^{\langle ij\rangle} - \frac{1}{2}D^{\langle ij\rangle}W \tag{33}$$

where W is a reduced chiral superfield.

• We then find

$$\delta J^{\langle j\rangle}_{\langle i\rangle\mu} = 0, \quad \delta\ell^{\mu} = 2\sqrt{2}\partial_\nu F^{\mu\nu}, \quad \delta J^{\langle i\rangle}_{\mu\alpha} = 2i\sigma_{\mu\nu\alpha}{}^{\beta}\partial^\nu\lambda^{\langle i\rangle}_{\beta},$$
$$\delta T_{\mu\nu} = -(\partial_\nu\partial_\mu - \eta_{\mu\nu})(\phi + \bar{\phi}) \tag{34}$$

Some simple example theories

• A theory of N massive hypermultiplets (considered by Sohnius)

$$\mathcal{L} = \int d^4\theta \left(\bar{Q}^i Q_i + \tilde{Q}^i \bar{\tilde{Q}}_i\right) + \left(\int d^2\theta \frac{1}{\sqrt{2}} M_i Q_i \tilde{Q}^i + h.c.\right) \qquad (35)$$

• Using the superconformal $U(2)_R$ charges of the hypermultiplets, we can construct the lowest component supercurrent operator

$$\hat{J} = \frac{1}{2} \sum_i \left(Q_i \bar{Q}^i + \bar{\tilde{Q}}_i \tilde{Q}^i\right). \qquad (36)$$

• We then find

$$X = \tfrac{2\sqrt{2}}{3} \textstyle\sum_i M_i Q_i \tilde{Q}^i,$$

$$L = -\frac{\sqrt{2}}{3} \sum_i M_i \left(Q_i \bar{Q}^i - \bar{\tilde{Q}}_i \tilde{Q}^i \right) . \qquad (37)$$

- SUSY is clearly unbroken.
- In the superconformal case the masses vanish and $X = L = 0$.

Some simple example theories (cont...)

- Another simple example: $\mathcal{N} = 2$ gauge field with a non-trivial prepotential

$$\mathcal{L} = \int d^2\theta_1 d^2\theta_2 \ \mathcal{F}(W) + h.c. \tag{38}$$

where

$$\mathcal{F}(W) = \frac{1}{4}W^2 + ... \tag{39}$$

- We find

$$\hat{J} = -\Phi\partial_{\bar{\Phi}}\bar{\mathcal{F}} - \bar{\Phi}\partial_{\Phi}\mathcal{F} - \left(\tilde{\mathcal{F}} + \bar{\tilde{\mathcal{F}}}\right) \tag{40}$$

- With the local shift term defined as follows

$$\partial_{\Phi}\tilde{\mathcal{F}} \equiv \partial_{\Phi}\mathcal{F} - \Phi\partial^2_{\Phi}\mathcal{F}. \tag{41}$$

- The anomaly multiplet is then

$$X = -\frac{1}{3}\left(\left(\partial_\Phi \mathcal{F} - \Phi \partial^2_\Phi \mathcal{F}\right) \bar{D}^2 \bar{\Phi} + 2\Phi \partial^3_\Phi \mathcal{F}\ W^2 + \bar{D}^2 \bar{\bar{\mathcal{F}}}\right)$$
$$L = \frac{\sqrt{2}}{3}\left(\bar{D}_{\dot\alpha}\left[\bar{W}^{\dot\alpha}\left(\partial_{\bar\Phi}\bar{\mathcal{F}} - \bar\Phi \partial^2_{\bar\Phi}\bar{\mathcal{F}}\right)\right] + D^\alpha\left[W_\alpha\left(\partial_\Phi \mathcal{F} - \Phi\partial^2_\Phi \mathcal{F}\right)\right]\right) \quad (42)$$

- SUSY is unbroken.

- When the theory has a free prepotential, we find $X = L = 0$.

Some not so simple theories

• $\mathcal{N} = 2$ SYM

$$D^{\langle ij \rangle}\mathcal{J} = \frac{c}{2} tr(D^{\langle ij \rangle}W^2 - \bar{D}^{\langle ij \rangle}\bar{W}^2) \qquad (43)$$

where $c = 8\pi i\beta$.

• And various other cousin theories with arbitrary gauge group and massive hypermultiplet flavors.

SUSY breaking

- First, assume $SU(2)_R$ is not spontaneously broken.

- In the deep IR we have

$$S_\alpha^{\mu\langle i\rangle} = \sqrt{2} f \sigma^\mu_{\alpha\dot\alpha} \bar{G}^{\langle i\rangle\dot\alpha} + \ldots \tag{44}$$

- So from above we see that

$$\begin{aligned} X_{IR} &= x_{IR} + \sqrt{2}\theta^\alpha G_{\langle 1\rangle\alpha} + \theta^2 F + \mathcal{O}(\theta\bar\theta) \\ L_{IR} &= \ell_{IR} - \frac{i}{\sqrt{2}}\theta^\alpha G_{\langle 2\rangle\alpha} + \frac{i}{\sqrt{2}}\bar\theta_{\dot\alpha}\bar{G}^{\langle 2\rangle\dot\alpha} + \theta\sigma^\mu\bar\theta\ell_{IR\mu} + \mathcal{O}(\theta^2\bar\theta, \bar\theta^2\theta) \end{aligned} \tag{45}$$

i.e., we have embedded the Goldstinos.

- However, x_{IR} and ℓ_{IR} must be composite.

