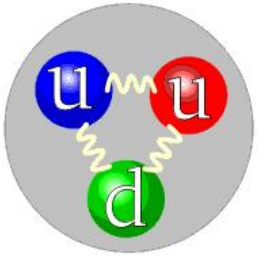


Lectures on High Energy QCD Scattering

from concepts to applications

Roman Pasechnik

- **The way: conceptual approach**
- **The ultimate goal: to initiate an interest**

"The greatest adventure my generation will ever have
- the confined field theory of QCD"

Bo Andersson

Contents

Lecture 1:
High energy scattering in QCD

Quark model;
Color charge;
Hard and soft QCD;
Feynman rules of QCD;
Kinematics 2→3 scattering;
Lorentz-invariant phase space
and the cross section;
Analyticity and unitarity;
Infrared safely;
QCD factorisation

Lecture 2:
Deep Inelastic Scattering and QCD factorization

DIS and Parton Model;
Bjorken scaling;
scaling violations;
Infinite momentum frame versus rest frame;
DIS in the Dipole Model;
DGLAP and BFKL evolution; PDFs ,
collinear and kt-factorization;
light cone coordinates;
virtual photon wave function;
Inclusive Drell-Yan

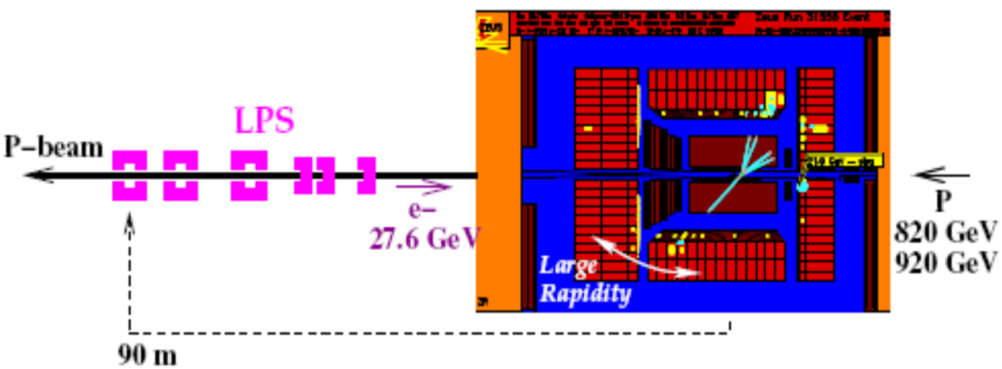
Lecture 3:
Diffractive scattering and soft QCD

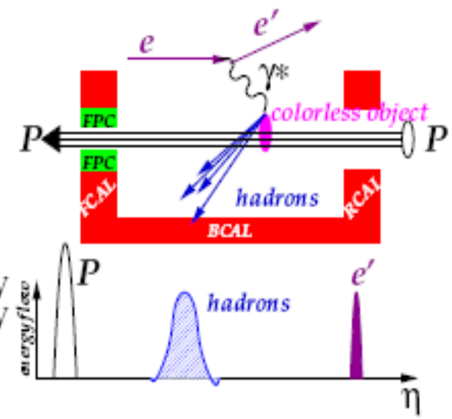
The QCD pomeron;
ladder diagrams;
Soft diffraction;
rapidity gaps;
diffractive vs non-diffractive;
Good-Walker formulation of diffraction;
diffractive DIS and DY;
soft gap survival and color screening effects.

Diffractive rapidity-gap events in ep at HERA

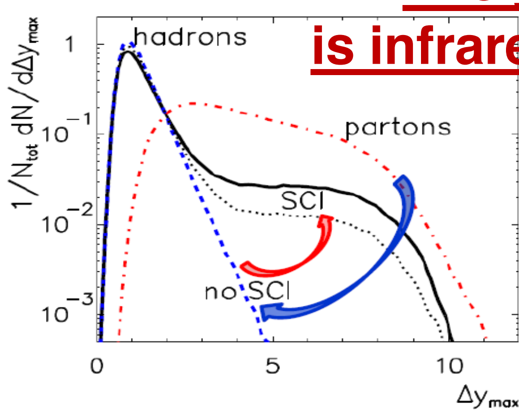
~ 10 % of gap events!!!

1. Diffractive scattering

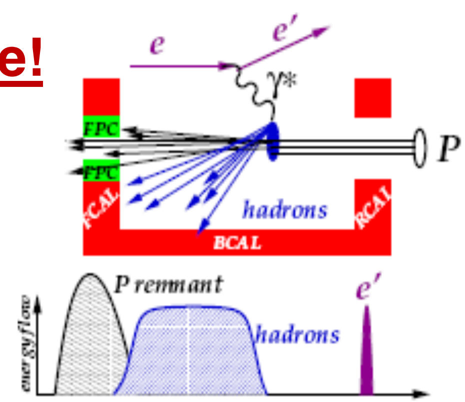




2. Non-diffractive scattering



The gap size is infrared-sensitive!



Diffraction Is a good probe for soft physics!

ZEUS Collab., Phys. Lett. **B315** (1993) 481; **B332** (1994) 228; Z. Phys. **C68** (1995) 569; Eur. Phys. J. **C1** (1998) 81; **C6** (1999) 43;
H1 Collab., Nucl. Phys. **B429** (1994) 477; Phys. Lett. **B348** (1995) 681; Nucl. Phys. **B472** (1996) 3; Z. Phys. **C69** (1995) 27; **C75** (1997) 607; **C76** (1997) 613.

Definition: What is the hadronic diffraction?

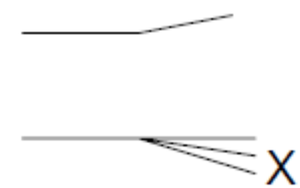
Two alternative definitions:

1. Diffraction is elastic or quasi-elastic scattering caused, via s-channel unitarity, by the absorption of components of the wave functions of the incoming particles

   e.g. $pp \rightarrow pp$,

   $pp \rightarrow pX$ (single proton dissociation, SD),

   $pp \rightarrow XX$ (both protons dissociate, DD)



quan.no. of p

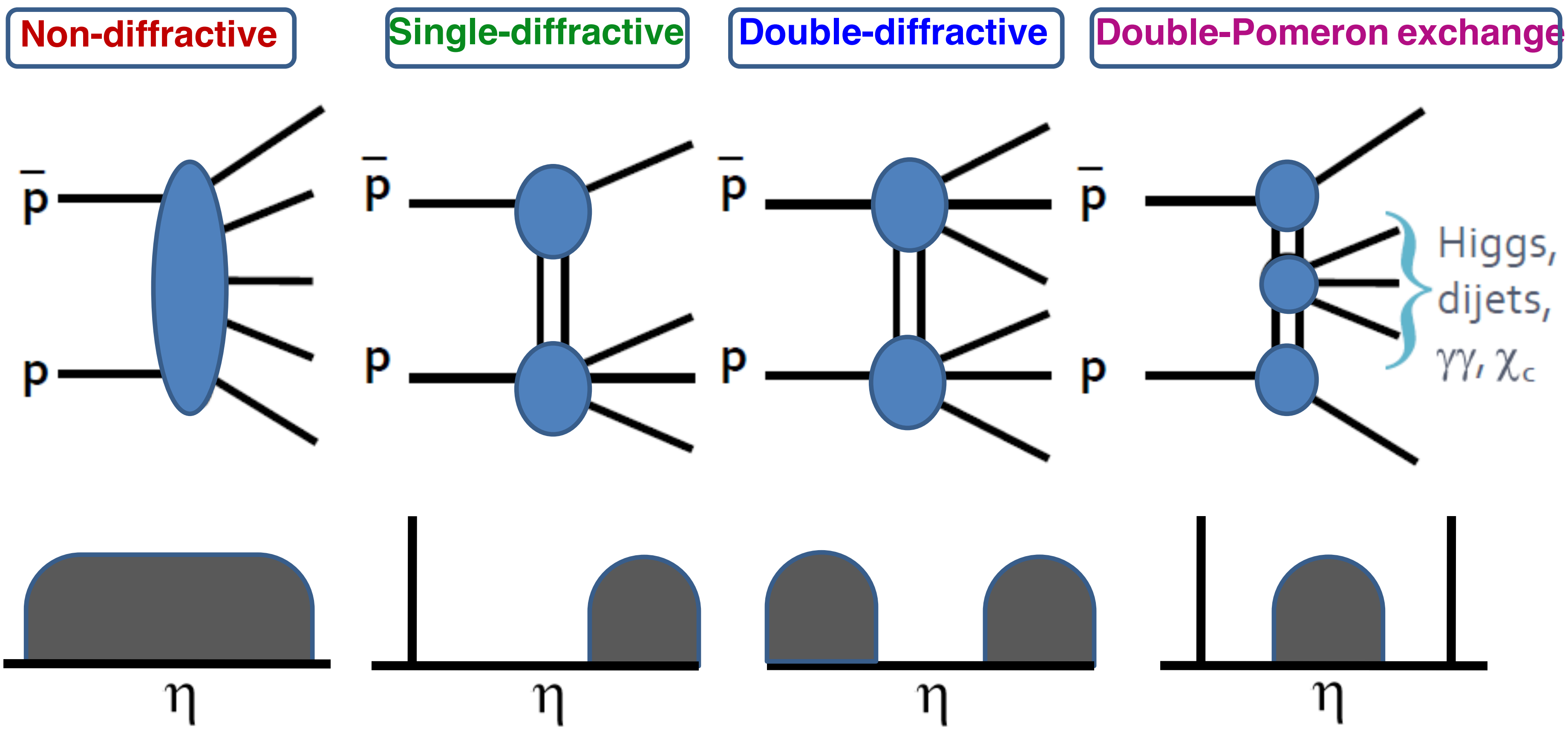
2. A diffractive process is characterized by a large rapidity gap (LRG), which is caused by t-channel Pomeron exch. (or, to be more precise, by the exchange corresponding to the rightmost singularity in the complex angular momentum plane with vacuum quantum numbers).

…according to A. Martin

Definition: diffractive reactions

Diffractive reactions – in which:

- no quantum numbers / significant momenta are exchanged
- a new diffractive state is produced

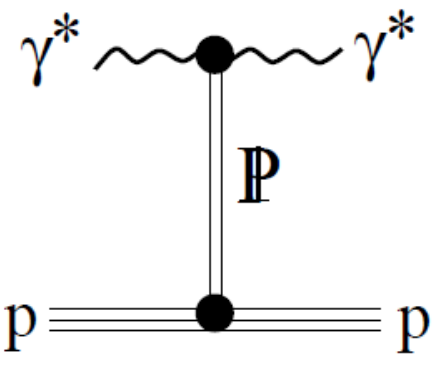


Soft vs Hard: Soft QCD and diffraction

Soft processes are characterized by the soft hadronic scale: $R \sim 1 \text{ fm}$

Hadronic diffraction ⟷ **predominantly soft phenomenon**

<u>Regge theory approach</u>



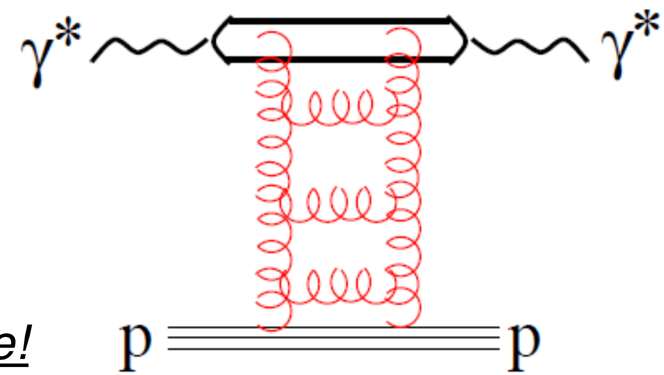
A. Donnachie, P.V. Landshoff, Nucl. Phys. **B231** (1984) 189.

Pomeron structure is still a mystery in QCD!

???

Continuous matching between soft and hard regimes is a big challenge!

<u>Perturbative QCD approach</u>



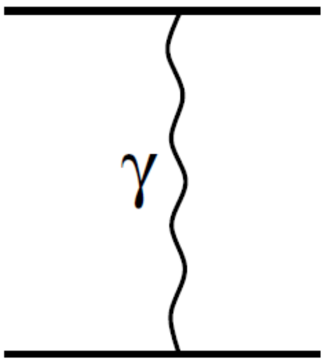
pQCD motivated models:

- Durham QCD mechanism
- Color Dipole Approach
- Soft Color Interactions model

QCD: Non-Abeliance and role of the color

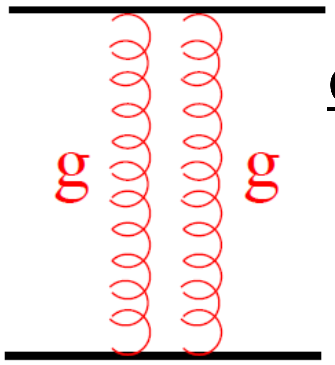
Born approximation for elastic scattering

Abelian



$$\mathrm{Im}\, f_{el} = 0$$

Non-Abelian



Color singlet exchange!

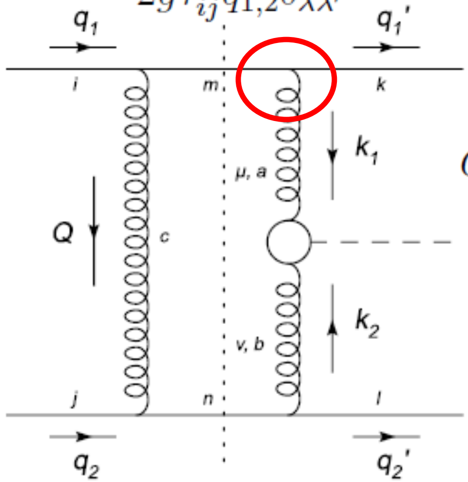
$$\boxed{\mathrm{Re}\, f_{el} = 0}$$

at asymptotically high energies!

Elastic amplitude is dominated by imaginary part → direct probe for non-Abeliance of the underlined theory!

Central exclusive Higgs production: Durham model

$$2g\tau^a_{ij}q_{1,2}\delta_{\lambda\lambda'}$$



$$\mathrm{Im}A^{ik}_{jl} = \frac{1}{2}\times 2\int d(PS)_2\delta((q_1-Q)^2)\delta((q_2+Q)^2)\frac{2gq_1^\alpha\, 2gq_{2\alpha}}{Q^2}\frac{2gq_1^\mu}{k_1^2}\frac{2gq_2^\nu}{k_2^2}V^{ab}_{\mu\nu}\,\tau^c_{im}\tau^c_{jn}\tau^a_{mk}\tau^b_{nl}$$

$$Q = \alpha q_1 + \beta q_2 + Q_t \qquad \int d(PS)_2 = \frac{s}{2}\int\frac{d^2\mathbf{Q}_t}{(2\pi)^2}d\alpha d\beta \qquad \alpha\approx-\beta\approx\mathbf{Q}_t^2/s\ll 1$$

$$k_i = x_i q_i + k_{it} \qquad x_i\sim m_H/\sqrt{s}$$

$$V^{ab}_{\mu\nu} = V\delta^{ab}\left(g_{\mu\nu}-\frac{k_1^\mu k_2^\nu}{(k_1k_2)}\right), \qquad V=\frac{m_H^2\alpha_s}{4\pi v}F(m_H^2/m_t^2) \qquad F\approx 2/3$$

$$\tau^c_{im}\tau^c_{jn}\tau^a_{mk}\tau^b_{nl}\to\frac{\delta^{ab}}{4N_c^2}$$

$$q_1^\mu V^{ab}_{\mu\nu}q_2^\nu\approx\frac{k_{1t}}{x_1}\frac{k_{2t}}{x_2}V^{ab}_{\mu\nu}\simeq\frac{s}{m_H^2}k^\mu_{1t}k^\nu_{2t}V^{ab}_{\mu\nu}, \qquad 2(k_1k_2)\approx x_1x_2s\approx m_H^2$$

In the forward limit $\epsilon_i \sim k_{it} \qquad Q_t = -k_{1t} = k_{2t}$

Jz=0 state is produced only!

Amplitude

$$\frac{\mathrm{Im}A}{s}\approx\frac{N_c^2-1}{N_c^2}\times 4\alpha_s^2V\int\frac{d^2\mathbf{Q}_t}{\mathbf{Q}_t^2\mathbf{k}_{1t}^2\mathbf{k}_{2t}^2}\frac{-(\mathbf{k}_{1t}\mathbf{k}_{2t})}{m_H^2}$$

Partonic cross section

$$\frac{d\sigma}{d^2\mathbf{q}'_{1t}d^2\mathbf{q}'_{2t}dy}\approx\left(\frac{N_c^2-1}{N_c^2}\right)^2\frac{\alpha_s^6}{(2\pi)^5}\frac{G_F}{\sqrt{2}}\left[\frac{2}{3}\int\frac{d^2\mathbf{Q}_t}{2\pi}\frac{(\mathbf{k}_{1t}\mathbf{k}_{2t})}{\mathbf{Q}_t^2\mathbf{k}_{1t}^2\mathbf{k}_{2t}^2}\right]^2$$

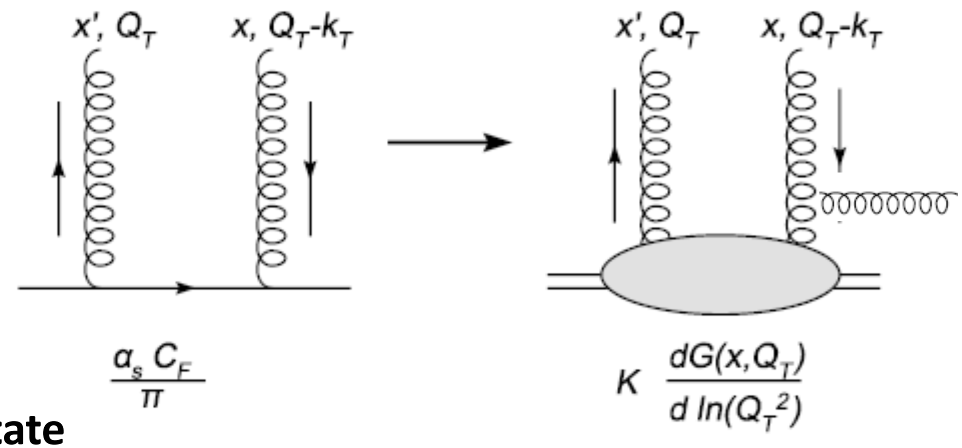
Gluon unintegrated densities and the Sudakov suppression

Standard gluon PDF (DGLAP evolution)

$$xG(x, Q_t^2)$$

UGDF (in the LLA; BFKL-like evolution)

$$f(x, Q_t^2) \sim \partial xG(x, Q_t^2)/\partial \ln Q_t^2$$



Probability to emit one gluon into the final state

$$\frac{C_A \alpha_s}{\pi} \int_{Q_t^2}^{m_H^2/4} \frac{dp_t^2}{p_t^2} \int_{p_t}^{m_H/2} \frac{dE}{E} \sim \frac{C_A \alpha_s}{4\pi} \ln^2 \left(\frac{m_H^2}{Q_t^2} \right)$$

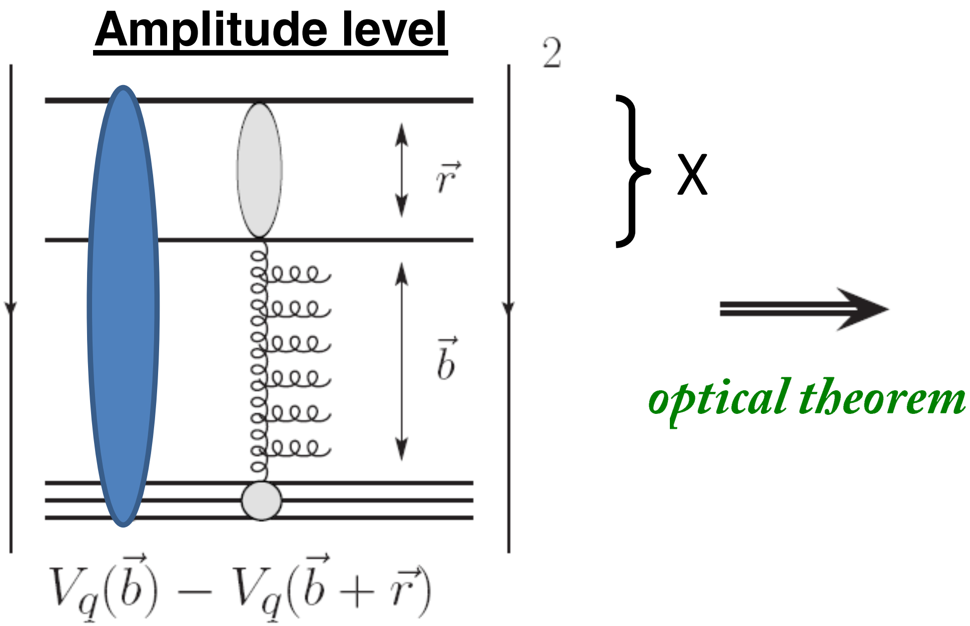
The sudakov suppression (for uneven longitudinal momentum sharing):
The first gluon completely screens the color charge of the second gluon, if the wavelength of soft radiation is sufficiently large, i.e. at $p_t < Q_t$

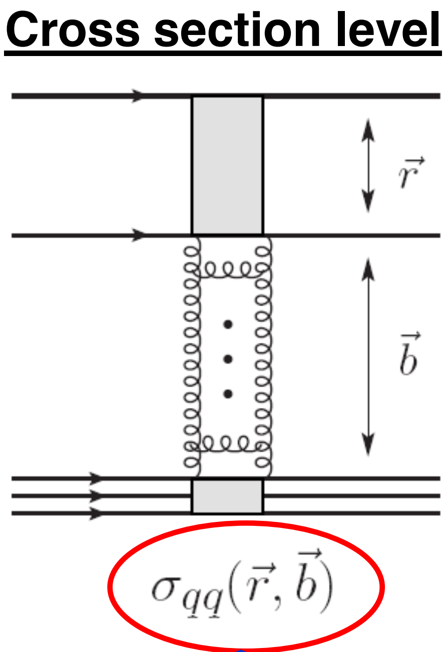
$Q_t \to 0$ *second t-channel "screening" gluon does not screen anymore and number of real gluons emitting to the final state blows up → resummation of soft emission!*

$$e^{-S} = \exp \left(- \frac{C_A \alpha_s}{\pi} \int_{Q_t^2}^{m_H^2/4} \frac{dp_t^2}{p_t^2} \int_{p_t}^{m_H/2} \frac{dE}{E} \right)$$

$$M \sim \int \frac{dQ_t^2}{Q_t^4} f(x_1, Q_t^2) f(x_2, Q_t^2) e^{-S}$$

Dipoles: Sudakov suppression and elastic scattering





Soft Color neutralization is required for diffraction

Hard gluon Bremsstrahlung contributes at larger M_X

soft gluons – a part of UGDF!

