



On the direct determination of $\sin^2\theta_{\text{eff}}(\text{leptonic})$ at hadron colliders

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Zürich, April 25th 2019
SM@LHC 2019

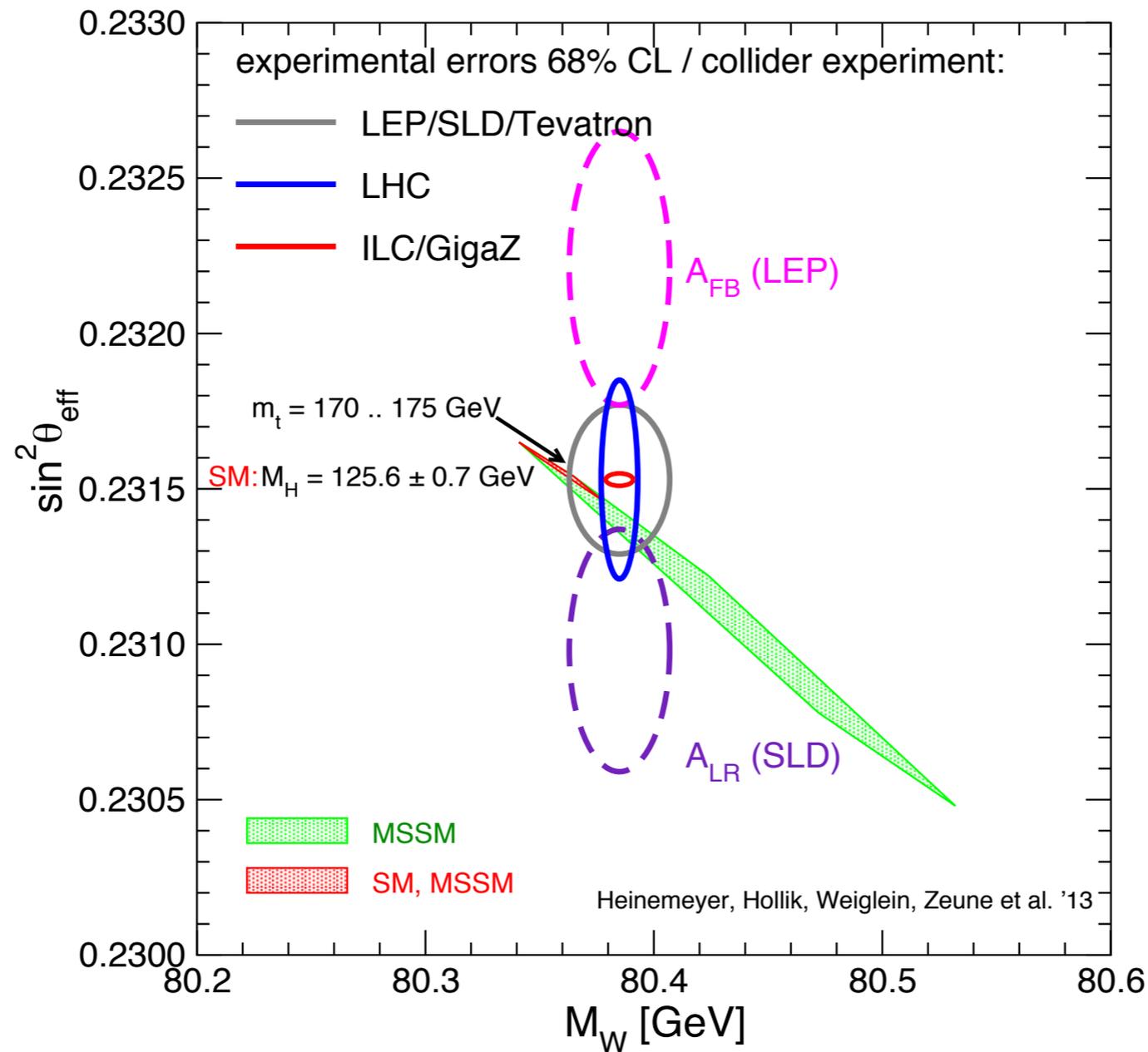
work in collaboration with Mauro Chiesa, Fulvio Piccinini and the POWHEG team

Plan of the talk

- introduction
 - motivations and definitions
 - the LEP legacy
 - measurements in the LHC framework
- an electroweak scheme with G_{μ} , M_Z , $\sin^2\theta_{\text{eff}}$ as inputs
 - one-loop renormalisation and counterterm evaluation
- numerical results for the lepton-pair invariant mass distribution and AFB
 - radiative corrections
 - comparison with different input schemes
 - parametric uncertainties
- prospects for the determination of $\sin^2\theta_{\text{eff}}$ at hadron colliders
 - template fit approach

Relevance of new high-precision measurement of EW parameters

Baak et al., arXiv:1310.6708, Snowmass 2013, EW WG



The precision measurement of M_W and $\sin^2\theta_{\text{eff}}$ with an error of 5 MeV and 0.00010 (formidable challenges!)

would offer a very stringent test of the SM likelihood

The weak mixing angle(s): definitions

- the prediction of the weak mixing angle can be computed in different renormalisation schemes differing for the systematic inclusion of large higher-order corrections

- on-shell** definition:
$$\sin^2 \theta_{OS} = 1 - \frac{m_W^2}{m_Z^2} \quad \text{definition valid to all orders}$$

- MSbar** definition:

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2} \longrightarrow \hat{s}^2 \hat{c}^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2 (1 - \Delta\hat{r})} \quad \hat{s}^2 \equiv \sin^2 \hat{\theta}$$

weak dependence on top-quark corrections

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weak dependence on top-quark corrections

- the **effective leptonic weak mixing** angle enters in the definition of the effective Z-f-fbar vertex at the Z resonance

$$\mathcal{M}_{Zl+l-}^{eff} = \bar{u}_l \gamma_\alpha [\mathcal{G}_v^f(m_Z^2) - \mathcal{G}_a^f(m_Z^2) \gamma_5] v_l \varepsilon_Z^\alpha \quad 4|Q_f| \sin^2 \theta_{eff}^\ell = 1 - \text{Re} \left(\frac{\mathcal{G}_v(m_Z^2)}{\mathcal{G}_a(m_Z^2)} \right)$$

and can be computed in the SM (or in other models) in different renormalisation schemes

$$\sin^2 \theta_{eff}^{lep} = \kappa(m_Z^2) \sin^2 \theta_{OS} = \hat{\kappa}(m_Z^2) \sin^2 \hat{\theta}$$

