

Recent results from proton intermittency analysis in nucleus-nucleus collisions from NA61/SHINE at CERN SPS

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1 QCD Phase Diagram and Critical Phenomena

2 Method of intermittency analysis

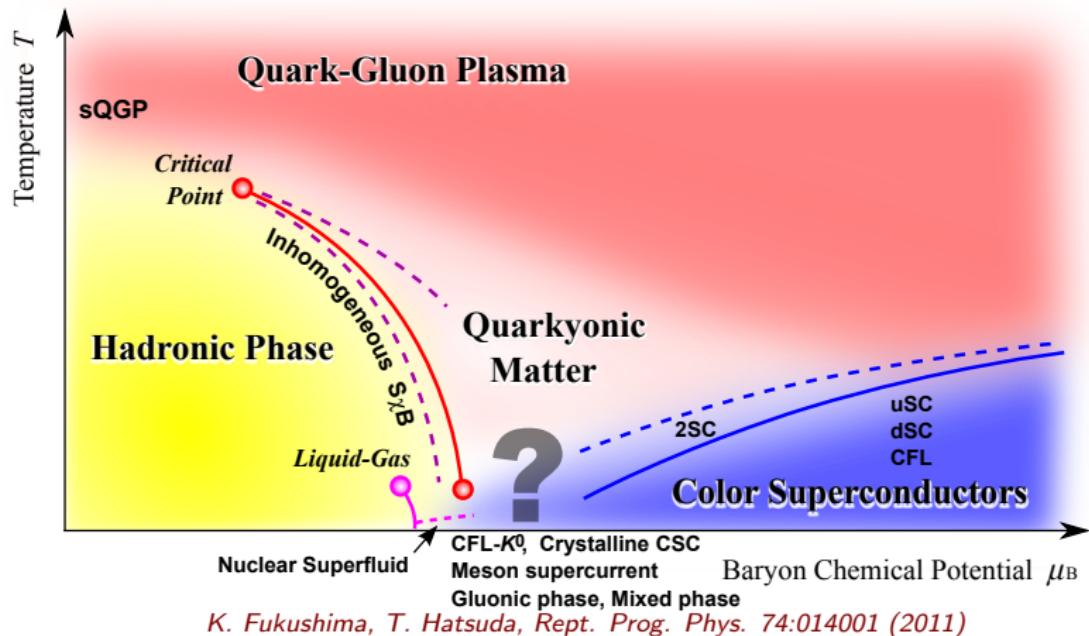
3 Previously released results at 150/158A GeV/c

4 New results on Ar+Sc at 150A GeV/c

5 Summary and outlook

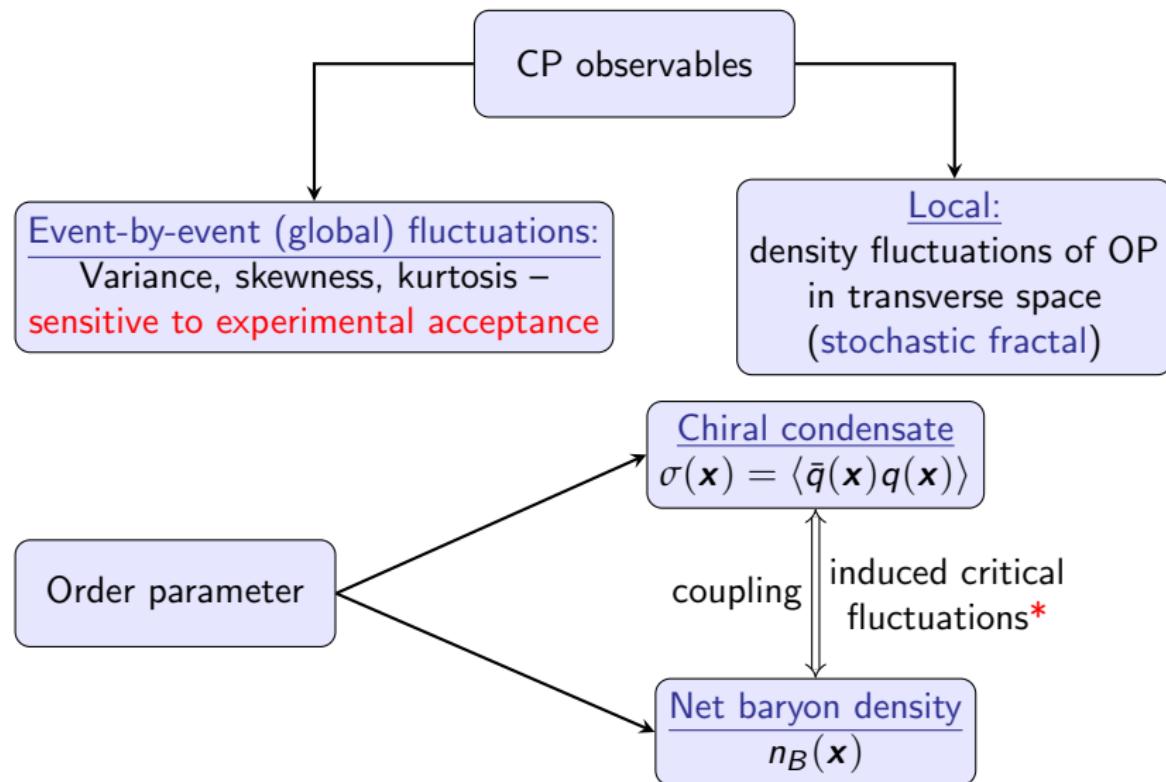
Phase diagram of QCD

- Objective: Detection / existence of the QCD Critical Point (CP)



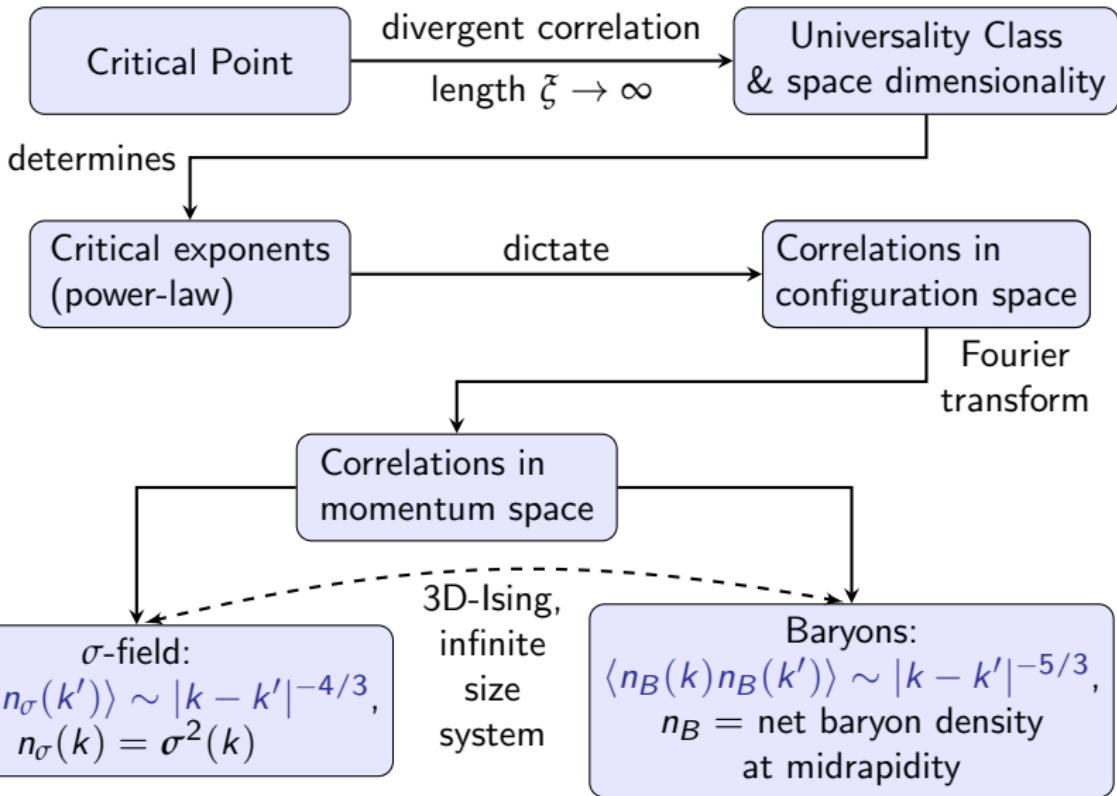
- Look for observables tailored for the CP; Scan phase diagram by varying energy and size of collision system.

