The Geometry of Minimal Flavour Violation
or
A Geometric Description of Flavour in the MSSM

Robert N. Hodgkinson

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Based on J. Ellis, RNH, J. S. Lee and A. Pilaftsis, JHEP **1002** (2010) 016 [arXiv:0911.3611]

Outline

Outline

SUSY Motivation

SUperSYmmetry

- The only possible extension of Poincaré symmetry of space-time
- Essential ingredient of String Theory description of nature
- Provides exactly solvable field theories

The Problem!

- Evidently not a symmetry at low energies → Must be broken at some scale

Why TeV-scale SUSY?

- Gives a technical solution to the hierarchy problem of the SM
- Predicts a light Higgs boson $m_h \lesssim 140$ GeV
- Helps to unify the gauge couplings → Grand Unification
- Provides a natural candidate for astrophysical Cold Dark Matter
- Can explain the discrepancy in $g_\mu - 2$

The Minimal Supersymmetric Standard Model

The Superpotential

$$W_{\text{MSSM}} = \widehat{U}^C \mathbf{h}_u \widehat{Q} \widehat{H}_u + \widehat{D}^C \mathbf{h}_d \widehat{H}_d \widehat{Q} + \widehat{E}^C \mathbf{h}_e \widehat{H}_d \widehat{L} + \mu \widehat{H}_u \widehat{H}_d$$

- Yukawa couplings for the quarks ($\mathbf{h}_{u,d}$) and leptons ($\mathbf{h}_e$)
- Higgs-mixing mass μ

Soft SUSY-breaking terms

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} \quad = \quad & \frac{1}{2}\left(M_1 \widetilde{B}\widetilde{B} + M_2 \widetilde{W}^i \widetilde{W}^i + M_3 \tilde{g}^a \tilde{g}^a + \text{h.c.}\right) \\
& + \widetilde{Q}^\dagger \widetilde{\mathbf{M}}^2_Q \widetilde{Q} + \widetilde{U}^\dagger \widetilde{\mathbf{M}}^2_U \widetilde{U} + \widetilde{D}^\dagger \widetilde{\mathbf{M}}^2_D \widetilde{D} + \widetilde{L}^\dagger \widetilde{\mathbf{M}}^2_L \widetilde{L} + \widetilde{E}^\dagger \widetilde{\mathbf{M}}^2_E \widetilde{E} \\
& + M^2_{H_u} H_u^\dagger H_u + M^2_{H_d} H_d^\dagger H_d + (B\mu H_u H_d + \text{h.c.}) \\
& + \left(\widetilde{U}^\dagger \mathbf{a}_u \widetilde{Q} H_u + \widetilde{D}^\dagger \mathbf{a}_d d H_d \widetilde{Q} + \widetilde{E}^\dagger \mathbf{a}_e H_d \widetilde{L} + \text{h.c.}\right)
\end{aligned}$$

- Masses for all scalars and gauginos
- Trilinear scalar couplings

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The Flavour Problem in Supersymmetry

The CKM picture of flavour

- In the SM all flavour violation from the Cabbibo-Kobayashi-Maskawa matrix $\rightarrow$ Charged Currents only (tree level)
- FCNCs highly suppressed by Glashow-Iliopoulos-Maiani mechanism
- Extremely accurate description of nature

SUSY FCNCs

- Generic squark mass matrices give unacceptably large FCNCs
- Implies *either*
  1) Very heavy squarks $\rightarrow$ tension with light Higgs boson?
  2) SUSY and SM flavour have common origin $\rightarrow$ Flavour Symmetries?
  3) CKM also controls SUSY flavour $\rightarrow$ Minimal Flavour Violation

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Minimal Flavour Violation

Maximal Flavour Group

- Neglecting Yukawa couplings and soft SUSY-breaking terms we have the large flavour symmetry group $[SU(3) \times U(1)]^5$

$$\widehat{Q}' = \mathbf{U}_Q \widehat{Q}, \quad \widehat{U}'^C = \mathbf{U}_U^* \widehat{U}^C, \quad \widehat{D}'^C = \mathbf{U}_D^* \widehat{D}^C, \quad \widehat{L}' = \mathbf{U}_L \widehat{L}, \quad \widehat{E}'^C = \mathbf{U}_E^* \widehat{E}$$