For diffractive M_X production Sudakov suppression is needed!

Includes all higher-order QCD and non-perturbative corrections!

Color neutralization is automatic,
no radiation into the gap,
small M_X is produced,
no need in Sudakov,
gap survival effects are there!

Theory: Quantum mechanics of diffractive excitation

R. J. Glauber, Phys. Rev. 100, 242 (1955).
E. Feinberg and I. Ya. Pomeranchuk, Nuovo. Cimento. Suppl. 3 (1956) 652.
M. L. Good and W. D. Walker, Phys. Rev. 120 (1960) 1857.

A hadron can be excited – not an eigenstate of interaction!

$$|h\rangle = \sum_{\alpha=1} C^h_\alpha |\alpha\rangle$$

$$\hat{f}_{el}|\alpha\rangle = f_\alpha |\alpha\rangle$$

Completeness and orthogonality relations

$$\langle h'|h\rangle = \sum_{\alpha=1} (C^{h'}_\alpha)^* C^h_\alpha = \delta_{hh'}$$

$$\langle \beta|\alpha\rangle = \sum_{h'} (C^{h'}_\beta)^* C^{h'}_\alpha = \delta_{\alpha\beta}$$

Elastic and single diffractive amplitudes

$$f^{h\to h}_{el} = \sum_{\alpha=1} |C^h_\alpha|^2 f_\alpha$$

$$f^{h\to h'}_{sd} = \sum_{\alpha=1} (C^{h'}_\alpha)^* C^h_\alpha f_\alpha$$

Single diffractive cross section

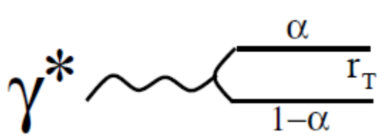
$$\sum_{h'\neq h} \left.\frac{d\sigma^{h\to h'}_{sd}}{dt}\right|_{t=0} = \frac{1}{4\pi}\left[\sum_{h'} |f^{hh'}_{sd}|^2 - |f^{hh}_{el}|^2\right]$$

$$= \frac{1}{4\pi}\left[\sum_\alpha |C^h_\alpha|^2 |f_\alpha|^2 - \left(\sum_\alpha |C^h_\alpha| f_\alpha\right)^2\right]$$

$$= \frac{\langle f^2_\alpha\rangle - \langle f_\alpha\rangle^2}{4\pi}$$

Dispersion of the eigenvalues distribution!

Eigenstates of interaction in QCD: color dipoles



Dipole:

- cannot be excited
- experience only elastic scattering
- have no definite mass, but only separation
- universal – elastic amplitude can be extracted in one process and used in another

B. Z. Kopeliovich, I. K. Potashnikova, I. Schmidt, Braz. J. Phys. **37**, 473-483 (2007).

Theory: Dipole cross section and color transparency

Eigenvalue of the total cross section is the universal dipole cross section

B.Z. Kopeliovich, L.I. Lapidus and A.B. Zamolodchikov, Sov. Phys. JETP Lett. **33** (1981) 595; Pisma v Zh. Exper. Teor. Fiz. **33** (1981) 612.

Single diffractive cross section

$$\sum_{h'} \left.\frac{d\sigma_{sd}^{h\to h'}}{dt}\right|_{t=0} = \sum_{\alpha=1} |C_\alpha^h|^2 \frac{\sigma_\alpha^2}{16\pi} = \int d^2 r_T \, |\Psi_h(r_T)|^2 \frac{\sigma^2(r_T)}{16\pi} = \frac{\langle \sigma^2(r_T) \rangle}{16\pi}$$

DIS cross section

$$\sigma_{tot}^{\gamma^* p}(Q^2, x_{Bj}) = \int d^2 r_T \int_0^1 dx \left|\Psi_{\gamma^*}(r_T, Q^2)\right|^2 \sigma_{\bar{q}q}(r_T, x_{Bj})$$

Theoretical calculation of the dipole CS is a challenge

GBW parameterization of HERA data

$$\sigma_{\bar{q}q}(r_T, x) = \sigma_0 \left[1 - e^{-\frac{1}{4} r_T^2 Q_s^2(x)}\right]$$

saturates at large separations $r_T^2 \gg 1/Q_s^2$

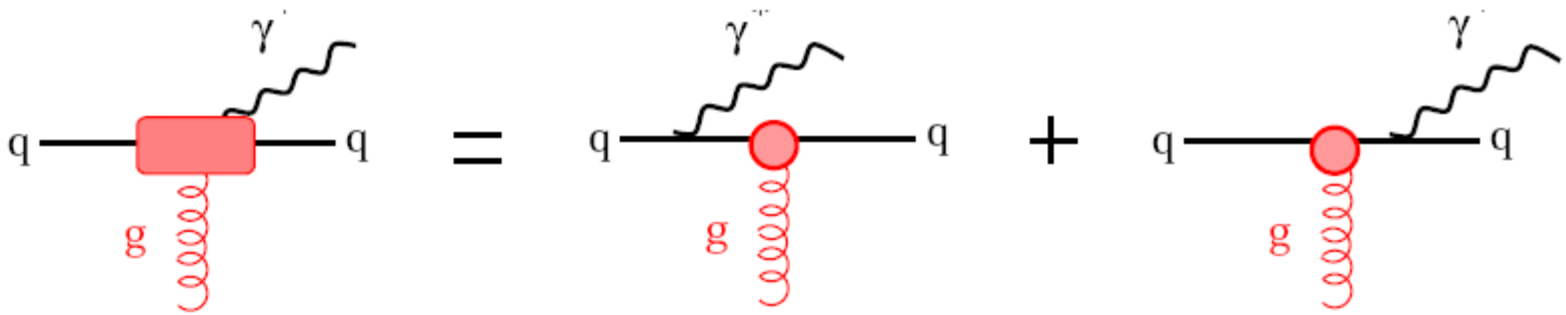
Color transparency: $\sigma_{\bar{q}q}(r_T) \propto r_T^2 \qquad r_T \to 0$

A point-like colorless object cannot interact with external color field!

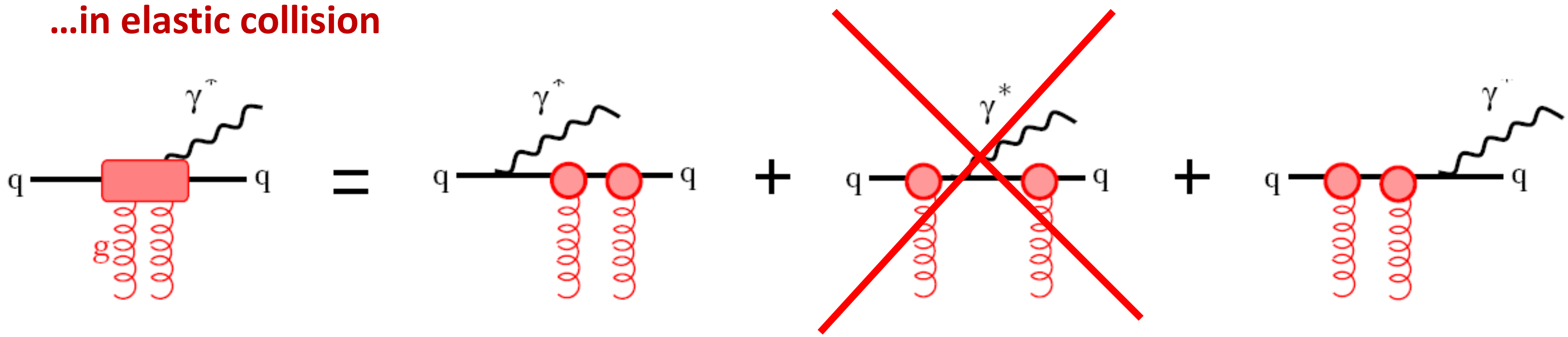
General property of any dipole cross section in QCD!

Drell-Yan off a quark: Forward Abelian radiation

...in inelastic collision



...in elastic collision

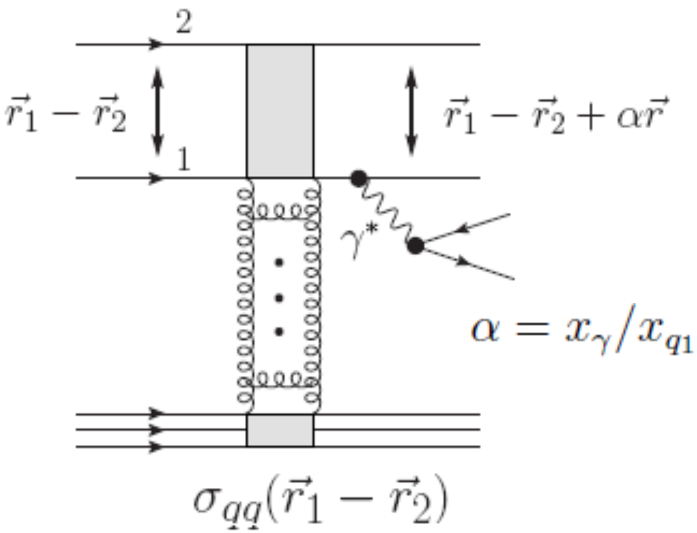


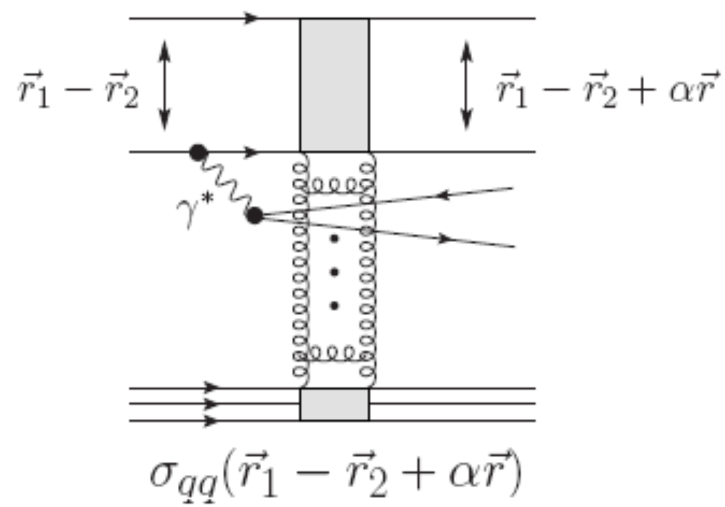
Landau-Pomeranchuk principle: non-accelerated charge does not radiate!

Radiation depends on the whole strength of the kick rather on its structure

No radiation from a quark at Pt=0!

Drell-Yan off a dipole: basics of forward diffraction





By optical theorem

$$2i\,\mathrm{Im}\,f_{el}(\vec{b},\vec{r}_p) = \frac{i}{N_c}\sum_X \sum_{c_f c_i} \left|V_q(\vec{b}) - V_q(\vec{b}+\vec{r}_p)\right|^2$$

$$\sigma_{\bar{q}q}(r_p) = \int d^2b\, 2\,\mathrm{Im} f_{el}(\vec{b},\vec{r}_p)$$

dipoles with different sizes interact differently!

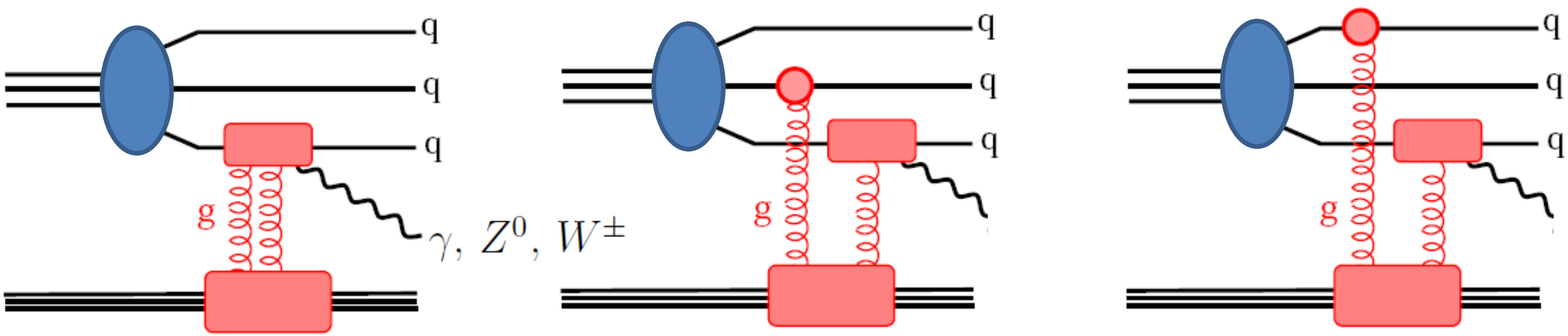
Amplitude of DDY in the dipole-target scattering

$$M_{qq}^{(1)}(\vec{b},\vec{r}_p,\vec{r},\alpha) = -2ip_i^0\sqrt{4\pi}\,\frac{\sqrt{1-\alpha}}{\alpha^2}\,\Psi^\mu_{\gamma^* q}(\alpha,\vec{r})\left[2\mathrm{Im}\,f_{el}(\vec{b},\vec{r}_p) - 2\mathrm{Im}\,f_{el}(\vec{b},\vec{r}_p+\alpha\vec{r})\right]$$

Model: Diffractive gauge bosons production in pp

R. Pasechnik, B. Kopeliovich, Eur. Phys. J. C71 (2011) 1827
B. Kopeliovich, I. Potashnikova, I. Schmidt and A. Tarasov, Phys. Rev. D74, (2006) 114024

..probing large distances in the proton



$$\frac{d^3\sigma_{\lambda_G}(pp\to pG^*X)}{d\ln\alpha\, d\delta_\perp^2}\bigg|_{\delta_\perp=0} = \frac{1}{64\pi}\sum_q \int d^2r_1 d^2r_2 d^2r_3 d^2r\, dx_{q1}dx_{q2}dx_{q3}$$

$$\times |\Psi^{\lambda_G}_{V+A}(\vec{r},\alpha,M)|^2 |\Psi_i(\vec{r}_1,\vec{r}_2,\vec{r}_3;x_{q1},x_{q2},x_{q3})|^2 \left[\int d^2b\,\Delta(\vec{r}_1,\vec{r}_2,\vec{r}_3;\vec{b};\vec{r},\alpha)\right]^2$$

$$\Delta \sim \mathrm{Im}\, f_{el}(\vec{b},\vec{r}_1-\vec{r}_2) - \mathrm{Im}\, f_{el}(\vec{b},\vec{r}_1-\vec{r}_2+\alpha\vec{r}) +$$
$$\mathrm{Im}\, f_{el}(\vec{b},\vec{r}_1-\vec{r}_3) - \mathrm{Im}\, f_{el}(\vec{b},\vec{r}_1-\vec{r}_3+\alpha\vec{r})$$

Consequences: QCD factorization breaking in DDY

Golec-Biernat-Wuestoff (GBW) dipole cross section

$$\sigma(r) = \sigma_0 \left(1 - e^{-r^2/R_0^2}\right)$$

Difference between two Fock states → Diffractive DY amplitude

$$\propto \sigma(\vec{R}) - \sigma(\vec{R} - \alpha\vec{r}) = \frac{2\alpha\,\sigma_0}{R_0^2(x_2)}\, e^{-R^2/R_0^2(x_2)} \underbrace{\left(\vec{r}\cdot\vec{R}\right)} + O(r^2)$$

$$r \sim 1/M \ll R_0(x_2)$$

Interplay between hard and soft scales

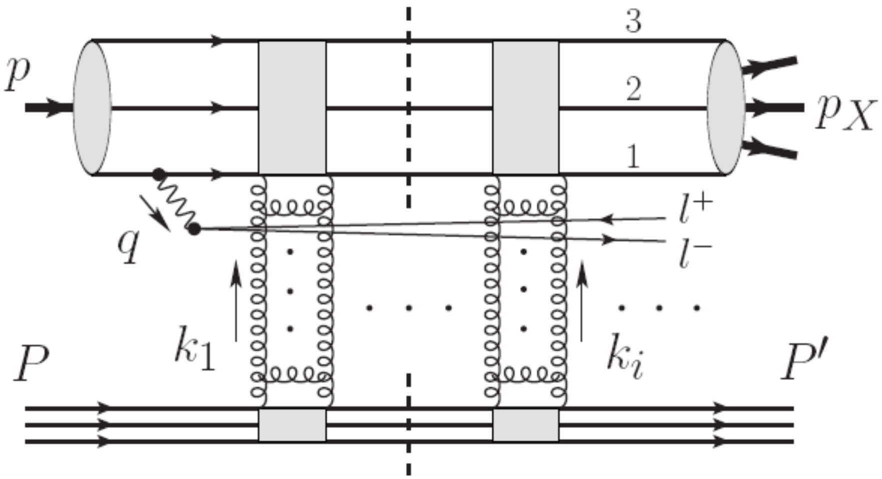
Diffractive DIS $\propto r^4 \propto 1/M^4$

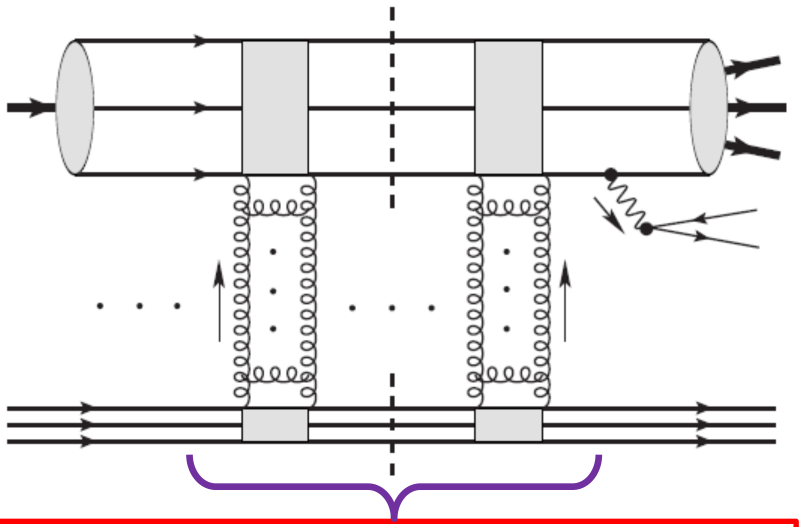
Diffractive DY $\propto r^2 \propto 1/M^2$

The QCD factorization holds!