Critical Observables; the Order Parameter (OP)



*[Y. Hatta and M. A. Stephanov, PRL91, 102003 (2003)]

Self-similar density fluctuations near the CP



Observing power-law fluctuations

Experimental observation of local, power-law distributed fluctuations



↓
Intermittency in transverse momentum space (net protons at mid-rapidity)
(Critical opalescence in ion collisions*)

- Net proton density carries the same critical fluctuations as the net baryon density, and can be substituted for it.
[Y. Hatta and M. A. Stephanov, PRL91, 102003 (2003)]
- Furthermore, antiprotons can be dropped to the extent that their multiplicity is much lower than of protons, and proton density analyzed.

[J. Wosiek, *Acta Phys. Polon. B* **19** (1988) 863-869]

[A. Bialas and R. Hwa, *Phys. Lett. B* **253** (1991) 436-438]

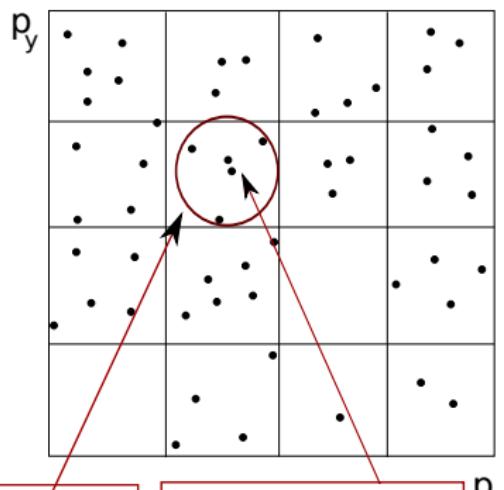
*[F.K. Diakonos, N.G. Antoniou and G. Mavromanolakis, PoS (CPOD2006) 010, Florence]

Observing power-law fluctuations: Factorial moments

- Transverse momentum space is partitioned into M^2 cells
- Calculate second factorial moments $F_2(M)$ as a function of cell size \Leftrightarrow number of cells M :

$$F_2(M) \equiv \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i - 1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^2},$$

where $\langle \dots \rangle$ denotes averaging over events.



Subtracting the background from factorial moments

- Experimental data is **noisy** \Rightarrow a **background** of non-critical pairs must be subtracted at the level of factorial moments.
- Intermittency will be revealed at the level of **subtracted moments** $\Delta F_2(M)$.

Partitioning of pairs into critical/background

$$\langle n(n-1) \rangle = \underbrace{\langle n_c(n_c - 1) \rangle}_{\text{critical}} + \underbrace{\langle n_b(n_b - 1) \rangle}_{\text{background}} + \underbrace{2\langle n_b n_c \rangle}_{\text{cross term}}$$

$$\underbrace{\Delta F_2(M)}_{\text{correlator}} = \underbrace{F_2^{(d)}(M)}_{\text{data}} - \lambda(M)^2 \cdot \underbrace{F_2^{(b)}(M)}_{\text{background}} - 2 \cdot \underbrace{\lambda(M)}_{\text{ratio}} \cdot \frac{\langle n_b \rangle}{\langle n_d \rangle}$$

- The **cross term** can be neglected under certain conditions (non-trivial! Justified by **Critical Monte Carlo*** simulations)

* [Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]

Scaling of factorial moments – Subtracting mixed events

For $\lambda \lesssim 1$ (background domination), two approximations can be applied:

- ① Cross term can be neglected
- ② Non-critical background moments can be approximated by (uncorrelated) mixed event moments; then,

$$\Delta F_2(M) \simeq \Delta F_2^{(e)}(M) \equiv F_2^{\text{data}}(M) - F_2^{\text{mix}}(M)$$

For a critical system, ΔF_2 scales with cell size (number of cells, M) as:

$$\Delta F_2(M) \sim (M^2)^{\varphi_2}$$

where φ_2 is the intermittency index.

Theoretical prediction for φ_2

universality class:
effective actions

$$\left\{ \begin{array}{l} \varphi_{2,cr}^{(p)} = \frac{5}{6} \quad (0.833\dots) \\ \text{net baryons (protons)} \\ \text{[N. G. Antoniou, F. K. Diakonos, A. S. Kapoyannis,} \\ \text{K. S. Kousouris, Phys. Rev. Lett. 97, 032002 (2006)]} \end{array} \right.$$

Statistical uncertainties & systematic effect estimation

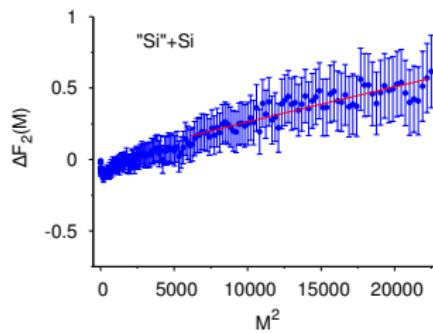
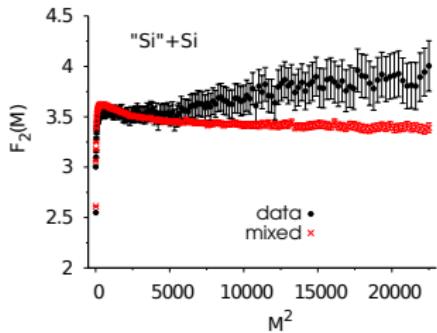
- Bootstrap method used to calculate statistical uncertainties
- Bootstrap samples of events created by sampling of events with replacement
- $\Delta F_2(M)$ calculated for each bootstrap sample; variance of sample values provides statistical error of $\Delta F_2(M)$

[W.J. Metzger, "Estimating the Uncertainties of Factorial Moments", HEN-455 (2004).]
- Distribution of φ_2 values, $P(\varphi_2)$, and confidence intervals for φ_2 obtained by fitting individual bootstrap samples

[B. Efron, *The Annals of Statistics* 7,1 (1979)]
- Systematic uncertainties arise from:
 - Misidentification of protons & detector effects (e.g. acceptance)
 - The fact that $F_2(M)$ are correlated for different bin sizes M
 - Selection of M -range to fit for power-law
- Bin correlations are partially handled by the bootstrap φ_2 distribution
- Other systematic uncertainties are estimated by varying proton and M -range selection

NA49: "C"+C, "Si"+Si, Pb+Pb at 158A GeV/c

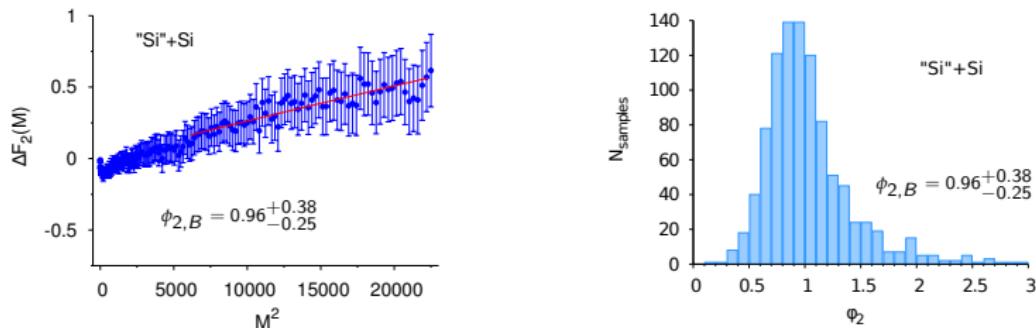
- 3 sets of NA49 collision systems were analysed, at 158A GeV/c
[T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]
- Factorial moments of proton transverse momenta analyzed at mid-rapidity
- Fragmentation beams used for C and Si ("C"=C,N ; "Si"=Si,Al,P) – components were merged to enhance statistics



- Fit with $\Delta F_2^{(e)}(M ; \mathcal{C}, \phi_2) = e^{\mathcal{C}} \cdot (M^2)^{\phi_2}$, for $M^2 \geq 6000$
- No intermittency detected in the "C"+C, Pb+Pb datasets.

