Karlsruher Institut für Technologie

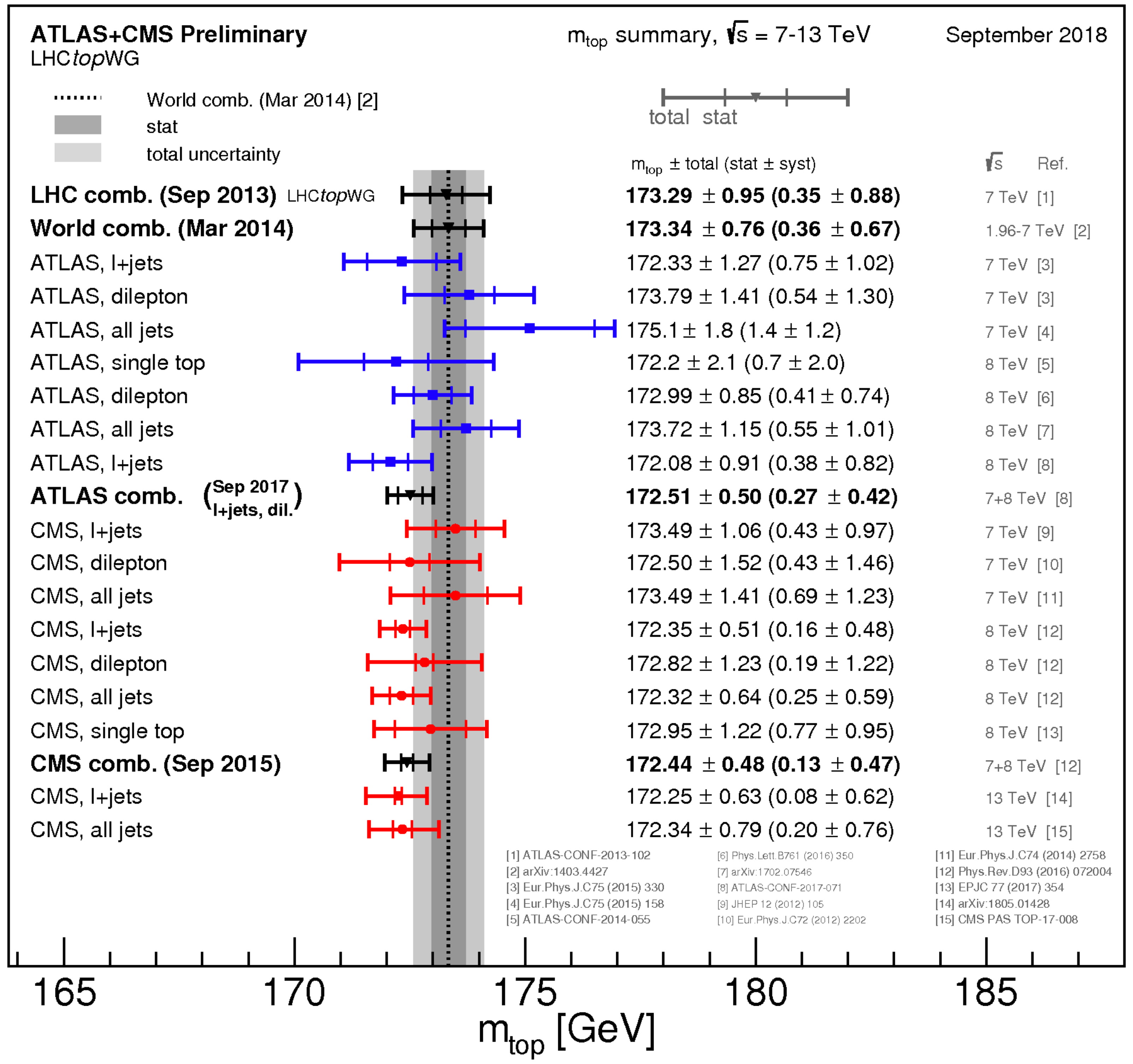
TRR 257 - Particle Physics Phenomenology after the Higgs Discovery

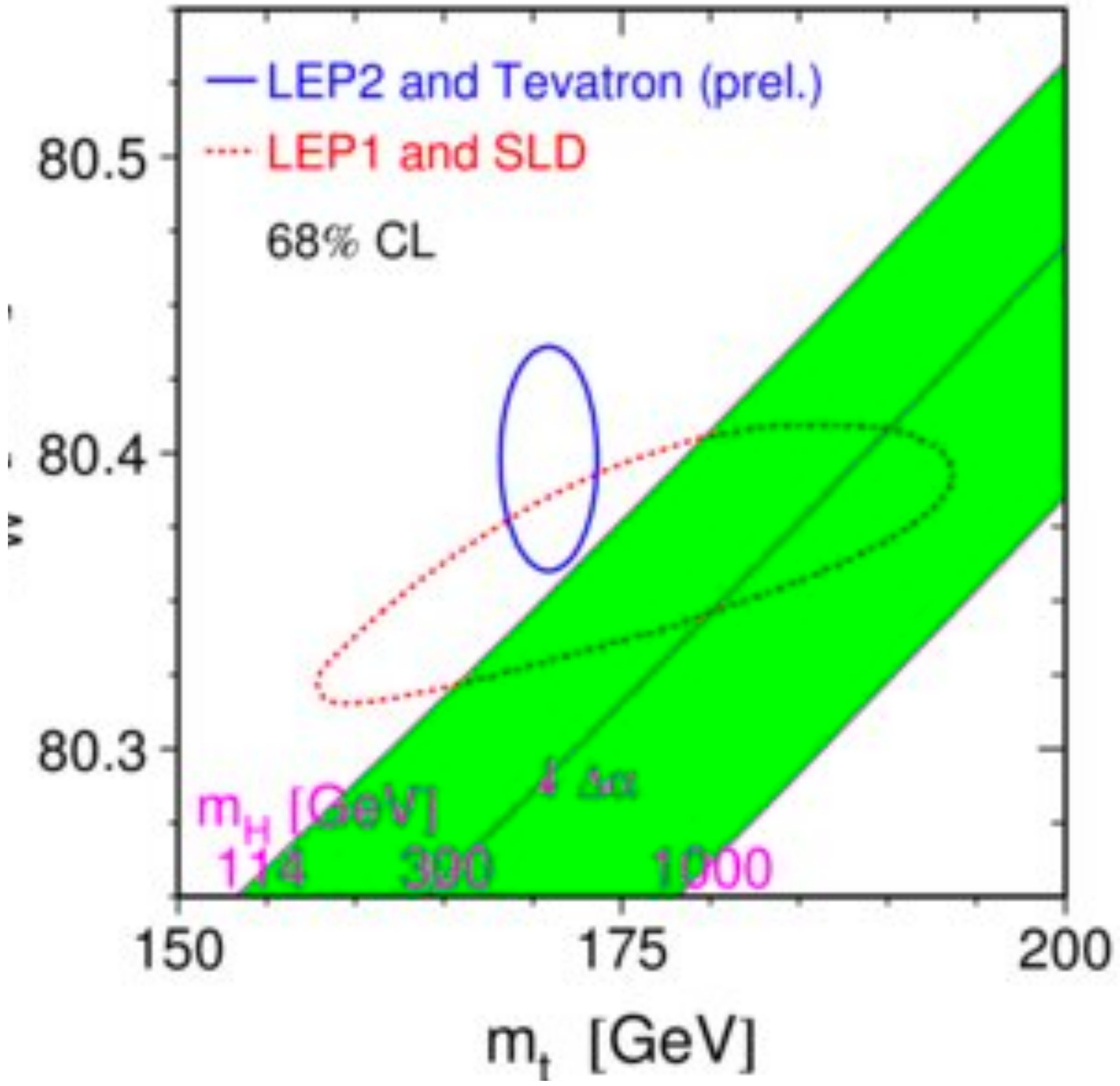
KIRILL MELNIKOV (TTP KIT)