Yukawa couplings and Spurion fields

- The superpotential is invarient under this transformation if we redefine the model parameters as

$$\mathbf{h}_{u,d} \to \mathbf{U}_{U,D}^\dagger \mathbf{h}_{u,d} \mathbf{U}_Q, \qquad \mathbf{h}_e \to \mathbf{U}_E^\dagger \mathbf{h}_e \mathbf{U}_L,$$

- Consider the Yukawa coupling matrices as VEVs of Spurion fields with the above transformation properties
- In Minimal Flavour Violation all flavour structures are built solely from couplings with the spurion Yukawa fields

Flavour Symmetry and SUSY-breaking terms

Extending the Flavour Symmetry

- The soft SUSY-breaking terms should transform as

$$\begin{aligned} \widetilde{\mathbf{M}}^2_{Q,U,D,L,E} &\rightarrow \mathbf{U}^\dagger_{Q,U,D,L,E}\widetilde{\mathbf{M}}^2_{Q,U,D,L,E}\mathbf{U}_{Q,U,D,L,E} \\ \mathbf{a}_{u,d} &\rightarrow \mathbf{U}^\dagger_{U,D}\mathbf{a}_{u,d}\mathbf{U}_Q \\ \mathbf{a}_e &\rightarrow \mathbf{U}^\dagger_E\mathbf{a}_e\mathbf{U}_L \end{aligned}$$

- Simple solution given by $\widetilde{\mathbf{M}}^2 \sim \mathbf{1}$, $\mathbf{a}_f \sim \mathbf{h}_f$
- Non-trivial structures generated by Renormalisation Group running

A Question

- Given these transformation properties, can generic soft SUSY-breaking masses be expressed wholly in terms of the Yukawa couplings?

Outline

1. Introduction- SuperSymmetry and the Flavour Problem
2. Flavour Geometry Decomposition
   - Theoretical Framework
   - Numerical Results
3. Conclusions

Outline

Left-handed Quark Decomposition

Basis Matrices

- At a given scale we can decompose

$$\widetilde{\mathbf{M}}^2_Q = \sum_{I=0}^{8} \tilde{m}_Q^{2,I}\ \mathbf{H}_I^Q\ ,$$

$$\begin{aligned}\left\{\mathbf{H}_I^Q\right\} \quad = \quad & \Big\{\mathbf{1}_3,\ \mathbf{h}_u^\dagger\mathbf{h}_u,\ \mathbf{h}_d^\dagger\mathbf{h}_d,\ \left(\mathbf{h}_u^\dagger\mathbf{h}_u\right)^2,\ \left(\mathbf{h}_d^\dagger\mathbf{h}_d\right)^2,\ \left[\mathbf{h}_u^\dagger\mathbf{h}_u, \mathbf{h}_d^\dagger\mathbf{h}_d\right]_+, \\ & i\left[\mathbf{h}_u^\dagger\mathbf{h}_u, \mathbf{h}_d^\dagger\mathbf{h}_d\right]_-,\ \mathbf{h}_u^\dagger\mathbf{h}_u\mathbf{h}_d^\dagger\mathbf{h}_d\mathbf{h}_u^\dagger\mathbf{h}_u,\ \mathbf{h}_d^\dagger\mathbf{h}_d\mathbf{h}_u^\dagger\mathbf{h}_u\mathbf{h}_d^\dagger\mathbf{h}_d\Big\}\end{aligned}$$

- $\mathbf{H}_I^Q$ furnish complete, LI set of basis vectors for $9D$ flavour space
- Basis is not unique
- This choice minimal in powers of $\mathbf{h}_{u,d}$; symmetric under $\mathbf{h}_u \leftrightarrow \mathbf{h}_d$

Inner Product Space

The Metric

- Take the Inner Product on the space to be the matrix trace
- Defines a metric

$$g^Q_{IJ} = \mathrm{Tr}\left(\mathbf{H}^Q_I \mathbf{H}^Q_J\right)$$

- I.P. automatically symmetric and distributive

Linear Independence of Basis Vectors

- g^Q_{IJ} non-singular provided the Jarlskog determinant is non-zero, i.e.

