Update on structures of Yukawa matrices and CP violation

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- *Textures zeroes* have been a very useful tool to descriminate different possibilities and to understand the hierarchy of mixing and masses, BUT for PRECISION tests EXACT expressions are neeeded
- On a more fundamental level: SM model completely forgets to address an explanation for the sctructure of Yukawa couplings and CP violation and to properly address the question we need to understand the nature of the mixing and mass hierarchies.

Outline

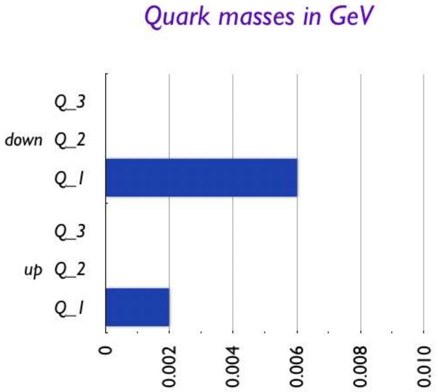
- Flash introduction
- What experimental information **[can we use]** do we know?
- Guidelines: CP violation and connection of mixing angles with fermion masses
- How to use this information?
- Two examples, which can be embedded respectively in S_3 and $SU(3)$
- What have we learnt?

Flash introduction

- The SM model completely forgets to address an explanation for the sctructure of Yukawa couplings and CP violation
- Extensions of the SM that attempt to solve the hierchy problem (between EW and GUT scales) do not by themselves address this problem ... they even worsten the flavour and CP problems
- the not-enough experimental constraints to determine the sctructures above we use the guidelines of Symmetries and CP violation
- For these guidelines and effective approach of postulating a determined symmetry requires a definitive Ansazt (Fritzch) of the form of Yukawa matrices [*Textures zeroes*]

What experimental information can we use?

- Masses



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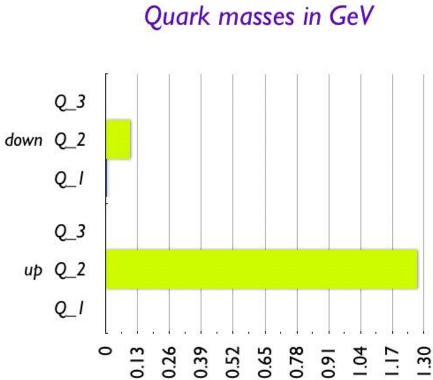

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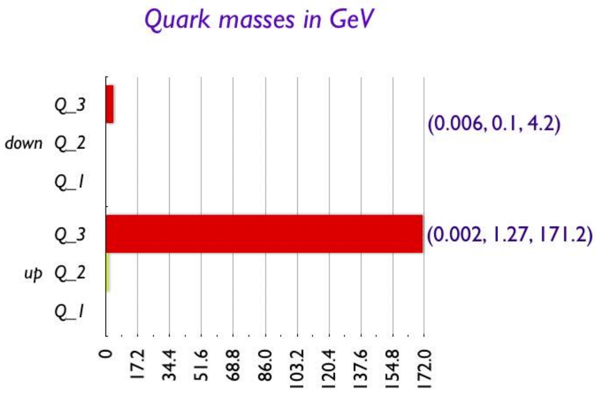

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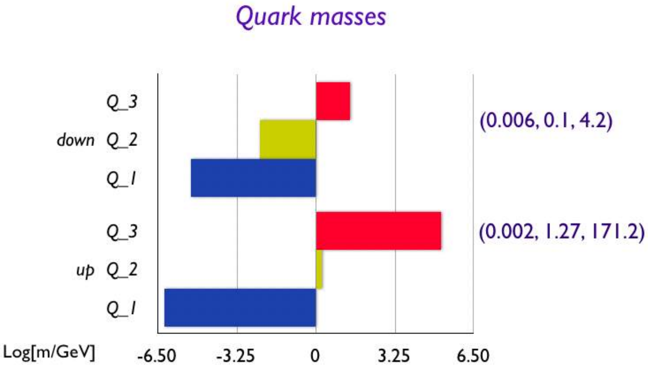

- Mixing

$$V_{CKM} = R_{23}P(-\delta,1,1)R_{13}P(\delta,1,1)R_{12}$$

$$= \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}s_{12}c_{23}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} \mathbf{(0.9739, 0.9751)} & \mathbf{(0.226, 0.228)} & \mathbf{(0.0029, 0.0045)} \\ \mathbf{(0.226, 0.228)} & \mathbf{(0.9730, 0.9744)} & \mathbf{(0.0416, 0.0423)} \\ \mathbf{(0.0075, 0.0085)} & \mathbf{(0.0409, 0.04173)} & \mathbf{(0.9990, 0.9992)} \end{pmatrix}$$

$$R = O, \quad (R_{ij})_{ij} = s_{ij}$$

$$s_{12} > s_{23} > s_{13} \quad [\lambda, A\lambda^2, A\lambda^3(\rho - i\eta)]$$

http://ckmfitter.in2p3.fr
http://www.utfit.org

What experimental information can we use?