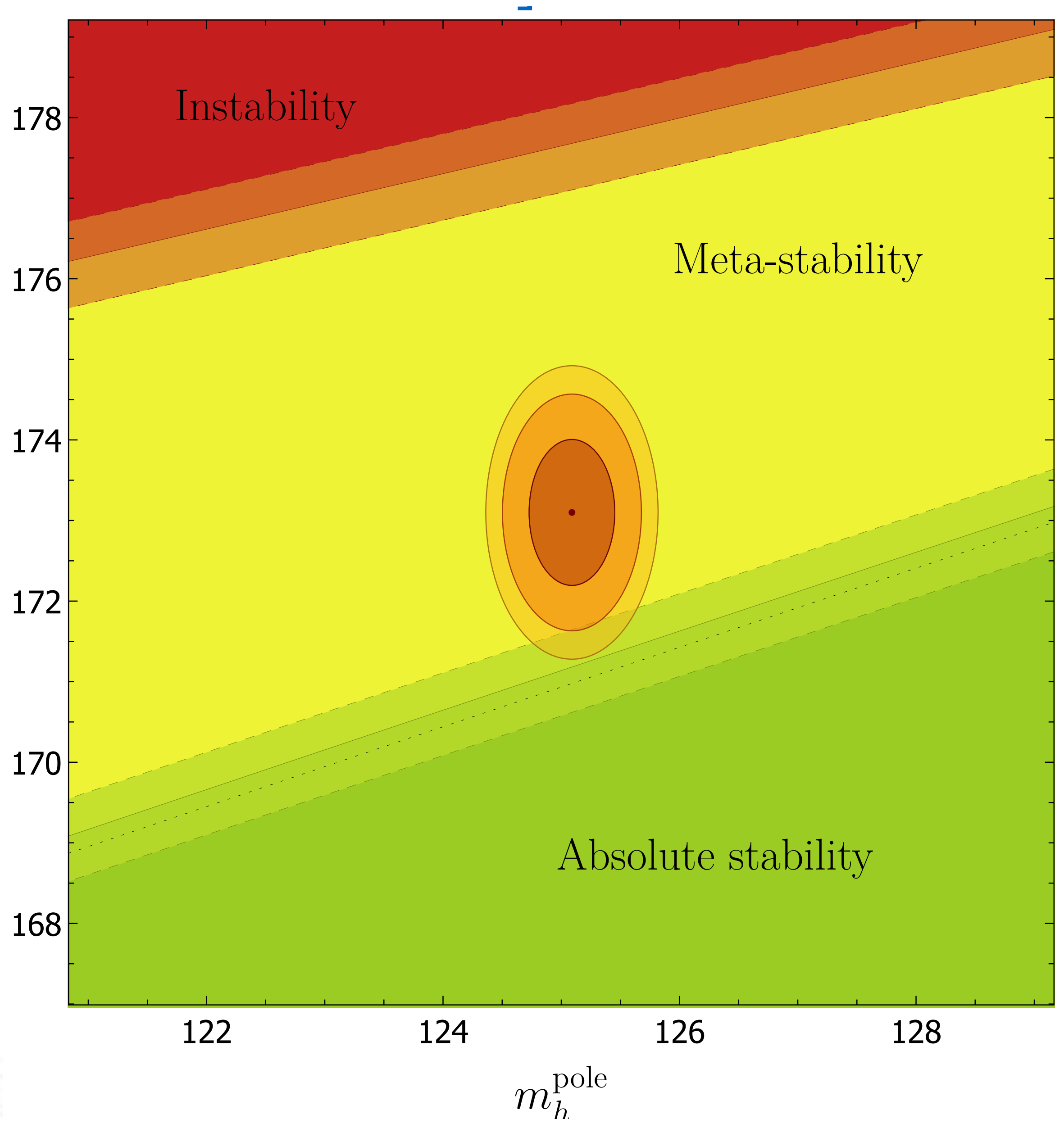
TOP MASS

SM@LHC 2019

Can we use results of CMS and ATLAS measurements to argue about the consistency of the SM and to discuss conclusively the stability of the electroweak vacuum?







CMS (talk by Defranchis)

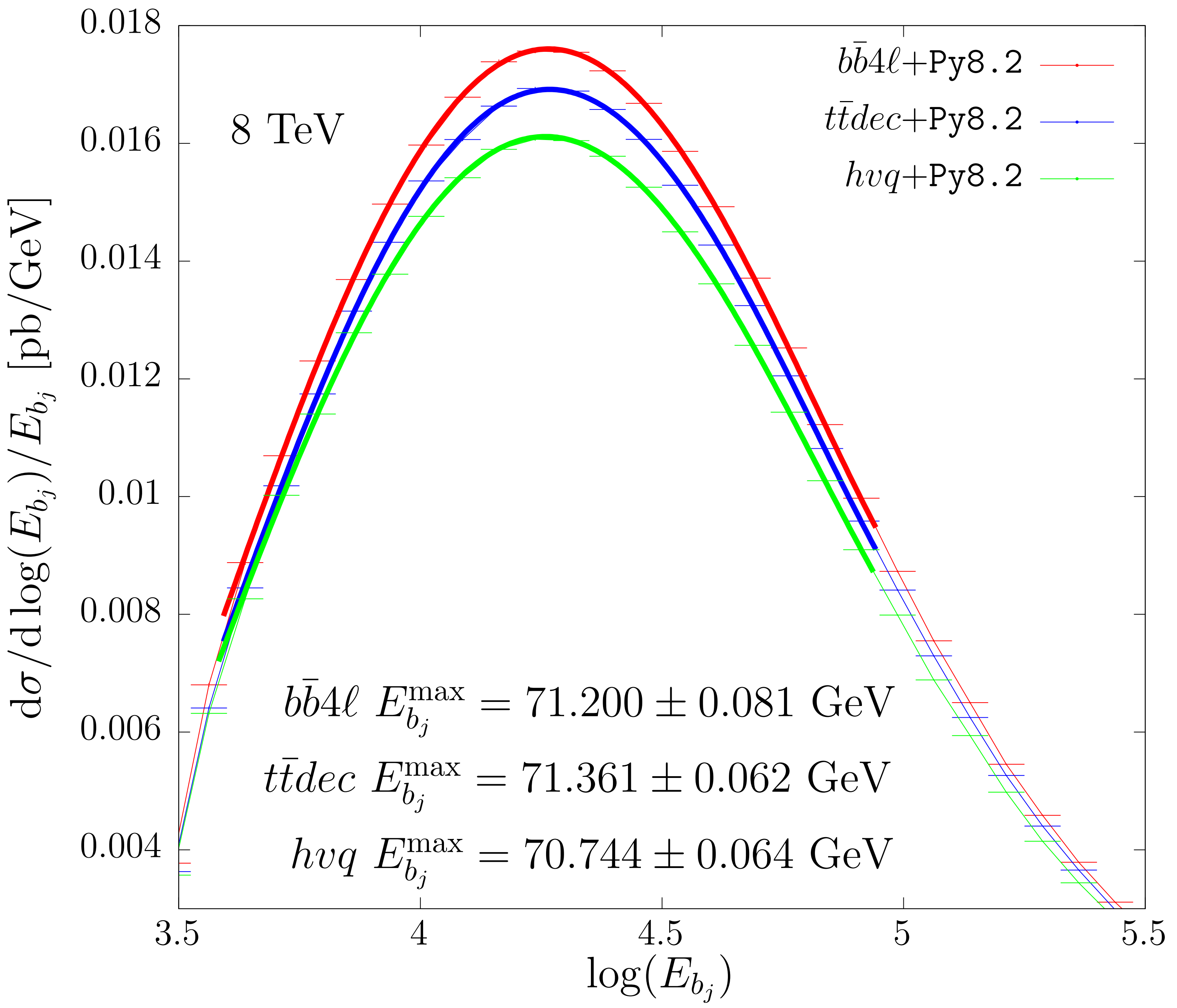
PDF set	m_t^{pole} [GeV]
ABMP16	$169.9 \pm 1.8\,(\text{fit} + \text{PDF} + \alpha_S)\,{}^{+0.8}_{-1.2}\,(\text{scale})$
NNPDF3.1	$173.2 \pm 1.9\,(\text{fit} + \text{PDF} + \alpha_S)\,{}^{+0.9}_{-1.3}\,(\text{scale})$
CT14	$173.7 \pm 2.0\,(\text{fit} + \text{PDF} + \alpha_S)\,{}^{+0.9}_{-1.4}\,(\text{scale})$
MMHT14	$173.6 \pm 1.9\,(\text{fit} + \text{PDF} + \alpha_S)\,{}^{+0.9}_{-1.4}\,(\text{scale})$

$$m_t = 173 \pm 0.76 \text{ GeV}$$

Stable EW vacuum requires $m_t < 171$ GeV

MODELLING TOP PRODUCTION AND DECAY

Top quark mass is extracted from measurements of various observables in processes with top quarks. Important to understand if the existing modelling of top production is up to the task. Can be studied by comparing available generators that are becoming very sophisticated. Moderate differences may be observed (except for PYTHIA/HERWIG for some observables).



