Froggatt-Nielsen Models from E8 in F-theory GUTs

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Flavour and String Theory

- Is it possible to explain the matter content and basic flavour patterns of the Standard Model in an elegant way from string theory?

- Is there an underlying unified symmetry behind all the interactions?

- String theory constructions are more constrained than field theory

- Would like to base the construction on a GUT model

- It is generally very difficult to calculate quantities such as Yukawa couplings even up to order 1 factors

- Perturbative type II string theory does not allow for a top Yukawa coupling

- F-theory GUTs present partial solutions to these 2 key problems

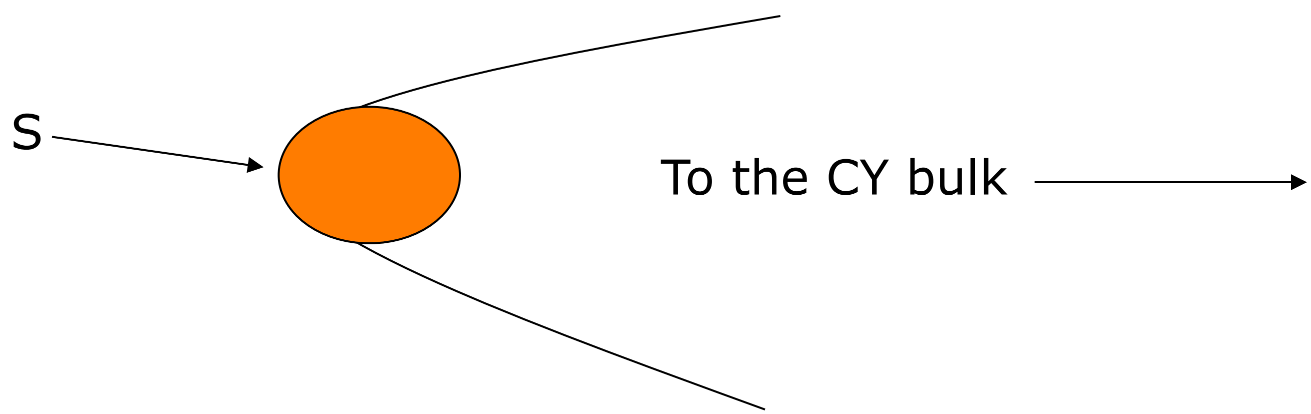
F-theory GUTs

Beasley, Heckman, Vafa; Donagi, Wijnholt; … '08-…

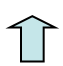
• F-theory describes strongly-coupled solutions of type IIB string theory.

• It is based on compactifying from 12 dimensions to 4 dimensions on an 8-dimensional Calabi-Yau manifolds with singularities.

• The geometry can be chosen such that there is an SU(5) gauge field that lives in four dimensional space time and on a four-dimensional surface S inside the CY