- CP violating quantities:

$$\left| J \sum_{m,n} \epsilon_{ikm}\epsilon_{jln} \right| = \left| \text{Im} \left[V_{ij} V_{kl} V_{il}^* V_{kj}^* \right] \right| = 3.05^{+0.19}_{-0.20} \times 10^{-5},$$

which in terms of the standard parameterization is

$$J = c_{13}^2 c_{12} c_{23} s_{13} s_{23} \sin\delta$$

however what is relevant, since they can be measured, are the angles arising from the unitary triange

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

What experimental information can we use?

- Thus we are left to be guided by

$$\Phi_1 = \beta = \left[-\frac{V_{cb}^* V_{cd}}{V_{td} V_{tb}^*}\right]$$

$$\Phi_2 = \alpha = \left[-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right]$$

$$\Phi_3 = \gamma = \left[-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right]$$

	CKM value $\pm 1\sigma C.L.$	$\pm 2\sigma$ C.L.	Direct exp. value $\pm 1\sigma C.L.$	$\pm 2\sigma$ C.L.
α	$90.6^{+3.8}_{-4.2}$	$^{+7.5}_{-6.3}$	$95.6.0^{+3.3}_{-8.8}$	$^{+5.2}_{-11.8}$
β	$21.58^{+0.91}_{-0.81}$	$^{+1.8}_{-1.4}$	$21.07^{+0.90}_{-0.88}$	$^{+1.8}_{-1.7}$
γ	$67.8^{+4.2}_{-3.9}$	$^{+6.3}_{-8.0}$	70.0^{+27}_{-30}	$^{+44}_{-41}$

Table: Relevant information from experiments and from the CKM fitter.

Guidelines: CP violation and connection of mixing angles with fermion masses*

- CP violation in the quark sector is a consequence of having complex Yukawa couplings
- In the quark sector the angles are hierarchical AND quark masses are hierarchical $\longrightarrow$ are these features related?
  $\longrightarrow$ The ideal solution would be to explain both in the same context: spontaneous CP violation?

* *I am leaving out for this presentation the connection to the lepton sector but the examples presented in the following sections can be embedded in models for which the proposed symmetry can account for the generation of masses and mixing both for leptons and quarks.*

Two examples [which can be embedded respectively in S_3 and $SU(3)$]

- Hermitian matrices: matrices that can be diagonalized by

$$U^f = O^{fT} P^f \; \rightarrow V_{\text{CKM}} = U^u U^{d\dagger} = O^{uT} P^u P^{d\dagger} O^d$$

idea: CP violation understood in terms of a single phase

how to explain it?: using a discrete symmetry: e.g. S_3

J. Kubo, A. & M. Mondragón and E. Rodriguez-Jauregui [hep-h/0302196]

- (Anti) Symmetric matrices with CKM-like diagonalizing matrices

$$U^f = \left[P_1^{f*} O_{23}^f P_2^{f*} O_{13}^f P_3^{f*} O_{12}^f \right]^T \rightarrow R_{23} = O_{23}^{uT} P_1^{u\dagger} P_1^d O_{23}^d$$

$$V_{CKM} = R_{23} P(-\delta, 1, 1) R_{13} P(\delta, 1, 1) R_{12}$$

idea: to link the hierarchy of fermion masses to that of the CKM matrix

how to explain it?: continuous symmetries with a strong breaking pattern: e.g. $SU(3)$

Hermitian matrices

- Suppose we have the Hermitian Yukawa matrix

$$Y = \begin{pmatrix} 0 & a & 0 \\ a^* & b & c \\ 0 & c & d \end{pmatrix} \to Y = P^\dagger \overline{Y} P$$

$\overline{Y}$ is a real symmetric matrix that may be diagonalized by orthogonal matrices:

$$\overline{Y} = O \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} O^T, \; \overline{Y} = \begin{pmatrix} 0 & \sqrt{\frac{\tilde{y}_1 \tilde{y}_2}{1-\delta}} & 0 \\ \sqrt{\frac{\tilde{y}_1 \tilde{y}_2}{1-\delta}} & \tilde{y}_1 - \tilde{y}_2 + \delta & \sqrt{\frac{\delta}{1-\delta} f_1 f_2} \\ 0 & \sqrt{\frac{\delta}{1-\delta} f_1 f_2} & 1-\delta \end{pmatrix}$$

$$\tilde{y}_1 = \frac{y_1}{y_3}, \quad \tilde{y}_2 = \frac{|y_2|}{y_3}$$

$$f_1 = 1 - \tilde{y}_1 - \delta, \quad f_1 = 1 + \tilde{y}_2 - \delta, \quad 0 < \delta < 1 - \tilde{y}_1$$