Dramatic breakdown of the QCD factorization!

Gap survival: Eikonalization of the diffractive DY amplitude





Accounts for soft and hard components on the same footing!

Diffractive DY (off dipole) eikonalized amplitude

$$M^{\mu}_{\bar{q}q}(\boldsymbol{b}, \boldsymbol{r}_p, \boldsymbol{r}, \alpha) \propto \left[e^{-2\mathrm{Im}\, f_{el}(\boldsymbol{b}, \boldsymbol{r}_p)} - e^{-2\mathrm{Im}\, f_{el}(\boldsymbol{b}, \boldsymbol{r}_p + \alpha \boldsymbol{r})} \right]$$

…reproduces the standard Regge-based gap survival suppression

$$\sigma_{DDY} = K \cdot \sigma^{bare}_{DDY}$$

$$K = 1 - \frac{1}{\pi} \frac{\sigma^{pp}_{tot}(s)}{B^{DY}_{sd}(s) + 2B^{pp}_{el}(s)} + \frac{1}{(4\pi)^2} \frac{[\sigma^{pp}_{tot}(s)]^2}{B^{pp}_{el}(s)[B^{DY}_{sd}(s) + B^{pp}_{el}(s)]}$$

B. Z. Kopeliovich, I. K. Potashnikova, I. Schmidt, Phys. Rev. **C73**, 034901 (2006). [hep-ph/0508277].

Absorption: Elastic amplitude and gap survival

Complete dipole elastic amplitude has **eikonal form**:

$$\mathrm{Im}\, f_{el}(\vec{b}, \vec{r}_1 - \vec{r}_2) = 1 - \exp\left[i\chi(\vec{r}_1) - i\chi(\vec{r}_2)\right],$$

$$\chi(b) = -\int_{-\infty}^{\infty} dz\, V(\vec{b}, z),$$

nearly imaginary at high energies!

Diffractive amplitude is proportional to

$$\mathrm{Im}\, f_{el}(\vec{b}, \vec{r}_1 - \vec{r}_2 + \alpha\vec{r}) - \mathrm{Im}\, f_{el}(\vec{b}, \vec{r}_1 - \vec{r}_2) = \boxed{\exp\left[i\chi(\vec{r}_1) - i\chi(\vec{r}_2)\right]} \exp\left[i\alpha\, \vec{r} \cdot \vec{\nabla}\chi(\vec{r}_1)\right]$$

Exactly the soft survival probability amplitude

vanishes in the black disc limit!

Absorption effect should be included into elastic amplitude parameterization (at the amplitude level)

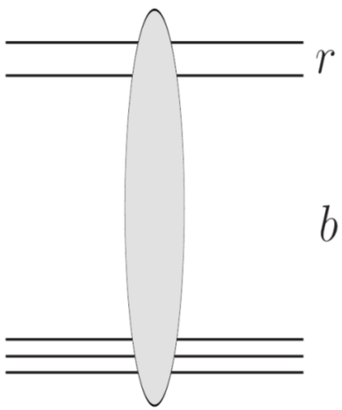
Hard vs soft: Small and large dipoles

small x (large Q^2)

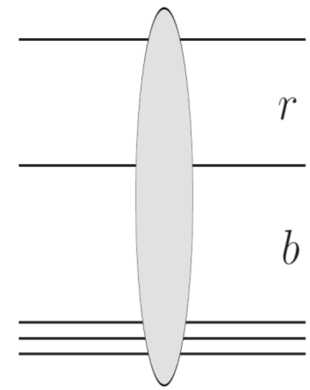
moderate and small Q^2

Fitted to DIS data

Fitted to soft data

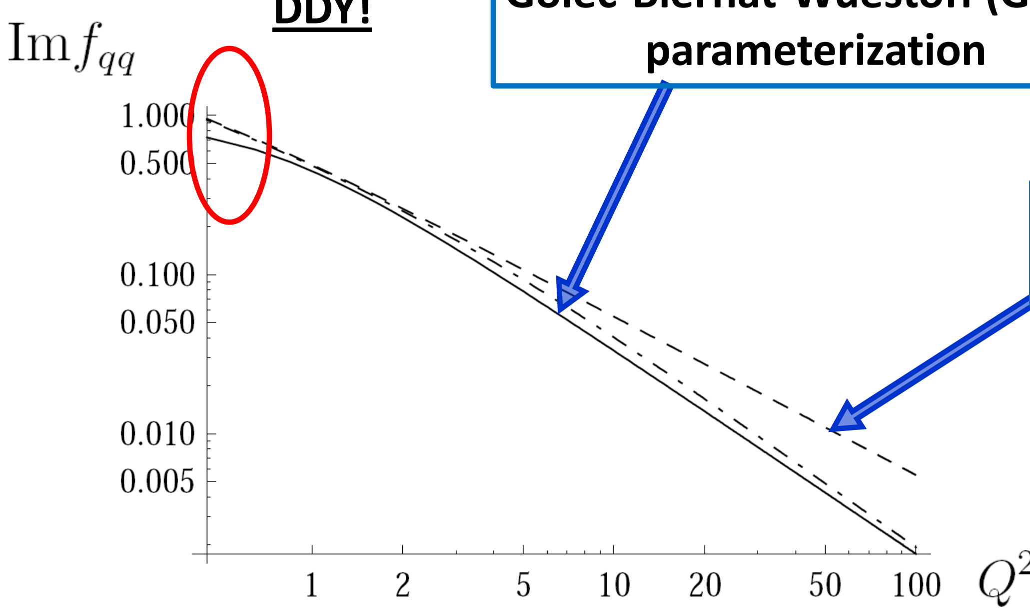


$$\mathrm{Im} f_{qq}(\vec{b},\vec{r}) = \frac{\sigma_0}{8\pi\mathcal{B}}\left\{ \exp\left[-\frac{[\vec{b}+\vec{r}(1-x_q)]^2}{2\mathcal{B}}\right] + \exp\left[-\frac{[\vec{b}+\vec{r}x_q]^2}{2\mathcal{B}}\right] - 2\exp\left[-\frac{r^2}{R_0^2} - \frac{[\vec{b}+\vec{r}(1/2-x_q)]^2}{2\mathcal{B}}\right]\right\},$$



DDY!

Golec-Biernat-Wuestoff (GBW) parameterization



Kopeliovich- Schafer -Tarasov (KST) parameterization

Phys. Rev. D 62 (2000) , 054022

fit to soft (pion-proton etc) data
absorption is there already!

Hard bremsstrahlung: bosons radiation wave functions

Vector WF: $$\Psi^{\mu}_{V}(\vec{r},\alpha,M) = \mathcal{C}^G_q g^G_{v,q}\,\alpha^3\sqrt{1-\alpha}\int\frac{d^2l_\perp}{(2\pi)^2}e^{-i\vec{l}_\perp\cdot\alpha\vec{r}}\,\frac{\bar{u}_{\sigma_2}(p_f)\gamma^\mu u_\sigma(p_2+q)}{\alpha^2 l_\perp^2+\eta^2},$$

Axial WF: $$\Psi^{\mu}_{A}(\vec{r},\alpha,M) = \mathcal{C}^G_q g^G_{a,q}\,\alpha^3\sqrt{1-\alpha}\int\frac{d^2l_\perp}{(2\pi)^2}e^{-i\vec{l}_\perp\cdot\alpha\vec{r}}\,\frac{\bar{u}_{\sigma_2}(p_2)\gamma^\mu\gamma_5 u_\sigma(p_2+q)}{\alpha^2 l_\perp^2+\eta^2}$$

$$\eta^2 = (1-\alpha)M^2+\alpha^2 m_q^2$$

wave functions products:

$$\Psi^T_V(\alpha,\vec{\rho}_1)\Psi^{T*}_V(\alpha,\vec{\rho}_2) = \sum_{\lambda=\pm1}\frac{1}{2}\sum_{\sigma_1,\sigma_2}\epsilon^*_\mu(\lambda)\Psi^\mu_V(\alpha,\vec{\rho}_1)\epsilon_\nu(\lambda)\Psi^{\nu*}_V(\alpha,\vec{\rho}_2)$$
$$= \frac{\mathcal{C}^2_q(g^G_{v,q})^2}{2\pi^2}\left\{m_q^2\alpha^4 \mathrm{K}_0(\eta\rho_1)\,\mathrm{K}_0(\eta\rho_2) + \left[1+(1-\alpha)^2\right]\eta^2\frac{\vec{\rho}_1\cdot\vec{\rho}_2}{\rho_1\rho_2}\mathrm{K}_1(\eta\rho_1)\,\mathrm{K}_1(\eta\rho_2)\right\},$$
$$\Psi^L_V(\alpha,\vec{\rho}_1)\Psi^{L*}_V(\alpha,\vec{\rho}_2) = \frac{1}{2}\sum_{\sigma_1,\sigma_2}\epsilon^*_\mu(\lambda=0)\Psi^\mu_V(\alpha,\vec{\rho}_1)\epsilon_\nu(\lambda=0)\Psi^{\nu*}_V(\alpha,\vec{\rho}_2)$$
$$= \frac{\mathcal{C}^2_q(g^G_{v,q})^2}{\pi^2}M^2(1-\alpha)^2\,\mathrm{K}_0(\eta\rho_1)\,\mathrm{K}_0(\eta\rho_2)\ .$$

$$\Psi^T_A(\alpha,\vec{\rho}_1)\Psi^{T*}_A(\alpha,\vec{\rho}_2) =$$
$$= \frac{\mathcal{C}^2_q(g^G_{a,q})^2}{2\pi^2}\left\{m_q^2\alpha^2(2-\alpha)^2 \mathrm{K}_0(\eta\rho_1)\,\mathrm{K}_0(\eta\rho_2) + \left[1+(1-\alpha)^2\right]\eta^2\frac{\vec{\rho}_1\cdot\vec{\rho}_2}{\rho_1\rho_2}\mathrm{K}_1(\eta\rho_1)\,\mathrm{K}_1(\eta\rho_2)\right\},$$
$$\Psi^L_A(\alpha,\vec{\rho}_1)\Psi^{L*}_A(\alpha,\vec{\rho}_2) = \frac{\mathcal{C}^2_q(g^G_{a,q})^2}{\pi^2}\frac{\eta^2}{M^2}\left\{\eta^2\mathrm{K}_0(\eta\rho_1)\,\mathrm{K}_0(\eta\rho_2) + \alpha^2 m_q^2\frac{\vec{\rho}_1\cdot\vec{\rho}_2}{\rho_1\rho_2}\mathrm{K}_1(\eta\rho_1)\,\mathrm{K}_1(\eta\rho_2)\right\}.$$

Signal: dilepton production cross section

Diffractive Drell-Yan pair production cross section

$$\frac{d^6\sigma_{L,T}(pp \to pl\bar{l}X)}{d^2q_\perp \, d\ln\alpha \, dM^2 \, d^2\delta_\perp} = \frac{\alpha_{em}}{3\pi M^2} \frac{d^5\sigma_{L,T}(pp \to p\gamma^* X)}{d^2q_\perp \, d\ln\alpha \, d^2\delta_\perp}$$

Diffractive W,Z production cross section in the leptonic channel

$$\frac{d^4\sigma_{L,T}(pp \to p(G^* \to l\bar{l},\, l\bar{\nu}_l)X)}{d^2q_\perp \, d\ln\alpha \, dM^2 \, d^2\delta_\perp} = \mathrm{Br}(G \to l\bar{l},\, l\nu_l)\, \rho_G(M) \frac{d^3\sigma_{L,T}(pp \to pG^* X)}{d^2q_\perp \, d\ln\alpha \, d^2\delta_\perp}$$

where **resonant invariant mass distribution**

$$\rho_G(M) = \frac{1}{\pi} \frac{M\,\Gamma_G(M)}{(M^2 - m_G^2)^2 + [M\,\Gamma_G(M)]^2}\,, \quad \Gamma_G(M)/M \ll 1$$

the narrow-width approximation

GB total decay widths

$$\Gamma_W(M) \simeq \frac{3\alpha_{em}\,M}{4\sin^2\theta_W}\,, \qquad \Gamma_Z(M) \simeq \frac{\alpha_{em} M}{6\sin^2 2\theta_W}\left[\frac{160}{3}\sin^4\theta_W - 40\sin^2\theta_W + 21\right]$$

Results: Diffractive GB production cross sections

The general result:

$$\frac{d^5\sigma_{\lambda_G}(pp\to pG^*X)}{d^2q_\perp d\ln\alpha\, d^2\delta_\perp} = \frac{1}{(2\pi)^2}\frac{1}{64\pi^2}\sum_q \int d^2r_1 d^2r_2 d^2r_3\, d^2r d^2r'\, d^2b d^2b'\, dx_{q_1}dx_{q_2}dx_{q_3}$$
$$\times\, \Psi^{\lambda_G}_{V-A}(\vec r,\alpha,M)\Psi^{\lambda_G *}_{V-A}(\vec r\,',\alpha,M)\,|\Psi_i(\vec r_1,\vec r_2,\vec r_3;x_{q_1},x_{q_2},x_{q_3})|^2$$
$$\times\, \Delta(\vec r_1,\vec r_2,\vec r_3;\vec b;\vec r,\alpha)\Delta(\vec r_1,\vec r_2,\vec r_3;\vec b\,';\vec r\,',\alpha)\, e^{i\vec\delta_\perp\cdot(\vec b-\vec b\,')}\, e^{i\vec l_\perp\cdot\alpha(\vec r-\vec r\,')}$$

$$\Delta = -2\mathrm{Im}\, f_{el}(\vec b,\vec r_1-\vec r_2) + 2\mathrm{Im}\, f_{el}(\vec b,\vec r_1-\vec r_2+\alpha\vec r)$$
$$-2\mathrm{Im}\, f_{el}(\vec b,\vec r_1-\vec r_3) + 2\mathrm{Im}\, f_{el}(\vec b,\vec r_1-\vec r_3+\alpha\vec r)$$

Proton wave function

$$|\Psi_i(\vec r_1,\vec r_2,\vec r_3;x_q,x_{q_2},x_{q_3})|^2 = \frac{3a^2}{\pi^2}e^{-a(r_1^2+r_2^2+r_3^2)}\rho(x_q,x_{q_2},x_{q_3})$$
$$\times\ \delta(\vec r_1+\vec r_2+\vec r_3)\delta(1-x_q-x_{q_2}-x_{q_3})$$

$$a = \langle r_{ch}^2\rangle^{-1}$$

Valence quark distribution

+ antiquarks!

$$\int dx_{q_2}dx_{q_3}\,\delta(1-x_q-x_{q_2}-x_{q_3})\rho(x_q,x_{q_2},x_{q_3}) = \rho_q(x_q)$$

⇨

$$\sum_q Z_q^2[\rho_q(x_q)+\rho_{\bar q}(x_q)] = \frac{1}{x_q}F_2(x_q)$$

In DDY we get an immediate access to the proton structure function at large x!

Results: Diffractive GB production cross sections

In the hard limit r<<R: $$\mathrm{Im}\, f_{el}(\vec{b},\vec{R}_{ij}+\alpha\vec{r}) - \mathrm{Im}\, f_{el}(\vec{b},\vec{R}_{ij}) \simeq \frac{\partial \mathrm{Im}\, f_{el}(\vec{b},\vec{R}_{ij})}{\partial \vec{R}_{ij}}\alpha\vec{r}$$

Forward diffractive CS:

$$\frac{d^4\sigma_{\lambda_G}(pp\to p\,G^*X)}{d^2q_\perp\,dx_{bos1}\,d\delta_\perp^2}\Big|_{\delta_\perp=0} = \frac{a^2}{24\pi^3}\frac{\sigma_0^2(s)}{R_0^4(s)}\frac{1}{A_2}\Big[\frac{2}{(A_2-4A_1)^2}+\frac{A_2^2}{(A_2^2-4A_3^2)^2}\Big]\times$$
$$\sum_q\int_{x_{bos1}}^1 d\alpha\,\Big[\rho_q\Big(\frac{x_{bos1}}{\alpha}\Big)+\rho_{\bar q}\Big(\frac{x_{bos1}}{\alpha}\Big)\Big]\int d^2r d^2r'\,(\vec{r}\cdot\vec{r}')\,\Psi^{\lambda_G}_{V-A}(\vec{r},\alpha,M)\Psi^{\lambda_G *}_{V-A}(\vec{r}',\alpha,M)\,e^{i\vec{q}_\perp\cdot(\vec{r}-\vec{r}')}$$

$$A_1=\frac{2a}{3}+\frac{2}{R_0^2(s)}\,,\qquad A_2=\frac{2a}{3}\,,\qquad A_3=\frac{2a}{3}+\frac{1}{R_0^2(s)}$$

Soft KST parametrization

$$R_0(s)=0.88\,\mathrm{fm}\,(s_0/s)^{0.14}$$
$$\sigma_0(s)=\sigma^{\pi p}_{tot}(s)\Big(1+\frac{3R_0^2(s)}{8\langle r^2_{ch}\rangle_\pi}\Big)$$
$$\sigma^{\pi p}_{tot}(s)=23.6(s/s_0)^{0.08}\ \mathrm{mb}$$
$$\langle r^2_{ch}\rangle_\pi = 0.44\ \mathrm{fm}^2$$

Due to exponential t-dependence

$$\frac{d\sigma(pp\to p\,G^*X)}{d^2q_\perp dx_{bos1}}=\frac{1}{B_{sd}(s)}\frac{d^3\sigma(pp\to p\,G^*X)}{d^2q_\perp dx_{bos1}\,d\delta_\perp^2}\Big|_{\delta_\perp=0}$$

with diffractive (Regge) slope

$$B_{sd}(s)\simeq\langle r^2_{ch}\rangle/3+2\alpha'_{I\!P}\ln(s/s_0)$$

Results: diffractive vs inclusive GB production

In the hard limit: $$\frac{1}{2}\Big\{\sigma(\alpha r)+\sigma(\alpha r')-\sigma(\alpha|\vec r-\vec r\,'|)\Big\}\simeq\frac{\alpha^2\bar\sigma_0}{\bar R_0^2(x)}\,(\vec r\cdot\vec r\,')$$

Inclusive production CS:

$$\frac{d^4\sigma_{\lambda_G}(pp\to G^*X)}{d^2q_\perp\,dx_{bos1}}=\frac{1}{(2\pi)^2}\frac{\bar\sigma_0}{\bar R_0^2(x)}\sum_q\int_{x_{bos1}}^1 d\alpha\,\Big[\rho_q\Big(\frac{x_{bos1}}{\alpha}\Big)+\rho_{\bar q}\Big(\frac{x_{bos1}}{\alpha}\Big)\Big]\times$$
$$\int d^2r d^2r'\,(\vec r\cdot\vec r\,')\,\Psi^{\lambda_G}_{V-A}(\vec r,\alpha,M)\Psi^{\lambda_G *}_{V-A}(\vec r\,',\alpha,M)\,e^{i\vec q_\perp\cdot(\vec r-\vec r\,')}\,.$$

with GBW parametrization:

$$\bar\sigma_0=23.03\,\mathrm{mb}\,,\quad R_0\equiv\bar R_0(x)=0.4\,\mathrm{fm}\times(x/x_0)^{0.144}\,,\quad x_0=3.04\times10^{-4}\,,$$