Ravasio, Jezo, Nason, Oleari

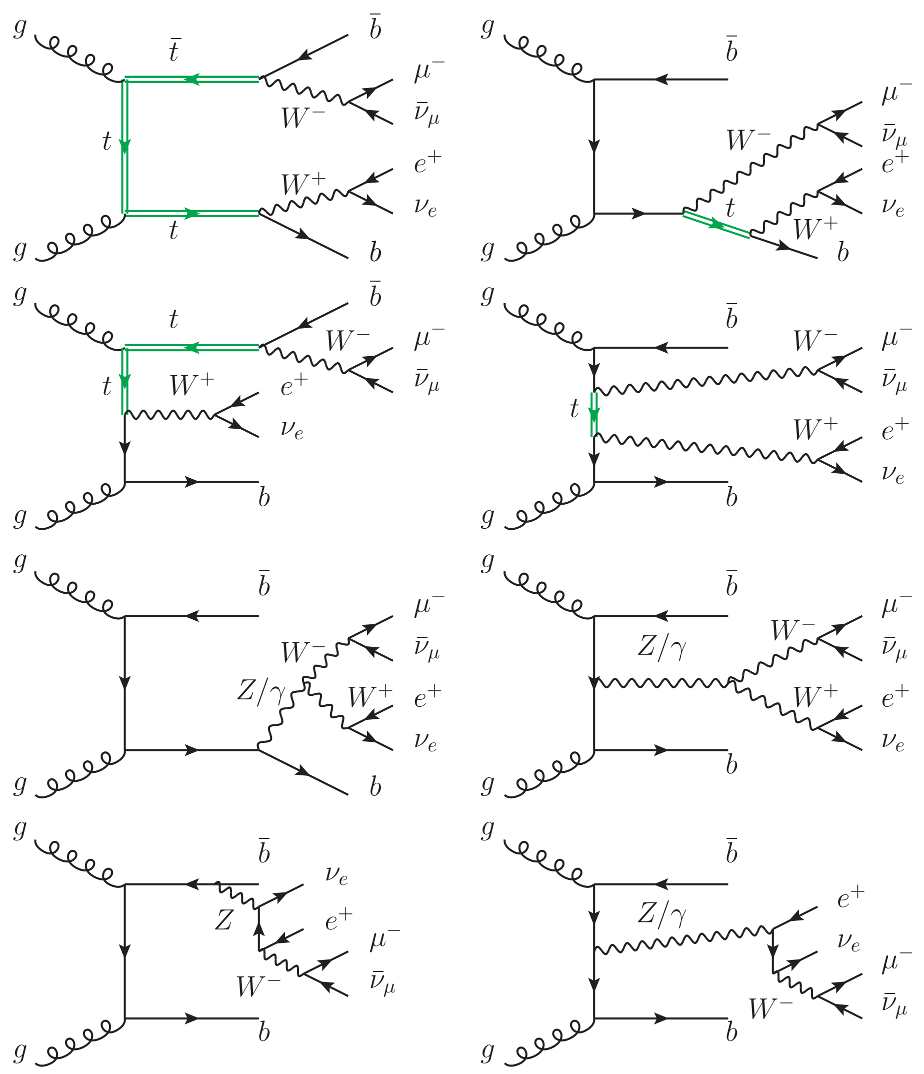
	$R = 0.4$	$R = 0.5$	$R = 0.6$
$b\bar{b}4\ell$	67.792 ± 0.089 GeV	71.200 ± 0.081 GeV	74.454 ± 0.076 GeV
$t\bar{t}dec - b\bar{b}4\ell$	$+365 \pm 110$ MeV	$+161 \pm 102$ MeV	$+75 \pm 97$ MeV
$hvq - b\bar{b}4\ell$	-563 ± 110 MeV	-456 ± 103 MeV	-323 ± 97 MeV

Generators

hvq: NLO in production

tTdec: NLO production and decay, narrow width

bB4l: full NLO, off-shell



$$E^{\max}_{b_j} = \frac{m_t^2 - m_W^2 + m_b^2}{2m_t}$$

K. Agashe, R. Francenschini, S. Hong, D. Kim

What exactly has been measured?

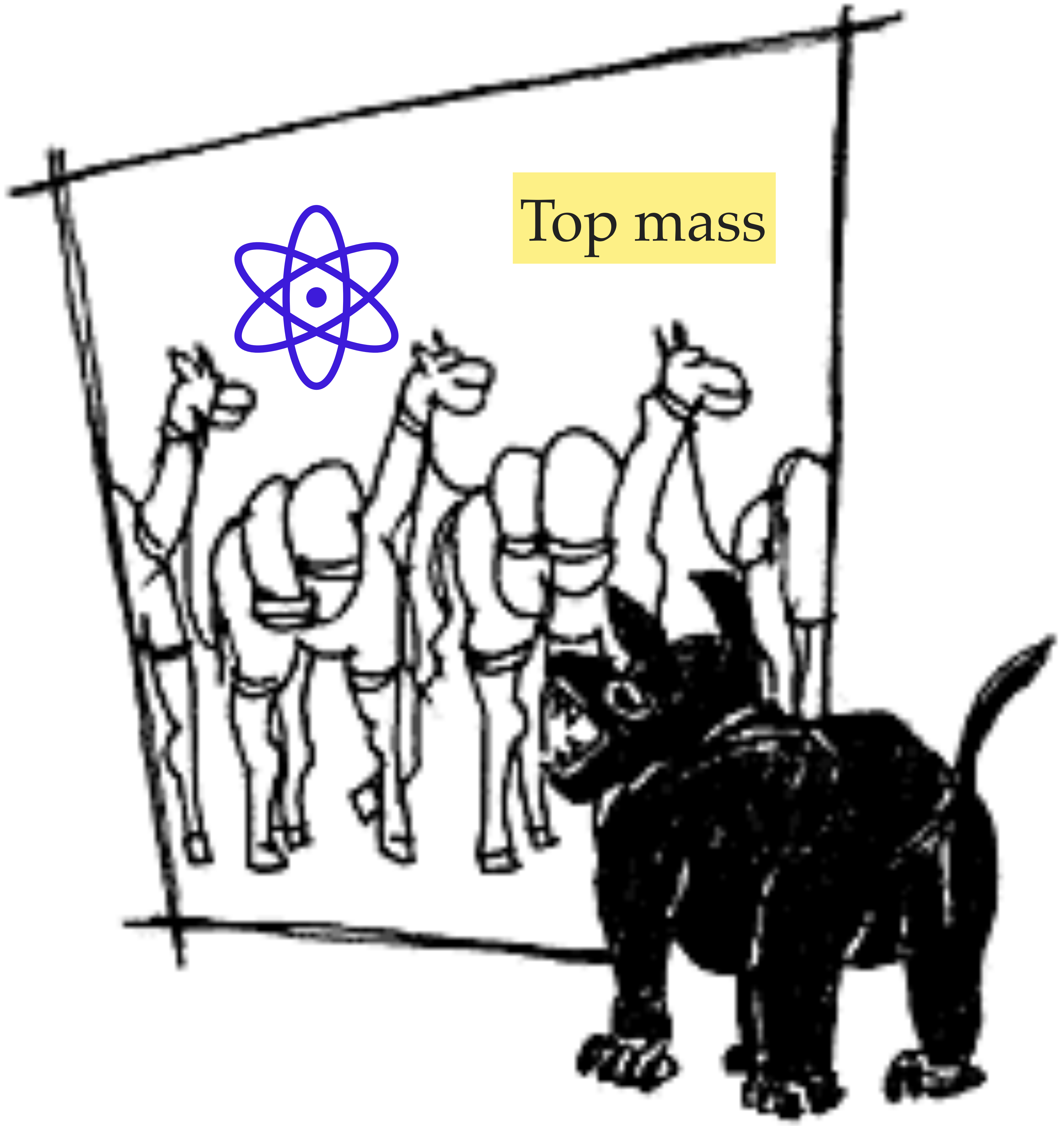
$$I(J^P) = 0(\tfrac{1}{2}^+)$$

Charge $= \frac{2}{3}\ e$ Top $= +1$

Mass (direct measurements) $m = 173.1 \pm 0.6$ GeV [a,b] $(S = 1.6)$
Mass from cross-section measurements) $m = 160^{+5}_{-4}$ GeV [a]
Mass (Pole from cross-section measurements) $m = 173.5 \pm 1.1$ GeV
$m_t - m_{\bar{t}} = -0.2 \pm 0.5$ GeV $(S = 1.1)$
Full width $\Gamma = 1.41^{+0.19}_{-0.15}$ GeV $(S = 1.4)$
$\Gamma(W\,b)/\Gamma(W\,q(q = b,\ s,\ d)) = 0.957 \pm 0.034$ $(S = 1.5)$

***t*-quark EW Couplings**

$F_0 = 0.685 \pm 0.020$
$F_- = 0.320 \pm 0.013$
$F_+ = 0.002 \pm 0.011$
$F_{V+A} < 0.29$, CL $= 95\%$



Top quark is the only quark (and the only particle for that matter) that officially got THREE(!) different masses according to PDG!