$$\det\left(i\left[\mathbf{h}_u^\dagger \mathbf{h}_u, \mathbf{h}_d^\dagger \mathbf{h}_d\right]_-\right) = \det \mathbf{H}^Q_6 \neq 0$$

- Necessary and sufficient condition for $\mathbf{H}^Q_I$ to form a linearly independant set
- Implies that ANY 3×3 hermitian matrix can be expanded in terms of $\mathbf{h}_{u,d}$ with real coefficients

Projection onto Coefficients

Inverse Metric and Projection

- Since g^{Q}_{IJ} is non-singular we can define an inverse $g^{Q,IJ}$

$$g^{Q,IJ}\ g^{Q}_{JK} = \delta^{I}{}_{K}$$

- The coefficients can be projected out as

$$\tilde{m}_{Q}^{2,I} = g^{Q,IJ}\mathrm{Tr}\left(\mathbf{H}^{Q}_{J}\widetilde{\mathbf{M}}^{2}_{Q}\right)$$

- Coefficients $\tilde{m}_{Q}^{2,I}$ real if $\widetilde{\mathbf{M}}^{2}_{Q}$ is Hermitian

MFV Philosophy

- For a generic matrix typically find very large coefficients
- 'Traditional' MFV in the limit

$$\tilde{m}_{Q}^{2,I}/\tilde{m}_{Q}^{2,0} \leq 1$$

- Flavour singlet term dominant

Trilinear Couplings

Expansion in $\mathbf{H}_I^Q$

- Trilinear couplings have same transformation properties as Yukawa matrices
- Can also be expanded using the left-handed basis as

$$\mathbf{a}_u = \sum_{I=0}^{8} a_u^I \mathbf{h}_u \mathbf{H}_I^Q \qquad \mathbf{a}_d = \sum_{I=0}^{8} a_d^I \mathbf{h}_d \mathbf{H}_I^Q$$

- Additional condition: $\mathrm{Det}(\mathbf{h}_f) \neq 0$

Alternate Notation

- Equivalently define $\mathbf{a}_f = \mathbf{h}_f \mathbf{A}_f$
- Expansion given by

$$\mathbf{A}_f = \sum_{I=0}^{8} a_f^I \mathbf{H}_I^Q$$

Other bases

Up- and Down-type Squark Mass Matrices

- Up- and Down-type squark mass matrices transform differently
- The Up-type squark mass matrix can be expanded using

$$\left\{\mathbf{H}_I^U\right\} = \Big\{\mathbf{1}_3,\ \mathbf{h}_u\mathbf{h}_u^\dagger,\ \mathbf{h}_u\mathbf{h}_d^\dagger\mathbf{h}_d\mathbf{h}_u^\dagger,\ \left(\mathbf{h}_u\mathbf{h}_u^\dagger\right)^2,\ \mathbf{h}_u\left(\mathbf{h}_d^\dagger\mathbf{h}_d\right)^2\mathbf{h}_u^\dagger,\ \mathbf{h}_u\left[\mathbf{h}_u^\dagger\mathbf{h}_u,\ \mathbf{h}_d^\dagger\mathbf{h}_d\right]_+\mathbf{h}_u^\dagger,\ i\mathbf{h}_u\left[\mathbf{h}_u^\dagger\mathbf{h}_u,\ \mathbf{h}_d^\dagger\mathbf{h}_d\right]_-\mathbf{h}_u^\dagger,\ \mathbf{h}_u\mathbf{h}_u^\dagger\mathbf{h}_u\mathbf{h}_d^\dagger\mathbf{h}_d\mathbf{h}_u^\dagger\mathbf{h}_u\mathbf{h}_u^\dagger,\ \mathbf{h}_u\mathbf{h}_d^\dagger\mathbf{h}_d\mathbf{h}_u^\dagger\mathbf{h}_u\mathbf{h}_d^\dagger\mathbf{h}_d\mathbf{h}_u^\dagger\Big\}$$

- Down-type basis $\mathbf{H}_I^D$ obtained by interchanging $\mathbf{h}_d \leftrightarrow \mathbf{h}_u$

Slepton Mass Matrices

- Same proceedure cannot be used for sleptons in the "pure" MSSM
- Analogous expansion can be performed if we include Yukawa couplings for ν_R