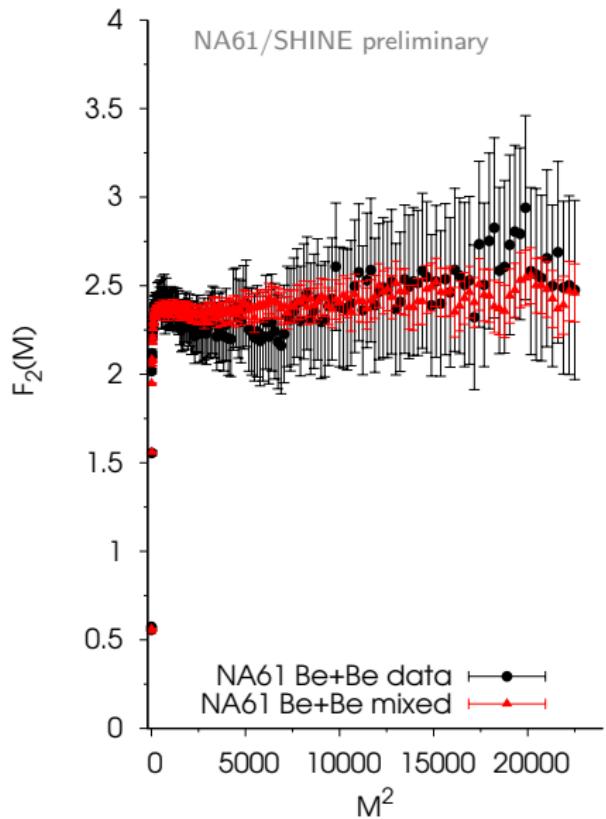
NA49: "C" + C, "Si" + Si, Pb + Pb at 158A GeV/c

- Evidence for intermittency in "Si" + Si – but large statistical errors.



- Bootstrap distribution of ϕ_2 values is highly asymmetric due to closeness of $F_2^{(d)}(M)$ to $F_2^{(m)}(M)$.
- Based on CMC simulation, we estimate a fraction of $\sim 1\%$ critical protons are present in the sample.
- Estimated intermittency index: $\phi_{2,B} = 0.96^{+0.38}_{-0.25}$ (stat.) ± 0.16 (syst.)
[T. Anticic et al., Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]

NA61/SHINE: Be+Be at 150A GeV/c

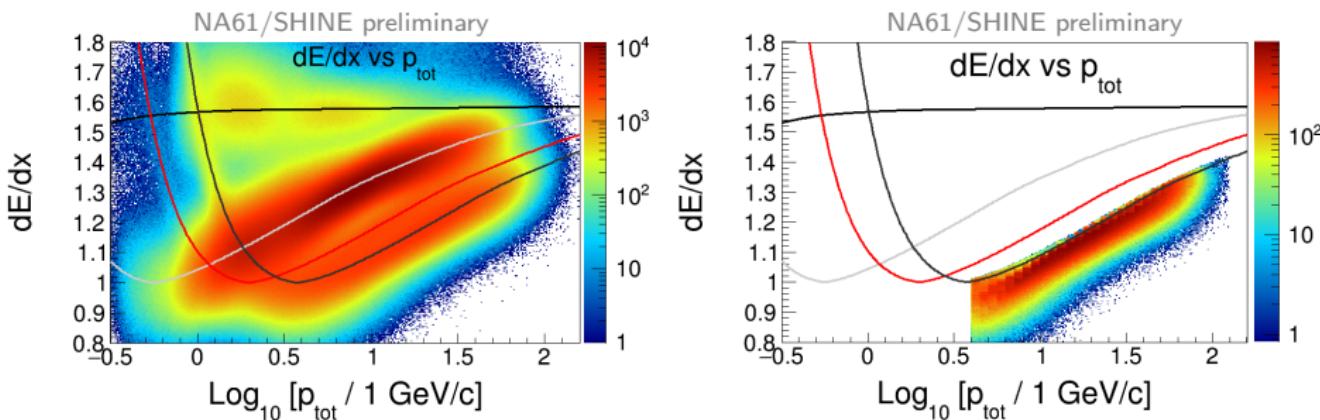


- $F_2(M)$ of data and mixed events overlap ⇒
 - Subtracted moments $\Delta F_2(M)$ fluctuate around zero ⇒
 - No intermittency effect is observed.
 - Preliminary analysis with CMC simulation indicates an upper limit of $\sim 0.3\%$ critical protons
- [PoS(CPOD2017) 054]

NA61/SHINE: $^{40}Ar + ^{45}Sc$ at 150A GeV/c

- First released results of preliminary analysis in Ar+Sc at 150A GeV/c – CPOD 2018.
- Intermittency analysis process:
 - Proton selection via particle energy loss dE/dx
 - Removal of split tracks – q_{inv} distribution & cut of proton pairs
 - Probe Δp_T distribution of proton pairs for power-law like behaviour in the limit of small p_T differences
 - Calculate factorial moments $F_2(M), \Delta F_2(M)$ for selected protons
 - Calculate intermittency index ϕ_2 (when possible) & estimate its statistical uncertainty
- Results were obtained for:
 - 0-5%, 5-10% and 10-15% centrality bins
 - 80%, 85% and 90% minimum proton purity selections

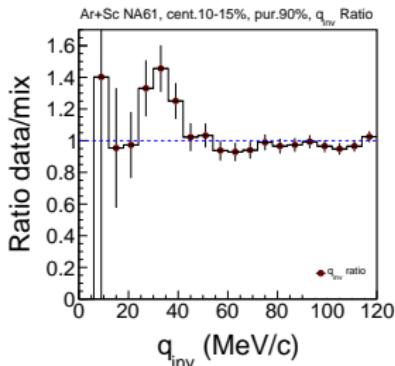
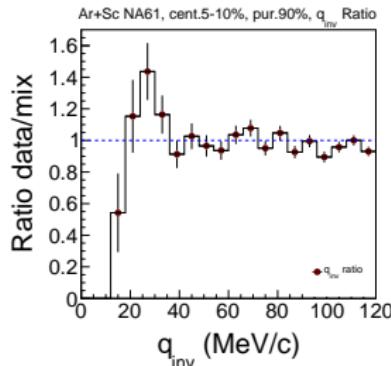
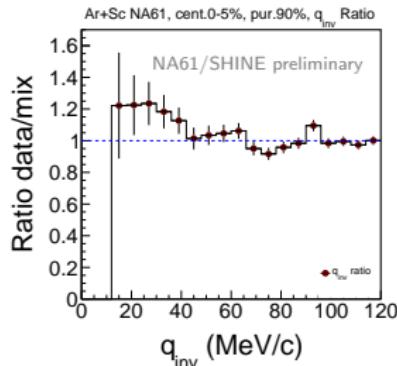
Proton selection



- Employ p_{tot} region where Bethe-Bloch bands **do not** overlap ($3.98 \text{ GeV}/c \leq p_{\text{tot}} \leq 126 \text{ GeV}/c$)
- Fit dE/dx distribution with **4-gaussian sum** for $\alpha = \pi, K, p, e$ – Bins: p_{tot}, p_T
- 30 Bins in $\text{Log}_{10}(p_{\text{tot}})$: $10^{0.6} \rightarrow 10^{2.1} \text{ GeV}/c$
- 20 Bins in p_T : $0.0 \rightarrow 2.0 \text{ GeV}/c$
- Proton purity: **probability** for a track to be a proton, $\mathcal{P}_p = p/(\pi + K + p + e)$
- **Additional cut along Bethe-Blochs**
(avoid low-reliability region between p and K curves)