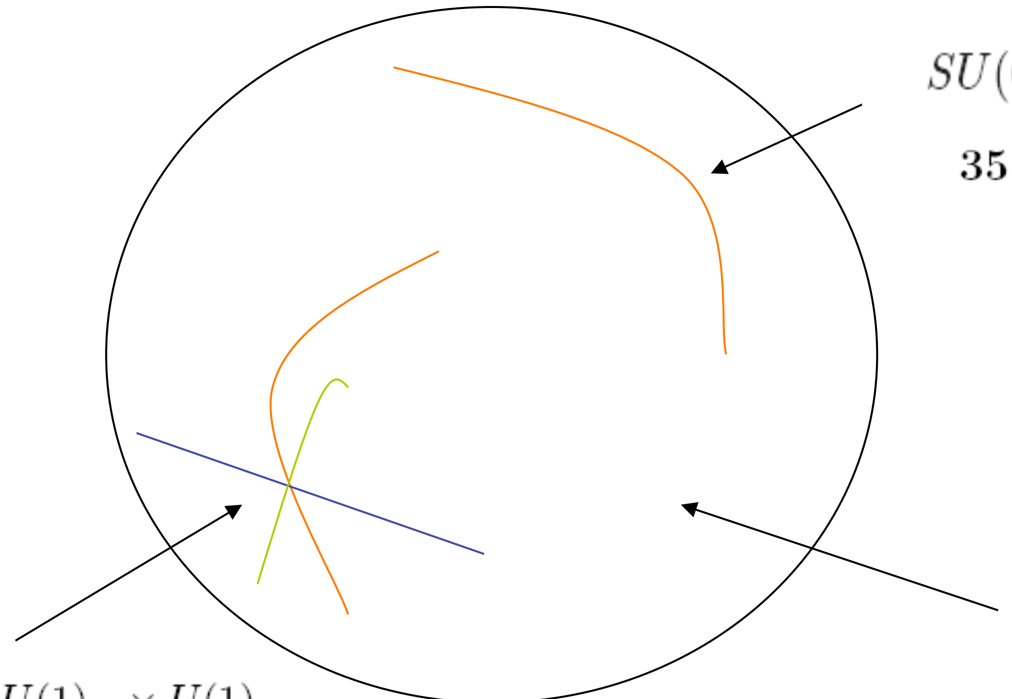


• Can decouple CY bulk as an expansion in GUT/Planck scale.

• Calculability 

F-theory GUTs

- In S there are curves on which the gauge group is enhanced
- In S there are points on which the gauge group is enhanced further



$$SU(6) \supset SU(5) \times U(1)$$

$$\mathbf{35} = \mathbf{24}_0 + \mathbf{5}_{-1} + \bar{\mathbf{5}}_1 + \mathbf{1}_0$$

SU(5)

$$E_6 \supset SU(5) \times U(1)_{a'} \times U(1)_{b'} \ ,$$

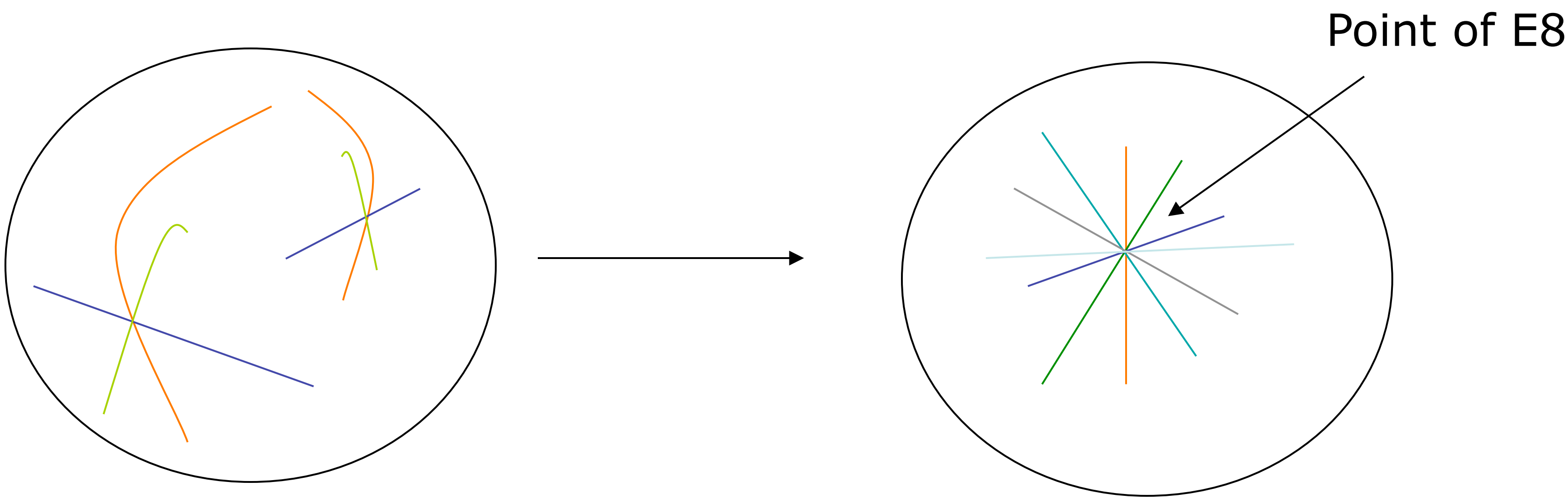
$$\mathbf{78} \to \mathbf{24}^{(0,0)} \oplus \mathbf{1}^{(0,0)} \oplus \mathbf{1}^{(0,0)} \oplus \mathbf{1}^{(-5,-3)} \oplus \mathbf{1}^{(5,3)} \oplus (\mathbf{5} \oplus \bar{\mathbf{5}})^{(-3,3)} \oplus (\mathbf{10} \oplus \bar{\mathbf{10}})^{(-1,-3)} \oplus (\mathbf{10} \oplus \bar{\mathbf{10}})^{(4,0)} \ .$$