J. Barranco, F. González Canales and A. Mondragón [hep-ph] 1004:3781
G. Couture, C. Hamzaoui, S. Lu, M. Toharia, [hep-ph] 0910:3132

Hermitian matrices

-

$$O = \begin{pmatrix} \left[\frac{\tilde{y}_1 f_1}{D_1}\right]^{1/2} & -\left[\frac{\tilde{y}_1 f_1}{D_2}\right]^{1/2} & \left[\frac{\tilde{y}_1 f_1}{D_3}\right]^{1/2} \\ \left[\frac{\tilde{y}_1 (1-\delta) f_1}{D_1}\right]^{1/2} & \left[\frac{\tilde{y}_2 (1-\delta) f_2}{D_2}\right]^{1/2} & \left[\frac{(1-\delta)\delta}{D_3}\right]^{1/2} \\ -\left[\frac{\tilde{y}_1 f_2 \delta}{D_1}\right]^{1/2} & -\left[\frac{\tilde{y}_2 f_1 \delta}{D_2}\right]^{1/2} & \left[\frac{\tilde{f}_1 f_2}{D_3}\right]^{1/2} \end{pmatrix}$$

$$\begin{aligned} D_1 &= (1-\delta_1)(\tilde{y}_1+\tilde{y}_2)(1-\tilde{y}_1), \\ D_2 &= (1-\delta_1)(\tilde{y}_1+\tilde{y}_2)(1-\tilde{y}_2), \\ D_3 &= (1-\delta_1)(1-\tilde{y}_1)(1+\tilde{y}_2), \end{aligned}$$

Hermitian matrices

-

$$V_{us} = -\sqrt{\frac{\tilde{y}_c\tilde{y}_d f_{u1} f_{d2}}{D_{u1}D_{d2}}} + \sqrt{\frac{\tilde{y}_u\tilde{y}_s}{D_{u1}D_{d2}}} g_1(\delta_u, \delta_d, f_{u1}, f_{d2}) e^{i\phi}$$

$$\rightarrow -\sqrt{\frac{y_d}{y_s}} + \sqrt{\frac{y_u}{y_c}} e^{i\phi}$$

$$V_{cb} = -\sqrt{\frac{\tilde{y}_u\tilde{y}_d\tilde{y}_s\delta_d f_{u2}}{D_{u1}D_{d2}}} + \sqrt{\frac{\tilde{y}_c}{D_{u2}D_{d3}}} g_2(\delta_u, \delta_d, f_{u1}, f_{d2}) e^{i\phi}$$

$$\rightarrow \mathcal{O}(\sqrt{\frac{y_c}{y_t}})$$

$$V_{ub} = \sqrt{\frac{\tilde{y}_c\tilde{y}_d\tilde{y}_s\delta_d f_{u1}}{D_{u1}D_{d3}}} + \sqrt{\frac{\tilde{y}_u}{D_{u1}D_{d3}}} g_3(\delta_u, \delta_d, f_{u1}, f_{d2}) e^{i\phi}$$

$$\rightarrow \sqrt{\frac{y_u}{y_c(1 + y_s/y_b)}} e^{i\phi}$$

$$\phi = 90^o$$

(Anti) Symmetric matrices with CKM-like diagonalizing matrices

- The procedure above can be adopted in general but instead of assuming Y Hermitian, for the combination $YY^\dagger$
  What is really different? $\rightarrow$ the sctructure of phases
- The formalism of using $YY^\dagger$ can be completely identified with the approach suggested by the idea of

$$
\begin{aligned}
U^f &= \left[P_1^{f*}O_{23}^f P_2^{f*}O_{13}^f P_3^{f*}O_{12}\right]^T \rightarrow R_{23} = O_{23}^{uT}P_1^{u\dagger}P_1^d O_{23}^d \\
V_{CKM} &= R_{23}P(-\delta,1,1)R_{13}P(\delta,1,1)R_{12}
\end{aligned}
$$

L. J. Hall and A. Rasin [hep-ph/9303303]
R. Roberts, A. Romanino, G. Ross and L. V-S [hep-ph/0104088]

(Anti) Symmetric matrices with CKM-like diagonalizing matrices

- Consider

$$Y = \begin{pmatrix} 0 & a_{12}\ \epsilon_f^3 & a_{13}\epsilon_f^3 \\ a_{21}\ \epsilon_f^3 & a_{22}\mathcal{Y}^f\ \epsilon_f^2 + a'_{22}\ \epsilon_f^3 & a_{23}\mathcal{Y}^f\ \epsilon_f^2 + a'_{23}\ \epsilon_f^3 \\ a_{31}\ \epsilon_f^3 & a_{32}\ \epsilon_f^2 + a'_{32}\ \epsilon_f^3 & a_{33} \end{pmatrix},$$

in principle all a_{ij} complex and $|a_{ij}| \approx |a_{ji}|$. One solution: just phases in a^u_{12} and a^d_{13}.

