Estimating Dynamic Structural Models in Economics using Open Science Grid

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CU Boulder, Department of Economics

Conference

HOW Meeting 2019 - Jefferson Lab
Corporate Default Rates and Credit Spreads

Estimating Dynamic Structural Models in Economics using Open Science Grid
What we do in our Project

- Paper: *Financial Development, Default Rates and Credit Spreads*

- Document disconnect between default rates and credit spreads

- Rationalize it with a theory of financial development

  - Firms increase their leverage: Default rates go up

  - Firms operate at a more efficient scale: recovery rates goes up

  - As a result Credit spreads barely move

- Heterogenous agents’ model quantitatively account for

  - disconnect between default rates and credit spreads

  - number of trends that have interested public firms during the last decades.
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What we do in Macroeconomics

- Study how changes in the environment $\theta$ affect the macroeconomy

- **How**: Heterogeneous Agents Model
  - Agents do the best they can (*Optimization*)
  - We aggregate agent decisions to get equilibrium outcomes
  - Find $\theta$ for which equilibrium outcomes closer to the data (*Optimization*)
  - Perform policy experiment

- **Computational challenges**:
  - Determining agents’ decisions (*Optimization* - Inner Loop)
  - Estimating the structural parameters (*Optimization* - Outer Loop)
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The Inner Loop
Determining Agents’ Decisions
Determining agents’ decisions

Agents do the best they can

\[ V(s) = \max_{s' \in \Gamma(s)} F(s, s') + \beta \cdot \int_{s'} V(s') Q(ds', s) \]
Optimization - Inner Loop

Determining agents’ decisions

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- More in general

\[ V(\cdot) = TV(\cdot) \]
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- Solution: Value Function Iteration (VFI)

\[
V^i(\cdot) = T^i V^0(\cdot)
\]
Determining agents’ decisions

Agents do the best they can Our problem

\[ V(s) = \max_{s' \in \Gamma(s)} F(s, s') + \beta \cdot \int_{s'} V(s')Q(ds', s) \]

Solution: Value Function Iteration (VFI)

\[ V^i(s) = T^i V^0(s) \]

Guess \( V^0 \)

\[ V^1(k, z) = \max_{k' \in K} F(k, z, k') + \beta \cdot \int_{z'} V^0(k', z') Q(z', z')dz' \]

New Guess

Guess

Get new Guess \( V^1 \)
Determining agents' decisions

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- **Solution**: Value Function Iteration (VFI)

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- Guess \( V^1 \)

\[ V^2(k, z) = \max_{k' \in K} F(k, z, k') + \beta \cdot \int_{z'} V^1(k', z') Q(z', z') dz' \]

- New Guess

- Get new Guess \( V^2 \)
Determining agents’ decisions

- Agents do the best they can

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- **Solution**: Value Function Iteration (VFI)

\[ V^i(s) = T^i V^0(s) \]

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\[ V^3(k, z) = \max_{k' \in K} F(k, z, k') + \beta \cdot \int_{z'} V^2(k', z') Q(z', z') dz' \]

- Get new Guess \( V^3 \)
Optimization - Inner Loop

Determining agents’ decisions

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- Guess \( V^{i-1} \)

\[ V^i(k, z) = \max_{k' \in K} F(k, z, k') + \beta \cdot \int_{z'} V^{i-1}(k', z') Q(z', z') dz' \]

- Get new Guess \( V^i \)

- Repeat \( V^i = T V^{i-1} \) till when \( ||V^i - V^{i-1}|| < \epsilon \)
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**Computational Challenges**:

- Peak-finding Algorithm
- Value Function Iteration
Given a particular economy $\theta$:

- Agents do the best they can (*Fixed Point Algorithm*):
  - Firms maximize dividends (given banks' interest rates)
  - Banks choose interest rates on loans (given firms' behavior)

(Other pieces of the puzzle: Entrants, Firms’ Invariant Distribution)

- We aggregate agent decisions to get equilibrium outcomes $m_{\text{Model}}(\theta)$

- Find $\theta$ for which equilibrium outcomes closer to the data (*Optimization - Outer Loop*)
Closing the Model

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The Outer Loop

Estimating the structural parameters $\theta$
## Estimating the structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta = 0.35$</td>
<td>Borrowing Costs</td>
<td>Avg Leverage</td>
<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>$\sigma_e = 0.20$</td>
<td>Uncertainty</td>
<td>Std Leverage</td>
<td>0.31</td>
<td>0.34</td>
</tr>
<tr>
<td>$\xi = 110.4$</td>
<td>Uncertainty 2</td>
<td>Default Rate</td>
<td>0.32%</td>
<td>0.36%</td>
</tr>
<tr>
<td>$k_e = 8.5$</td>
<td>Capital Entrants</td>
<td>Investment Rate</td>
<td>0.63%</td>
<td>0.54%</td>
</tr>
<tr>
<td>$\gamma = 0.09$</td>
<td>Equity Issuance</td>
<td>Share Dividends</td>
<td>83%</td>
<td>82%</td>
</tr>
</tbody>
</table>
Estimating the Structural Parameters

$$\theta^* = \arg\min_{\theta \in \Theta} L ( m^{\text{Model}}(\theta), m^{\text{Data}} )$$

- **Problem 1**: Computing $m^{\text{Model}}(\theta)$ takes time
  - Estimating Bellman Equation via Value Function Iteration

- **Problem 2**: Loss Function
  - Local Minima may not be Global Minima
  - **Solution**: Global Search

- **Problem 3**: The Parameter Space: $\Theta$
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Quest for Computing Power

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- Amazon AWS
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  - RMACC Summit supercomputer
  - XSEDE
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Global Search over Parameter Space in OSG

- **Perfect Fit:** Model Economy $\theta$
  - independent of each other
  - not memory intensive (<2GB)
  - run within 1-12 hours
  - portable (C++)

- DAGMan HTC Condor’s workflow
  - Easy to use
  - Low Monitoring Costs

- Opportunistic usage and sharing of resources (No SU)
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Incumbents’ Value Function

\[
V_i(\omega_i, z_i) = \max_{\phi_{D,i} \in \{0,1\}} (1 - \phi_{D,i}) V_i^c(\omega_i, z_i)
\]

\[
V_i^c(\omega_i, z_i) = \max_{d_i, k_i', l_i'} d_i + \beta \sum_{z_i'} \sum_{\xi_i'} \Gamma_{z_i, z_i'} \Gamma_{\xi_i} V_i(\omega_i', z_i')
\]

s.t. \( (1 + \gamma \mathbb{I}_{\{d_i<0\}}) d_i = \omega_i + l_i' - k_i' \)

\[
\omega_i' = \pi_i' + (1 - \delta) k_i' - \left( [1 + r_i'] \mathbb{I}_{\{l_i'>0\}} + [1 + r_F] \mathbb{I}_{\{l_i'<0\}} \right) l_i'
\]

\[(z_i, k_i', l_i', r_i') \in H(z_i, k_i, l_i')\]