Outline

Framework

Renormalization Group Effects

- RG running generates flavour structures at low energy
- We examine MCPMFV model with GUT scale parameters

$$\widetilde{\mathbf{M}}^2_{Q,U,D,L,E} = \widetilde{M}^2_{Q,U,D,L,E}\ \mathbf{1}_3\ , \qquad \mathbf{A}_{u,d,e} = A_{u,d,e}\ \mathbf{1}_3$$

Initial Conditions

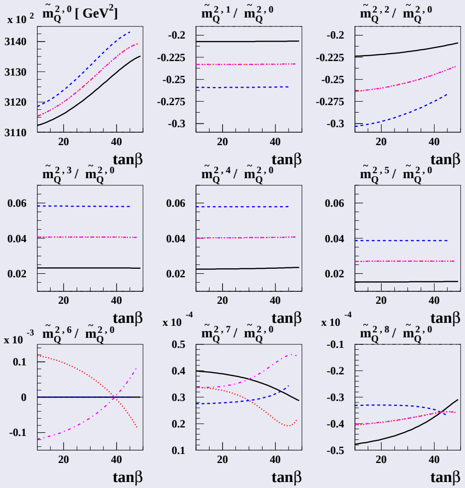
- We take the input values

$$|M_{1,2,3}| = 250\ \mathrm{GeV}\ ,$$
$$M^2_{H_u} = M^2_{H_d} = \widetilde{M}^2_Q = \widetilde{M}^2_U = \widetilde{M}^2_D = \widetilde{M}^2_L = \widetilde{M}^2_E = (100\ \mathrm{GeV})^2\ ,$$
$$|A_u| = |A_d| = |A_e| = 100\ \mathrm{GeV}, \qquad \Phi_{A_u} = \Phi_{A_d} = \Phi_{A_e} = 0^\circ$$

- Allow $\tan\beta$ to vary $10 \sim 50$
- Take common CP-violating gaugino phases $\Phi_M = 0^\circ, 90^\circ, 180^\circ, 270^\circ$