- the parameterization of the full two-loop EW calculation is

$$\begin{aligned} \sin^2 \theta_{eff}^f = & s_0 + d_1 L_H + d_2 L_H^2 + d_3 L_H^4 + d_4 (\Delta_H^2 - 1) + d_5 \Delta_\alpha \\ & + d_6 \Delta_t + d_7 \Delta_t^2 + d_8 \Delta_t (\Delta_H - 1) + d_9 \Delta_{\alpha_s} + d_{10} \Delta_Z, \end{aligned}$$

f	e, μ, τ	$\nu_{e, \mu, \tau}$	u, c	d, s
s_0	0.2312527	0.2308772	0.2311395	0.2310286
d_1 [10^{-4}]	4.729	4.713	4.726	4.720
d_2 [10^{-5}]	2.07	2.05	2.07	2.06
d_3 [10^{-6}]	3.85	3.85	3.85	3.85
d_4 [10^{-6}]	-1.85	-1.85	-1.85	-1.85
d_5 [10^{-2}]	2.07	2.06	2.07	2.07
d_6 [10^{-3}]	-2.851	-2.850	-2.853	-2.848
d_7 [10^{-4}]	1.82	1.82	1.83	1.81
d_8 [10^{-6}]	-9.74	-9.71	-9.73	-9.73
d_9 [10^{-4}]	3.98	3.96	3.98	3.97
d_{10} [10^{-1}]	-6.55	-6.54	-6.55	-6.55

Awramik, Czakon, Freitas, hep-ph/0608099

The LEP/SLD legacy: two distinct approaches

- SM prediction of cross sections and asymmetries and comparison with data (SM test)
SM prediction of xsecs and asym. computed as a function of $(\alpha, G_{\mu}, M_Z; m_t, M_H)$
 m_t and M_H fit to the data to maximise the agreement
with best values for the input parameters,
 $\sin^2\theta_{\text{eff}}$ has been computed in the SM using Zfitter/TOPAZ0

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- SM prediction of cross sections and asymmetries and comparison with data (SM test)
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mt and MH fit to the data to maximise the agreement
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- Extraction of $\sin^2\theta_{\text{eff}}$ from pseudo-observables introduced to describe the Z resonance
parameterisation of xsecs and asym in terms of pseudoobservables

$$m_Z, \Gamma_Z, \sigma_{had}^0, R_e^0, R_{\mu}^0, R_{\tau}^0, A_{FB}^{0,e}, A_{FB}^{0,\mu}, A_{FB}^{0,\tau}$$

fit of the model to the data → experimental values of the pseudoobservables

tree-level relation between the Z decay widths and the ratio g_V/g_A → effective angle

→ algebraic solution for $\sin^2\theta_{\text{eff}}$

- The $\sin^2\theta_{\text{eff}}$ determination from pseudo-observables depended on:
deconvolution of large universal QED/QCD corrections (Zfitter/TOPAZ0)
subtraction of SM non-factorisable contributions (Zfitter/TOPAZ0)
checked to be small compared to the LEP/SLD precision target
→ factorised expression (initial)x(final) form factors
high precision in the measurement of the xsec e^+e^- → hadrons
separation of individual flavours

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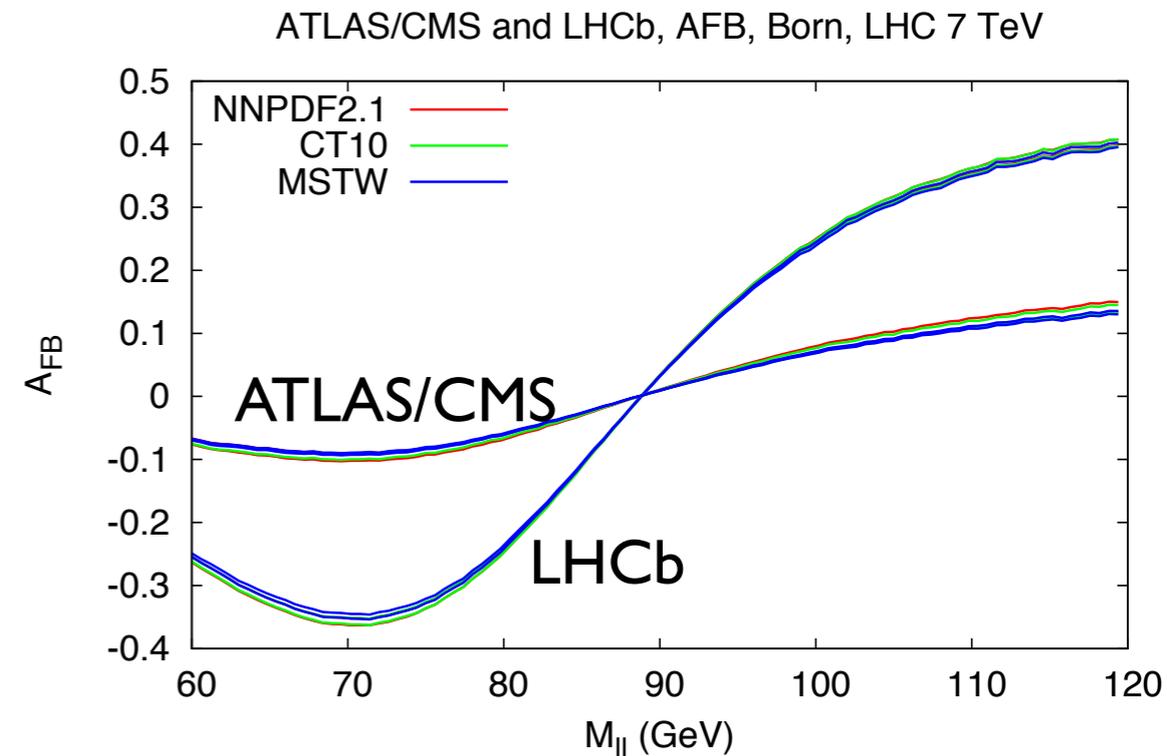
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Measurement of $\sin^2\theta_{\text{eff}}$ in the LHC framework

invariant mass forward-backward asymmetry

$$A_{FB}(M_{l+l-}) = \frac{F(M_{l+l-}) - B(M_{l+l-})}{F(M_{l+l-}) + B(M_{l+l-})}$$

$$F(M_{l+l-}) = \int_0^1 \frac{d\sigma}{d\cos\theta^*} d\cos\theta^* \quad B(M_{l+l-}) = \int_{-1}^0 \frac{d\sigma}{d\cos\theta^*} d\cos\theta^*$$