- Top Yukawa: Uniqueness ⇧

The point of E8

Heckman, Tavanfar, Vafa. `08

- The highest possible enhancement is to E8



$$E_8 \supset SU(5) \times SU(5)_\perp :$$

$$\mathbf{248} \rightarrow (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus (\mathbf{10},\mathbf{5}) \oplus (\bar{\mathbf{5}},\mathbf{10}) \oplus (\bar{\mathbf{10}},\bar{\mathbf{5}}) \oplus (\mathbf{5},\bar{\mathbf{10}})$$

- Interactions can be studied by just looking at a single point in the CY
- Uniqueness ⇧
- Calculability ⇧⇧⇧⇧⇧⇧

A closer look at the Point of E8

- The 4-dimensional gauge interactions are $E_8 \rightarrow SU(5) \times U(1)^4$

- The U(1) charges $t_1 + t_2 + t_3 + t_4 + t_5 = 0$.

- The U(1)s become massive through Green-Schwartz mechanism and leave behind Global Symmetries which the matter is charged under

$$\begin{aligned} \Sigma_{\mathbf{10}\oplus\bar{\mathbf{10}}} &: \quad t_i = 0 \,, \\ \Sigma_{\mathbf{5}\oplus\bar{\mathbf{5}}} &: \quad -t_i - t_j = 0, \quad i \neq j \,, \\ \Sigma_{\mathbf{1}} &: \quad \pm\left(t_i - t_j\right) = 0, \quad i \neq j \,. \end{aligned}$$

Interactions are required to be neutral under the full E8, e.g. Yukawas:

$$\mathbf{5}_1\mathbf{10}_1\mathbf{10}_2 = -\mathrm{t}_1 - \mathrm{t}_2 + \mathrm{t}_1 + \mathrm{t}_2 \qquad \bar{\mathbf{5}}_1\bar{\mathbf{5}}_2\mathbf{10}_3 = \mathbf{t}_1 + \mathbf{t}_2 + \mathbf{t}_3 + \mathbf{t}_4 + \mathbf{t}_5$$

Quark flavour in E8

- Consider the point of E8 and use the U(1) symmetries to generate flavour structure a la Froggat-Nielsen.
- Take each generation to come from a different curve.
- Use U(1)s to forbid Yukawas for all but the top generation.
- Generate corrections through higher dimension operators involving the singlets after giving them a vev.

$$\mathbf{10}_{\rm top} = \{\mathbf{t_1}, \mathbf{t_2}\}$$
$$\mathbf{10}_{\rm chm} = \mathbf{t_3}$$
$$\mathbf{5}_{\rm higgs} = -\mathbf{t_1} - \mathbf{t_2}$$
$$\mathbf{1}_{\rm FN} = \{\mathbf{t_1} - \mathbf{t_3}, \mathbf{t_2} - \mathbf{t_3}\}$$

$$\mathbf{5}_{\rm higgs}\mathbf{10}_{\rm top}\mathbf{10}_{\rm top}$$
$$\frac{<\mathbf{1}_{\rm FN}>}{M_*}\mathbf{5}_{\rm higgs}\mathbf{10}_{\rm top}\mathbf{10}_{\rm chm}$$
$$\left(\frac{<\mathbf{1}_{\rm FN}>}{M_*}\right)^2\mathbf{5}_{\rm higgs}\mathbf{10}_{\rm chm}\mathbf{10}_{\rm chm}$$

Point of E8 model building

- Model build on a point!

- Must specify 2 things that would be determined by a global embedding:

  1) The monodromy action: some t's can be identified

  2) The multiplicity/chirality on each curve (Flux content)

- Once these are specified the theory includes all operators that are allowed by the U(1) selection rules – follows from point intersection.

- Note the difference to field theory models:

1) U(1) fields already included in the constructions
2) Frogatt-Nielsen fields (GUT singlets) already included
3) Matter charges are fixed