1. A mass parameter extracted from a measurement depends mostly on an observable rather than a simulation tool.

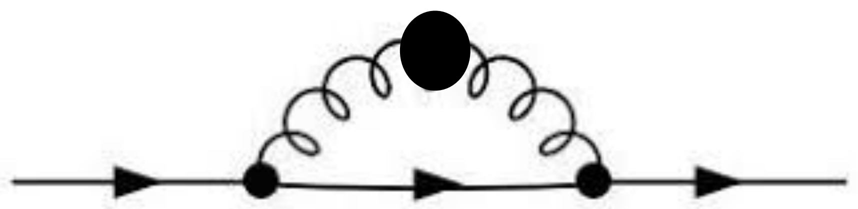
2. It is very important to understand non-perturbative effects and how they may affect the mass measurements. Resurrection of renormalons.

3. Another important issue is to understand modelling limitations of parton shower event generators. A recent study suggests that the peak of a thrust distribution of boosted top-initiated jet is shifted relative to the top quark mass squared. The shift itself may be mass-dependent. What are the consequences of this observation?

4. The top quark width should soften the IR sensitivity. Are there implications of top instability for top quark dynamics (point 3) and for sensitivities of various observables to non-perturbative physics?

Normally, we associate particle masses with poles in propagators. This does not work for quarks beyond fixed-order perturbation theory. Obscure issue for the top quark since it is very heavy and very unstable.

$$G(p,m) \sim \frac{1}{p^2 - m^2}$$



$$m_{\rm pole} = m_{\rm bare} + \frac{4}{3}\int\limits_0^\infty \frac{{\rm d}^3\vec{k}}{4\pi^2}\frac{\alpha_s(|\vec{k}|)}{\vec{k}^2} w(|\vec{k}|,m) \qquad w(|\vec{k}|,m) \approx 1, \quad |\vec{k}| \le m \qquad \alpha_s(|\vec{k}|) \approx \frac{\Lambda_{\rm QCD}^2}{\vec{k}^2 - \Lambda_{\rm QCD}^2}$$

$$\delta m_{\rm pole} \sim \Lambda_{\rm QCD}$$

We think about the top quark mass as a parameter of the Lagrangian and define it according to a particular renormalization ``scheme''. Depending on the choice of scale and the exact definition, we get different mass parameters from MS-bar to low-scale short-distance masses (kinetic, potential-subtracted, 1S etc.).

$$m(\mu) = m_{\rm bare} + \frac{4}{3}\int\limits_\mu^\infty \frac{{\rm d}^3\vec{k}}{4\pi^2}\frac{\alpha_s(|\vec{k}|)}{\vec{k}^2} w(|\vec{k}|,m) \qquad \Lambda_{\rm QCD} \ll \mu \qquad m_{\rm pole} = m(\mu) + \frac{4}{3}\int\limits_0^\mu \frac{{\rm d}^3\vec{k}}{4\pi^2}\frac{\alpha_s(|\vec{k}|)}{\vec{k}^2}$$

$$m_{\rm pole} = m(\mu) + \frac{4}{3}\alpha_s(\mu)\,\mu$$

In theory, all mass definitions (schemes) are equivalent provided that one works to all orders in perturbation theory. This implies that one can extract a mass parameter in any scheme from any observable.

$$\text{Measured} = F(m_{R_1}, \alpha_s, R_1, \Lambda_{\text{QCD}}) = F(m_{R_2}, \alpha_s, R_2, \Lambda_{\text{QCD}})$$

$$F(m_R, \alpha_s, R, \Lambda_{\text{QCD}}) = F^{\text{PT}}(m_R, \alpha, R) + F^{\text{NPT}}(m_R, \alpha_s, R, \Lambda_{\text{QCD}})$$

In reality, the practicality of the above statement depends on how fast perturbative expansion of an observable converges when a particular mass scheme is used.

$$F^{\text{PT}}(m_{R_1}) = \sum^{N_1} \alpha_s^k c_1(m_{R_1})$$

Moreover, different observables have different sensitivity to non-perturbative effects. Understanding this sensitivity is central for extracting the correct value of the mass parameter in any scheme.

$$F^{\text{NPT}} \sim \mathcal{O}(\Lambda_{\text{QCD}}^n)$$

Non-perturbative corrections to semileptonic decay rate of a B-meson are strongly suppressed; the rate is also quite sensitive to the mass of a heavy quark. Ideal short-distance observable!

$$\Gamma^{\rm sl}_{B\to X_u} = \Gamma^{\rm sl}_{b\to u}\left[1+\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m_b^2}\right)\right]$$

The pole mass: gives us a (technically) useful formula but is not relevant otherwise.

$$\Gamma^{\rm sl}_{\rm b\to u} \sim m_b^5\left(1+\frac{\alpha_s}{\pi}\left(\frac{25}{6}-\frac{2}{3}\pi^2\right)\right) \sim m_b^5\left(1-2.41\frac{\alpha_s}{\pi}\right)$$

The MS-mass: can be extracted from the measured decay rate but requires (too) accurate theory prediction.

$$m_b = \bar{m}_b\left(1+\frac{4}{3}\frac{\alpha_s}{\pi}\right) \qquad \Gamma^{\rm sl}_{\rm b\to u} \sim \bar{m}_b^5\left(1+\frac{\alpha_s}{\pi}\left(\frac{25}{6}-\frac{2}{3}\pi^2+\frac{20}{3}\right)\right) \sim \bar{m}_b^5\left(1+3.9\frac{\alpha_s}{\pi}\right)$$

The kinetic mass: can be extracted from the measured decay rate; requires tree-level theory prediction.

$$m_b = m_k(\mu)\left(1+\frac{16\alpha_s(\mu)}{9\pi}\frac{\mu}{m}\right) \qquad \Gamma^{\rm sl}_{b\to u} = m_k^5\left(1+\frac{\alpha_s}{\pi}\left(\frac{25}{6}-\frac{2\pi^2}{3}+5\frac{16}{9}\frac{\alpha_s(\mu)}{\alpha_s(m)}\frac{\mu}{m_k}\right)\right) = m_k^5\left(1+\frac{\alpha_s}{\pi}\left(-2.41+2.92\,\frac{\mu}{1\rm GeV}\right)\right)$$

The bottom line: we can extract both the MS-mass and the kinetic mass from semileptonic B-decays. In case of MS mass, we need at least the two-loop theory prediction; in case of the kinetic mass, we only need the tree-level.

$$\left(\frac{m_k}{\bar{m}_b}\right)^5 = \left(\frac{4.6}{4.2}\right)^5 = 1.6 \approx 1+3.9\frac{\alpha_s}{\pi}+\mathcal{O}(0.3)$$

If we perform a similar analyses for the top decay width, the result is different in that there is no clear winner! All corrections are small (few percent) which means that, in order to determine the mass properly, one needs to control theoretical description of the width at the percent level.