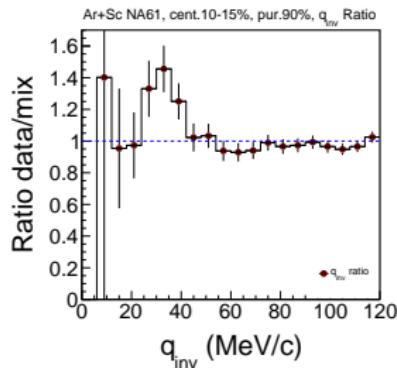
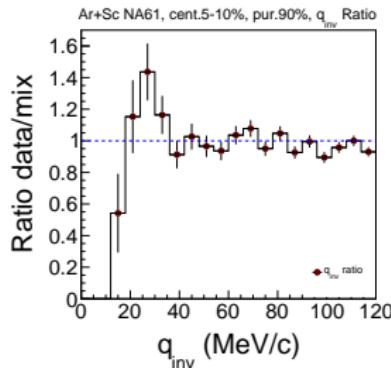
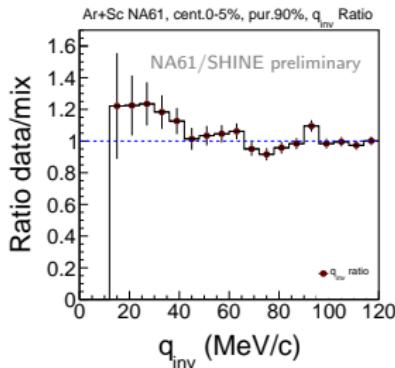
Split tracks & the q_{inv} cut

- Events may contain **split tracks**: sections of the same track erroneously identified as **a pair of tracks** that are close in momentum space.
- Three cuts to root them out:
 - Ratio of points / potential points in a track (removes most)
 - Minimum track distance in the detector (pair cut)
 - q_{inv} cut (pair cut, physics-significant)
- q_{inv} distribution of track pairs probed in order to root the rest out:
$$q_{inv}(p_i, p_j) \equiv \frac{1}{2} \sqrt{-(p_i - p_j)^2}, p_i : \text{4-momentum of } i^{\text{th}} \text{ track.}$$
- We calculate the ratio of $q_{inv}^{data} / q_{inv}^{mixed}$.



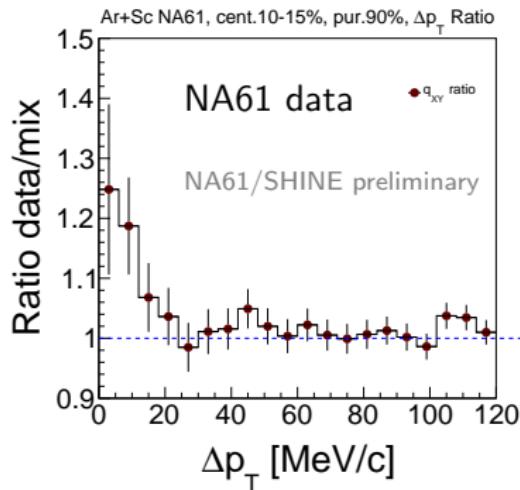
Split tracks & the q_{inv} cut

- A peak at low q_{inv} (below 20 MeV/c) indicates a possible split track contamination that must be removed.
- Anti-correlations due to F-D effects and Coulomb repulsion must be removed before intermittency analysis \Rightarrow “dip” in low q_{inv} , peak predicted around 20 MeV/c [Koonin, PLB 70, 43-47 (1977)]
- Universal cutoff of $q_{inv} > 7$ MeV/c applied to all sets before analysis.

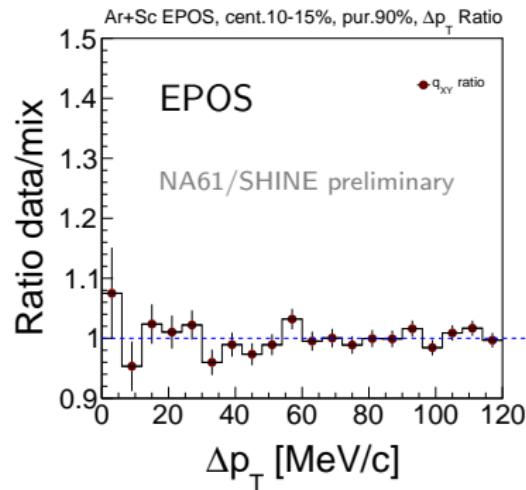


Δp_T distributions: NA61 data vs EPOS

- Ar+Sc at 150A GeV/c: $\Delta p_T = 1/2 \sqrt{(p_{X_1} - p_{X_2})^2 + (p_{Y_1} - p_{Y_2})^2}$
distributions of protons selected for intermittency analysis



Significant peak
at $\Delta p_T \rightarrow 0$

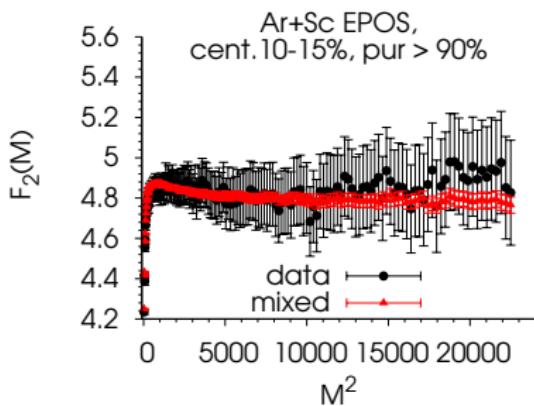
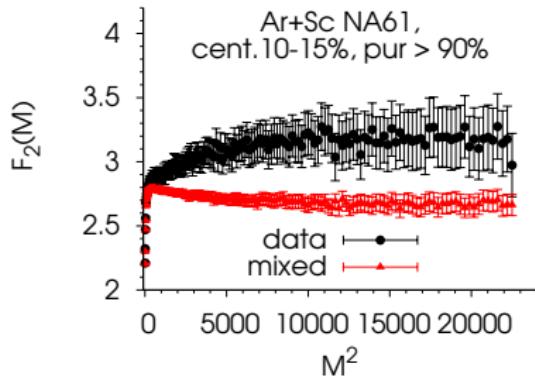
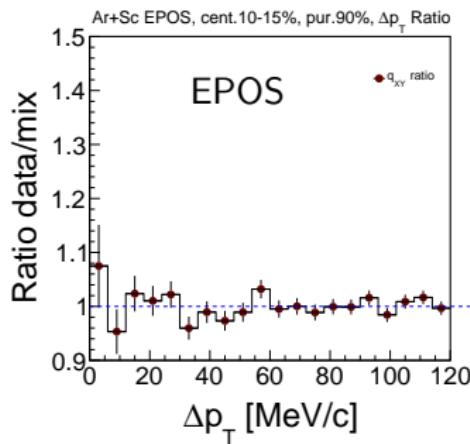
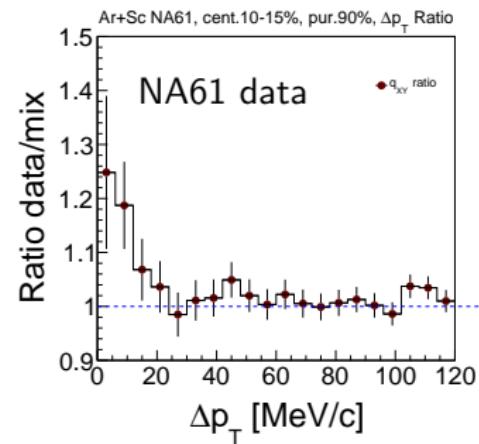


Flat distribution

- In NA61 data, we see strong correlations in $\Delta p_T \rightarrow 0 \Rightarrow$ indication of intermittent behaviour