(Anti) Symmetric matrices with CKM-like diagonalizing matrices

- Then

$$V_{us} \approx \left|\frac{|Y^d_{12}|}{Y^d_{22}} - \frac{|Y^u_{12}|}{Y^u_{22}} e^{i\Phi_1}\right|$$
$$\to -\sqrt{\frac{y_d}{y_s}} + \sqrt{\frac{y_u}{y_c}} e^{i\phi}$$
$$V_{cb} \approx |s^Q_{23}| = \quad s^Q_{23} = \frac{a^d_{23}}{a^d_{33}}\epsilon_d^2 - \frac{a^u_{23}}{a^u_{33}}\epsilon_u^2 \to \left|\frac{y_c}{y_t} - \frac{y_s}{y_b}\right|$$
$$V_{ub} \approx \frac{Y^d_{13}}{Y^d_{33}} e^{i\phi_2} - \frac{|Y^u_{12}|}{Y^u_{22}} s^Q_{23} e^{i\Phi_1} \to \mathcal{O}\left(\frac{y_d}{y_b}\right) - \mathcal{O}\left(\sqrt{\frac{y_u}{y_c}} s^Q_{23}\right),$$

(Both phases Φ_1 and Φ_2 are important)
We can then identify this with the traditional picture of describing the angles as

$$\mathsf{s}_{12} \approx \frac{a_{12}}{(a_{22} - a_{23}a_{32}\epsilon^2)}, \quad \mathsf{s}_{13} \approx \frac{a_{13}}{a_{33}}\epsilon^3, \quad \mathsf{s}_{23} \approx \frac{a_{23}}{a_{33}}\epsilon^2$$

Other examples

- $$Y_u = \begin{pmatrix} a_u & b_u & 0 \\ b_u & c_u & d_u \\ x & x & e_u \end{pmatrix}, \quad Y_d = \begin{pmatrix} 0 & ib_d 0 & \\ \pm ib_d & c_d & d_d \\ x & x & e_d \end{pmatrix}$$

$$V_{us} \approx -\sqrt{\frac{y_d}{y_s}} + \sqrt{\frac{y_u}{r_u y_c}} e^{i\phi}$$

S. Antusch, S. King, M. Malinsky, M. Spinrath, [hep-ph] 0910.5127 I. Masina and C. Savoy [hep-ph] 0603101

How to make the fit to fermion masses and mixings?

- Decide which is the scale at which Y is effective [10^{17} orders of magnitude possible $\rightarrow$ running involved]
- The nature of the extension of the SM incorporating an explanation of Y [details of the parameters describing this extension involved]
- The **exact expressions** from matrices $U^\dagger$ obtained from the diagonalization of Y (or equivalently $YY^\dagger$) **must be used for a fit**.

What have we learnt?

$$Y = \begin{pmatrix} 0 & a & 0 \\ a^* & b & c \\ 0 & c & d \end{pmatrix}, \qquad M_{eff} = M_{EW} \text{ Consistent at } 2\sigma$$

Problems: V_{td}, V_{cb}

Consistent: J. Barranco, F. González Canales and A. Mondragón [hep-ph] 1004:3781

$$Y = \begin{pmatrix} 0 & a & a' \\ a & b & c \\ a' & c & d \end{pmatrix}, \ (a^u, a'^d \text{complex}) \left\{ \begin{array}{c} M_{eff} = M_{EW} \\ \text{Out of the } 3\sigma \text{region} \\ M_{eff} = M_{GUT} \text{ and} \\ \tan\beta \text{ large} \\ \text{Consistent at } 1\sigma \end{array} \right.$$

Problems at M_{EW}: V_{us}, α

L. C, E.J. Chun and L. V-S, [hep-ph]1005.5563

These an other examples in detail to appear soon

What have we learnt?

- Textures zeroes and its understanding in terms of simple expressions of mixing elements and fermion masses are a great guideline of postulating symmetries BUT:
- Do not forget the well known fact that experimental information is not enough to describe the structure of Yukawa matrices i.e we need assumptions
- The precision that the measurements involved in the determination of the CKM matrix require the use of the exact formulas for our assumptions
- Without these guidelines, the classification of textures for Yukawa matrices cannot make much progress... and so the construction of family symmetries