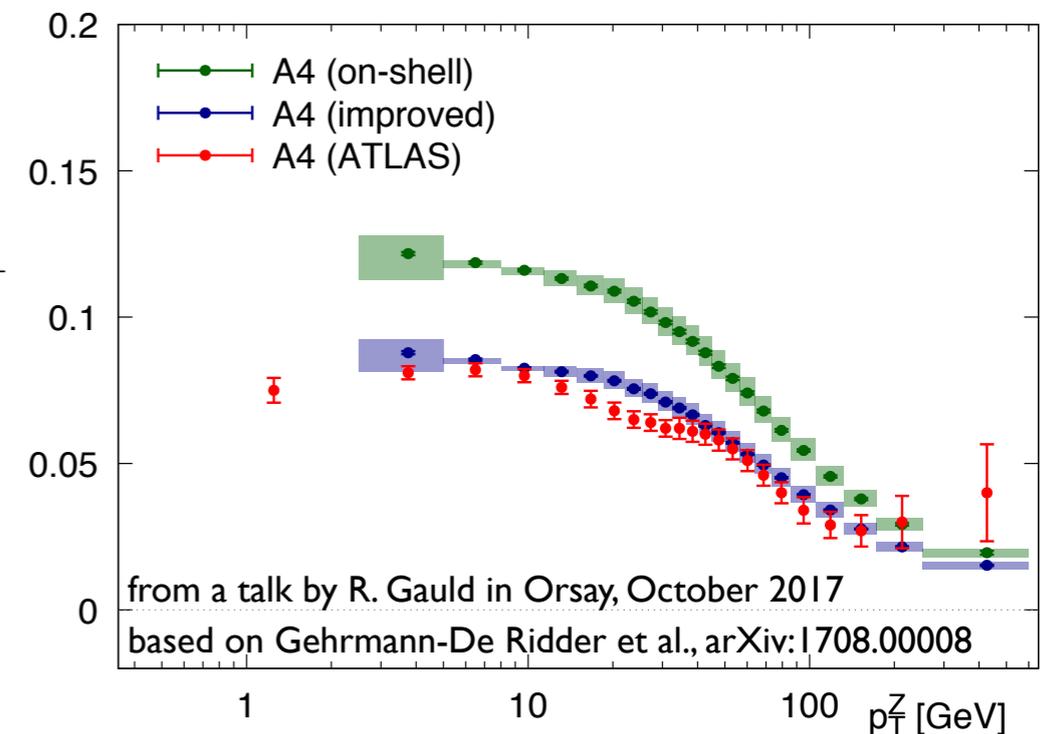
lepton-pair angular decomposition: A4 coefficient

$$\frac{d\sigma}{d^4q d\cos\theta d\phi} = \frac{3}{16\pi} \frac{d\sigma^{unpol}}{d^4q} \left\{ 1 + \cos^2\theta + \right.$$

$$A_0(1 - \cos^2\theta) + A_1 \sin(2\theta) \cos\phi + \frac{1}{2} A_2 \sin^2\theta \cos(2\phi) +$$

$$A_3 \sin\theta \cos\phi + A_4 \cos\theta +$$

$$\left. A_5 \sin^2\theta \sin(2\phi) + A_6 \sin(2\theta) \sin\phi + A_7 \sin\theta \sin\phi \right\}$$



Measurement of $\sin^2\theta_{\text{eff}}$ in the LHC framework

A few differences compared to the LEP measurement and analysis framework

- the initial state is a mixture, weighted by PDFs, of different quark flavours
 - PDF uncertainty + problems to disentangle individual Z decay widths
- the precision on the Z peak cross section is lower than the one at LEP for $e^+e^- \rightarrow \text{hadrons}$
 - σ_{had} was at LEP an important constraint of the pseudo-observable fit
- the experimental analysis might involve an invariant mass window (instead of only $q^2=M_Z^2$)
 - non-factorisable contributions might spoil the factorisation (initial)x(final) form factors

→ it is not obvious that the LEP approach in terms of pseudo-observables can be pursued at LHC

→ a template fit approach in the full SM to analyse the data would offer a well defined procedure to extract $\sin^2\theta_{\text{eff}}$

to assign the associated theoretical uncertainties

it requires some technical developments

What can we measure in a scattering process?

Cross sections and asymmetries are observables based on a counting procedure → always available

The determination of other parameters (masses, coupling constants) requires

- the choice of a model (e.g. the SM)
- the choice of an input scheme linking

the renormalised lagrangian parameters to experimental observables

→ only the input parameters can be measured in a fit to the data,
because they are the only quantities that can be freely varied

→ any derived quantity is computable in terms of the input param's

→ is fixed → can not be measured

Commonly used electroweak input schemes

$$(g, g', v; \lambda) \quad + 9 \text{ yukawa couplings} + 4 \text{ CKM param's} \quad \lambda \rightarrow m_H = v \sqrt{\lambda/2}$$

The gauge sector is parameterised by 3 independent couplings (g, g', v) .

Any other observable can/must be computed in terms of these 3 couplings.

Different possibilities to express (g, g', v) in terms of measured quantities.

$(g, g', v) \rightarrow (\alpha_0, G_\mu, m_Z)$ LEP scheme: minimal parametric uncertainty in the predictions
Z and γ diagrams have their “natural” coupling
MW and $\sin^2\theta_w$ are predictions, can not be fitted

$\rightarrow (G_\mu, m_W, m_Z)$ Gmu scheme: MW is a free parameter which can be fitted

independent of light-quark masses
it reabsorbs large logarithmic corrections

α and $\sin^2\theta_w$ are predictions, can not be fitted

$\rightarrow (\alpha_0, m_W, m_Z)$ α_0 scheme: dependent on the light-quark masses
receives large logarithmic corrections

In these schemes the weak mixing angle is not an input, is predicted \rightarrow is fixed \rightarrow can not be measured
 \rightarrow we need a scheme with $\sin^2\theta_{\text{eff}}$ among the input param's

An electroweak scheme with $(G_{\mu}, M_Z, \sin^2\theta_{\text{eff}})$ as inputs

The weak mixing angle is related to the left- and right-handed (vector and axial-vector) couplings of the Z boson to fermions

$$\sin^2 \theta_{eff}^l = \frac{I_3^l}{2Q_l} \left(1 - \frac{g_V^l}{g_A^l} \right) = \frac{I_3^l}{Q_l} \left(\frac{-g_R^l}{g_L^l - g_R^l} \right)$$

The radiative corrections (expressed with bare constants) yield left- and right-handed form factors; we focus on the scale $q^2=M_Z^2$

$$\sin^2 \theta_0 = \frac{I_3^f}{Q_f} \text{Re} \left(\frac{-\mathcal{G}_R^f(M_Z^2)}{\mathcal{G}_L^f(M_Z^2) - \mathcal{G}_R^f(M_Z^2)} \right) \Big|_0$$

We introduce the counterterms and collect their effects together with the one of the diagrams in $\delta g_{L,R}$

$$\sin^2 \theta_{eff}^l + \delta \sin^2 \theta_{eff}^l = \frac{I_3^l}{Q_l} \text{Re} \left(\frac{-g_R^l - \delta g_R^l}{g_L^l - g_R^l + \delta g_L^l - \delta g_R^l} \right)$$

The request that the tree-level relation holds to all orders fixes the counterterm for $\sin^2\theta_{\text{eff}}$

$$\delta \sin^2 \theta_{eff}^l = -\frac{1}{2} \frac{g_L^l g_R^l}{(g_L^l - g_R^l)^2} \text{Re} \left(\frac{\delta g_L^l}{g_L^l} - \frac{\delta g_R^l}{g_R^l} \right)$$

The renormalised angle is identified with the LEP leptonic effective weak mixing angle

The Z mass is defined in the complex mass scheme.