Δp_T distributions & $F_2(M)$: NA61 data vs EPOS

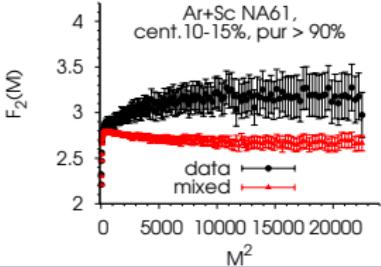
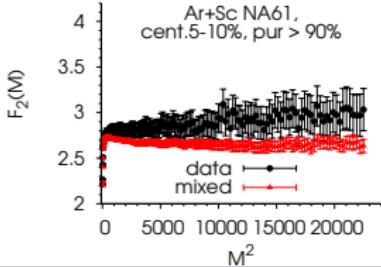
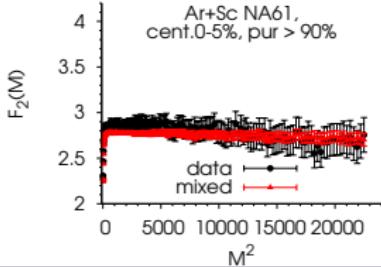
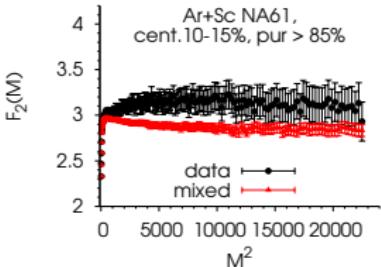
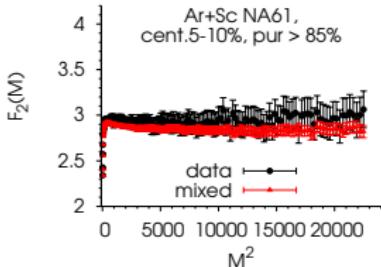
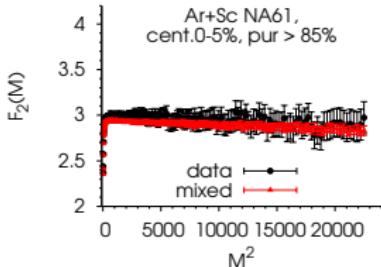
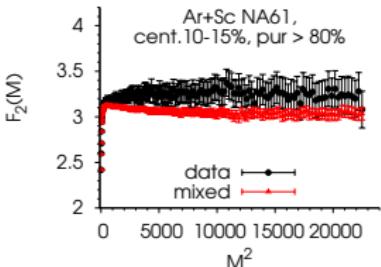
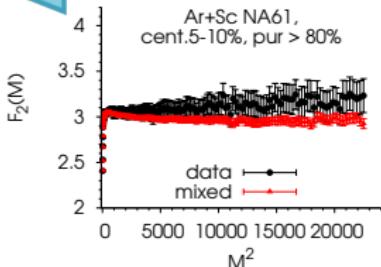
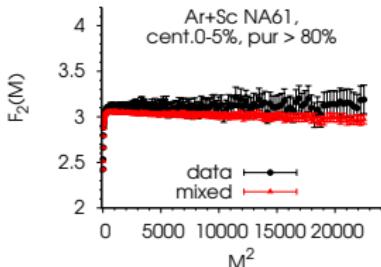
NA61/SHINE preliminary



NA61/SHINE: Ar+Sc at 150A GeV/c: $F_2(M)$

NA61/SHINE preliminary

centrality



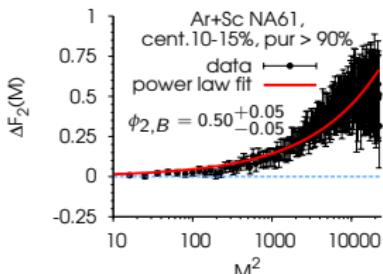
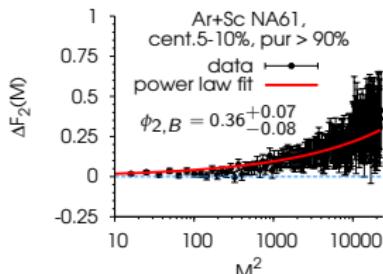
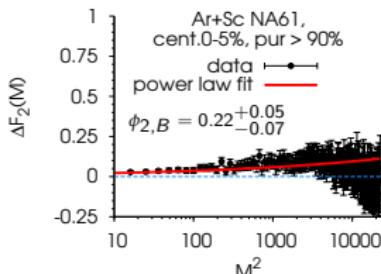
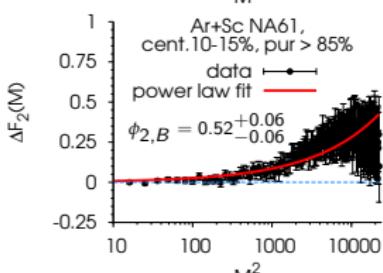
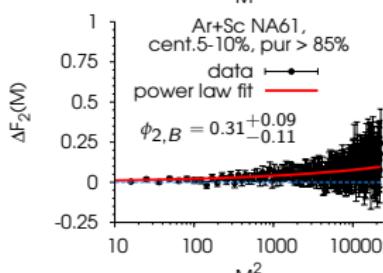
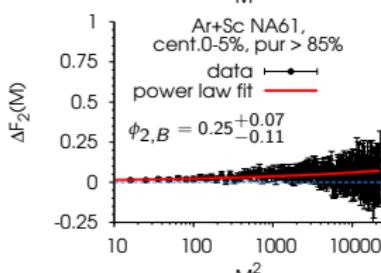
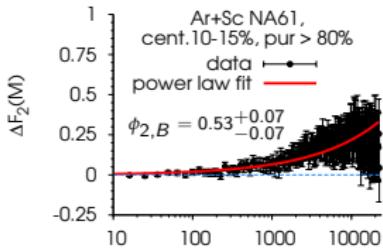
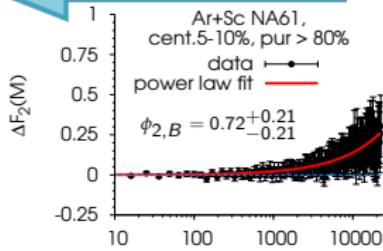
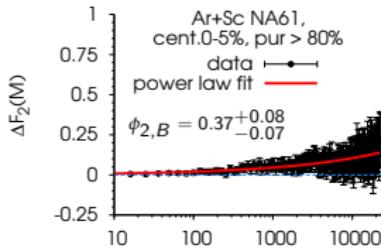
purity