The candidate model

Field	Curve	Charges/Orbit
Chiral spectrum		
$\mathbf{5}_{H_u}$	$\mathbf{5}_{H_u}$	$-t_1 - t_2$
$\bar{\mathbf{5}}_{H_d}$	$\bar{\mathbf{5}}_5$	$t_3 + t_5$
$\mathbf{10}_t$	$\mathbf{10}_1$	$\{t_1, t_2\}$
$\mathbf{10}_c$	$\mathbf{10}_3$	t_4
$\mathbf{10}_u$	$\mathbf{10}_2$	t_3
$\bar{\mathbf{5}}_b$	$\bar{\mathbf{5}}_2$	$\{t_1 + t_4, t_2 + t_4\}$
$\bar{\mathbf{5}}_s$	$\bar{\mathbf{5}}_1$	$\{t_1 + t_3, t_2 + t_3\}$
$\bar{\mathbf{5}}_d$	$\bar{\mathbf{5}}_4$	$t_3 + t_4$
N_1	$\bar{1}_6$	$-t_4 + t_5$
N_2	$\bar{1}_5$	$-t_3 + t_5$
N_3	$\bar{1}_3$	$\{-t_1 + t_5, -t_2 + t_5\}$
X_1	$\bar{1}_4$	$-t_3 + t_4$
X_2	1_2	$\{t_1 - t_4, t_2 - t_4\}$
X_3	$\bar{1}_1$	$\{-t_1 + t_3, -t_2 + t_3\}$
Non-Chiral spectrum		
-	$\mathbf{10}_4$	t_5
-	$\mathbf{5}_3$	$\{-t_1 - t_5, -t_2 - t_5\}$
-	$\mathbf{5}_6$	$-t_4 - t_5$
-	1_7	$t_1 - t_2$

Chiral interactions	
$\mathbf{5}_{H_u}\mathbf{10}_i\mathbf{10}_j$	$\begin{pmatrix} \epsilon_2^2\epsilon_{\bar4}^2 & \epsilon_2^2\epsilon_{\bar4} & \epsilon_2\epsilon_{\bar4} \\ \epsilon_2^2\epsilon_{\bar4} & \epsilon_2^2 & \epsilon_2 \\ \epsilon_2\epsilon_{\bar4} & \epsilon_2 & 1 \end{pmatrix}$
$\bar{\mathbf{5}}_{H_d}\bar{\mathbf{5}}_i\mathbf{10}_j$	$\begin{pmatrix} \epsilon_2^2\epsilon_{\bar4}^2 & \epsilon_2\epsilon_{\bar4}^2 & \epsilon_2\epsilon_{\bar4} \\ \epsilon_2^2\epsilon_{\bar4} & \epsilon_2\epsilon_{\bar4} & \epsilon_2 \\ \epsilon_2\epsilon_{\bar4} & \epsilon_{\bar4} & 1 \end{pmatrix}$
$K \supset \mathbf{5}_{H_d}\bar{\mathbf{5}}_i N_j$	$\begin{pmatrix} \epsilon_2 & 1 & \epsilon_{\bar1}\epsilon_2 \\ \epsilon_{\bar1}\epsilon_2 & \epsilon_{\bar1} & \epsilon_{\bar1}\epsilon_{\bar1}\epsilon_2 \\ 1 & \epsilon_{\bar1}\epsilon_{\bar4} & \epsilon_{\bar1} \end{pmatrix}$
$\beta\mathbf{5}_{H_u}\bar{\mathbf{5}}_i$	$(\epsilon_{\bar4}\epsilon_2^2, \epsilon_{\bar4}\epsilon_2, \epsilon_2)$
$\mathbf{5}_{H_u}\bar{\mathbf{5}}_i N_j$	0
MN_iN_j	0
$\bar{\mathbf{5}}_i\bar{\mathbf{5}}_j\mathbf{10}_k$	0
$\mathbf{10}_i\mathbf{10}_j\mathbf{10}_k\bar{\mathbf{5}}_l$	0
$\mu\mathbf{5}_{H_u}\bar{\mathbf{5}}_{H_d}$	0

- If relax some constraints can find more models King, Leontaris, Ross ‘10

The masses and mixing

- We have 3 real parameters that are the 3 vevs which we take as

$$\epsilon_2 = \lambda^2 \, , \epsilon_{\bar{4}} = \lambda \quad \epsilon_{\bar{1}} \simeq 0.1$$

$$\lambda \simeq 0.2.$$

- The quark sector physics:

$$Y^U \simeq \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \, , \quad Y^D \simeq \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ \lambda^5 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda & 1 \end{pmatrix} \, , \quad V_{CKM} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$m_u \sim 6 \times 10^{-6}\ \text{TeV} \, , \quad m_c \sim 2 \times 10^{-4}\ \text{TeV} \, , \quad m_t \sim 0.1\ \text{TeV} \, ,$$
$$m_d \sim 4 \times 10^{-8}\ \text{TeV} \, , \quad m_s \sim 5 \times 10^{-6}\ \text{TeV} \, , \quad m_b \sim 6 \times 10^{-4}\ \text{TeV} \, .$$