Δr is evaluated with $\sin^2\theta_{\text{eff}}$ as input and differs from the usual (α, MW, MZ) expression

Numerical results: setup and definitions

Input parameters chosen as in S.Alioli et al, arXiv:1606.02330

No acceptance cuts on the leptons, but $M_{ll} > 50$ GeV

Simulation obtained with the POWHEG QCD+EW code simulating neutral current Drell-Yan process

directory POWHEG-BOX-V2/Z_ew-BMNNPV

the public release of the implementation is forthcoming

Comparison of the results obtained in two input schemes:

$(G_{\mu}, M_Z, \sin^2\theta_{\text{eff}})$ vs (G_{μ}, M_W, M_Z)

Comparison of the results obtained in two approximations:

NLO-EW and NLO-EW+higher orders (universal corrections to the ρ parameter)

The plots show the effect of the **purely weak** virtual corrections

(gauge invariant separation between photonic and weak corrections)

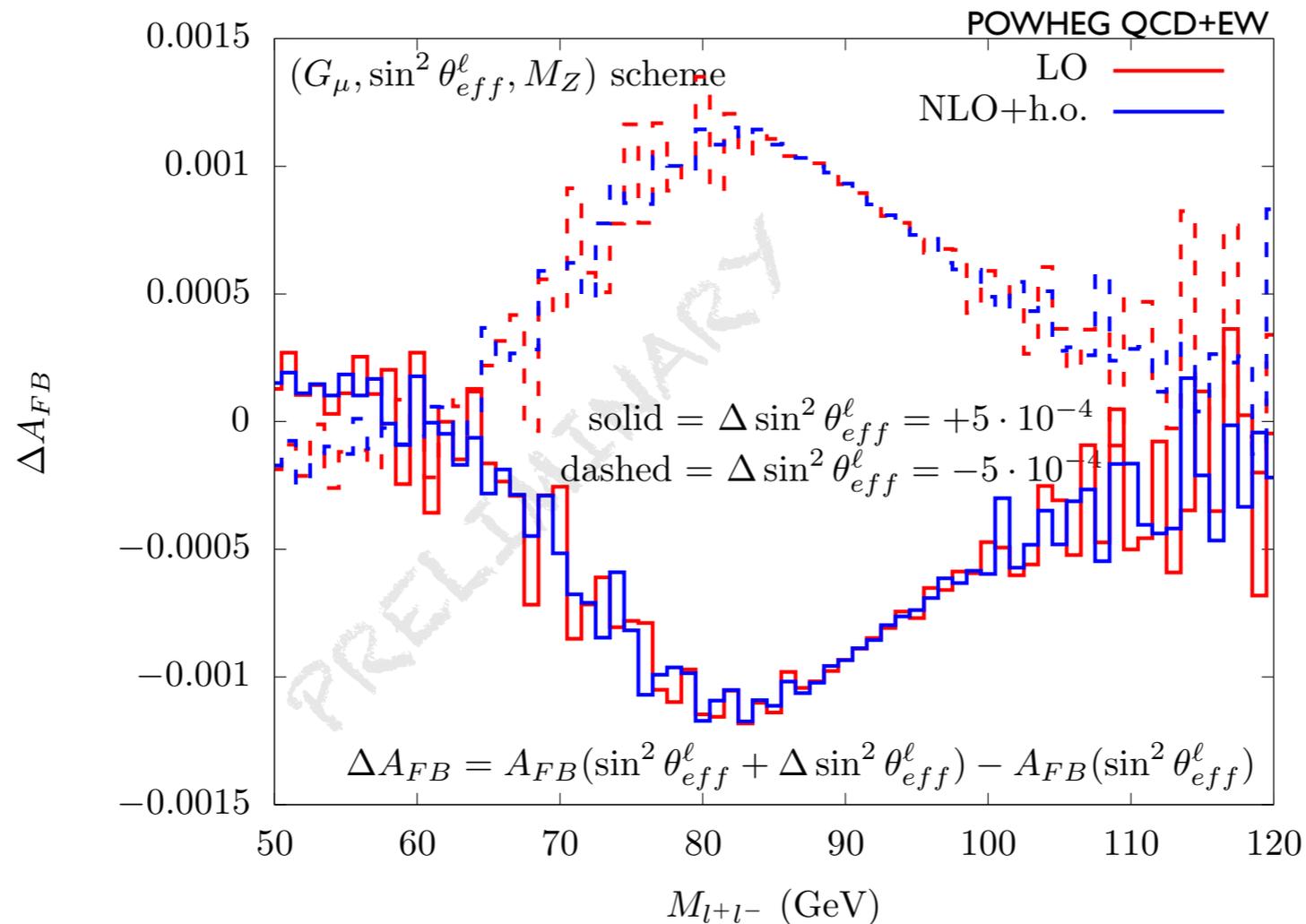
the photonic corrections modify by the same amount left- and right-handed couplings

The plots show the percentage correction (invariant mass distribution) of the absolute shift (AFB)

Huge cancellations affect the convergence of the MC simulations

All the results are still preliminary!