NA61/SHINE: Ar+Sc at 150A GeV/c: $\Delta F_2(M)$

NA61/SHINE preliminary

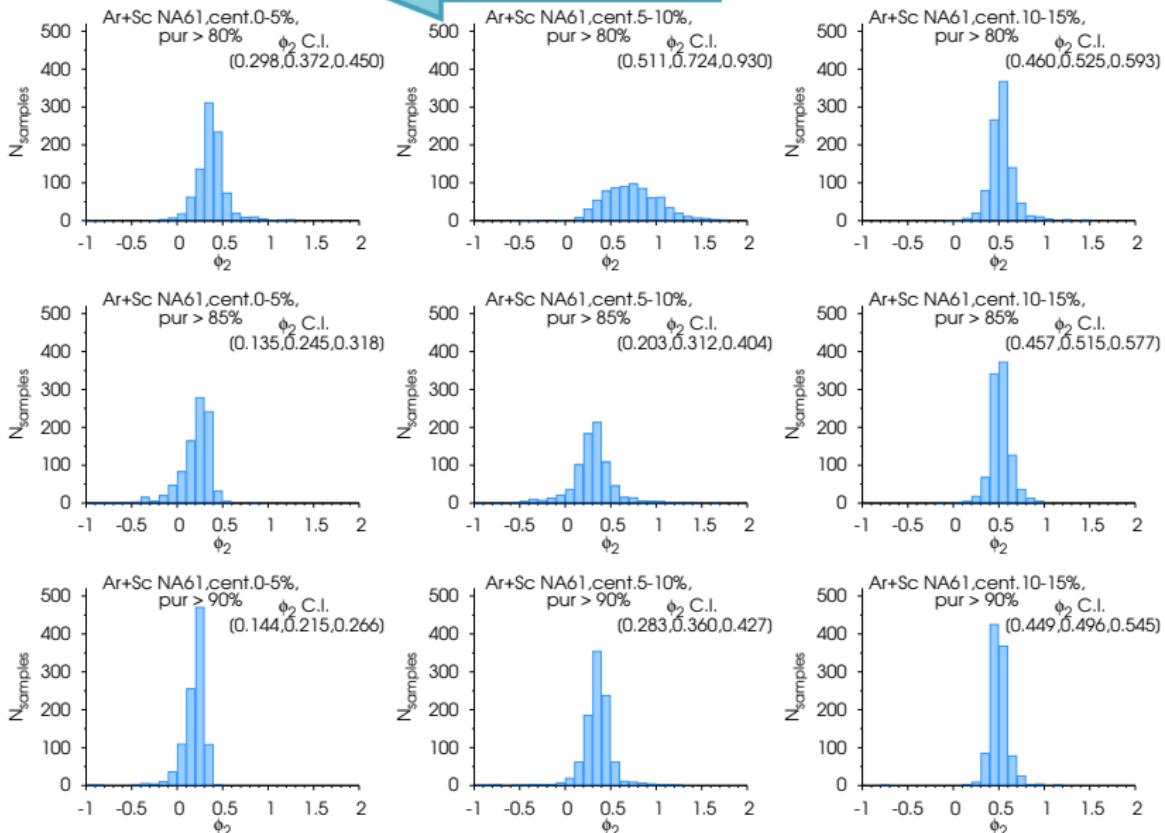
centrality

purity

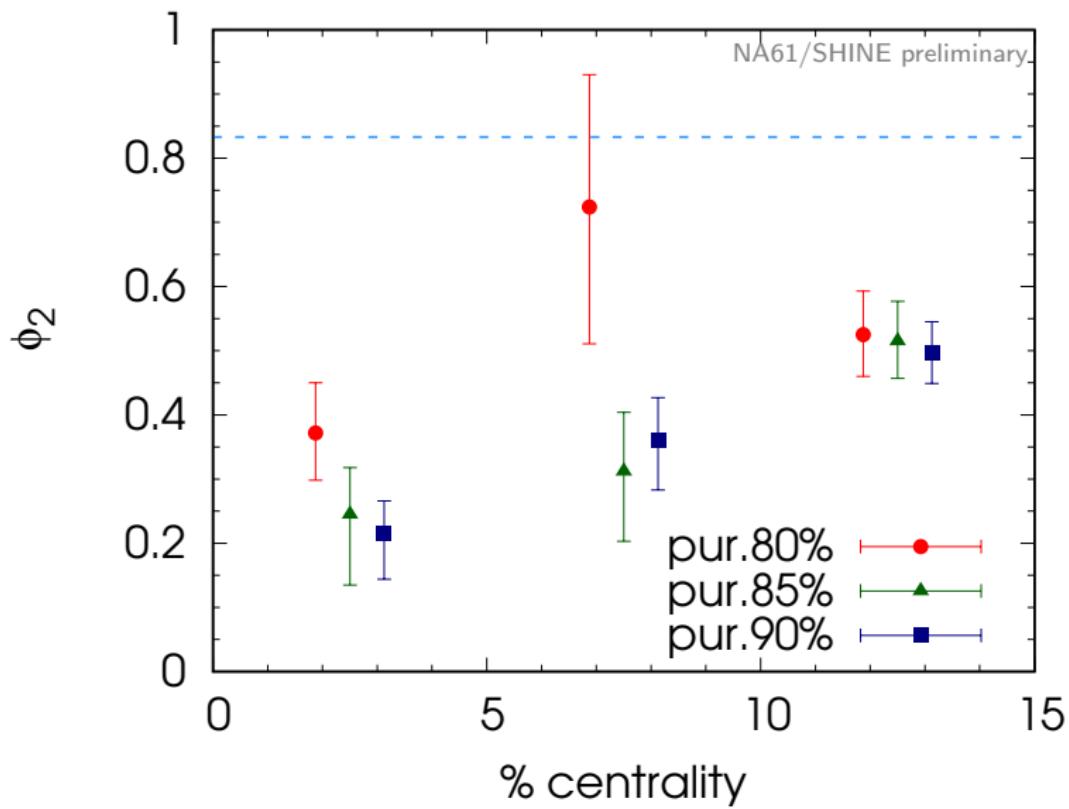


NA61/SHINE: Ar+Sc at 150A GeV/c: ϕ_2 bootstrap dist.