The pole mass $$\Gamma_t \sim m_t^3 \left(1 + \frac{\alpha_s}{\pi}\left(\frac{5}{3} - \frac{4\pi^2}{9}\right)\right) \sim m_t^3 \left(1 + \frac{\alpha_s}{\pi}(-2.72)\right)$$

The MS-mass $$m_t = \bar{m}_t \left(1 + \frac{4}{3}\frac{\alpha_s}{\pi}\right) \qquad \Gamma_t \sim \bar{m}_t^3 \left(1 + \frac{\alpha_s}{\pi}\left(\frac{5}{3} - \frac{4\pi^2}{9} + 4\right)\right) \sim \bar{m}_t^3 \left(1 + \frac{\alpha_s}{\pi}(-2.72 + 4)\right)$$

The kinetic mass $$m_t = m_k(\mu)\left(1 + \frac{16\alpha_s(\mu)}{9}\frac{\mu}{m_k}\right) \qquad \Gamma_t \sim m_k^3 \left(1 + \frac{\alpha_s}{\pi}\left(\frac{5}{3} - \frac{4\pi^2}{9} + 3\frac{16}{9}\frac{\alpha_s(\mu)}{\alpha_s(m_k)}\frac{\mu}{m_t}\right)\right) \sim m_k^3 \left(1 + \frac{\alpha_s}{\pi}\left(-2.72 + 0.1\frac{\mu}{1\mathrm{GeV}}\right)\right)$$

For other observables, such as e.g. the total cross section, the benefit of using one mass over the other is also not very obvious. The series for the pole and the MS-scheme read (approximately) $1 + 0.4_{\mathrm{NLO}} + 0.05_{\mathrm{NNLO}}$ and $1 + 0.3_{\mathrm{NLO}} - 0.07_{\mathrm{NNLO}}$, respectively. The second series is slightly better than the first one but only slightly.

Table 6: Extraction of $m_t(m_t)$ at NNLO from $\sigma_{t\bar{t}}$ using different PDF sets.

PDF set (NNLO)	$m_t(m_t)$ [GeV]
ABMP16	$161.6 \pm 1.6\ (\mathrm{fit} + \mathrm{PDF} + \alpha_S)\ ^{+0.1}_{-1.0}\ (\mathrm{scale})$
NNPDF3.1	$164.5 \pm 1.5\ (\mathrm{fit} + \mathrm{PDF} + \alpha_S)\ ^{+0.1}_{-1.0}\ (\mathrm{scale})$
CT14	$165.0 \pm 1.7\ (\mathrm{fit} + \mathrm{PDF}) \pm 0.6\ (\alpha_S)\ ^{+0.1}_{-1.0}\ (\mathrm{scale})$
MMHT14	$164.9 \pm 1.7\ (\mathrm{fit} + \mathrm{PDF}) \pm 0.5\ (\alpha_S)\ ^{+0.1}_{-1.1}\ (\mathrm{scale})$

Table 7: Extraction of m_t^{pole} at NNLO from $\sigma_{t\bar{t}}$ using different PDF sets.

PDF set (NNLO)	m_t^{pole} [GeV]
ABMP16	$169.1 \pm 1.8\ (\mathrm{fit} + \mathrm{PDF} + \alpha_S)\ ^{+1.3}_{-1.9}\ (\mathrm{scale})$
NNPDF3.1	$172.4 \pm 1.6\ (\mathrm{fit} + \mathrm{PDF} + \alpha_S)\ ^{+1.3}_{-2.0}\ (\mathrm{scale})$
CT14	$172.9 \pm 1.8\ (\mathrm{fit} + \mathrm{PDF}) \pm 0.7\ (\alpha_S)\ ^{+1.4}_{-2.0}\ (\mathrm{scale})$
MMHT14	$172.8 \pm 1.7\ (\mathrm{fit} + \mathrm{PDF}) \pm 0.6\ (\alpha_S)\ ^{+1.3}_{-2.0}\ (\mathrm{scale})$

CMS collaboration

If the top quark mass is extracted from an observable that receives non-perturbative corrections, the extracted value will be shifted accordingly. However, only linear effects are important. No reliable theory of non-perturbative effects !

$$\sigma_t = \sigma_0 \left(\frac{m_0}{m_t}\right)^5 \left(1 + c_{\rm np}\frac{\Lambda_{\rm QCD}}{m_t}\right) \quad \Rightarrow \quad m_t \to m_t - \frac{c_{\rm np}}{5}\Lambda_{\rm QCD}.$$

We can check the degree of infra-red sensitivity of various observables and identify this infrared sensitivity with the power of power corrections; this is a subject of ``renormalon'' calculus. An alternative is to run parton showers and study non-perturbative effects there.

$$\frac{-ig_{\mu\nu}}{k^2} \to \frac{-ig_{\mu\nu}}{k^2-\lambda^2} \qquad \alpha_s(\mu)\int \frac{{\rm d}^4k}{(2\pi)^4} \Rightarrow \int \frac{{\rm d}^4k}{(2\pi)^4}\alpha_s(|k|) \Rightarrow \int \frac{{\rm d}^4k}{(2\pi)^4}\, \pi\Lambda_{\rm QCD}^2\delta(|k|^2-\Lambda_{\rm QCD}^2)$$

A Landau pole

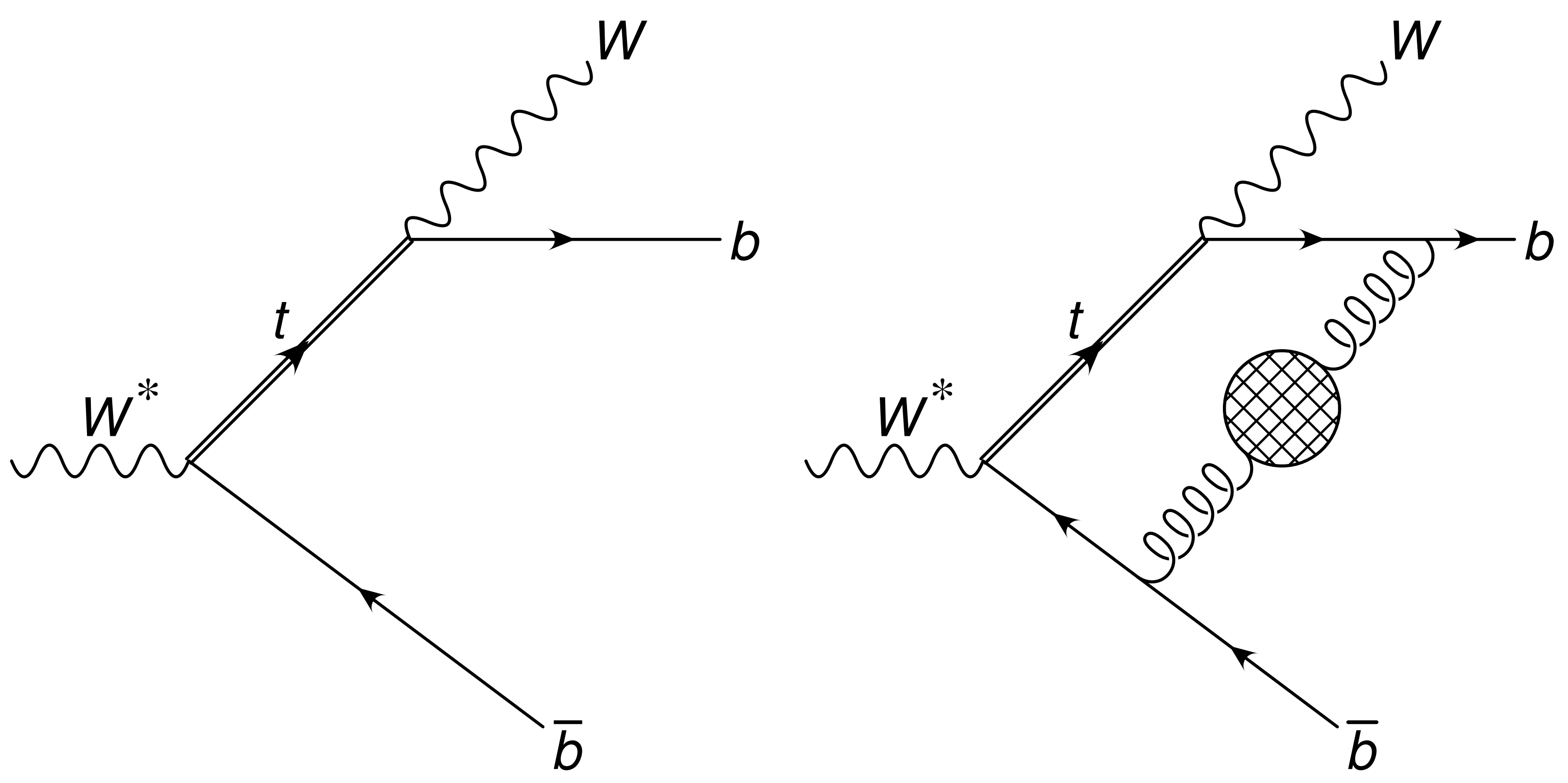
It is natural to ask how much we know about the infra-red sensitivity of various observables used for the determination of the top quark mass? To an extent that I can judge — not much.