AFB sensitivity to the weak mixing angle value



The AFB sensitivity to the precise $\sin^2 \theta_{eff}$ value sets the experimental precision scale for a measurement at hadron colliders competitive with LEP

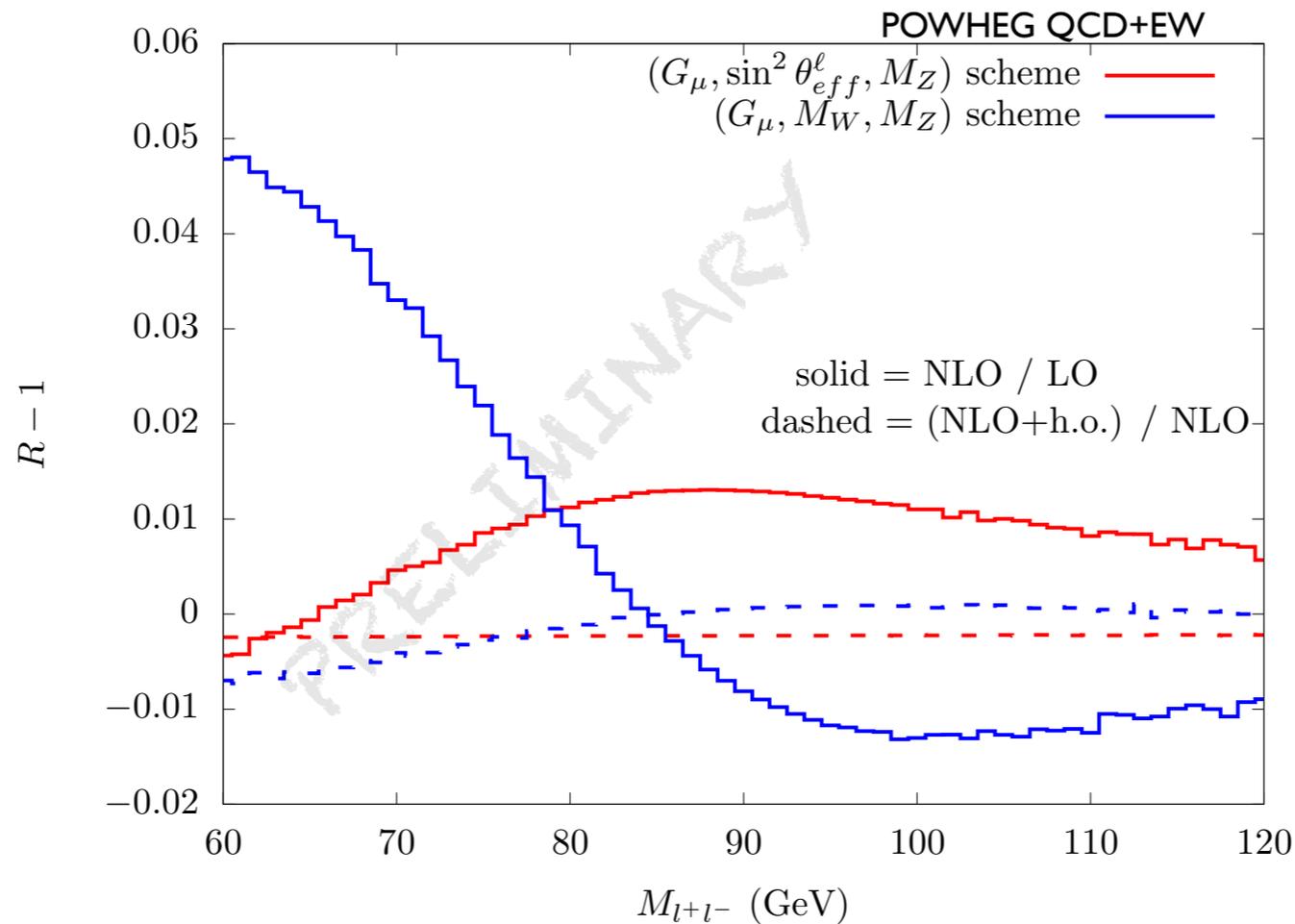
LEP final result: $0.23153(16)$

Without acceptance cuts, stronger variation ΔA_{FB} (almost a factor 2) under a given $\Delta \sin^2 \theta_{eff}$ change

We consider relevant all the effects yielding a $\Delta A_{FB} \sim 1 \times 10^{-4}$

The sensitivity is dominated by the LO behaviour

NLO-EW and higher-order corrections: invariant mass distribution

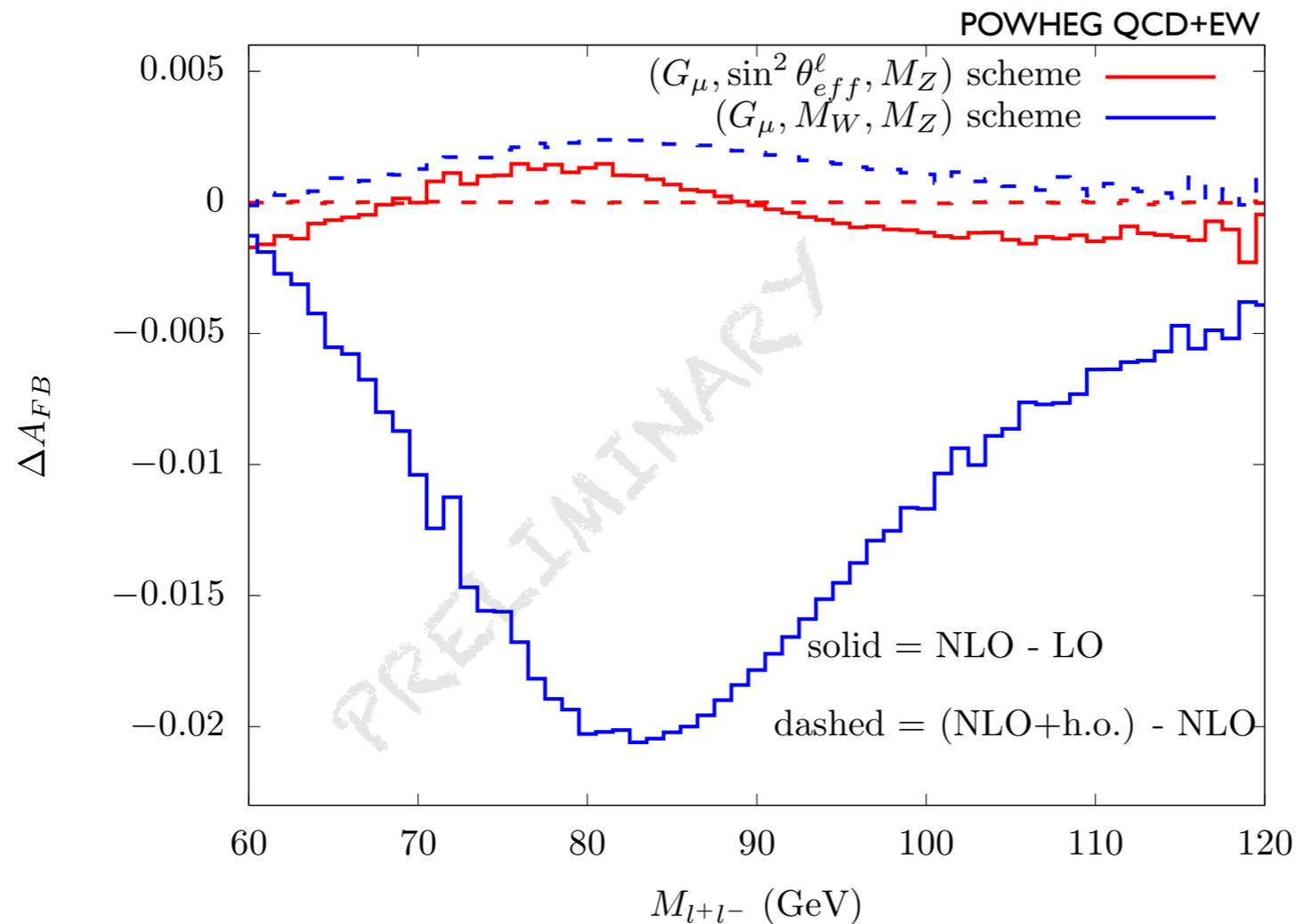


The radiative corrections in the $(G_\mu, M_Z, \sin^2 \theta_{eff})$ scheme are smaller in size than in the (G_μ, M_W, M_Z) scheme