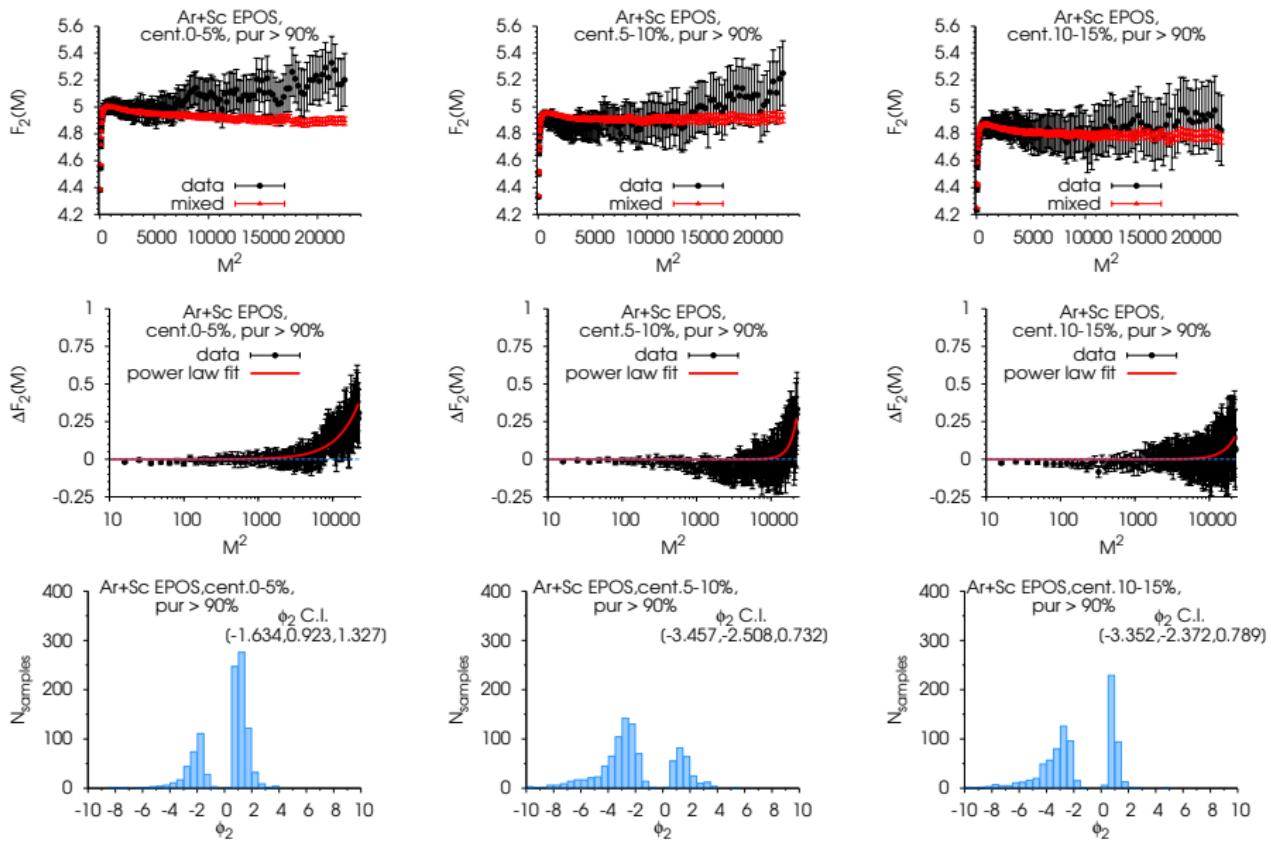
NA61/SHINE preliminary



NA61/SHINE: Ar+Sc at 150A GeV/c: Summary

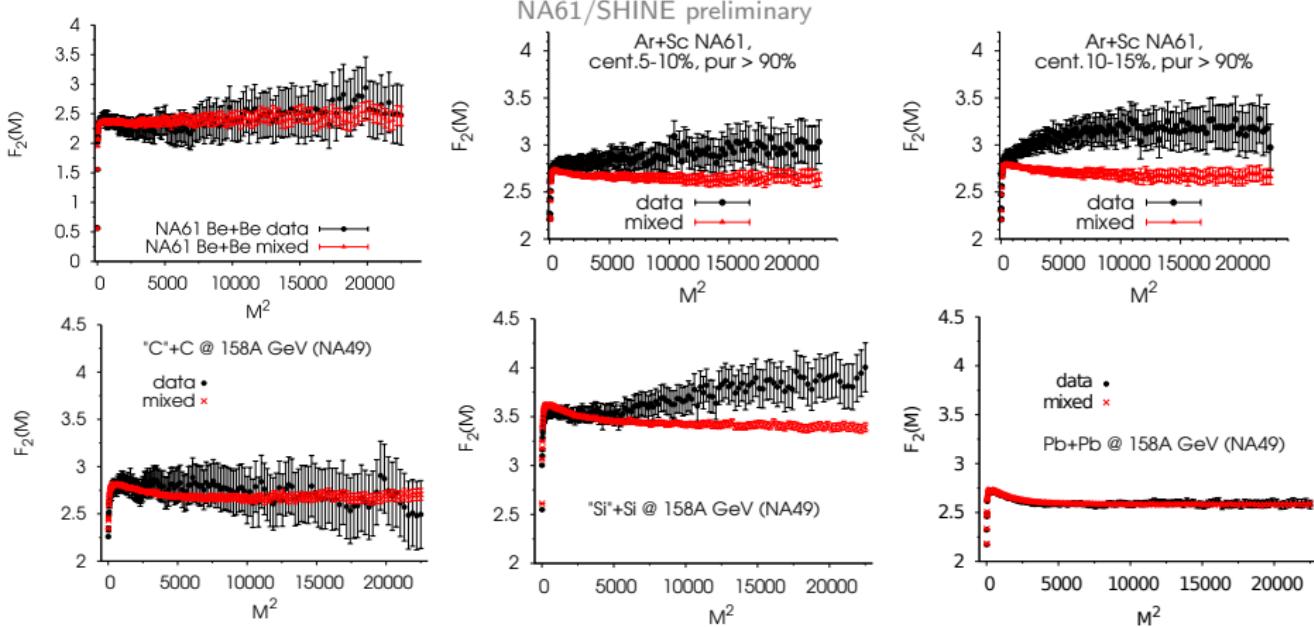


Ar+Sc EPOS: $F_2(M)$, $\Delta F_2(M)$, ϕ_2 bootstrap distribution



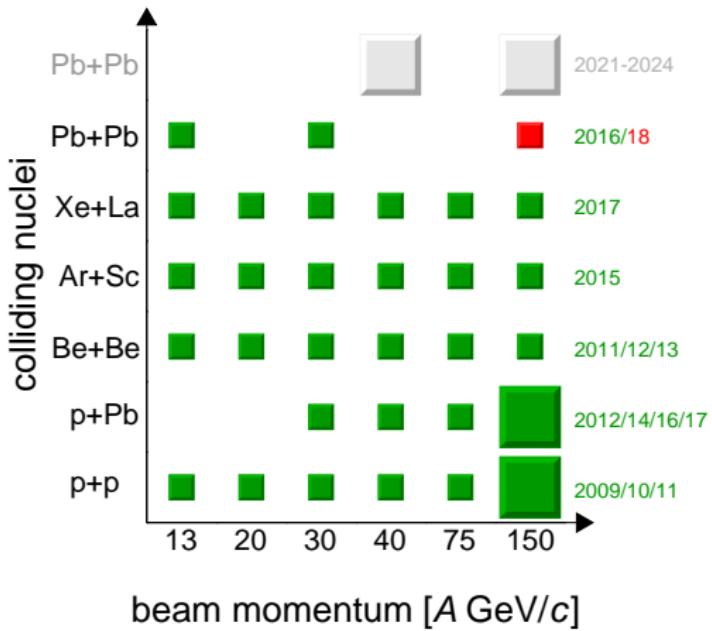
NA61/SHINE preliminary

Intermittency analysis at 150/158A GeV/c: Summary



- Indication of intermittency effect in middle-central NA61/SHINE Ar+Sc collisions
- First possible evidence of CP signal in NA61/SHINE
- Effect quality increases with increased proton purity selection, up to 90% proton purity; EPOS does not reproduce observed effect.

Outlook



- Expanding the analysis to other NA61/SHINE systems (Xe+La, Pb+Pb) and SPS energies (Ar+Sc) will hopefully lead to a **more reliable interpretation** of the observed intermittency signal in terms of the **critical point**.

Thank you!

Acknowledgements

This work was supported by the National Science Centre, Poland
under grant no. 2014/14/E/ST2/00018.

Back Up Slides

Scaling of factorial moments – Subtracting mixed events

For $\lambda \lesssim 1$ (background domination), $\Delta F_2(M)$ can be approximated by:

$$\Delta F_2^{(e)}(M) = F_2^{\text{data}}(M) - F_2^{\text{mix}}(M)$$

For a critical system, ΔF_2 scales with cell size (number of cells, M) as:

$$\Delta F_2(M) \sim (M^2)^{\varphi_2}$$

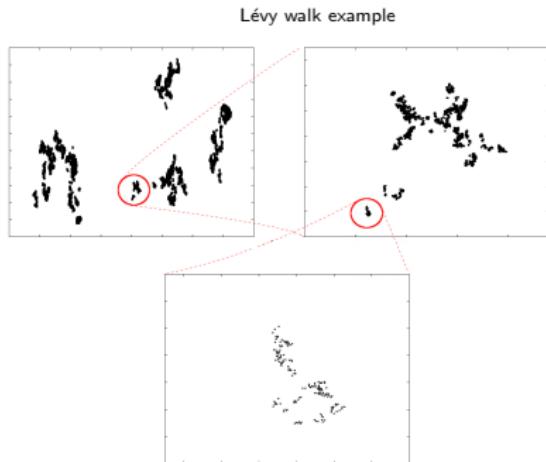
where φ_2 is the intermittency index.

Theoretical predictions for φ_2

$$\left\{ \begin{array}{ll} \varphi_{2,cr}^{(\sigma)} = \frac{2}{3} \ (0.66\dots) & \varphi_{2,cr}^{(p)} = \frac{5}{6} \ (0.833\dots) \\ \text{sigmas (neutral isoscalar dipions)} & \text{net baryons (protons)} \\ \text{[N. G. Antoniou et al, Nucl. Phys. A 693, 799 (2001)]} & \text{[N. G. Antoniou, F. K. Diakonos, A. S. Kapoyannis, K. S. Kousouris, Phys. Rev. Lett. 97, 032002 (2006)]} \end{array} \right.$$

Critical Monte Carlo (CMC) algorithm for baryons

- Simplified version of CMC* code:
 - Only protons produced
 - One cluster per event, produced by random Lévy walk: $\tilde{d}_F^{(B,2)} = 1/3 \Rightarrow \phi_2 = 5/6$
 - Lower / upper bounds of Lévy walks $p_{min,max}$ plugged in.
 - Cluster center exponential in p_T , slope adjusted by T_c parameter.
 - Poissonian proton multiplicity distribution.



Input parameters

Parameter	p_{min} (MeV)	p_{max} (MeV)	λ_{Poisson}	T_c (MeV)
Value	$0.1 \rightarrow 1$	$800 \rightarrow 1200$	$\langle p \rangle_{\text{non-empty}}$	163

* [Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]

NA61/SHINE data analysis – $^{40}Ar + ^{45}Sc$ at 150A GeV/c

- NA49 analysis encourages us to look for intermittency in medium-sized nuclei, in the NA61 experiment.
- Intermittency analysis requires:
 - Large event statistics $\Rightarrow \sim 100K$ events min., ideally $\sim 1M$ events.
 - Reliable particle ID \Rightarrow proton purity should be $\sim 80\% - 90\%$.
 - Central collisions.
 - Adequate mean proton multiplicity in midrapidity (≥ 2)
- A preliminary analysis for Be+Be data at 150A GeV/c was previously performed [PoS(CPOD2017) 054]; no intermittency signal was observed.
- We now expand on it with our preliminary analysis in Ar+Sc at 150A GeV/c.
- Simulation through EPOS* (detector effects included) would suggest:

$$\left. \frac{dN_p}{dy} \right|_{|y_{CM}| \leq 0.75, p_T \leq 1.5} \sim 1.6 - 2$$

for $\sim 0 - 15\%$ centrality; adequate for an intermittency analysis

- We perform a 2D scan in proton purity (80-90%) and centrality of collisions

*[K. Werner, F. Liu, and T. Pierog, Phys. Rev. C 74, 044902 (2006)]

$^{40}Ar + ^{45}Sc$ – data set overview

- Production used: Ar_Sc_150_15/026_17c_v1r8p0_pA_slc6_phys_PP (miniSHOE, **unofficial**)
- Runs: 20328 - 20345 , 20368 - 20380
- **Bad runs rejected** – almost 2/3rds of total!
- miniSHOE files with Potential Point information provided by B. Maksiak – **not an official production yet**
- SHINE code to select events (primary vertex charged particles)
- Event & Track cuts based on Maciej Lewicki's and Michal Naskret's h^- analysis.
- Non-bias event cuts: used Andrey Seryakov's NonBiasEventCutsArSc class.
- 0%-20% most central events in 5% bin intervals selected via cut in energy sum of PSD selected modules (based on Andrey Seryakov's Moscow meeting presentation on centrality determination).