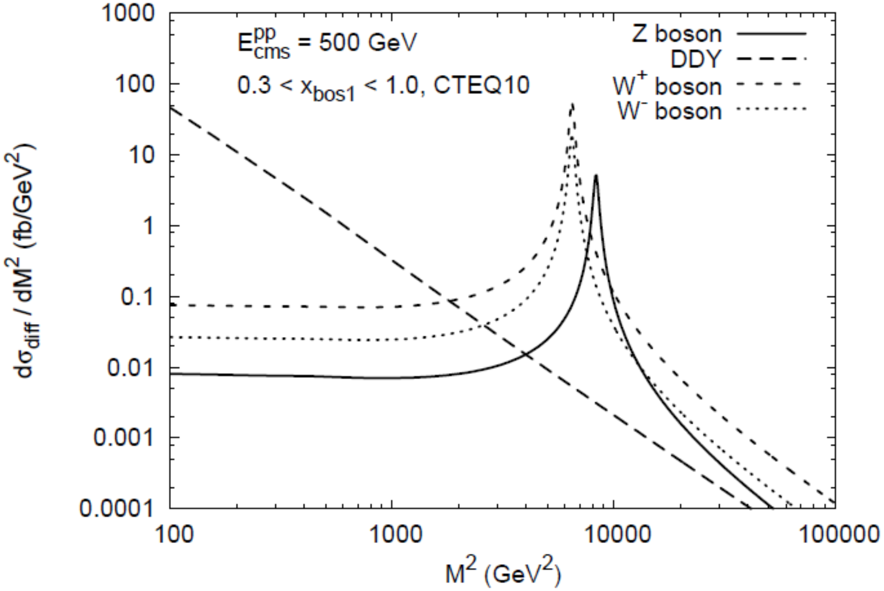
So, the diffraction-to-inclusive ratio:

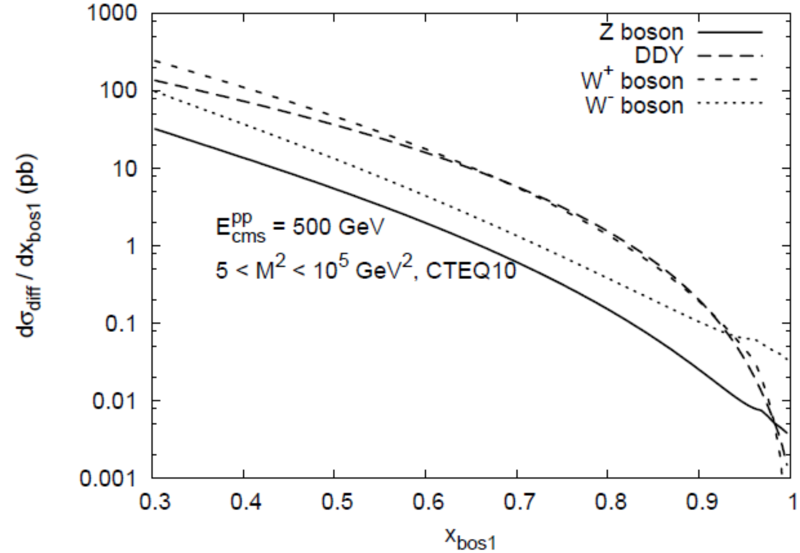
$$\frac{d\sigma^{sd}_{\lambda_G}/d^2q_\perp\,dx_{bos1}\,dM^2}{d\sigma^{incl}_{\lambda_G}/d^2q_\perp dx_{bos1}\,dM^2}=\frac{a^2}{6\pi}\frac{\bar R_0^2(M_\perp^2/x_{bos1}s)}{B_{sd}(s)\,\bar\sigma_0}\frac{\sigma_0^2(s)}{R_0^4(s)}\frac{1}{A_2}\Big[\frac{2}{(A_2-4A_1)^2}+\frac{A_2^2}{(A_2^2-4A_3^2)^2}\Big]$$

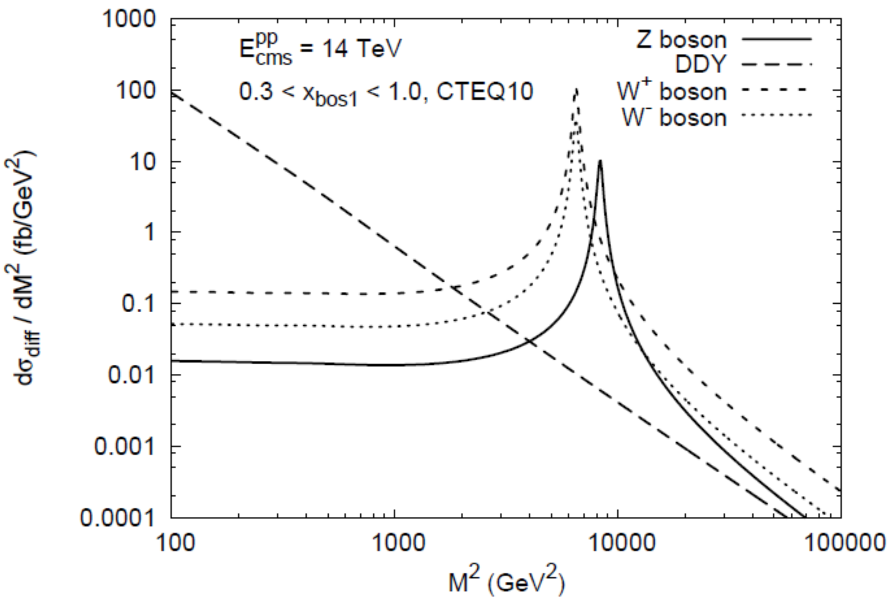
$$M_\perp^2\equiv M^2+|\vec q_\perp|^2=x_{bos1}\,x\,s$$

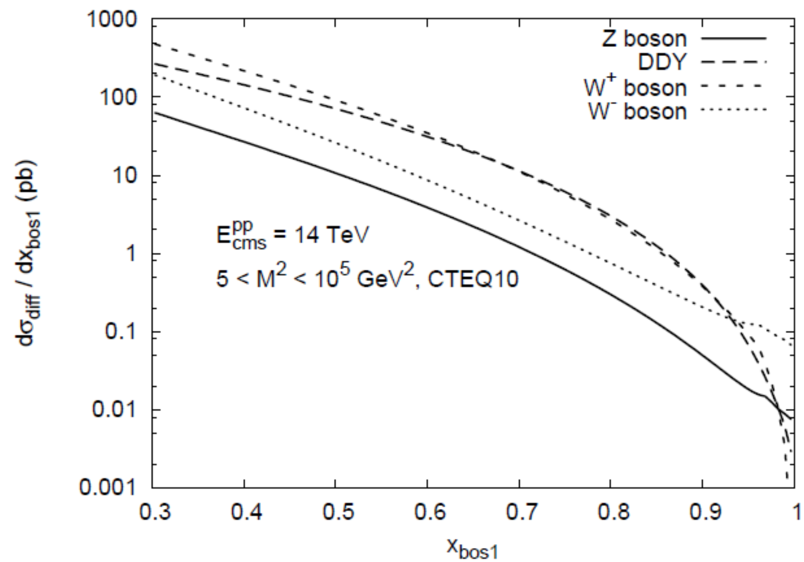
ratio does not depend on type of the boson!

Results: diffractive GB production cross sections

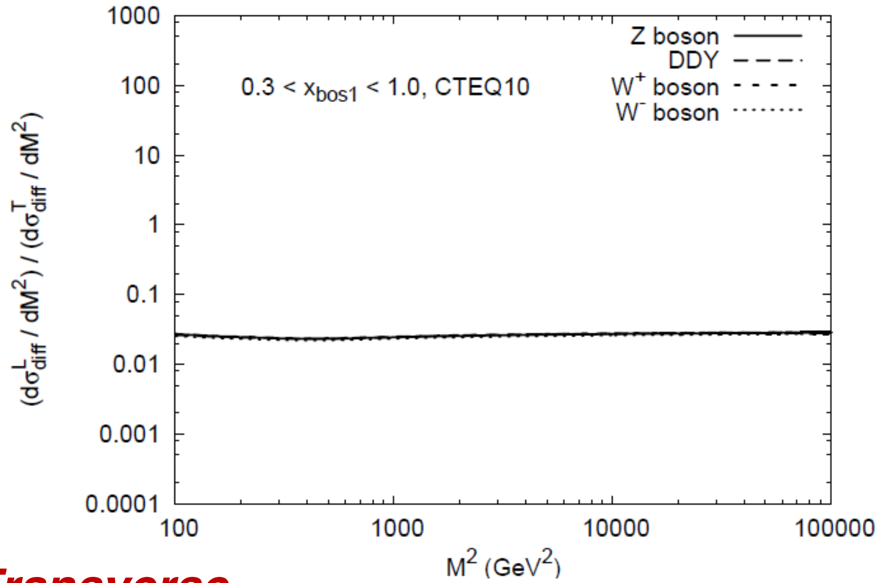


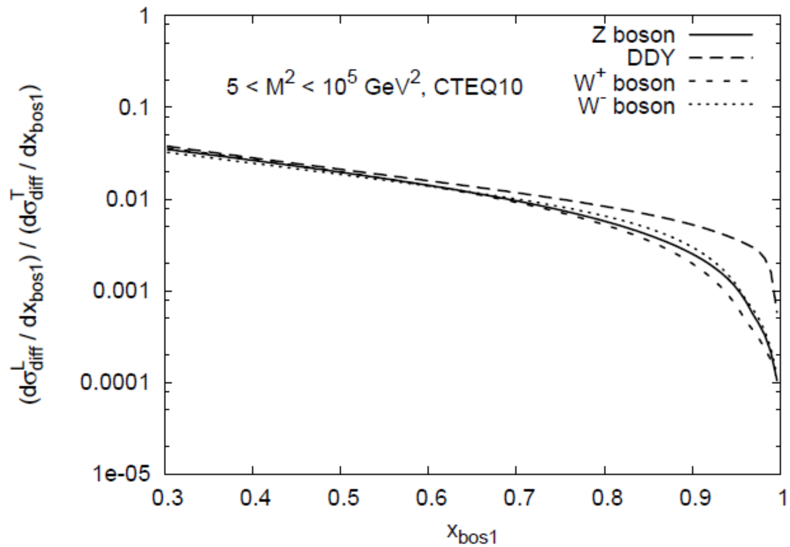






Results: longitudinal vs transverse polarisation





Transverse

$$\frac{d^4\sigma_T(pp \to p\,G^* X)}{d^2q_\perp dx_{bos1}} = \frac{1}{B_{sd}(s)} \frac{a^2}{24\pi^3} \frac{\sigma_0^2(s)}{R_0^4(s)} \frac{1}{A_2} \left[\frac{2}{(A_2-4A_1)^2} + \frac{A_2^2}{(A_2^2-4A_3^2)^2}\right] \times$$
$$\sum_q \frac{(\mathcal{C}_q^G)^2}{2\pi^2} \int_{x_{bos1}}^1 d\alpha \left[\rho_q\left(\frac{x_{bos1}}{\alpha}\right) + \rho_{\bar q}\left(\frac{x_{bos1}}{\alpha}\right)\right] \Big\{ m_q^2\alpha^2 \left[(g_{v,q}^G)^2\alpha^2 + (g_{a,q}^G)^2(2-\alpha)^2\right] J_1 +$$
$$\left[(g_{v,q}^G)^2 + (g_{a,q}^G)^2\right]\left[1+(1-\alpha)^2\right]\eta^2 J_2 \Big\},$$

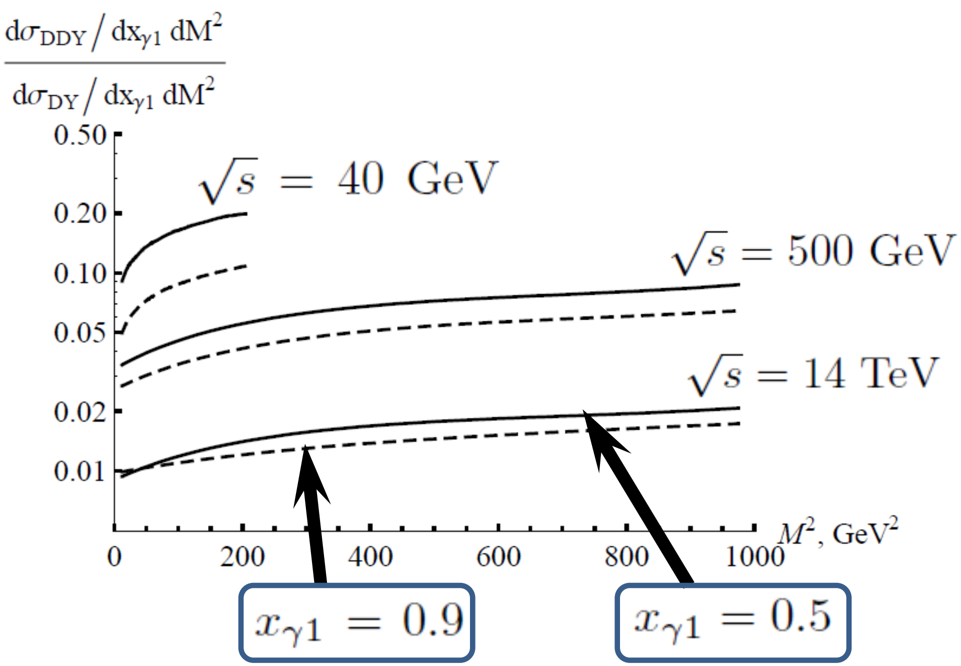
$$J_1(q_\perp, \eta) \equiv 16\pi^2 \frac{q_\perp^2}{(\eta^2+q_\perp^2)^4},$$

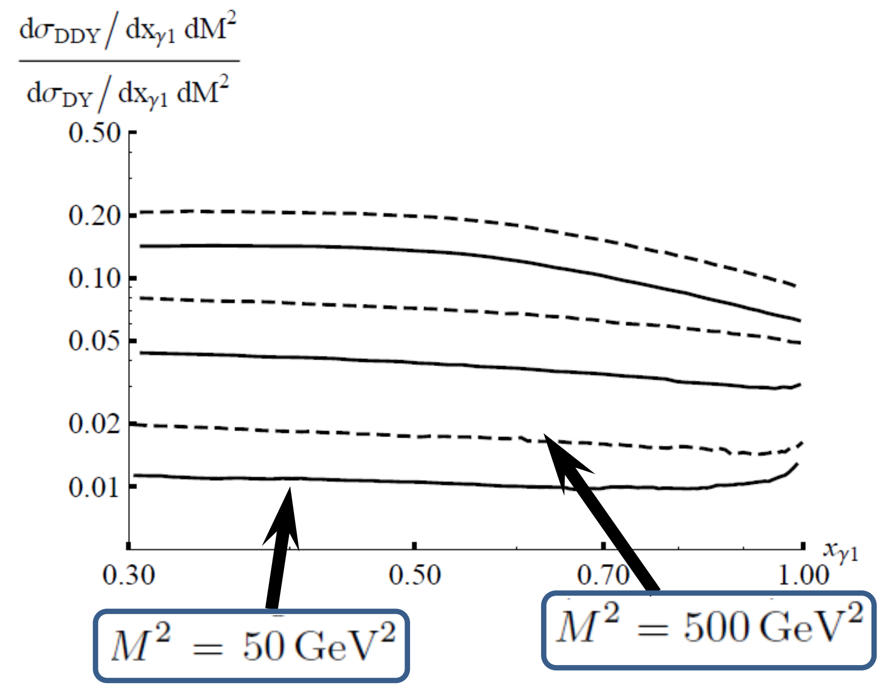
$$J_2(q_\perp, \eta) \equiv 8\pi^2 \frac{\eta^4+q_\perp^4}{\eta^2(\eta^2+q_\perp^2)^4}$$

Longitudinal

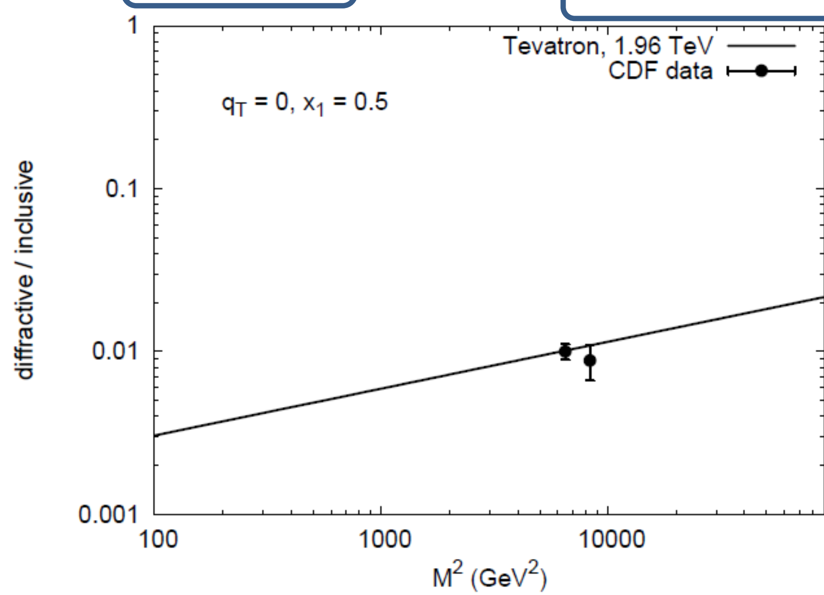
$$\frac{d^4\sigma_L(pp \to p\,G^* X)}{d^2q_\perp dx_{bos1}} = \frac{1}{B_{sd}(s)} \frac{a^2}{24\pi^3} \frac{\sigma_0^2(s)}{R_0^4(s)} \frac{1}{A_2} \left[\frac{2}{(A_2-4A_1)^2} + \frac{A_2^2}{(A_2^2-4A_3^2)^2}\right] \times$$
$$\sum_q \frac{(\mathcal{C}_q^G)^2}{\pi^2} \int_{x_{bos1}}^1 d\alpha \left[\rho_q\left(\frac{x_{bos1}}{\alpha}\right) + \rho_{\bar q}\left(\frac{x_{bos1}}{\alpha}\right)\right] \Big\{ \left[(g_{v,q}^G)^2 M^2 (1-\alpha)^2 + (g_{a,q}^G)^2 \frac{\eta^4}{M^2}\right] J_1 +$$
$$(g_{a,q}^G)^2 \alpha^2 m_q^2 \frac{\eta^2}{M^2} J_2 \Big\}.$$

Results: diffractive vs inclusive





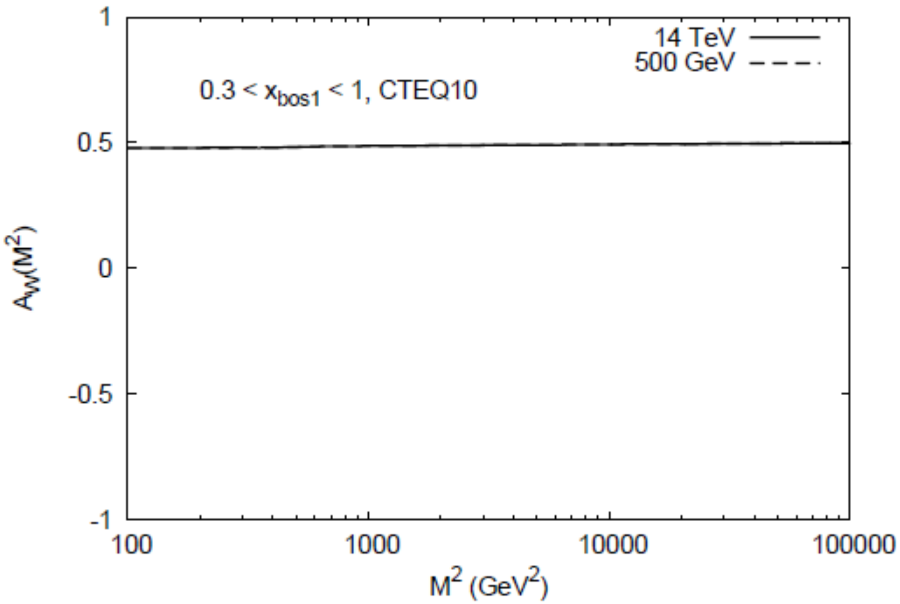
Energy/scale behavior is opposite from predicted in diffractive factorisation-based approaches

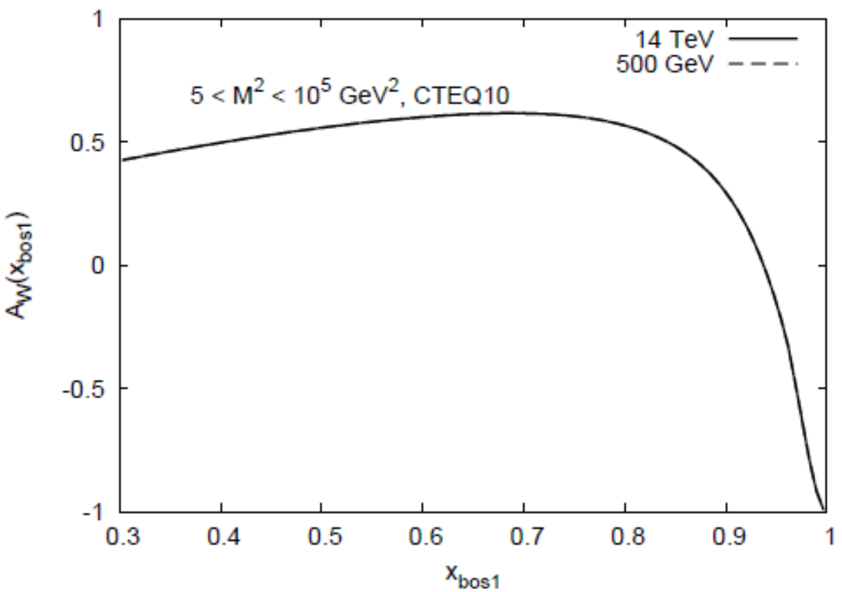


Agrees well with the Tevatron data!

