Accelerator Physics Exercises No. 2

- Work to be handed in on 24 October 2018

Question 2.1

Solve Hill's equation

$$y'' + K(s)y = 0$$

by substituting

$$y = \alpha\sqrt{\beta(s)}cos[\phi(s) + \phi_0]$$

with

$$\phi' = \frac{1}{\beta(s)}$$

demonstrating that a necessary condition is

$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + K\beta^2 = 1$$

Question 2.2

In "weak focusing" accelerators, the field gradient is defined as

$$k = -\frac{1}{(B_0\rho_0)}\frac{\partial B_z}{\partial x}$$

Show that the equation of motion of a particle in the horizontal degree of freedom is

$$\frac{1}{p_0}\frac{d}{ds}\left(p_0\frac{dx}{ds}\right) + \left(\frac{1}{\rho^2} - k\right)x = 0$$

and in the vertical plane it is

$$\frac{1}{p_0}\frac{d}{ds}\left(p_0\frac{dz}{ds}\right)+kz=0$$

Vertical oscillations are stable as long as $k > 0$. Discuss the limit on k for horizontal stability.

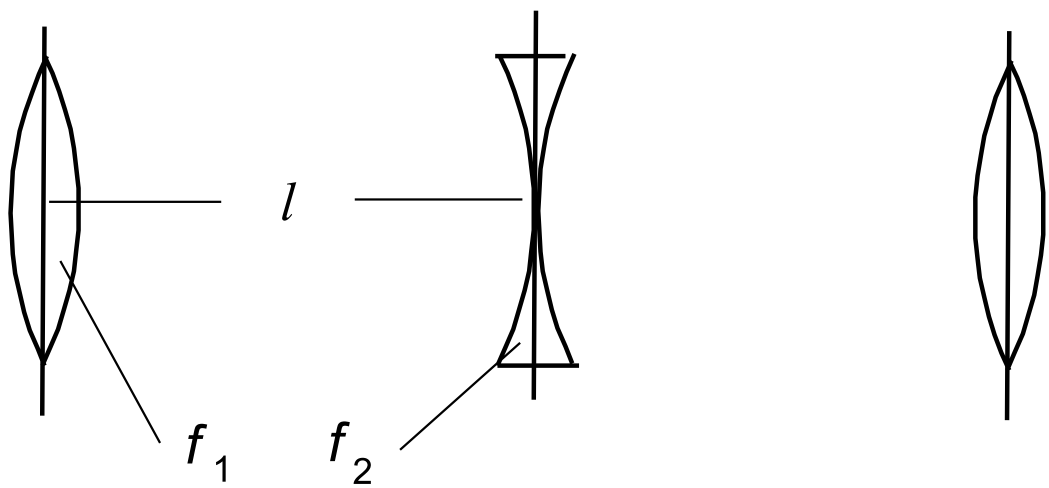
Question 2.3

a) A quadrupole doublet (half of the FODO cell from the mid-point of the F to the mid-point D) consists of two lenses of focal length f_1 and f_2 separated by a drift length of l m. Assume that the lenses are thin and show, by writing the three matrices for the lenses, that the product matrix is:

$$M=\begin{pmatrix}1-l/f_1 & \ell \\ -1/f^* & 1-l/f_2\end{pmatrix}$$

where:

$$\frac{1}{f^*}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{l}{f_1 f_2}$$



(b) A FODO cell (from mid-F, through D and to the mid-point of the next F) may be considered to be one such matrix with f_1 = +2f and f_2 = -2f followed (and multiplied) by another matrix with f_1 = -2f and f_2 = +2f. Using the result of the last question, write down these two matrices and show by taking their product that the matrix for a FODO cell from mid-F to mid-F quadrupole is

$$=\begin{pmatrix}1-l^2/2f^2 & 2l(1+l/2f) \\ -l/2f^2(1-l/2f) & 1-l^2/2f^2\end{pmatrix}$$

(c) The matrix for a FODO period must have the form:

$$M = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

Take the trace of this matrix and equate it to the result of (b) to obtain an expression for μ, the phase advance per period as a function of l and f.

Now equate m_{12} to find β at the mid-plane of the F quadrupole.

You are given the following data for the new PS2 ring:

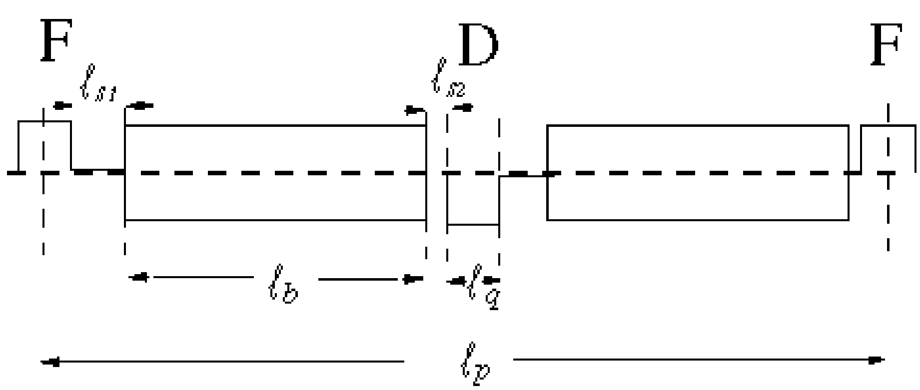
Magnetic rigidity (Bρ) at 50 GeV = 169.88 Tm.

Quadrupole half-length ($l_{quad}/2$) = (1.49/2) m.

Quadrupole gradient = 17 T/m.

Distance between quadrupole centres l = 11.605 m.

Substitute the data to obtain the numerical value of μ and β.



Question 2.4

The emittance of a proton beam at injection in the CERN SPS is 2 mm mrad.

a) Calculate the half-width of the beam at an F quadrupole where β = 108 m.

b) What is the maximum value of the divergence in the beam if the β at a D quadrupole is 18 m. and α = 0?

c) What is the normalized emittance of this beam if the above data refer to a proton momentum of 10 GeV/c?

d) If this normalized emittance is accelerated to 400 GeV/c, what will be the half-width of the beam at an F quadrupole (β = 109 m.)?

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