Recently, the infra-red sensitivity of observables used in the top quark measurements was discussed in a toy model where the top quark is produced in a decay of a hypothetical vector boson.

Ravasio, Nason, Oleari

A number of different observables are studied:

1) total cross section: no infra-red sensitivity if the result is written in terms of short-distance masses; gets reflected in the convergence of perturbative series;
2) jet selection introduces linear infra-red sensitivity (1/R) ; independent of the mass definition used; this is known and anticipated effect; the jet energy/momentum is affected by the hadronization in that way;
3) W-b invariant mass has linear infra-red sensitivity due to a final state jet, irrespective of the mass definition used;
4) Average energy of the W-boson: exhibits linear infra-red sensitivity in the narrow width limit, regardless of the mass used (boost to top rest frame); however, if the calculation is done keeping the width of the top quark, there is no linear infra-red sensitivity in case the short-distance mass is used.



$$\frac{d\omega}{\omega} \Rightarrow \frac{d\omega\ \omega}{\omega^2+\Gamma^2}$$

$$\langle\omega\rangle = \int_\lambda \frac{d\omega}{\omega}\,\omega \sim \lambda \Rightarrow \int_\lambda \frac{d\omega\ \omega}{\omega^2+\Gamma^2}\omega \sim \lambda\,\frac{\lambda^2}{\Gamma^2}$$

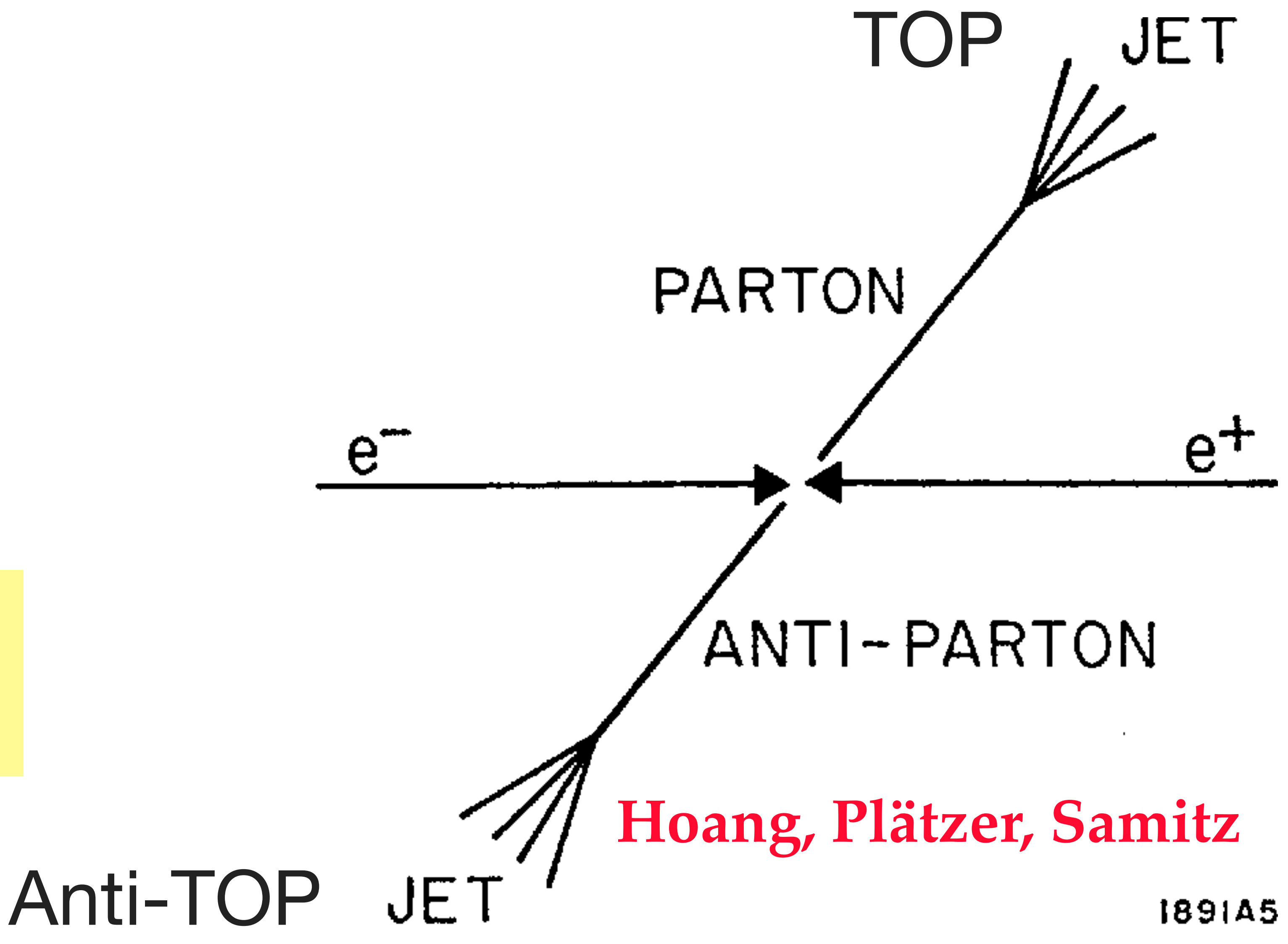
It is quite obvious that parton shower Monte-Carlo programs provide a) approximate description of the actual physics and b) work with observables that almost certainly receive linear non-perturbative corrections. Ideally, when we deal with the top quark measurement, we need to explain why, within quoted errors, both issues are not a problem.

Unfortunately, I am not aware of a clear discussion of these questions.

An analytic study of a parton-shower description of a mass-sensitive observable may help to develop answers to this and similar questions and, perhaps, question established practices. A recent study of a boosted top jets by Hoang et al. is an excellent example of that.

$$\frac{d\sigma}{d\tau}\Big|_{\rm NP} = \int {\rm d}l \, \frac{d\sigma}{d\tau}\left(\tau - \frac{l}{Q}\right) S_{\rm mod}(l)$$

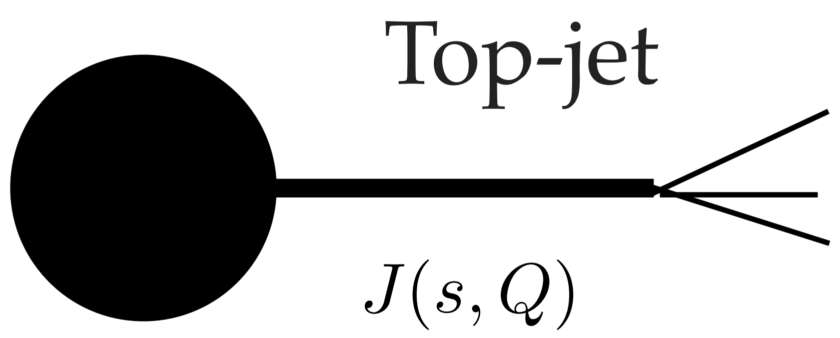
Since it is quite difficult to explain the details of what they do in the short period of time, I will discuss a simplified example.

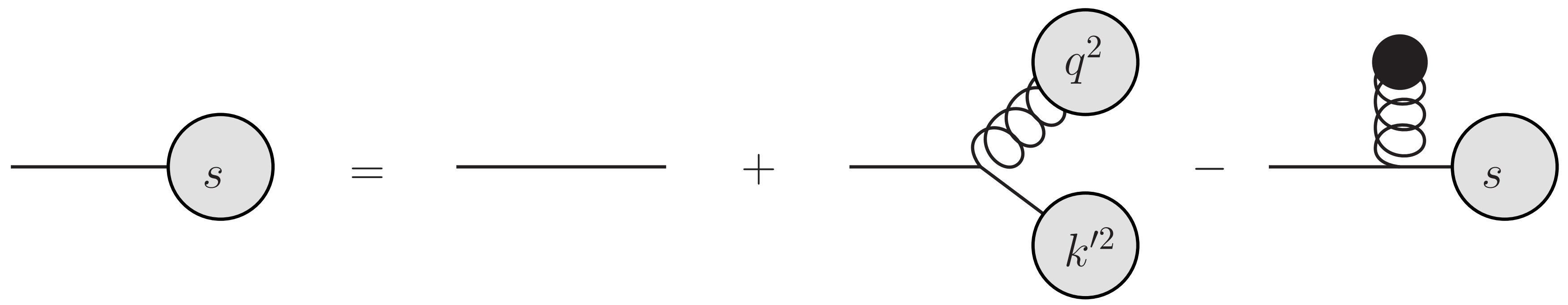


Hoang, Plätzer, Samitz

Consider production of highly-boosted top quarks that appear as top-jets. Obviously, an observable sensitive to the top quark mass is an average invariant mass of a top-jet.