- The neutrino sector physics: (PMNS anarchic, random example shown)

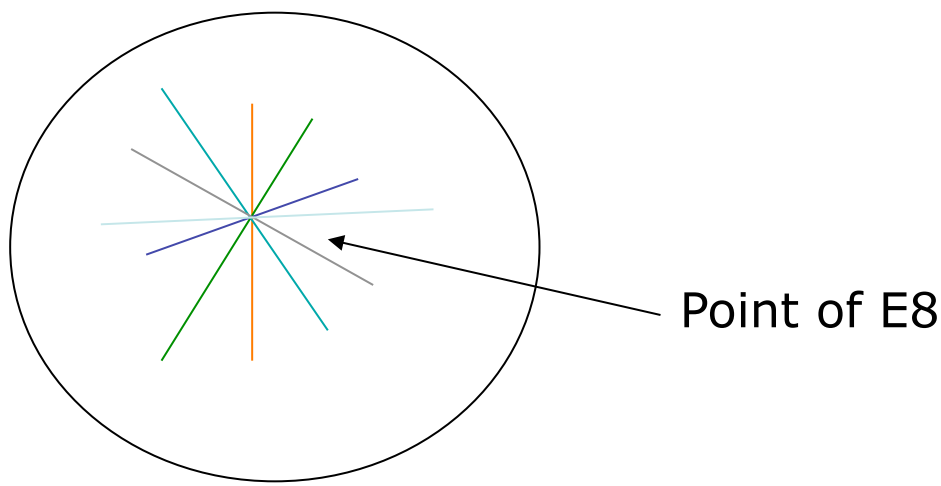
$$\begin{aligned} m_{\nu_1} &\simeq m_{\nu_2} \simeq 10^{-2}\ \text{eV}, \\ m_{\nu_3} &\simeq 4 \times 10^{-3}\ \text{eV} \, . \end{aligned}$$

$$U_{PMNS} \simeq \begin{pmatrix} -0.05 & -0.49 & -0.91 \\ 0.94 & 0.30 & -0.14 \\ -0.11 & 0.91 & -0.35 \end{pmatrix}$$

Summary

• F-theory GUTs are a promising arena for flavour physics: top Yukawa, GUT, locality.

• It is possible to explain the flavour structure observed in the SM through a simple and elegant model based on a type of E8 unification



• Global issues still unrealised/unresolved: matter spectrum, moduli stabilisation, supersymmetry breaking…

Working the U(1)s

- The U(1) symmetries should not just be used for flavour:
  - Forbid a mu term in the superpotential

$\longrightarrow$ Naturally generate via Giudice-Masiero $\int d^4\theta \, Y^\dagger \mathbf{5}_{H_u} \bar{\mathbf{5}}_{H_d}$

  - Forbid dim 4 and dim 5 proton decay operators
  - Forbid neutrino superpotential Dirac masses *
  - Forbid neutrino superpotential Majorana masses *

$\longrightarrow$ Naturally generated via operator

Arkani-Hamed et al. '00

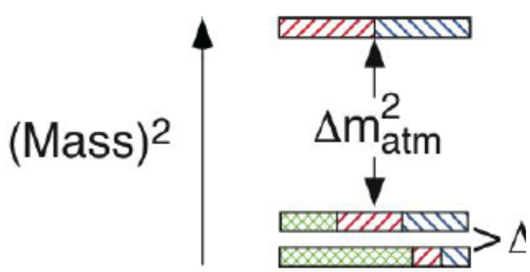
$$\int d^4\theta \frac{D_K \, (H_d)^\dagger \, L \, N}{M_*} \quad = \int d^2\theta \, \frac{\mu}{M_*} v D_K L N$$

- Unable to forbid the operator $\beta_i \bar{\mathbf{5}}^i_M \mathbf{5}_{H_u}$ $\longrightarrow$ R-parity ?