Results: W charge asymmetry

Does not depend on energy and invariant mass!

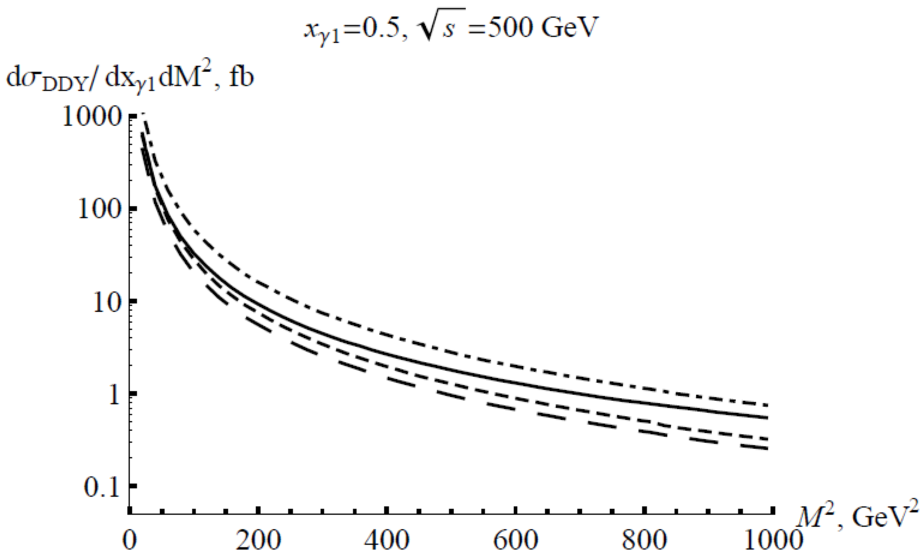


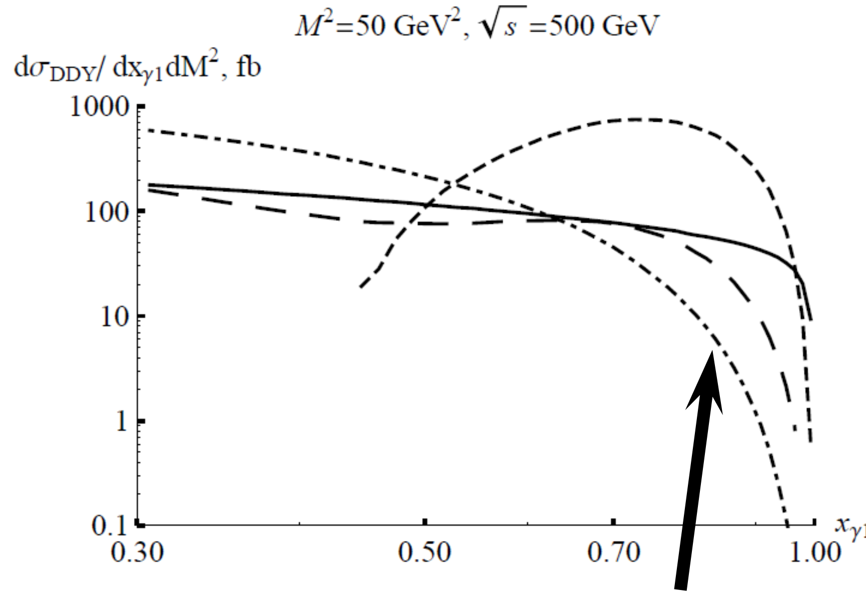


A good probe for QCD diffractive mechanism and soft interactions!

Results: theory uncertainties

Curves are given for different parametrizations of the proton structure function F2





Huge sensitivity to F2 parameterizations at small Q0 and large x!

DDY measurement can improve our understanding of the proton structure in the non-perturbative region

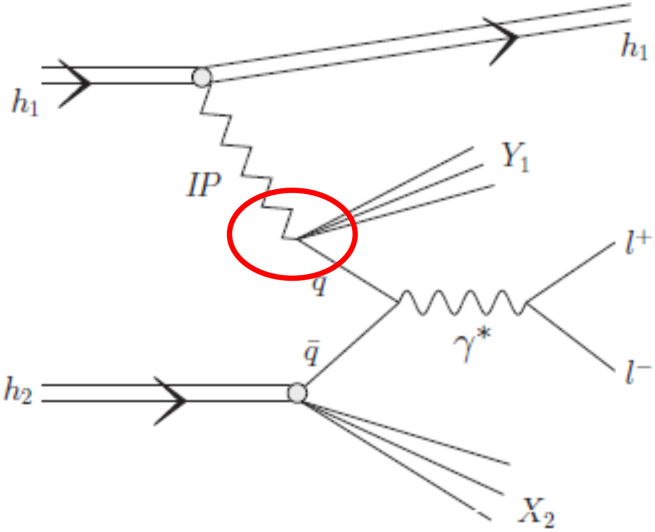
Other models: QCD factorisation approach to diffractive DY

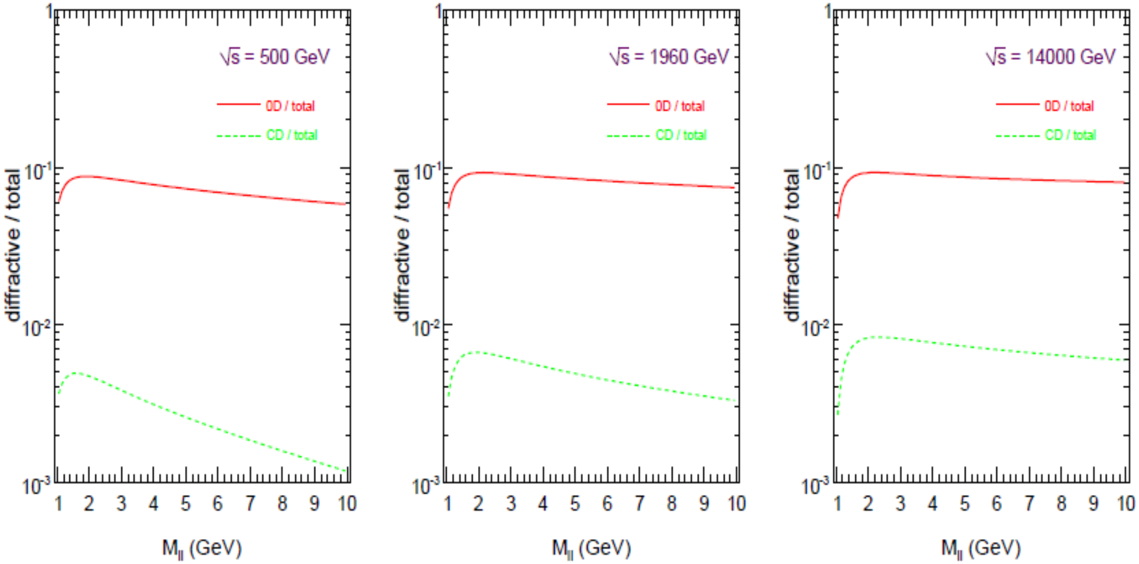
by G. Kubasiak and A. Szczurek, Phys.Rev.D84:014005,2011

Ingelman-Schein mechanism → QCD/Regge factorisation

Diffractive quark density

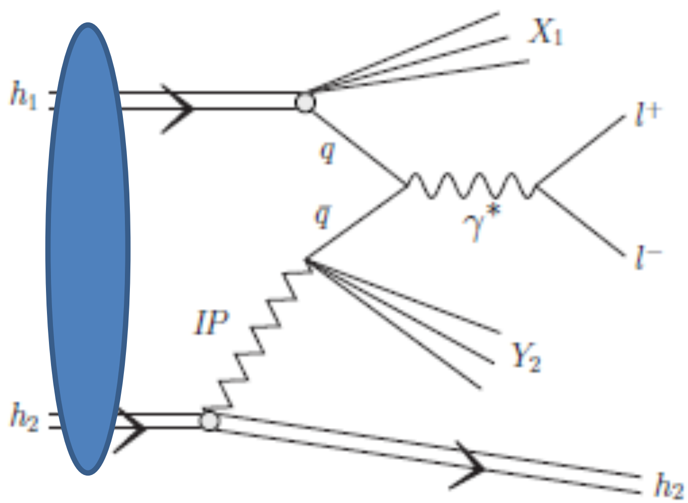
$$q_f^D(x,\mu^2) = \int_x^1 \frac{dx_{I\!P}}{x_{I\!P}} f_{I\!P}(x_{I\!P}) q_{f/I\!P}\left(\frac{x}{x_{I\!P}},\mu^2\right)$$

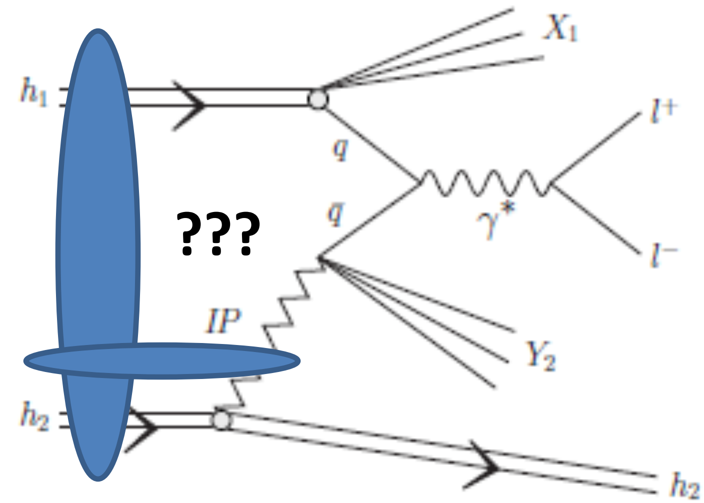




Regge factorization breaking and "enhanced" corrections

Absorptive effects destroy diffractive factorization in hadron-hadron scattering!





without the IS factorisation breaking:

Diffractive Z,W / Inclusive Z,W **~ 30 %**

Gay-Ducati et al Phys. Rev. D75, 114013 (2007)

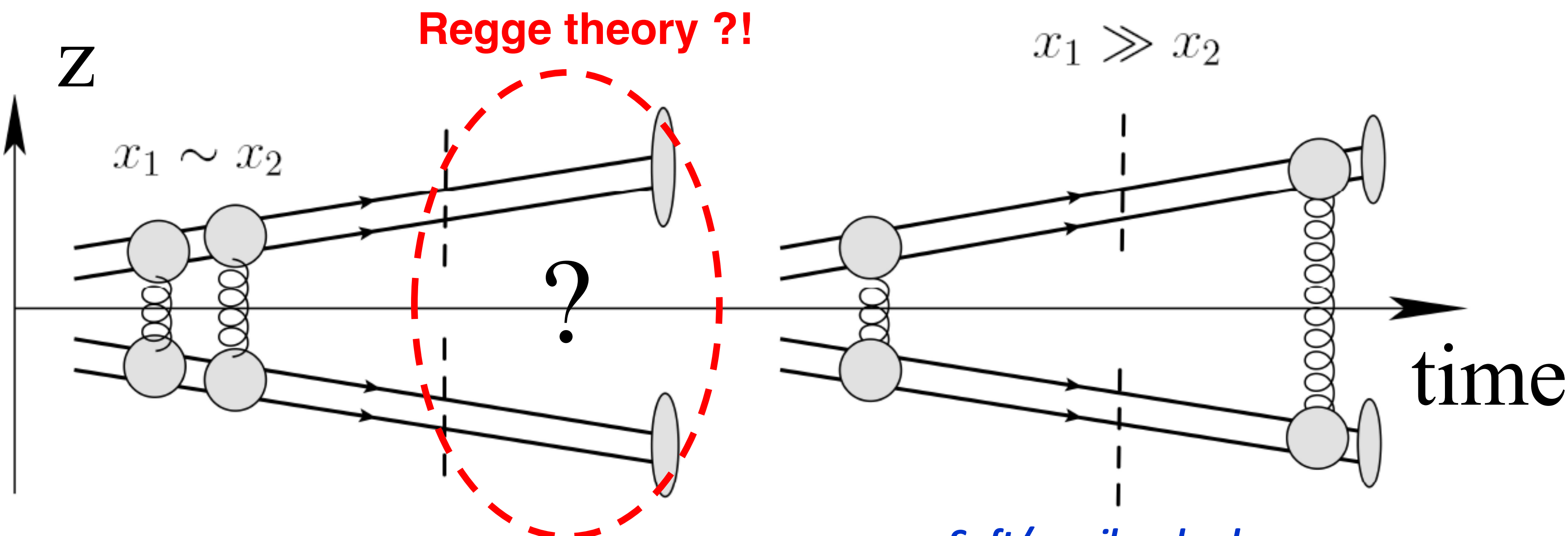
with the IS factorisation breaking:

Diffractive Z,W / Inclusive Z,W **~ 1 %**

Theoretical calculation of the dipole CS is still a challenge

Different models are applied!

Color screening



Perturbative color neutralization

Lack of absorptive effects!

Soft/semihard color neutralization

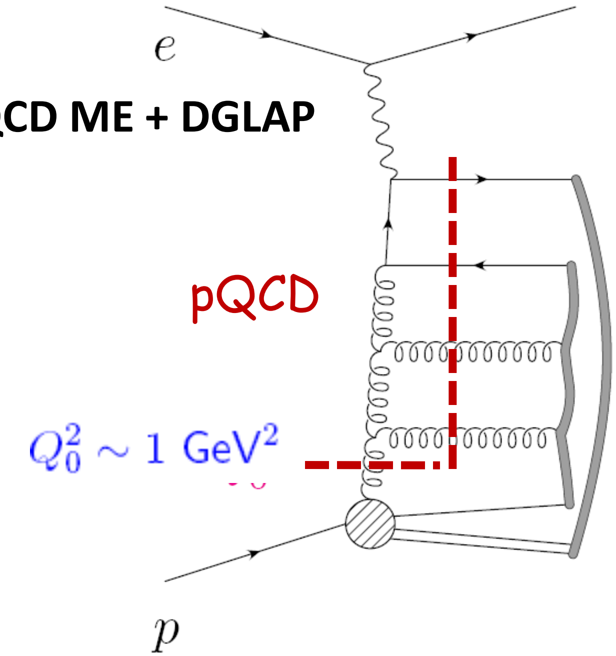
- *Both include unitarity corrections and color neutralisation*
- *Both diffractive – nondiffractive processes*

Diffractive Deep Inelastic Scattering: the NP color screening

G. Ingelman, A. Edin, J. Rathsman, Comput. Phys. Commun. **101**, 108-134 (1997).
A. Edin, G. Ingelman, J. Rathsman, Z. Phys. **C75**, 57-70 (1997).

✓ **The success of Soft Color Interaction (SCI) model**

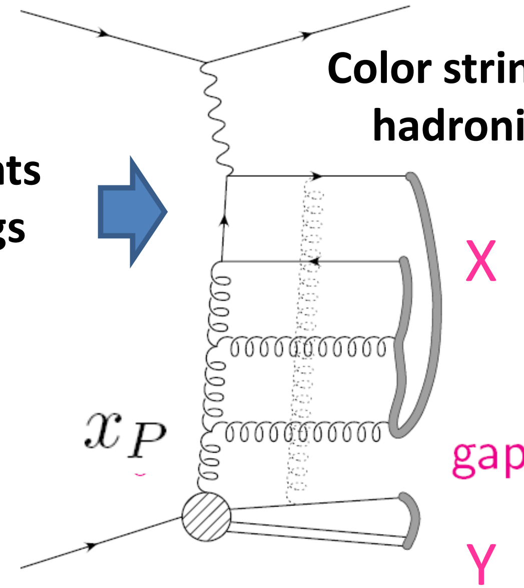
pQCD ME + DGLAP



diffractive ↔ nondiffractive

Color rearrangements modify colour strings topology

Color string (Lund) hadronisation



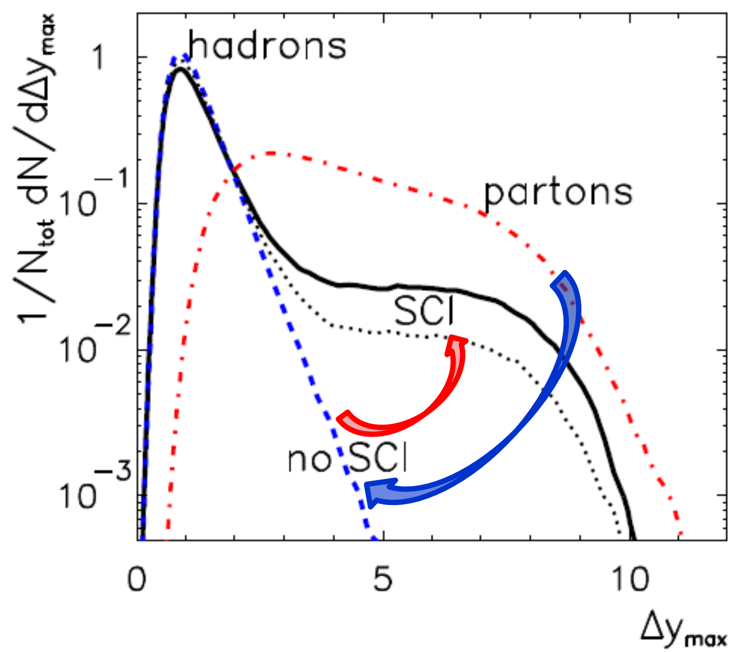
- ***Soft interactions among the final state partons and proton remnants (=> proton color field) at small momentum transfers < 1 GeV***
- ***Hard pQCD part (small distances) is not affected by soft interactions (large distances)***
- ***Single parameter - probability for soft colour-anticolour (gluon) exchange***
- ***Single model describing all final states: both diffractive and nondiffractive***

Soft Colour Interaction model (SCI)

Add-on to Lund Monte Carlo's LEPTO (ep) and PYTHIA ($p\bar{p}$)

ME + DGLAP PS $> Q_0^2$	$\rightarrow$	SCI model	$\rightarrow$	String hadronisation $\sim \Lambda$
colour ordered parton state		rearranged colour order		modified final state

Size Δy_{max} of largest gap in DIS events



SCI $\Rightarrow$ plateau in Δy_{max}
characteristic for diffraction

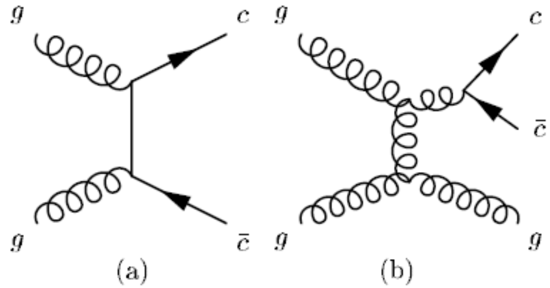
Small parameter sensitivity
— $P = 0.5$
$\cdots$ $P = 0.1$

Gap-size is infrared sensitive observable !