To compute the average invariant mass, we recall that, in the context of coherent branching formalism, boosted massive jet can be described by a jet function. The jet function depends on two variables: the jet mass and the ``broadness'' (maximal emission angle). It satisfies the evolution equation and and the unitarity constraint.





Catani, Trentadue, Turnock, Webber

$$\int \mathrm{d}s\; J(s,Q) = 1$$

The simplified version (soft limit, gluon virtuality neglected, boosted) of the evolution equation for the jet function reads

$$J(s,Q^2,m^2) = \delta(s-m^2) + \int_{m^2}^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_0^1 \mathrm{d}z\; P_{qq}\left[\alpha_s((1-z)\tilde{q}), z, \frac{m^2}{\tilde{q}^2}\right] \times \left[J\left(s+m^2(1-z)-\tilde{q}^2(1-z),\tilde{q}^2,m^2\right) - J(s,\tilde{q}^2,m^2)\right]$$

$$P_{QQ}\left(\alpha_s, z, \frac{m^2}{\tilde{q}}^{2}\right) = \frac{C_F\alpha_s}{2\pi}\left[\frac{1+z^2}{1-z} - \frac{2m^2}{z(1-z)\tilde{q}^2}\right]$$

I would like to compute the average jet mass of a top-jet and use it to determine the top quark mass

$$\langle s \rangle = \int \mathrm{d}s\; s\, J(s,Q^2,m^2)$$

It is convenient to use the Laplace transform to compute the average jet mass.

$$J(\nu, Q, m^2) = \int \mathrm{d}s\, e^{-s\nu} J(s, Q, m^2) \qquad \langle s \rangle = -\frac{\partial}{\partial \nu} J(\nu, Q, m^2)|_{\nu=0} = -\frac{\partial}{\partial \nu} \ln J(\nu, Q, m)|_{\nu=0}$$

We solve for the jet function using Laplace transform and, upon differentiating with respect to ν, we obtain

$$\ln J(\nu, Q) = -\nu m^2 + \int_{m^2}^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_0^1 \mathrm{d}z\, P_{QQ}\left(\alpha_s((1-z)\tilde{q}), z, \frac{m^2}{\tilde{q}^2}\right) \left[e^{-\nu \tilde{q}^2/(1-z)} - 1\right]$$

$$\langle s \rangle = m^2 + \int_{m^2}^{Q^2} \mathrm{d}\tilde{q}^2 \int_0^1 \frac{\mathrm{d}z}{(1-z)} P_{QQ}\left(\alpha_s((1-z)\tilde{q}), z, \frac{m^2}{\tilde{q}^2}\right) \qquad P_{QQ}\left(\alpha_s, z, \frac{m^2}{\tilde{q}}^2\right) = \frac{C_F \alpha_s}{2\pi}\left[\frac{1+z^2}{1-z} - \frac{2m^2}{z(1-z)\tilde{q}^2}\right]$$

Since the parton shower operates with a cut-off on the transverse momentum of daughters in the branching process, we modify the above equation to account for the cut-off

$$q_\perp^2 = z^2(1-z)^2\tilde{q}^2 - (1-z)^2 m^2 > Q_0^2 \quad \Rightarrow \tilde{q}^2 - m^2 > \frac{Q_0^2}{1-z} \qquad I(Q_0) = \int_{m^2}^{Q^2} \mathrm{d}\tilde{q}^2 \int_0^1 \frac{\mathrm{d}z}{(1-z)} P_{QQ}\left(\alpha_s((1-z)\tilde{q}), z\frac{m^2}{\tilde{q}^2}\right) \theta(Q_0^2 - q_\perp^2)$$

$$\langle s \rangle = m^2 + I(Q_0) + \int_{m^2}^{Q^2} \mathrm{d}\tilde{q}^2 \int_0^1 \frac{\mathrm{d}z}{(1-z)} P_{QQ}\left(\alpha_s((1-z)\tilde{q}, z\frac{m^2}{\tilde{q}^2}\right) \theta(q_\perp^2 - Q_0^2)$$

mass with a cut-off? **Shower with the cut-off**

Because of the complexity of the constraint, the mass-shift integral receives two contributions that correspond to different integration regions over q.

$$I(Q_0) = I_1(Q_0) + I_2(Q_0) \qquad I(Q_0) = \int_{m^2}^{Q^2} d\tilde{q}^2 \int_0^1 \frac{dz}{(1-z)} P_{QQ}\left(\alpha_s((1-z)\tilde{q}), z\frac{m^2}{\tilde{q}^2}\right) \theta(Q_0^2 - q_\perp^2)$$

$$I_1(Q_0) = \frac{C_F}{2\pi}\int_0^{Q_0^2} dq^2 \alpha_s(q) \int_{1-q/m}^{1-q/Q} dz \left[\frac{2}{(1-z)^2} - \frac{2m^2}{q^2}\right] = \frac{2C_F\alpha_s(Q_0)}{\pi} Q_0(Q-2m)$$

$$I_2(Q_0) = \frac{C_F}{2\pi}\int_{Q_0^2}^{m^2} dq^2 \alpha_s(q) \int_{1-q/m}^{1-\sqrt{q^2-Q_0^2}/m} dz \left[\frac{2}{(1-z)^2} - \frac{2m^2}{q^2}\right] = \frac{2C_F\alpha_s(Q_0)}{\pi} mQ_0\left(2-\frac{\pi}{2}\right)$$

$$\Bigg\} \quad I = \frac{2C_F\alpha_s(Q_0)}{\pi} Q_0 Q - C_F\alpha_s(Q_0) m Q_0$$

Recall that the average jet-mass is

$$\langle s\rangle = m^2 + I(Q_0) + \int d\tilde{q}^2 \int \frac{dz}{(1-z)} P_{QQ}\left(\alpha_s((1-z)\tilde{q}, z\frac{m^2}{\tilde{q}^2}\right) \theta(q_\perp^2 - Q_0^2)$$

Rewriting $I(Q_0)$ to isolate the dependence on the mass, we arrive at

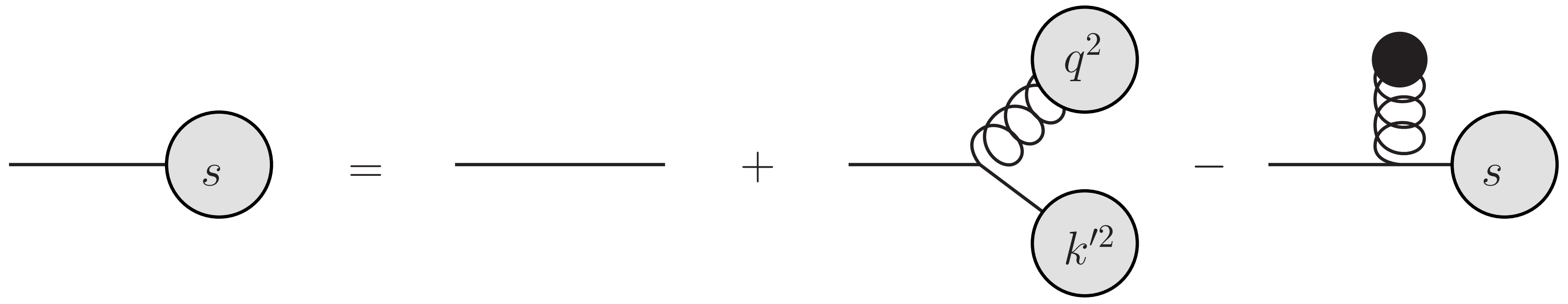
$$\langle s\rangle = \langle s_{\text{soft}}\rangle + m_{\text{CB}}^2(Q_0) + \int_{m^2}^{Q^2} d\tilde{q}^2 \int_0^1 \frac{dz}{(1-z)} P_{QQ}\left(\alpha_s((1-z)\tilde{q}), z\frac{m^2}{\tilde{q}^2}\right) \theta(q_\perp^2 - Q_0^2)$$

$$\langle s_{\text{soft}}\rangle = \frac{2C_F\alpha_s(Q_0)}{\pi} Q_0 Q \qquad m_{\text{CB}}(Q_0) = m - \frac{C_F}{2}\alpha_s(Q_0) Q_0$$

Note the appearance of the coherent-branching mass parameter that, in effect, accounts for the contribution of the region removed by imposing the cut.