• Up to zero derivatives, must have

$$X_{IR} = \frac{G_{\langle 1\rangle}G_{\langle 1\rangle}}{2F} + \frac{\bar{G}^{\langle 2\rangle}\bar{G}^{\langle 2\rangle}}{2\bar{F}} + \sqrt{2}\theta G_{\langle 1\rangle} + \theta^2 F + \mathcal{O}(\partial^\mu),$$
$$L_{IR} = -i\frac{G_{\langle 1\rangle}G_{\langle 2\rangle}}{2F} + i\frac{\bar{G}^{\langle 1\rangle}\bar{G}^{\langle 2\rangle}}{2\bar{F}} - \frac{i}{\sqrt{2}}\theta G_{\langle 2\rangle} + \frac{i}{\sqrt{2}}\bar{\theta}\bar{G}^{\langle 2\rangle} + \mathcal{O}(\partial^\mu) \quad (46)$$

• However, find a contradiction when imposing proper transformations under the full SUSY algebra.

• $\left[\bar{\xi}\bar{Q}_{\langle 1\rangle}, x_{IR}\right] = 0$ implies:

$$x_{IR} = \frac{G_{\langle 1\rangle}G_{\langle 1\rangle}}{2F} + \frac{\bar{G}^{\langle 2\rangle}\bar{G}^{\langle 2\rangle}}{2\bar{F}} - \frac{i}{2|F|^2}\left(G_{\langle 1\rangle}\sigma^\mu\bar{G}^{\langle 1\rangle} + G_{\langle 2\rangle}\sigma^\mu\bar{G}^{\langle 2\rangle}\right).$$

$$
\begin{aligned}
&\cdot \quad \partial_\mu \left(\frac{\bar{G}^{\langle 2 \rangle} \bar{G}^{\langle 2 \rangle}}{\bar{F}} \right) + g^{\mu}_{(0)\dot{\alpha}}(G_{\langle i \rangle}, \bar{G}^{\langle 2 \rangle}) \partial_\mu \left(\frac{\bar{G}^{\langle 1 \rangle \dot{\alpha}}}{\bar{F}} \right) \\
&+ \quad g_{(1)}(G_{\langle i \rangle}, \bar{G}^{\langle 2 \rangle}) + \mathcal{O}(\partial^2), \qquad (47)
\end{aligned}
$$

• On the other hand, $\left[\xi Q^{\langle 2 \rangle}, x_{IR}\right] = 0$ implies

$$
\begin{aligned}
x_{IR} \;=\;& \frac{G_{\langle 1 \rangle} G_{\langle 1 \rangle}}{2F} + \frac{\bar{G}^{\langle 2 \rangle} \bar{G}^{\langle 2 \rangle}}{2\bar{F}} + \frac{i}{2|F|^2} \left(G_{\langle 1 \rangle} \sigma^\mu \bar{G}^{\langle 1 \rangle} + G_{\langle 2 \rangle} \sigma^\mu \bar{G}^{\langle 2 \rangle} \right) \cdot \\
&\cdot \quad \partial_\mu \left(\frac{G_{\langle 1 \rangle} G_{\langle 1 \rangle}}{F} \right) + \tilde{g}^{\alpha}_{(0)\mu}(G_{\langle 1 \rangle}, \bar{G}^{\langle i \rangle}) \partial^\mu \left(\frac{G_{\langle 2 \rangle \alpha}}{F} \right) \\
&+ \quad \tilde{g}_{(1)}(G_{\langle 1 \rangle}, \bar{G}^{\langle i \rangle}) + \mathcal{O}(\partial^2) \qquad (48)
\end{aligned}
$$

• These two forms contradict each other and so SUSY cannot be broken in this case.

The IR effective action

• Suppose that $SU(2)_R$ is spontaneously broken. Then x_{IR} and ℓ_{IR} can contain the corresponding R-axions.

• Assuming that the theory is weakly coupled (after integrating out strong dynamics), we must have

$$\int d^4\theta(X_{IR}\bar{X}_{IR} - 2L_{IR}^2) + (\int d^2\theta f X_{IR} + h.c.) + ... \qquad (49)$$

where the ellipses contain weak interactions and higher-order corrections.

• Can write a supercurrent for this theory and compute the IR anomaly. We find

$$\bar{D}^2\hat{\mathcal{J}} = 8fX_{IR} + 2\bar{D}^2(L_{IR}^2),$$

$$
\begin{aligned}
D^{\alpha}\mathcal{J}_{\alpha} &= 2iD^{\alpha}L_{IR}D_{\alpha}X_{IR} - 8ifL_{IR}, \\
\bar{D}^2\mathcal{J}_{\alpha} &= -2i\bar{D}^2\left(L_{IR}D_{\alpha}X_{IR}\right), \\
\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} &= \tfrac{8}{3}D_{\alpha}\left(fX_{IR} - \bar{D}_{\dot{\alpha}}L_{IR}\bar{D}^{\dot{\alpha}}L_{IR}\right) \\
&+2\bar{D}^2D_{\alpha}\left(L_{IR}^2\right) + 2\bar{D}_{\dot{\alpha}}D_{\alpha}X_{IR}\bar{D}^{\dot{\alpha}}\bar{X}_{IR}.
\end{aligned} \tag{50}
$$

• No shift in $\mathcal{J}$ gives back the (RG evolved) anomaly of the original theory. This is a contradiction, and so SUSY cannot be broken!

• **Consequence:** No SUSY breaking in this class of theories ($\mathcal{N} = 2$ SYM, etc.).

Conclusions and open problems

• Using very general techniques we have understood that a large class of theories cannot break $\mathcal{N} = 2$ SUSY.

• This statement is clearly related to the fact that they have an underlying conserved $SU(2)_R$ symmetry.

• What about theories that do not have a conserved $SU(2)_R$?

• Relation to recent hypermultiplet no-go theorems? [Jacot and Scrucca].