Large gaps at parton level
normally string across $\rightarrow$ hadrons fill up
SCI $\rightarrow$ new string topologies, some with gaps

Gap events not 'special', but
fluctuation in colour/hadronisation

Soft Colour Interaction model → prompt charmonium



pert. QCD

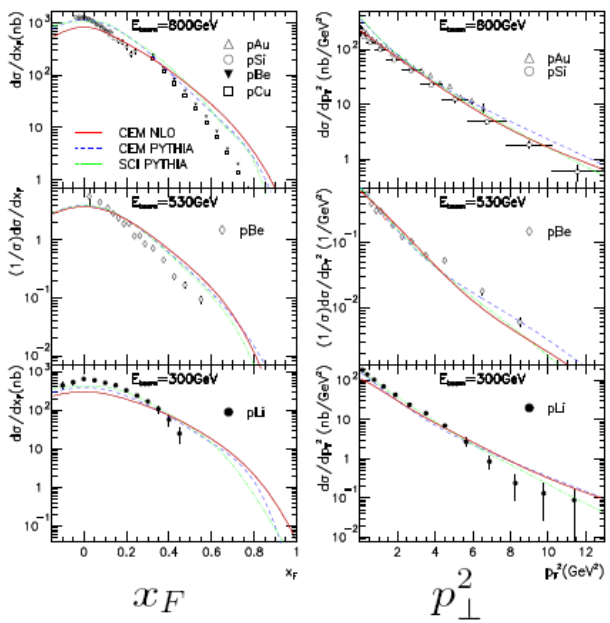
↓

$c\bar{c}$ pair

Colour octet $c\bar{c}$ turned into singlet $c\bar{c}$

$m_{c\bar{c}} < 2m_D$ mapped on charmonium states

with spin statistics (+ soft smearing)

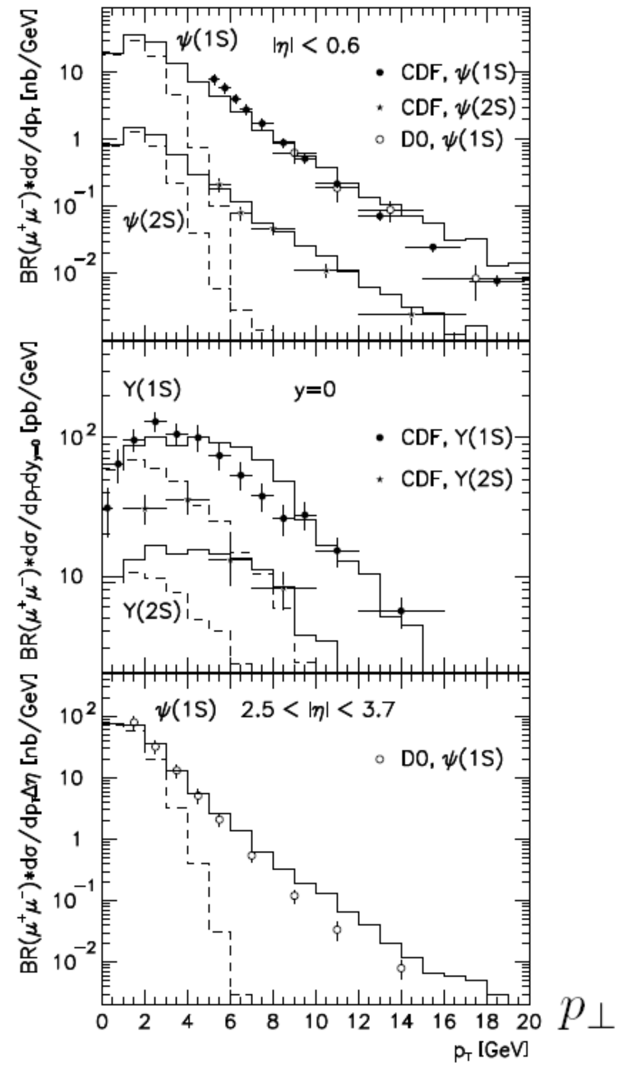


pA @

800 GeV

530 GeV

300 GeV



J/ψ, ψ' in fixed target πA, pA is OK

High-$p_\perp$ J/ψ, ψ', Υ at Tevatron is OK

A. Edin, G. Ingelman, J. Rathsman, Phys. Rev. **D56**, 7317-7320 (1997)

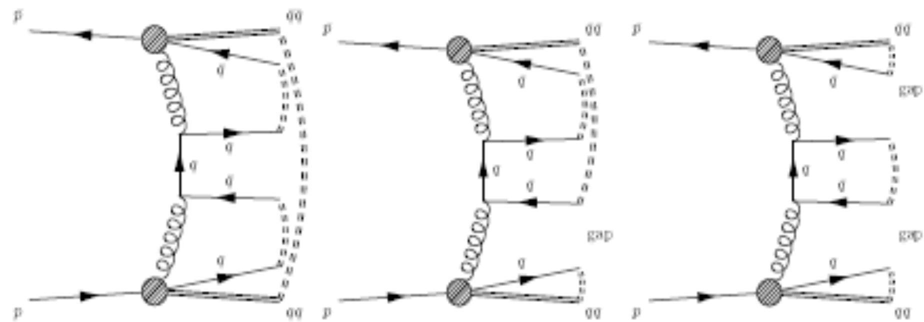
Jets, W, Z, $b\bar{b}$, J/ψ in diffractive gap events at the Tevatron

$$R_{\text{hard}} = \frac{1}{\sigma_{\text{hard}}^{\text{tot}}} \int_{x_{F\min}}^{1} dx_F \, \frac{d\sigma_{\text{hard}}}{dx_F}$$

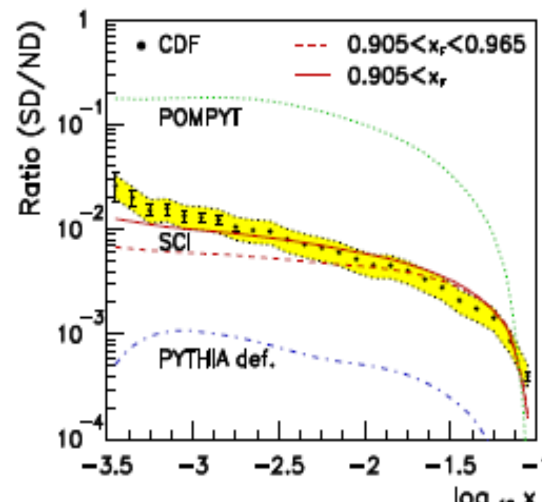
$R_{\text{hard}}[\%]$	Exp.	observed	SCI
dijets	CDF	0.75 ± 0.10	0.7
W	CDF	1.15 ± 0.55	1.2
W	DØ	$1.08\,^{+0.21}_{-0.19}$	1.2
$b\bar{b}$	CDF	0.62 ± 0.25	0.7
Z	DØ	$1.44\,^{+0.62}_{-0.54}$	1.0
J/ψ	CDF	1.45 ± 0.25	1.4

↑ predictions

SCI → gap & $c\bar{c}$ colour octet → singlet → J/ψ



R_{dijets} vs x of parton in $\bar{p}$



SCI model OK, also for two-gap (DPE) events

Pomeron model too high, PYTHIA too low

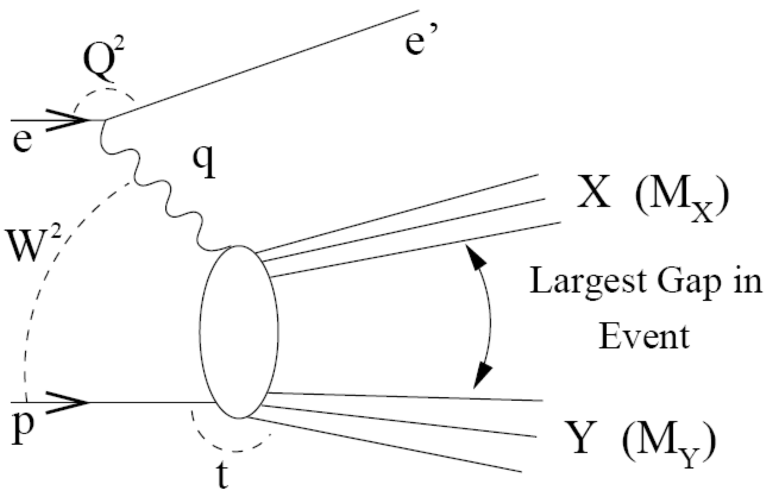
SCI model phenomenologically successful — Why ?

Captures most essential QCD dynamics ⇒ theory emerging . . .

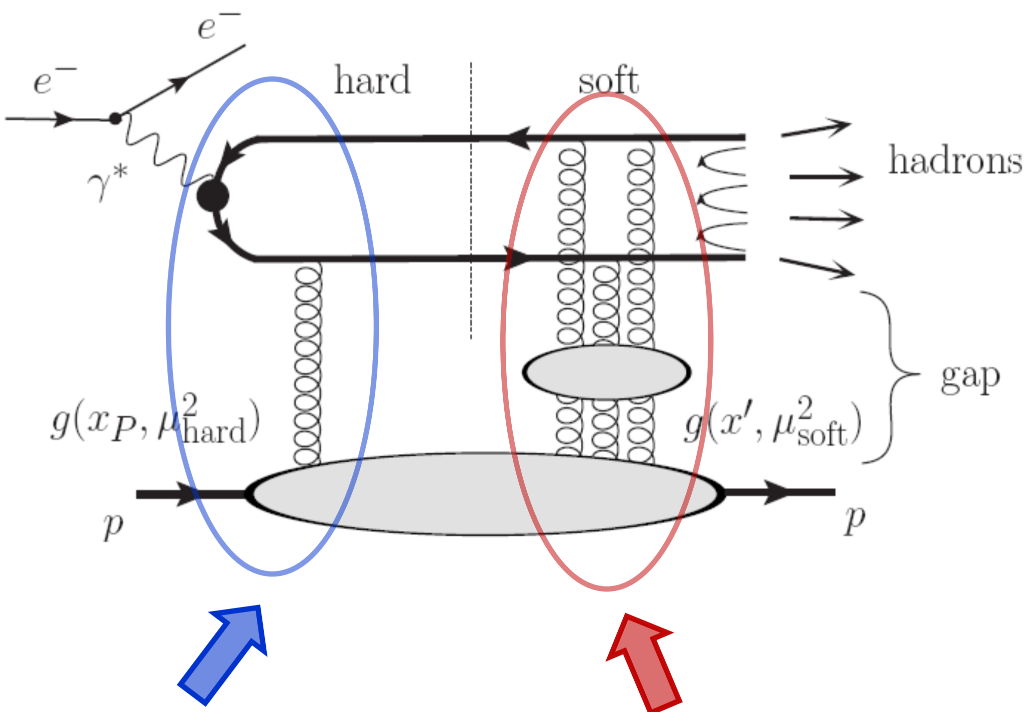
Pasechnik, Enberg, Ingelman, Phys.Rev. D82 (2010) 054036

QCD rescattering theory: the dipole model

Diffractive DIS at HERA



QCD rescattering model

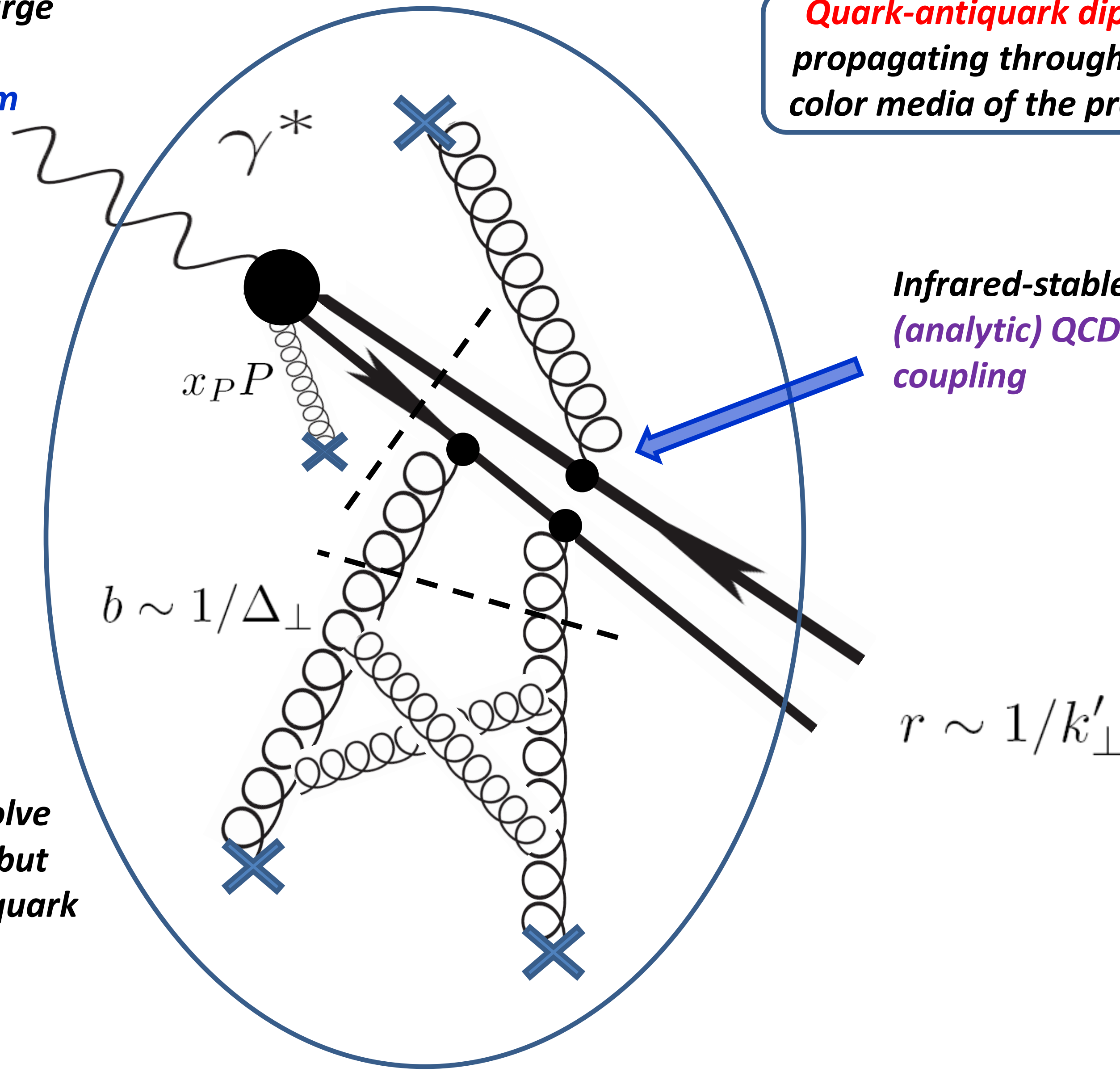


Hard part
conventional
(small distance)

Soft part:
color-screening (octet)
multigluon exchange
(large distance)

Diffractive DIS in the dipole picture

Compensation of the large photon virtuality by longitudinal momentum transfer in the single hard interaction



Quark-antiquark dipole propagating through the color media of the proton

Infrared-stable (analytic) QCD coupling

t- and s-channel factorisation

Soft gluons cannot resolve quarks dynamically → but they always couple to quark current!

Diffractive DIS at HERA: kinematics

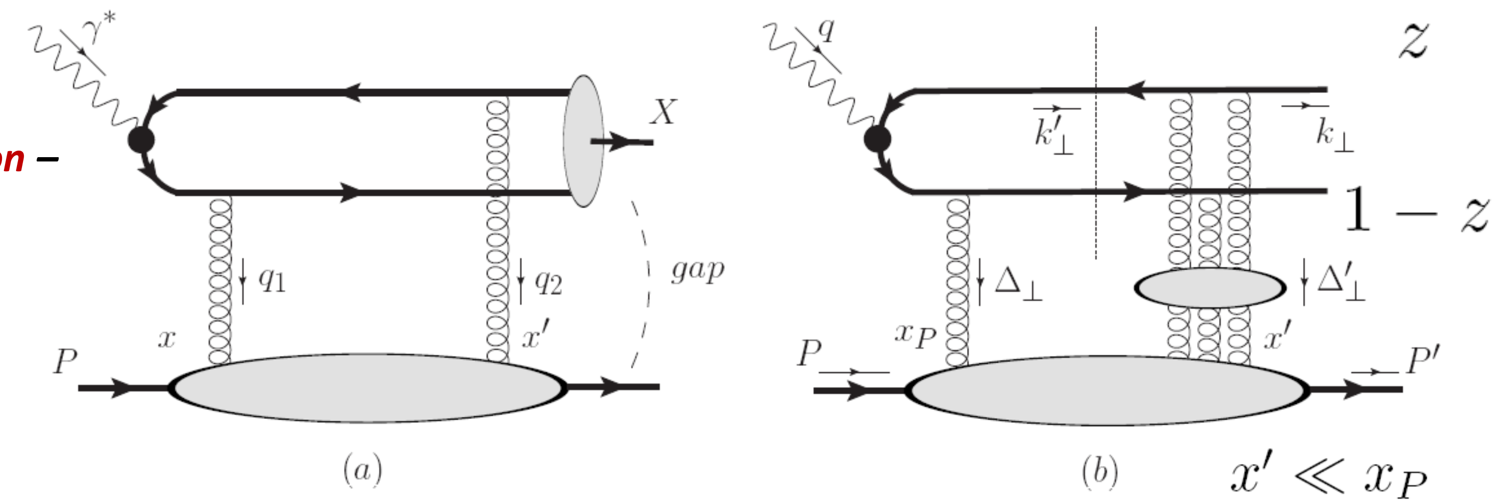
Basic variables:

$$x \equiv \frac{Q^2}{2Pq} = \frac{Q^2}{Q^2 + W^2}, \quad \beta = \frac{Q^2}{Q^2 + M_X^2}, \quad x_P = \frac{x}{\beta}, \quad t = (P' - P)^2$$

$$Q^2 = -q^2$$

Leading contribution – by quark dipole

$$\beta = x/x_P \to 1$$



$$x' \ll x_P$$

Invariant mass of X system and c.m.s energy

$$M_X^2 = \frac{1-\beta}{\beta} Q^2, \quad W^2 \simeq \frac{Q^2}{x_P \beta},$$

$$\varepsilon^2 = z(1-z)Q^2 + m_q^2, \quad k_\perp^2 = z(1-z)M_X^2 - m_q^2$$

The hard QCD factorization scale = quark virtuality!