Patterns in Flavour

$$m_u \sim 5\times 10^{-7}\ \mathrm{TeV}\ ,\quad m_c \sim 2\times 10^{-4}\ \mathrm{TeV}\ ,\quad m_t \sim 0.1\ \mathrm{TeV}\ ,$$
$$m_d \sim 5\times 10^{-7}\ \mathrm{TeV}\ ,\quad m_s \sim 10^{-5}\ \mathrm{TeV}\ ,\quad m_b \sim 6\times 10^{-4}\ \mathrm{TeV}\ .$$

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97419\pm 0.00022 & 0.2257\pm 0.0010 & 0.00359\pm 0.00016 \\ 0.2256\pm 0.0010 & 0.97334\pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407\pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{bmatrix}.$$



$\sin^2(2\theta_{12}) = 0.87 \pm 0.03$
$\Delta m^2_{21} = (7.59 \pm 0.20)\times 10^{-5}\ \mathrm{eV}^2$
$\sin^2(2\theta_{23}) > 0.92$ [i]
$\Delta m^2_{32} = (2.43 \pm 0.13)\times 10^{-3}\ \mathrm{eV}^2$ [j]
$\sin^2(2\theta_{13}) < 0.19,\ \mathrm{CL} = 90\%$

Babu 0910.2948
PDG `08

Flavour on E8 (the competition)

- Take 3 generations to come from one curve.

Heckman, Vafa `08

- The corresponding Yukawa matrix is rank 1.

Cecotti, Cheng, Heckman, Vafa `09

Conlon, EP `09

- Corrections can be induced by non-perturbative effects.

Marchesano, Martucci`09

- These can also be viewed as H-flux.

Cecotti, Cheng, Heckman, Vafa `09;

Baumann et al. '09

- The resulting Yukawa matrix can potentially look like hierarchical

$$(W_{\text{Yuk}}) \sim \begin{pmatrix} 1 & \hbar_\theta & \hbar_\theta^2 \\ \hbar_\theta & \hbar_\theta^2 & \hbar_\theta^3 \\ \hbar_\theta^2 & \hbar_\theta^3 & \hbar_\theta^4 \end{pmatrix} .$$

- Much yet to be understood…

(global effects, size of corrections, backreaction…)

Other candidate model

$$K = \begin{pmatrix} 0 & 0 & 0 \\ \epsilon_2 & 1 & \epsilon_4 \\ 1 & 0 & 0 \end{pmatrix}, \; W = \begin{pmatrix} 0 & \epsilon_2^2\epsilon_{-4}^2 & \epsilon_2^2\epsilon_{-4} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; M = \begin{pmatrix} \epsilon_2^2\epsilon_{-4}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

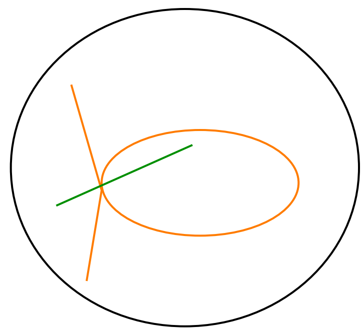
$$\mathcal{L} \supset \frac{v^2}{M_*}\left(\epsilon_2^2\epsilon_{-4}^2\nu_\mu^2 + 2\epsilon_2^2\epsilon_{-4}\nu_\mu\nu_\tau + \epsilon_2^2\nu_\tau^2\right) + \frac{v\mu}{M_*}\left[\left(\epsilon_2\nu_e + \nu_\mu + \epsilon_{-4}\nu_\tau\right)N_2 + \nu_e N_1\right] .$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_2 \\ N_1 \end{pmatrix} = \begin{pmatrix} -0.4 \\ -0.6 \\ -0.1 \\ -0.6 \\ -0.4 \end{pmatrix}, \begin{pmatrix} -0.4 \\ -0.6 \\ -0.1 \\ 0.6 \\ 0.4 \end{pmatrix}, \begin{pmatrix} 0.6 \\ -0.4 \\ -0.1 \\ -0.4 \\ 0.6 \end{pmatrix}, \begin{pmatrix} -0.6 \\ 0.4 \\ 0.1 \\ -0.4 \\ 0.6 \end{pmatrix}, \begin{pmatrix} 10^{-8} \\ 0.2 \\ -1 \\ 0.001 \\ 10^{-5} \end{pmatrix},$$

$$v^2/M_* \sim 10^{-3}\text{eV}, \; (1, -1, 1, -1, 10^{-3}).$$

Monodromies

This corresponds to the fact that 2 curves intersecting at the E8 point P may actually be the same curve when extended away from P:



In this case the localised wavefunctions are related by a monodromy about P.

• Field theory: Some of the U(1)s may be identified if they come from the same bulk brane

The matter curves that have the same charges under this equivalence are also identified

$$\mathbf{10_1 = t_1 \leftrightarrow t_2 = 10_2} \qquad \mathbf{10_1 = \{t_1, t_2\}} \quad \text{Orbit}$$