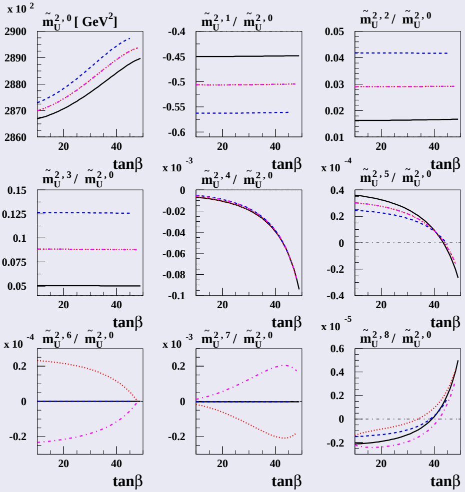
Quark Mass coefficients

Left-handed Squark Mass Matrix



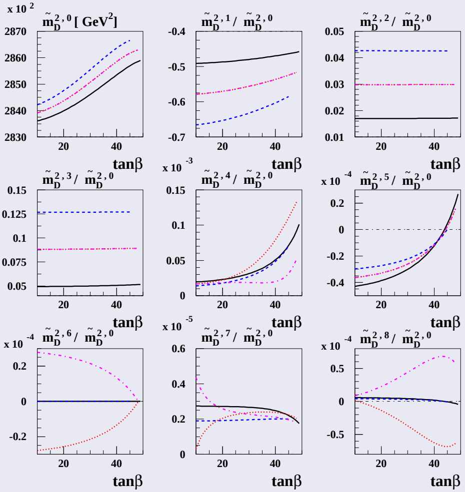
Quark Mass coefficients

Up-type Squark Mass Matrix



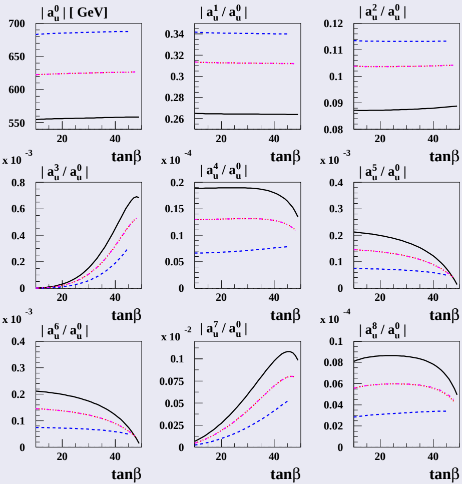
Quark Mass coefficients

Down-type Squark Mass Matrix



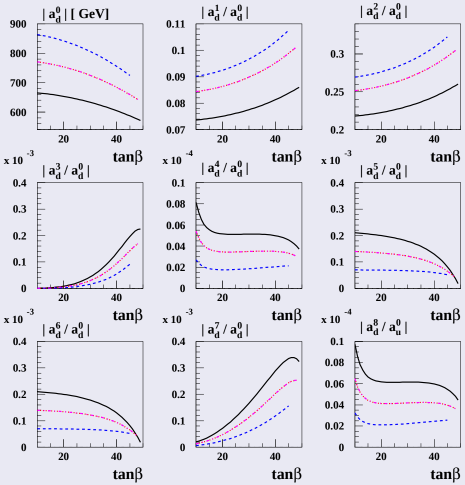
Squark Trilinear coefficients

Up-type Squark Trilinear Couplings



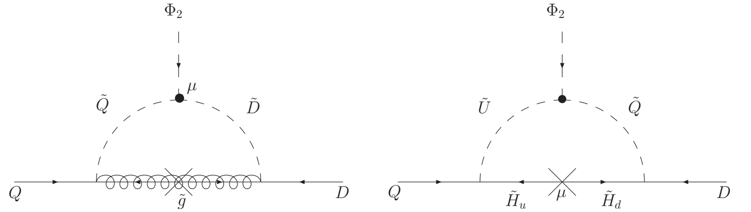
Squark Trilinear coefficients

Down-type Squark Trilinear Couplings



Contribution to Effective Couplings

Threshold Corrections



Insertion Expansion

- Flavour Singlet Masses

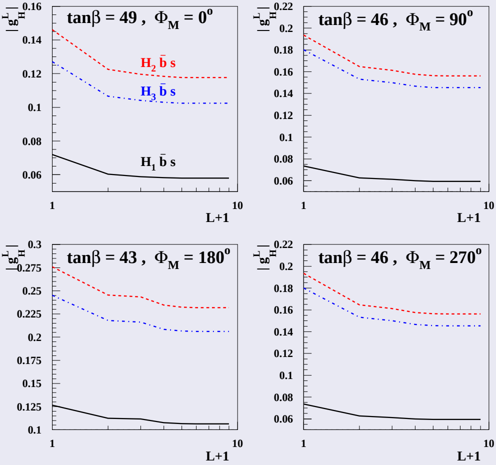
$$g^H_{d_i d_j} \sim \frac{\mathbf{h}_u^\dagger \mathbf{h}_u}{16\pi^2} \frac{\mu A_u}{\widetilde{M}^2}$$

- Non-singlet corrections

$$g^H_{d_i d_j} \sim \frac{2\alpha_S}{3\pi} \frac{\mu}{\widetilde{M}^2} \left[\delta \widetilde{\mathbf{M}}^2_Q + \mathbf{h}_d^{-1} \delta \widetilde{\mathbf{M}}^2_D \mathbf{h}_d \right] , \qquad \delta \widetilde{\mathbf{M}}^2 = \widetilde{\mathbf{M}}^2 - \widetilde{M}^2 \mathbf{1}_3$$

Higgs-mediated FCNCs

SUSY contribution



Outline

Summary and Outlook

Summary

- We have presented a geometric description of flavour in soft SUSY-breaking
- Completeness of the expansion depends on a single condition
  $\rightarrow$ non-vanishing Jarlskog determinant
- Any flavour structure can be described by MFV interactions if we allow large couplings
- The decomposition in the lepton sector requires the introduction of right-handed neutrinos with corresponding Yukawa coupling

Outlook

- Our framework offers a simple, systematic approach to flavour violation calculations
- Allows a physically-motivated interpretation of limits from FCNCs
  $\rightarrow$ Departure from 'traditional' MFV in terms of large couplings

References

'Geometric' approach

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'Algebraic' approach

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