Working domain of interest:

$$x_P \ll 1, \quad M_X \ll W$$

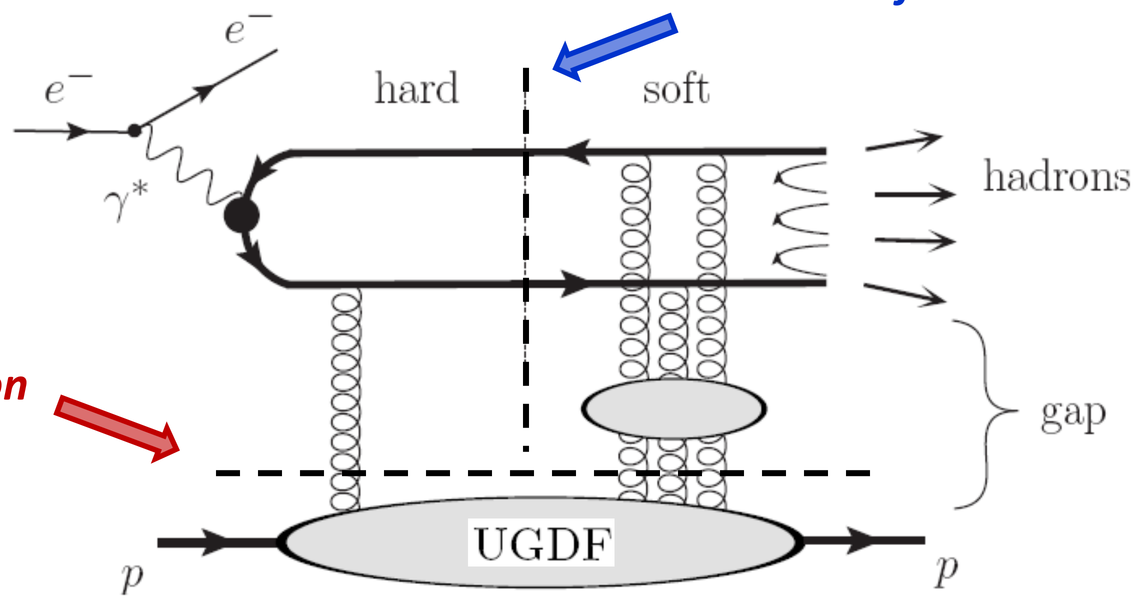
$$|t| \ll Q^2, \, M_X^2$$

$$\mu_F^2 = k_\perp^2 + \varepsilon^2 = z(1-z)\frac{Q^2}{\beta}$$

Hard-soft factorization scheme

s-channel factorisation

t-channel factorisation



loop integration + cutting rules

✓ The total **amplitude**

$$\delta \equiv \sqrt{-t} = |\mathbf{\Delta}_\perp + \mathbf{\Delta}'_\perp|$$

$$M(\delta) \sim \int \frac{d^2\Delta_\perp}{(2\pi)^2} \cdot M^{hard}(\mathbf{\Delta}_\perp) \cdot M^{soft}(\boldsymbol{\delta} - \mathbf{\Delta}_\perp)\ \mathcal{F}_g^{\rm off}$$

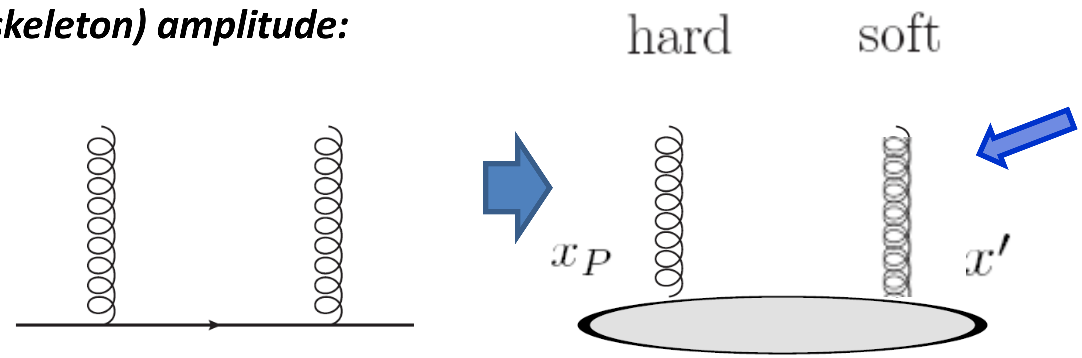
factorisation of the b-dependence

to the **impact parameter representation** →

$$M(\delta, \mathbf{k}_\perp) \sim \int d^2r d^2b e^{-i\mathbf{k}_\perp \mathbf{r}} e^{-ik\mathbf{b}} \hat{M}^{\rm hard}(\mathbf{b}, \mathbf{r})\, \hat{M}^{\rm soft}(\mathbf{b}, \mathbf{r})\, \mathcal{V}(\mathbf{b}, \mathbf{r})$$

Generalized (skewed) unintegrated gluon density

Partonic (skeleton) amplitude:



"fat" soft gluon – ***effectively carries*** ***color octet charge*** ***in the limit of small*** $r \sim 1/k'_\perp$

$C_F \alpha_s / \pi$ $\qquad$ $f_g^{\rm off}(x_P, x', \Delta_\perp^2, \Delta'^2_\perp, \mu_F^2)$

Notion of "hardness" is different w.r.t. the standard one:

* *"hard" gluon in our case – the gluon which takes the largest* ***longitudinal*** *momentum, compensating the quark virtuality*

* *"hard" scale is related* ***with longitudinal momentum transfer*** *given by* x_P *(similarity with Durham model for CEP of Higgs)*

Off-diagonal UGDF currently unknown → ***different models are applied!***

UGDF model and impact parameter representation

B. Pire, J. Soffer and O. Teryaev, Eur. Phys. J. C **8**, 103 (1999)

The skewedness effect in UGDF using positivity constraints:

$$\mathcal{F}_g^{\rm off} \simeq \sqrt{\mathcal{F}_g(x_P, \Delta_\perp^2, \mu_F^2)\mathcal{F}_g(x', {\Delta'_\perp}^2, \mu_{\rm soft}^2)}\,,$$

Infrared behavior:

$$\frac{f_g(x, \Delta_\perp^2)}{\Delta_\perp^2} \equiv \mathcal{F}(x, \Delta_\perp^2) \to {\rm const}\,, \qquad \Delta_\perp^2 \to 0$$

Impact parameter representation of UGDF:

$$\mathcal{V}(\mathbf{b}, \mathbf{r}) = \frac{1}{\alpha_s(\mu_{\rm soft}^2)} \int \frac{d^2\Delta_\perp}{(2\pi)^2} \sqrt{x_P}\, \mathcal{F}_g^{\rm off} \times \left\{e^{-i\mathbf{r}\boldsymbol{\Delta}_\perp} - e^{i\mathbf{r}\boldsymbol{\Delta}_\perp}\right\} e^{i\mathbf{b}\boldsymbol{\Delta}_\perp}.$$

Gluon at very small-*x'* unknown, fit

Gaussian Ansatz:

$$\sqrt{x_P}\mathcal{F}_g^{\rm off} \simeq \sqrt{x_P g(x_P, \mu_F^2)\, x' g(x', \mu_{\rm soft}^2)}\, f_G(\Delta_\perp^2),$$

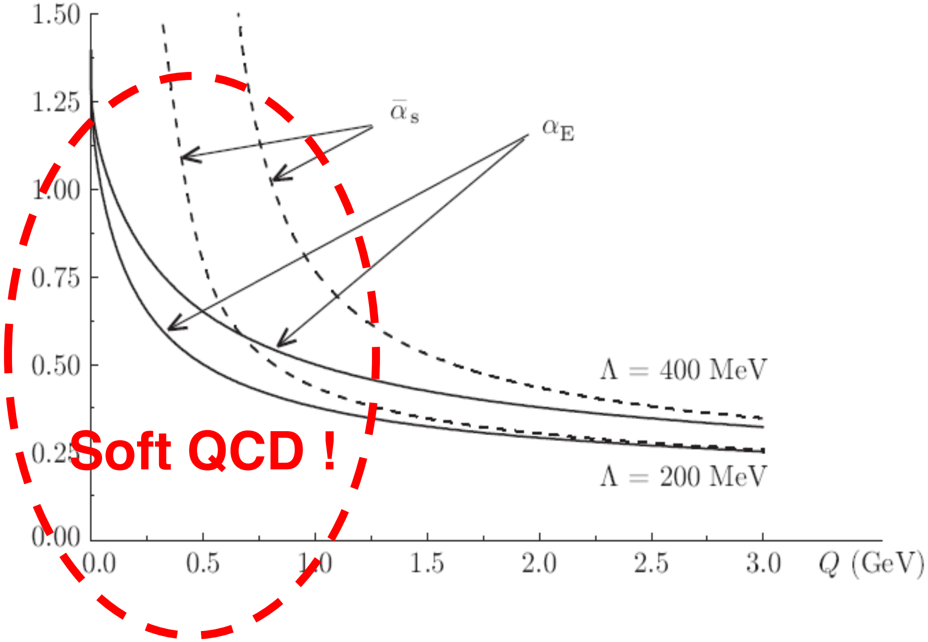
$$f_G(\Delta_\perp^2) = 1/(2\pi\rho_0^2)\, \exp\left(-\Delta_\perp^2/2\rho_0^2\right),$$

Soft hardronic scale – transverse proton radius $\quad r_p \sim 1/\rho_0$

Diffractive slope (ZEUS) – given by a soft scale (proton radius)

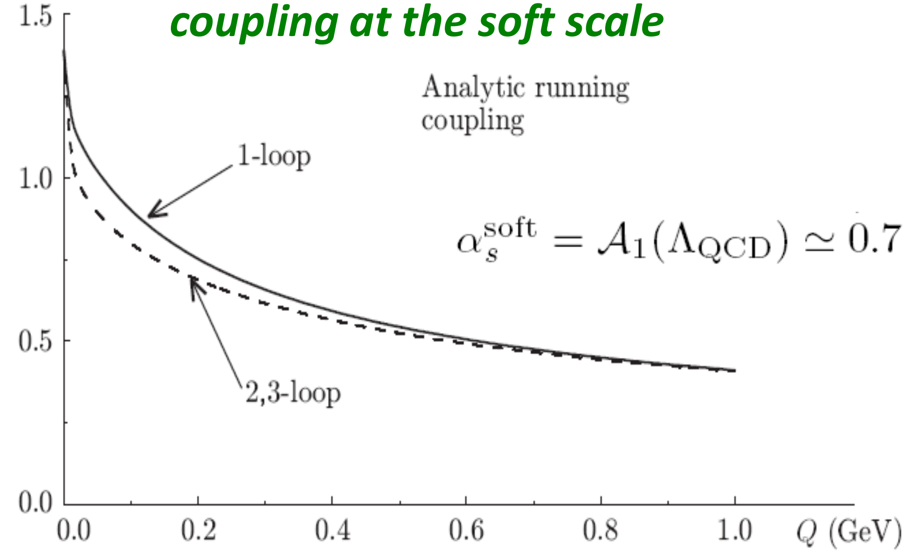
$$\sim \exp(B_D t) \qquad B_D = 1/\rho_0^2 \simeq 6.9 \pm 0.2\ {\rm GeV}^2 \qquad \rho_0 \simeq 380\ {\rm MeV}$$

QCD coupling at low scales



One-loop analytic coupling

Analytic (Shirkov-Solovtsev) coupling at the soft scale



$$\alpha_{\rm E}(Q^2) = \frac{1}{\pi}\int_0^\infty d\sigma \, \frac{\rho(\sigma)}{\sigma + Q^2},$$

$$\alpha_{\rm E}^{(1)}(Q^2) = \frac{1}{\beta_0}\left[\frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2}\right].$$

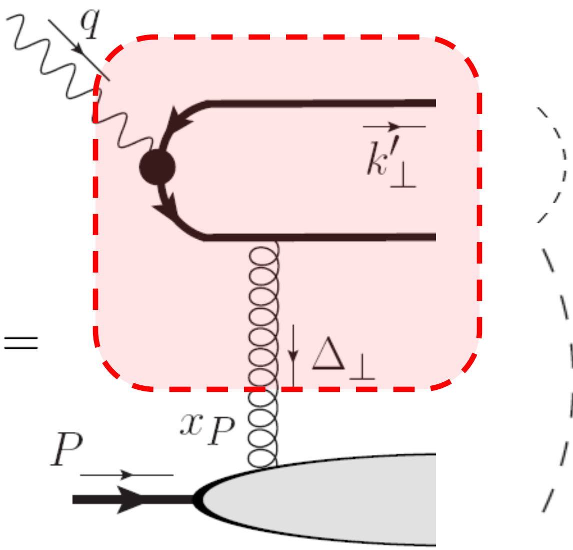
$$\rho_k(\sigma) = {\rm Im}\, \bar{\alpha}_{\rm s}^k(-\sigma - i\epsilon)$$

D.V. Shirkov, I.L. Solovtsov, JINR Rapid Comm. No.2 76-96 (1996) 5; Phys. Rev. Lett. 79 (1997) 1209, arXiv:hep-th/9704333;
K.A. Milton, I.L. Solovtsov, Phys. Rev. D 55 (1997) 5295, arXiv:hep-ph/9611438.

The hard scattering amplitude

✓ **Hard part**

$$M_{L,T}^{hard}(\Delta_\perp, k'_\perp) =$$



$$r \sim 1/k'_\perp$$

$$= \int d^2\mathbf{r} d^2\mathbf{b}\, \hat{M}_{L,T}^{hard}(\mathbf{b}, \mathbf{r}) e^{-i\mathbf{r}\mathbf{k}'_\perp} e^{-i\mathbf{b}\boldsymbol{\Delta}_\perp}$$

$$b \sim 1/\Delta_\perp$$

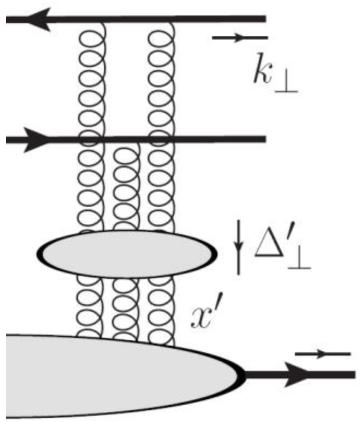
$$\hat{M}_L^{\rm hard} = i\mathcal{C}\, \alpha_s(\mu_F^2)\sqrt{\beta}\, W^3 z^{3/2}(1-z)^{3/2}\, K_0(\varepsilon r)$$

$$\hat{M}_{T,\pm}^{\rm hard} = i\mathcal{C}\alpha_s(\mu_F^2)\sqrt{\frac{2\beta}{1-\beta}}\, \frac{1}{\sqrt{x_P}} W^2 z^{1/2}(1-z)^{3/2}\, \varepsilon K_1(\varepsilon r)\frac{r_x \pm i r_y}{r}$$

Soft gluon scattering and "exponentiation"

✓ **Soft gluon exchanges generate only the phase shifts – to be resummed to all orders!**

$$M^{soft}(\Delta'_\perp, k_\perp) =$$



the large N_c limit – planar diagrams only!

$$e^{-i\mathbf{r}\mathbf{k}'_\perp} M_1^{soft} = \mathcal{A}\, e^{-i\mathbf{r}\mathbf{k}_\perp} \frac{1}{\Delta'^2_\perp}\left[e^{-i\mathbf{r}\mathbf{\Delta}'_\perp} - 1\right],$$

$$e^{-i\mathbf{r}\mathbf{k}'_\perp} M_2^{soft} = \frac{\mathcal{A}^2}{2!} e^{-i\mathbf{r}\mathbf{k}_\perp} \times$$

$$\int \frac{d^2\Delta'_{2\perp}}{(2\pi)^2} \frac{1}{\Delta'^2_{1\perp}\Delta'^2_{2\perp}} \left[e^{-i\mathbf{r}\mathbf{\Delta}'_\perp} - e^{-i\mathbf{r}\mathbf{\Delta}'_{2\perp}} - e^{-i\mathbf{r}\mathbf{\Delta}'_{1\perp}} + 1\right]$$

etc ...

Fourier transform →

$$e^{-i\mathbf{r}\mathbf{k}'_\perp} \hat{M}_1^{soft} = e^{-i\mathbf{r}\mathbf{k}_\perp} \mathcal{A}\cdot\mathcal{W}(\mathbf{b},\mathbf{r}),$$

$$e^{-i\mathbf{r}\mathbf{k}'_\perp} \hat{M}_2^{soft} = e^{-i\mathbf{r}\mathbf{k}_\perp} \frac{\mathcal{A}^2\cdot\mathcal{W}(\mathbf{b},\mathbf{r})^2}{2!}, \quad \ldots$$

Summing series
↓
rescattering amplitude

Soft gluon rescattering amplitude →

$$e^{-i\mathbf{r}\mathbf{k}'_\perp} \hat{M}^{soft}(\mathbf{b},\mathbf{r}) = -e^{-i\mathbf{r}\mathbf{k}_\perp}\left(1 - e^{\mathcal{A}\cdot\mathcal{W}(\mathbf{b},\mathbf{r})}\right)$$

$$\mathcal{A} = i g_s^2 C_F/2 \qquad \mathcal{W}(\mathbf{b},\mathbf{r}) = \frac{1}{2\pi}\ln\frac{|\mathbf{b}-\mathbf{r}|}{|\mathbf{b}|}$$

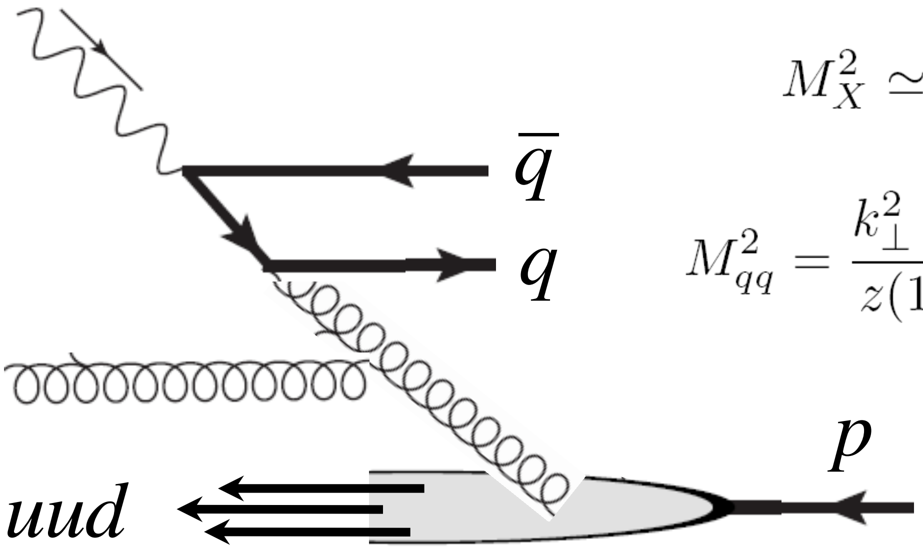
Analytic perturbation theory → coupling at soft scale

$$\alpha_s^{\rm soft} = \mathcal{A}_1(\Lambda_{\rm QCD}) \simeq 0.7$$

Inspired by Brodsky et al, PRD65, 114025 (2002)

Gluonic contribution @ large M_X

Gluon radiated from "hard" gluon is far away in *p*-space from $q\bar{q}$
$\rightarrow$ leading contribution to large M_X



$$M_X^2 \simeq M_{qq}^2 + \frac{p_\perp^2}{z'} \rightarrow \infty,$$

$$M_{qq}^2 = \frac{k_\perp^2 + m_q^2}{z(1-z)}, \quad z_g = \frac{z'}{x_P} + \beta\left(1 + \frac{M_{qq}^2}{Q^2}\right)$$

$$z' \ll z \qquad p_\perp \sim k_{1,\perp} \gg k_{2,\perp}$$

$\rightarrow$ Altarelli-Parisi splitting $\otimes$ $q\bar{q}$-dipole $\otimes$ multiple gluon exchange

$$x_P F_{q\bar{q}g}^{D(4)} \simeq \frac{1}{N_c^2} \int \frac{dt_g dz_g}{t_g + m_g^2} P_{gg}(z_g) \frac{\alpha_s(t_g)}{2\pi} x_P F_{q\bar{q}}^{D(4)}$$

Diffractive DIS cross section

$$x_P \sigma_r^{D(3)} = x_P F_{q\bar{q},T}^{D(3)} + \frac{2-2y}{2-2y+y^2} x_P F_{q\bar{q},L}^{D(3)} + x_P F_{q\bar{q}g}^{D(3)} \qquad y = Q^2/(s x_B) \leq 1$$