The $(G_\mu, M_Z, \sin^2 \theta_{eff})$ is very stable against additional higher orders

The Z resonance is naturally parameterised in terms of a normalisation factor (G_μ), the peak position (M_Z) and a line-shape parameter ($\sin^2 \theta_{eff}$)
→ the experimental values reabsorb a large fraction of the radiative corrections at $q^2 = M_Z^2$, leaving a small remainder away from the peak
→ this scheme is well suited to describe neutral current Drell-Yan

NLO-EW and higher-order corrections: AFB

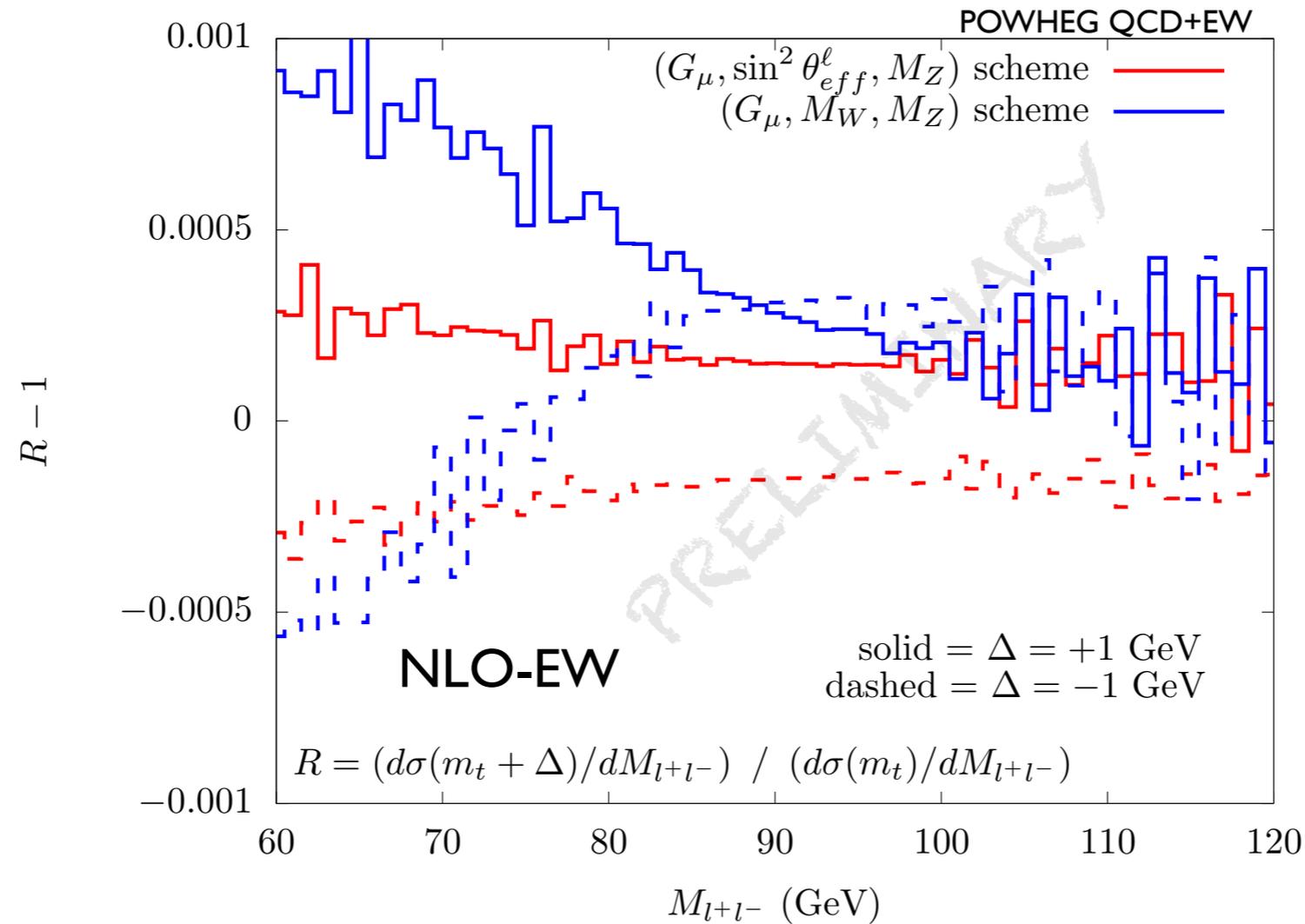


The radiative corrections in the $(G_\mu, M_Z, \sin^2 \theta_{eff})$ scheme are one order of magnitude smaller than in the (G_μ, M_W, M_Z) scheme

The $(G_\mu, M_Z, \sin^2 \theta_{eff})$ is very stable against additional higher orders