We can go back now and write the evolution equation in the following way (I ignore the mass-independent shift of the average jet mass which must be ``calibrated away'' by comparing heavy and light jets)

$$J(s,Q^2,m_{\rm CB}^2) = \delta\left(s - m_{\rm CB}^2(Q_0)\right) + \int\limits_{m_{\rm CB}^2}^{Q^2} \frac{{\rm d}\tilde{q}^2}{\tilde{q}^2} P_{QQ}\left(\alpha_s((1-z)\tilde{q}), z\frac{m_{\rm CB}^2}{\tilde{q}^2}\right)\theta(q_\perp^2 - Q_0^2)$$
$$\times\left[J(s+m_{\rm CB}^2(1-z) - \tilde{q}^2(1-z), \tilde{q}^2, m_{\rm CB}^2) - J(s,\tilde{q}^2,m_{\rm CB}^2)\right]$$



$$m_{\rm CB}(Q_0) = m - \frac{C_F}{2}\alpha_s(Q_0)Q_0$$

The above equation suggests that a parton shower with a cut-off (the evolution) combined with the propagator with the cut-off dependent mass reproduces the parton shower without a cut-off. This observation is further supported by e.g computing higher moments of the jet function that are all proportional to cut-off dependent mass.

$$\langle s^n\rangle \sim \left(m_{\rm CB}(Q_0)\right)^{2n}$$

Is this a miracle? Perhaps not. If we infer the mass of an object by studying the amount of radiation that this object produces and if we restrict the amount of radiation that is allowed (cut-off), we should expect linear dependence of what we infer (reconstructed invariant mass and the mass proper) on the cut-off. This is a ``kinematic'' effect.

I was hoping that the result of this construction can be phrased in the following way:

if a parton shower is written in terms of the coherent branching mass parameter, it does not exhibit linear infra-red sensitivity.

Unfortunately, I don't see this happening (it may be that I am doing something wrong).

Indeed, the infra-red renormalon appears in I_1 but not in I_2. Easy to compute it replacing the running coupling constant with a delta-function.

$$\langle s \rangle = m^2 + I(Q_0) + \int_{m^2}^{Q^2} \mathrm{d}\tilde{q}^2 \int_0^1 \frac{\mathrm{d}z}{(1-z)} P_{QQ}\left(\alpha_s((1-z)\tilde{q}, z\frac{m^2}{\tilde{q}^2}\right) \theta(q_\perp^2 - Q_0^2)$$

$$I(Q_0) = I_1(Q_0) + I_2(Q_0)$$

$$I_2(Q_0)|_{\mathrm{NP}} \to 0$$

$$I_1(Q_0)|_{\mathrm{NP}} = \frac{C_F}{2\pi} \int_0^{Q_0^2} \mathrm{d}q^2 \, [\alpha_s(q)]_{\mathrm{NP}} \int_{1-q/m}^{1-q/Q} \mathrm{d}z \left[\frac{2}{(1-z)^2} - \frac{2m^2}{q^2}\right] \to C_F \Lambda_{\mathrm{QCD}}(Q-2m)$$

$$[\alpha_s(q)]_{\mathrm{NP}} = \pi \Lambda_{\mathrm{QCD}}^2 \delta(q^2 - \Lambda_{\mathrm{QCD}}^2)$$

A similar non-perturbative contribution to the pole mass reads $m|_{\mathrm{NP}} = \dfrac{C_F \Lambda_{\mathrm{QCD}}}{2}$

$$\left[m^2 + I(Q_0)\right]_{\mathrm{NP}} = 2\, m\, m_{\mathrm{NP}} + I_1|_{\mathrm{NP}} \to m C_F \Lambda_{\mathrm{QCD}} + C_F \Lambda_{\mathrm{QCD}}(Q-2m) = C_F \Lambda_{\mathrm{QCD}} Q - m C_F \Lambda_{\mathrm{QCD}}$$

The mass-dependent term in the above formula is a problem, in my opinion.

What is the numerical difference between the coherent-branching mass and the pole mass? Taking Q_0 equal to the mass of a tau-lepton (1.7 GeV) and the value of the strong coupling constant measured in tau-decays (0.34) we find

$$m_{\rm CB}(Q_0) = m - \frac{C_F}{2}\alpha_s(Q_0)Q_0 \qquad\qquad m_{\rm pole} - m_{\rm CB}(1.7\ {\rm GeV}) \approx 0.4\ {\rm GeV}$$

In practice, life is more complicated. Indeed, in an actual parton showers Q_0 is a fit parameter; it should account (partially) for non-perturbative effects and for the radiation removed because of the cut. So, what parton shower does is not obvious.

However, since linear infra-red effects, both perturbative and non-perturbative, are mass-independent, they should, in effect, be indentical for top and bottom jets. This should give the following result where the difference between the two b-masses accounts for both perturbative (Q_0) and non-perturbative effects.

$$m^t_{\rm MC} - m^b_{\rm MC} = m^t_{\rm pole} - m^b_{\rm pole}$$

Other issue that one has to discuss is the potential impact of the top quark width since the top quark width must soften the infra-red sensitivity making everything less sensitive to the infra-red cut.

Indeed, we know that the sensitivity to Q_0 that we just discussed arises from soft emissions; we also know that the spectrum of soft emissions is affected by the width.

$$\langle \omega \rangle = \int_{\lambda} \frac{\mathrm{d}\omega}{\omega}\, \omega \sim \lambda \;\; \Rightarrow \;\; \int_{\lambda} \frac{\mathrm{d}\omega \;\; \omega}{\omega^2 + \Gamma^2} \omega \sim \lambda \, \frac{\lambda^2}{\Gamma^2}$$

Something similar should occur in the computation of the average jet mass

$$\langle s \rangle = m^2 + c_1 m\Gamma + 2\mathrm{Re} \int_{m^2}^{Q^2} \frac{\mathrm{d}\tilde{q}^2 \mathrm{d}z \; (1-z)}{\tilde{q}^2(1-z) + i\Gamma m} \frac{C_F \alpha_s(\tilde{q}(1-z))}{2\pi} \left[\frac{1}{1 - z - i\frac{\Gamma m}{Q^2}} - \frac{m^2}{(1-z)\tilde{q}^2 - i\Gamma m} \right] \times \frac{\tilde{q}^2}{1-z}$$

The infra-red sensitivity of the r.h.s. is strongly reduced compared to the case of a stable top quark

$$\langle s \rangle = m^2 + c_1 m\Gamma + \mathcal{O}\left(\frac{\Lambda^3 (Q-m)}{\Gamma^2} \right)$$

I am not sure what to make out of this result except that it does show how the width of the top affects earlier considerations.

1) The mass parameter that one extracts from a measurement depends on an observable and not on a tool used.

2) Credible statements about precision with which the top quark mass can be extracted can only be made once non-perturbative contributions to observables are understood (c.f. the b-physics case). Some of these non-perturbative contributions disappear if a short-distance mass parameter is used, but there are definitely other non-perturbative effects that are not affected by using the short-distance mass parameter.

3) If you infer mass (energy) of an object by measuring total amount of energy that it radiates and if you restrict this energy by imposing a cut-off, you will get a linear shift in what you try to infer. I think this is kinematics.

4) Parton showers cut the soft radiation off at some scale Q_0. This leads to linear shifts proportional to Q_0 in some mass-related observables. Since, in principle, the results should be Q_0-independent, one can introduce matching shifts into physical parameters, e.g. the mass.

5) However, what this means in the context of actual parton showers isn't clear since parton-shower cut is a fitting parameter that should absorb perturbative and non-perturbative effects and add the missing radiation back in a non-perturbative fashion. In principle, this should replace the question about dependences on Q0 with the question of how relevant observables depend on non-perturbative effects; this takes us to point 2).

6) Numerically, we estimate the differences between the coherent-branching mass and the pole mass to be around 0.5 GeV although this estimate is quite imprecise.