$q\bar{q}$ dipole contribution:

$$x_P F_L^{D(4)} = \mathcal{S}\, Q^4 M_X^2 \int_{z_{min}}^{\frac{1}{2}} dz (1-2z)\, z^2 (1-z)^2 |J_L|^2$$

$$x_P F_T^{D(4)} = 2\mathcal{S}\, Q^4 \int_{z_{min}}^{\frac{1}{2}} dz (1-2z) \left\{(1-z)^2 + z^2\right\} |J_T|^2$$

with $\mathcal{S} = \sum_q e_q^2/(2\pi^2 N_c^3)$ and amplitudes:

$$J_L = i\alpha_s(\mu_F^2) \int d^2\mathbf{r} d^2\mathbf{b}\, e^{-i\boldsymbol{\delta}\mathbf{b}} e^{-i\mathbf{r}\mathbf{k}_\perp} K_0(\varepsilon r)\, \mathcal{V}(\mathbf{b},\mathbf{r}) \left[1 - e^{\mathcal{A}\mathcal{W}}\right]$$

$$J_T = i\alpha_s(\mu_F^2) \int d^2\mathbf{r} d^2\mathbf{b}\, e^{-i\boldsymbol{\delta}\mathbf{b}} e^{-i\mathbf{r}\mathbf{k}_\perp} \varepsilon K_1(\varepsilon r) \frac{r_x \pm i r_y}{r} \mathcal{V}(\mathbf{b},\mathbf{r}) \left[1 - e^{\mathcal{A}\mathcal{W}}\right]$$

where

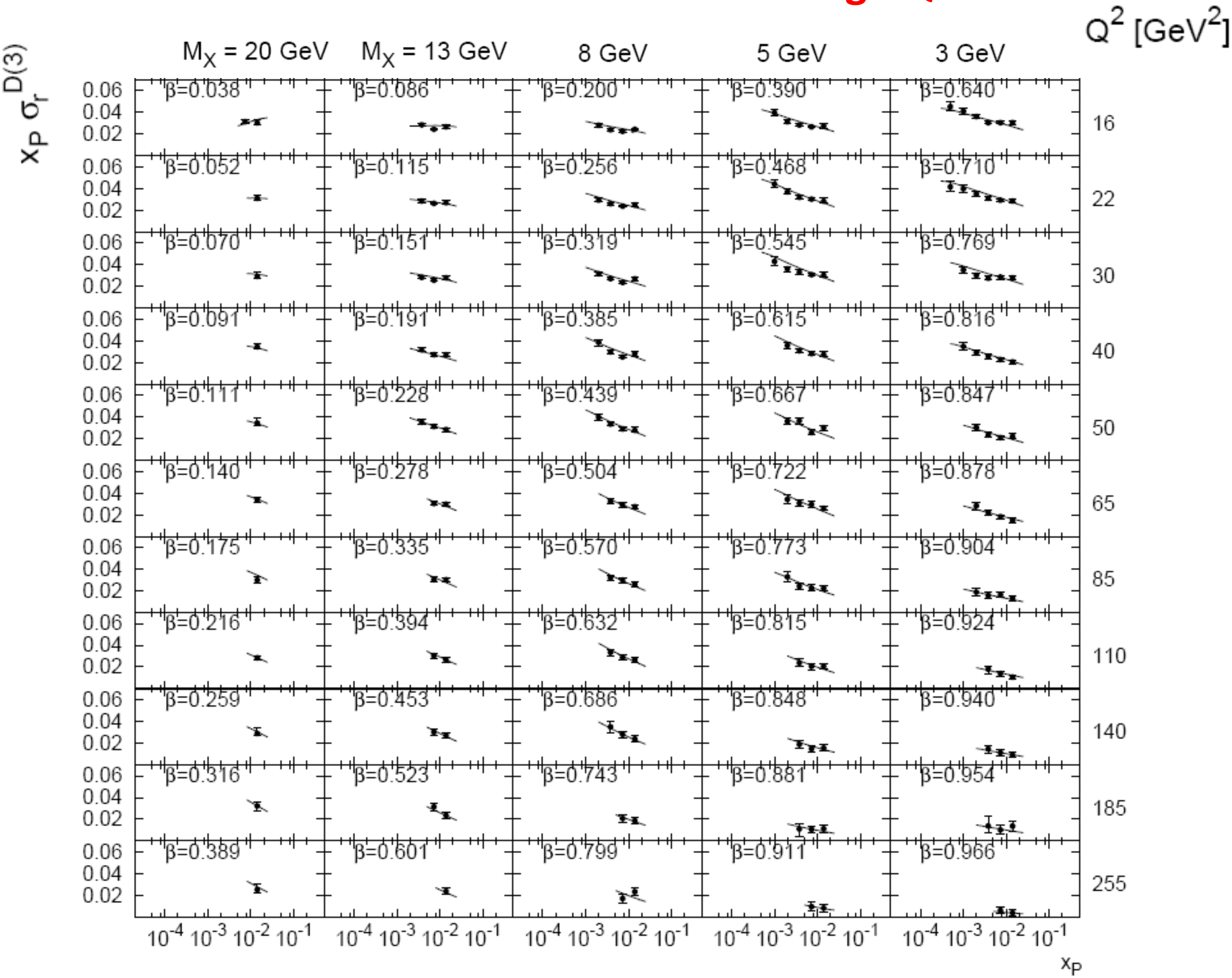
$$\mathcal{V}(\mathbf{b},\mathbf{r}) = \frac{1}{\alpha_s(\mu_{\rm soft}^2)} \frac{\bar{R}_g(x')}{(2\pi)^2} \sqrt{x_P g(x_P, \mu_F^2)} \left[e^{-\frac{\rho_0^2}{2}|\mathbf{b}-\mathbf{r}|^2} - e^{-\frac{\rho_0^2}{2}|\mathbf{b}+\mathbf{r}|^2}\right]$$

$$\mathcal{A} = i g_s^2 C_F/2 \qquad \mathcal{W}(\mathbf{b},\mathbf{r}) = \frac{1}{2\pi} \ln \frac{|\mathbf{b}-\mathbf{r}|}{|\mathbf{b}|}$$

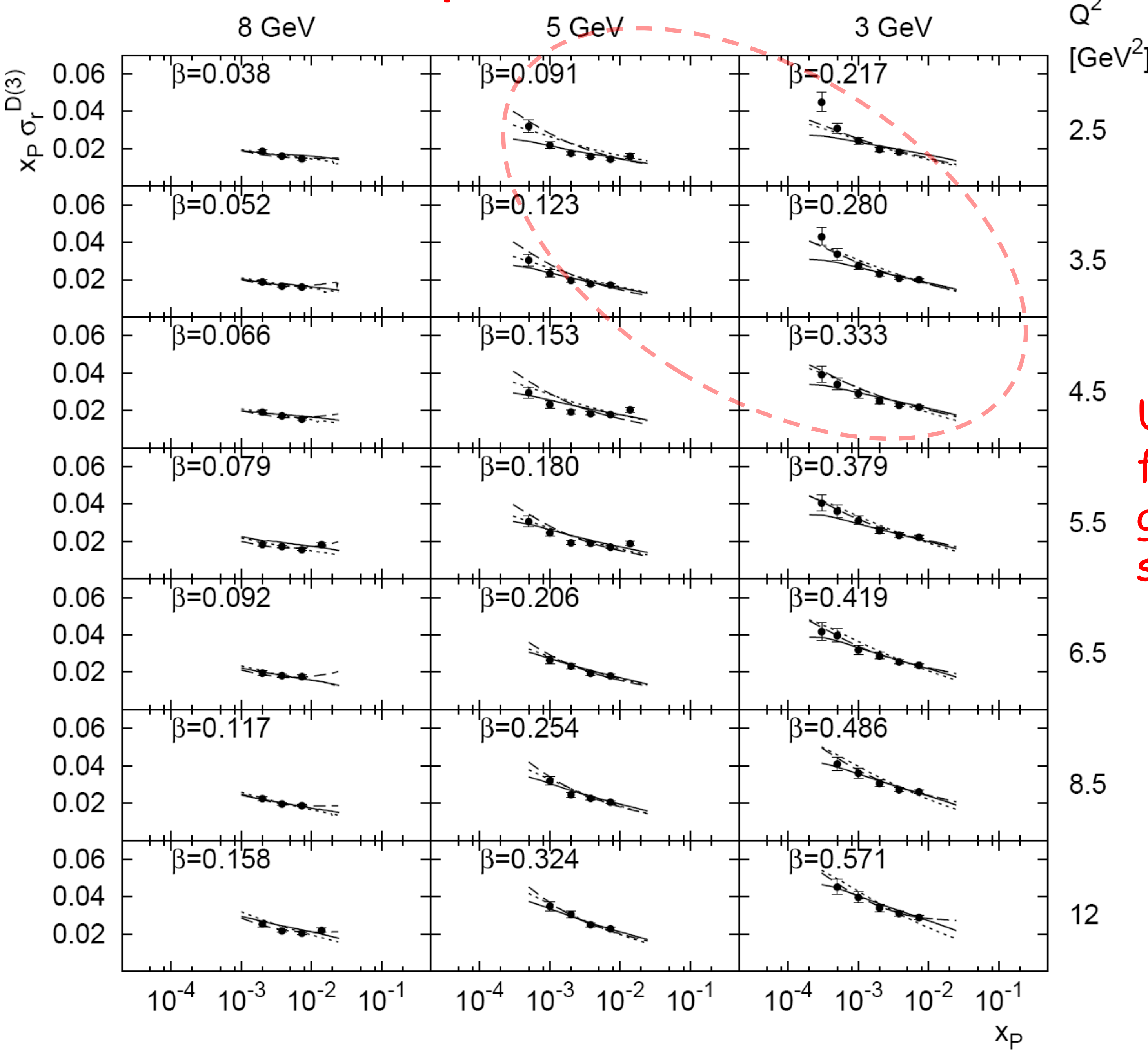
gluonic dipole contribution:

$$x_P F_{q\bar{q}g}^{D(4)} \simeq \frac{1}{N_c^2} \int \frac{dt_g dz_g}{t_g + m_g^2} P_{gg}(z_g) \frac{\alpha_s(t_g)}{2\pi} x_P F_{q\bar{q}}^{D(4)}$$

Diffractive structure function: large Q^2



Comparison to HERA data: low Q^2

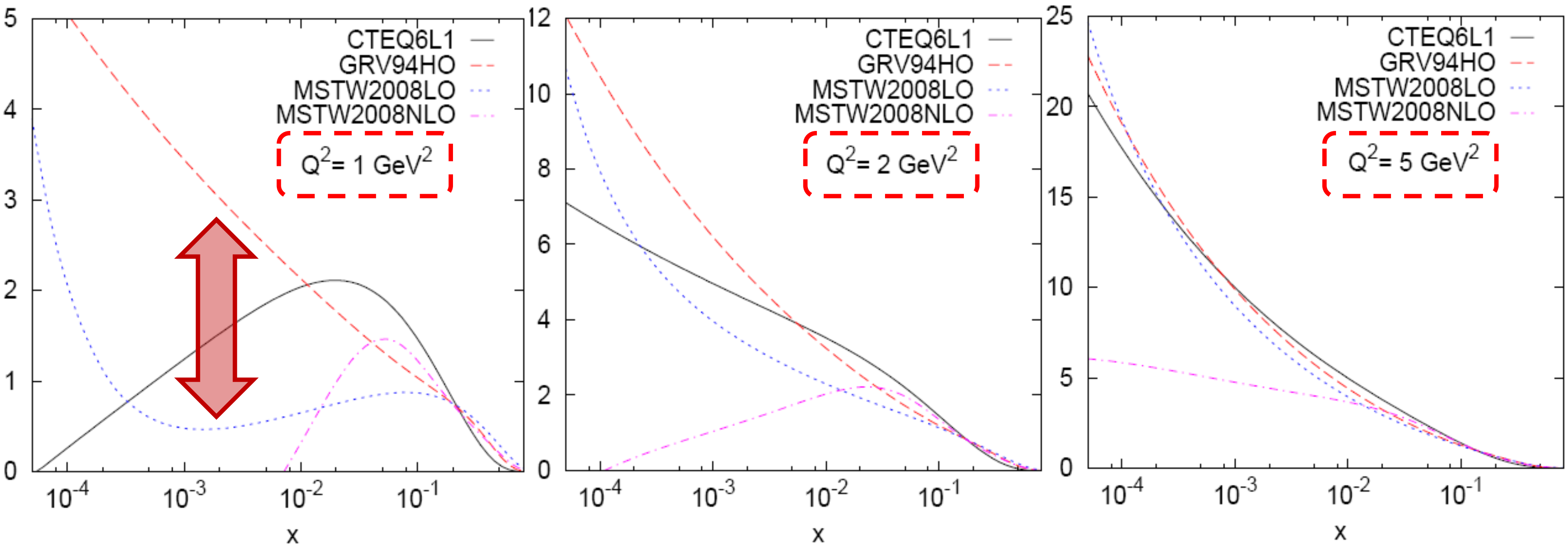


Curves for different $xg(x,\mu)$ parametrizations

Uncertainty from unknown gluon density at small x & scale

Possibility to extract $g(x,\mu_F)$ at $x\sim10^{-4}$ $\mu_F < \sim1$ GeV

Gluon density parametrizations at low-*x* and low Q^2

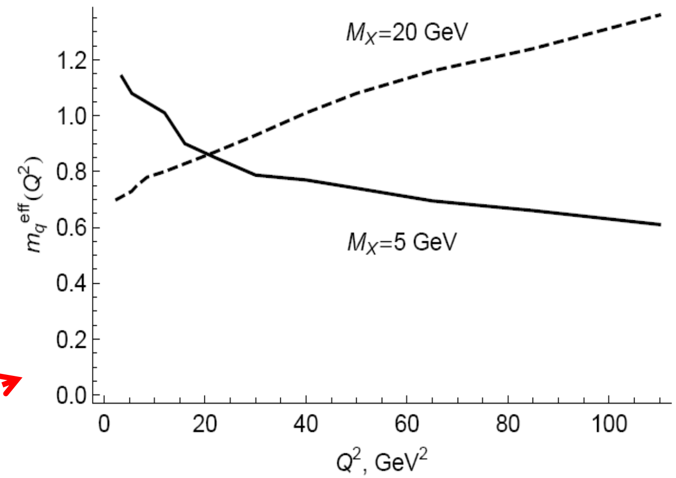


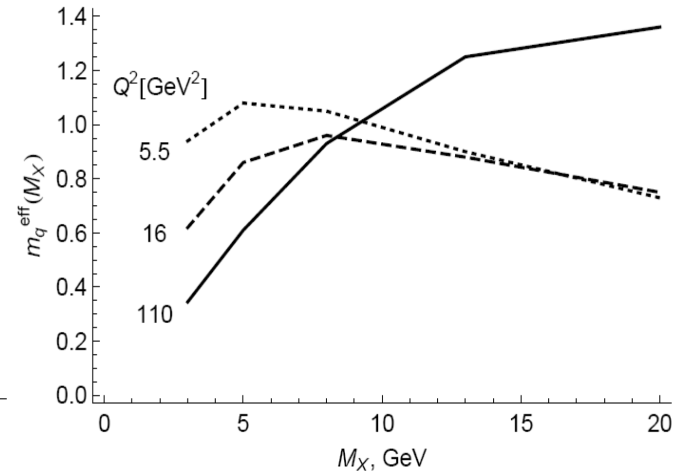
Large differences at $x<\sim 10^{-2}$ and $Q^2<\sim 2\ GeV^2$!!

→ Unknown gluon density in this region !!!

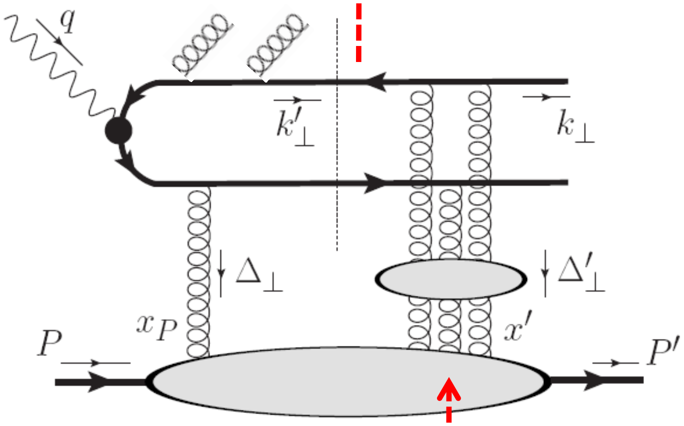
Model parameters

Effective mass m_{eff} of 0.3—1.2 GeV of quark in dipole from dressing-up with gluon radiation



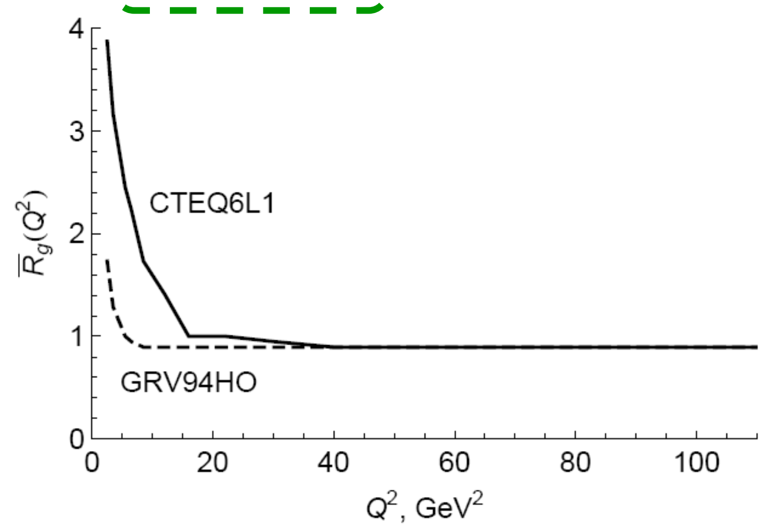


$$m_q \to m_{eff}$$



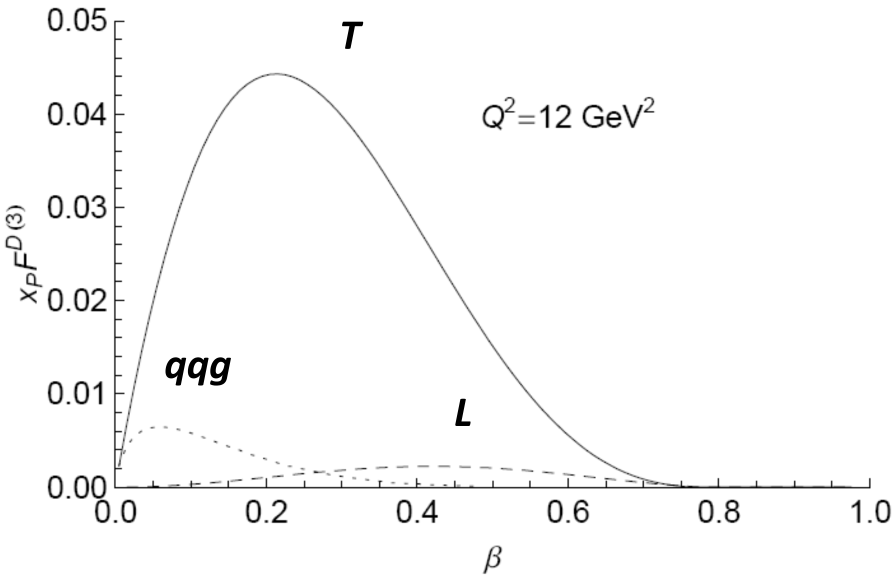
We fit unknown gluon at very small-*x’*

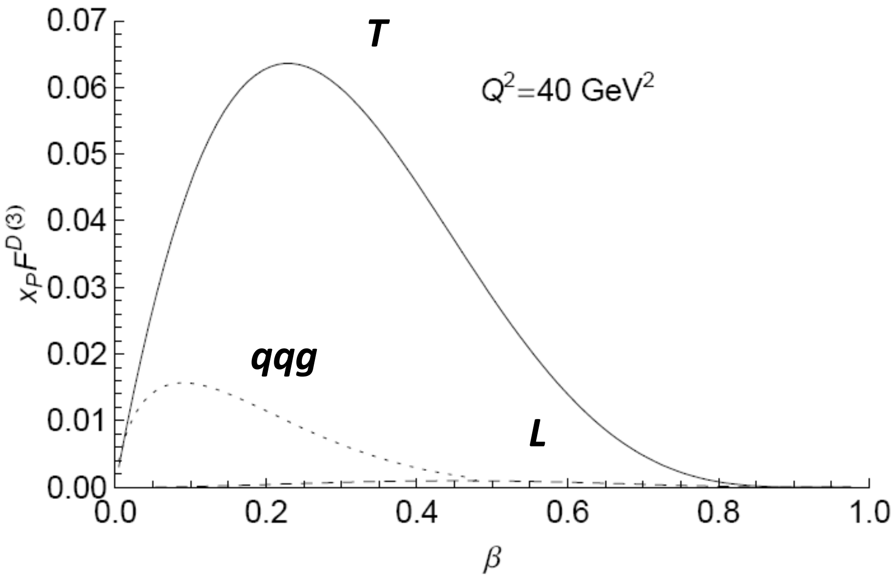
$$\sqrt{x_P}\mathcal{F}_g^{\rm off} \simeq \bar{R}_g(x', \mu^2_{\rm soft})\sqrt{x_P g(x_P, \mu^2_F)}\, f_G(\Delta^2_\perp)$$



Soft gluon density function R_g = constant ≈ 1, except at small Q^2

Photon polarization contributions and mass spectrum





Gluonic contribution increases at high M_X and Q^2 !

The main points we touched upon today…

- Definition of diffractive scattering
- The QCD Pomeron
- Good-Walker formulation of diffraction
- Gap survival
- The Sudakov form factor
- The Durham Model: Higgs CEP
- Soft Color Interactions model: DDIS
- Diffractive Drell-Yan
- Breakdown of diffractive factorisation