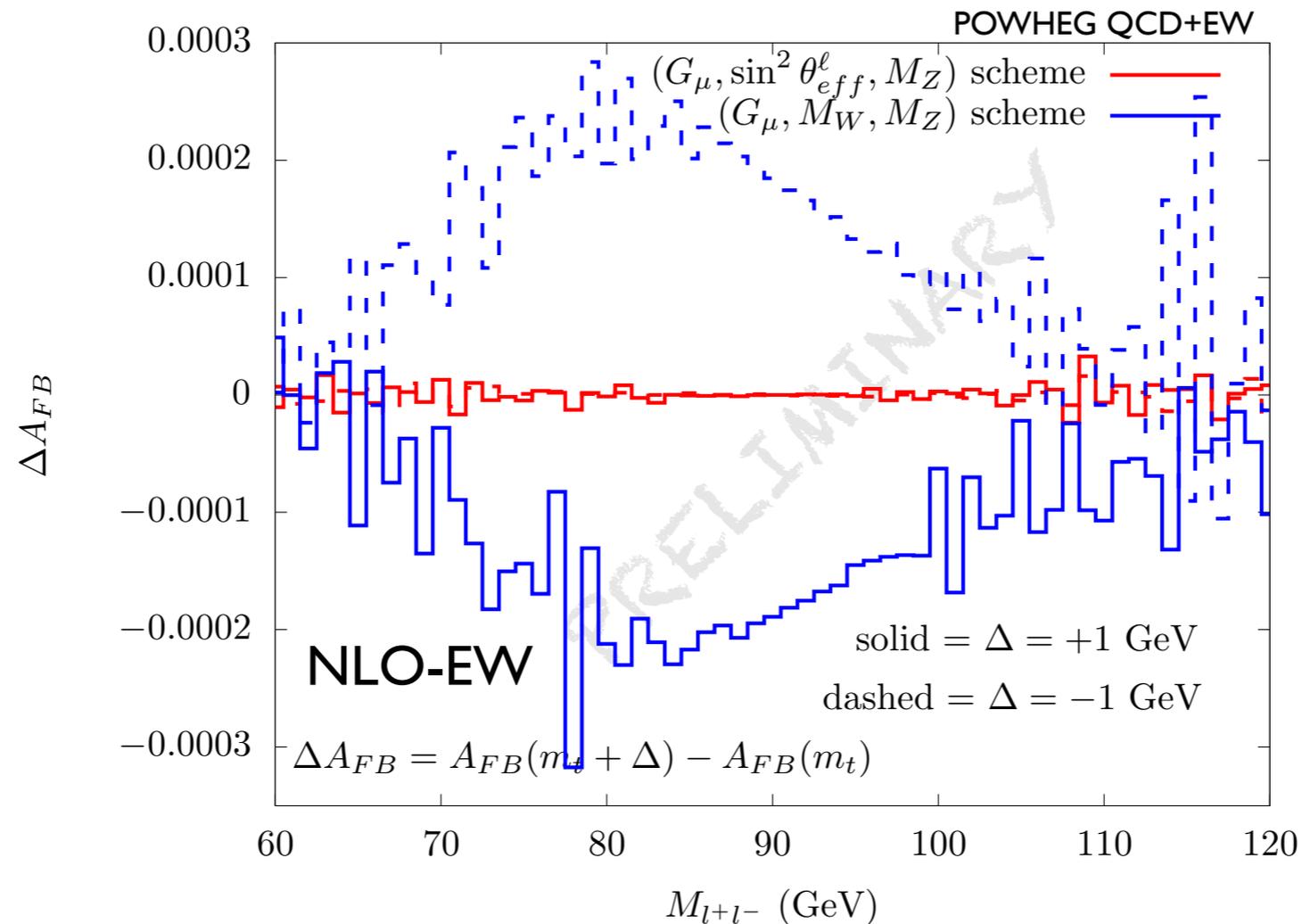
→ any determination of $\sin^2 \theta_{eff}$ using this scheme will suffer of small theoretical systematic uncertainties due to missing higher-order corrections

Parametric uncertainties with m_{top} : invariant mass distribution



- The m_t dependence in the $(G_\mu, M_Z, \sin^2 \theta_{eff})$ scheme is weaker than in the (G_μ, M_W, M_Z) scheme
- because of the absence of m_t^2 enhancement factors in the radiative corrections after renormalisation
 - m_t^2 factors are present instead in the (G_μ, M_W, M_Z) scheme via δM_W^2

Parametric uncertainties with $m_{\text{top}}: A_{FB}$



The m_t dependence in the $(G_\mu, M_Z, \sin^2 \theta_{eff}^l)$ scheme is weaker than in the (G_μ, M_W, M_Z) scheme

- because of the absence of m_t^2 enhancement factors in the radiative corrections after renormalisation
- m_t^2 factors are present instead in the (G_μ, M_W, M_Z) scheme via δM_W^2

→ any determination of $\sin^2 \theta_{eff}^l$, using this scheme, will suffer of small parametric uncertainties

Prospects: studies of theoretical systematics on $\sin^2\theta_{\text{eff}}$ determination

The presence of $\sin^2\theta_{\text{eff}}$ among the input parameters allows

- 1) to look for the best $\sin^2\theta_{\text{eff}}$ value maximising the agreement with the data
- 2) to perform a clean assessment of theoretical systematics on the final result

The definition of a model to fit the data involves several choices

e.g. PDF set, QCD accuracy, EW accuracy, matching with QCD/QED Parton Shower

can we quantify the result spread w.r.t. different equivalent choices

the error due to the missing inclusion of a set of higher-order corrections ?

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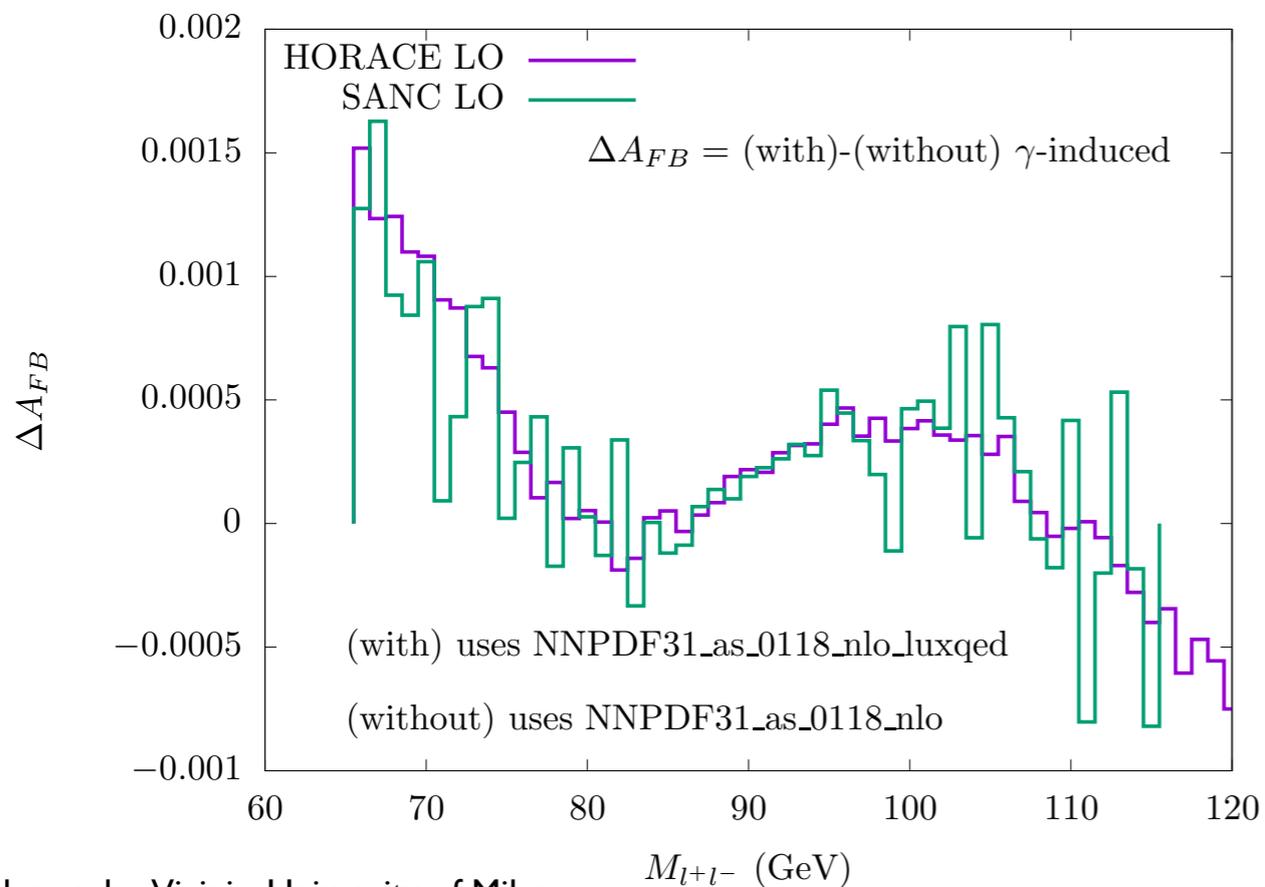
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The CERN EW WG subgroup on Drell-Yan precision measurement is addressing a survey of all higher-order effects relevant for the $\sin^2\theta_{\text{eff}}$ determination



example: the role of γ -induced subprocesses

- 1) evidence of a non-trivial distortion
- 2) need to translate into a $\sin^2\theta_{\text{eff}}$ shift
- 3) add to the th. error in any analysis not including γ -induced contributions and PDFs

Conclusions

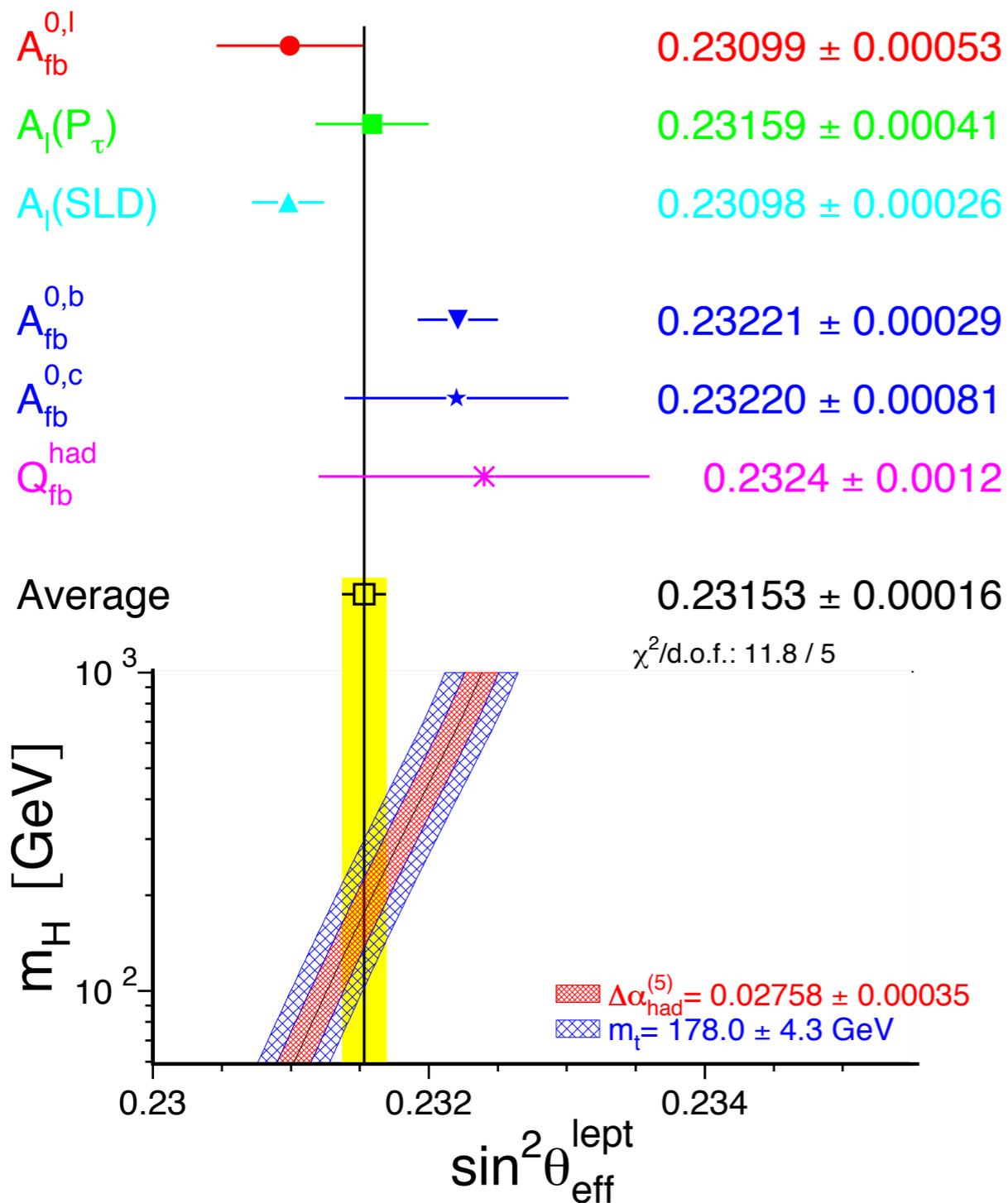
NLO-EW renormalization in a scheme with $(G_{\mu}, M_Z, \sin^2\theta_{\text{eff}})$ as inputs has been presented

The $(G_{\mu}, M_Z, \sin^2\theta_{\text{eff}})$ scheme offers a possibility to perform a consistency test of the SM
finding the best $\sin^2\theta_{\text{eff}}$ value compatible with the data
via template fit of the kinematical distributions in the full SM

This scheme uses as input the effective leptonic weak mixing angle according to the LEP definition
reabsorbs in the inputs large radiative corrections relevant at the Z resonance
has a weak dependence on m_t
allows to quantify the impact of residual theoretical uncertainties on the $\sin^2\theta_{\text{eff}}$ determination
will soon be available in POWHEG QCD+EW

Backup slides

Results from LEP and SLC: $\sin^2\theta_{\text{eff}}^{\text{leptonic}}$



- good sensitivity to the Higgs mass value
- tension between SLD and LEP results
- tension between leptonic and b-quark asymmetries

an independent measurement at hadron colliders can help to test the likelihood of the SM