Ar+Sc at 150A GeV/c: NA61 data vs EPOS

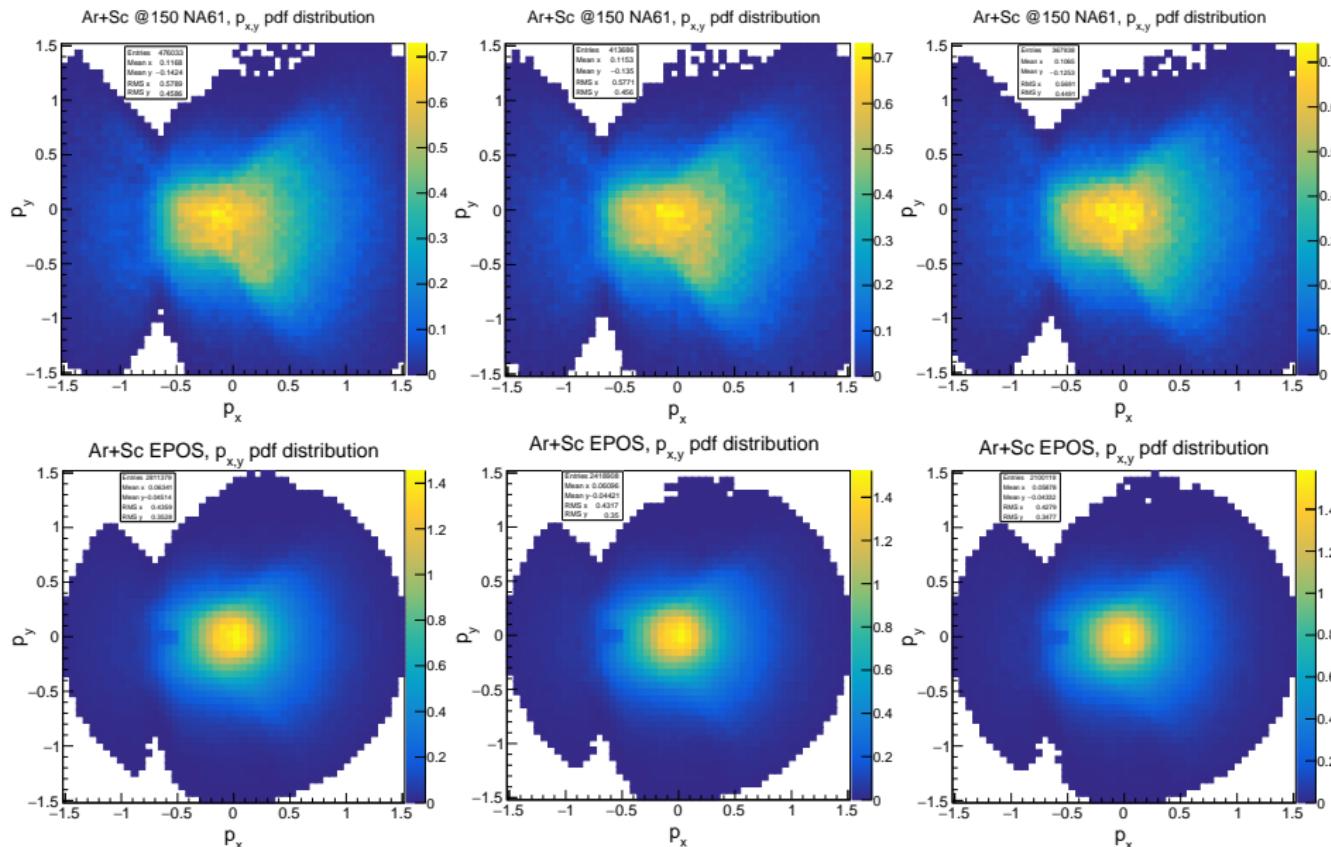
EPOS – proton p_T statistics

Centrality	#events	$\langle p \rangle_{ p_T \leq 1.5 \text{ GeV}, y_{CM} \leq 0.75}$ Non-empty	$\langle p \rangle_{ p_T \leq 1.5 \text{ GeV}, y_{CM} \leq 0.75}$ With empty	$\Delta p_{x,y}$
0- 5%	293,412	3.06 ± 1.60	2.89 ± 1.70	0.35 - 0.43
5-10%	252,362	2.72 ± 1.45	2.49 ± 1.58	0.35 - 0.43
10-15%	274,072	2.45 ± 1.33	2.16 ± 1.48	0.35 - 0.43

$^{40}\text{Ar} + ^{45}\text{Sc}$ NA61 data – proton p_T statistics

Centrality	#events	$\langle p \rangle_{ p_T \leq 1.5 \text{ GeV}, y_{CM} \leq 0.75}$ Non-empty	$\langle p \rangle_{ p_T \leq 1.5 \text{ GeV}, y_{CM} \leq 0.75}$ With empty	$\Delta p_{x,y}$
0- 5%	144,362	3.44 ± 1.79	3.30 ± 1.89	0.46 - 0.58
5-10%	148,199	3.00 ± 1.61	2.79 ± 1.73	0.46 - 0.58
10-15%	142,900	2.81 ± 1.53	2.58 ± 1.66	0.45 - 0.57

$p_{x,y}$ spectra comparison – NA61 vs EPOS (0 – 15%)



Event & Track cuts

Event cuts

- Target IN/OUT,
- BPD status,
- WFA particles ($4.5 \mu s$),
- WFA interaction ($25 \mu s$),
- BPD3X(Y) charge,
- S5 ($0 \rightarrow 170$),
- **T2 trigger (eAll)**,
- Vertex track fitted to the main vertex,
- Vertex fit quality = ePerfect,
- Fitted vertex position -580 ± 10 cm,
- PSD Module Energy Sum cut
(inner/outer),
- Centrality 0-20% (based on PSD)
- **$n\text{TracksFit}/n\text{TracksAll} > 0.25$ if
 $n\text{TracksFit} \leq 50$ (Andrey)**

Track cuts

- Track status,
- Charge ± 1 ,
- Impact point $[\pm 4\text{cm}; \pm 2\text{cm}]$,
- Total number of clusters ≥ 30 ,
- VTPCs clusters ≥ 15 ,
- **NO GTPC clusters**,
- dE/dx clusters ≥ 30 ,
- **$0.5 \leq \frac{\# \text{Points}}{\# \text{Potential Points}} \leq 1.0$**
- TTD cut > 2 cm
- **$dE/dx \leq 1.8$ (dE/dx fit issue)**
- proton selection (scan)
- **$3.98 \text{ GeV}/c \leq p_{tot} \leq 126 \text{ GeV}/c$**
(for dE/dx proton ID – scan)

$^{40}\text{Ar} + ^{45}\text{Sc}$ – EPOS MC production overview

- Production used: `Simulation/Ar_Sc_150_15/15_011_v14e_v1r2p0_pA_slc6_phys/EPOS_with_potential_points/`
- An estimated $\sim 300\text{K}$ simulated events per 5% centrality bin.
- Potential Point information included **for limited events subset**.
- SHINE code to select events (primary vertex charged particles)
- Event & Track cuts (hastily) adapted to match Ar+Sc @150 data analysis (where applicable).
- **No PSD simulation** – centrality selection based on # of forward spectators, `nFSpec = 40 - simEvent.GetPrimaryInteraction().GetProjectileParticipants()` (see [Andrey Seryakov's centrality determination information on twiki](#)).

Event & Track cuts – EPOS

Event cuts

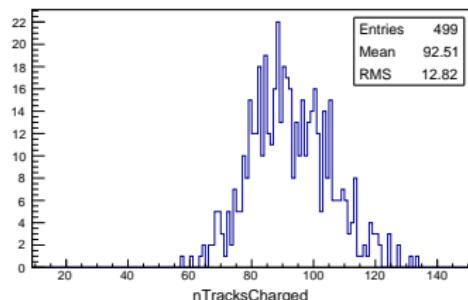
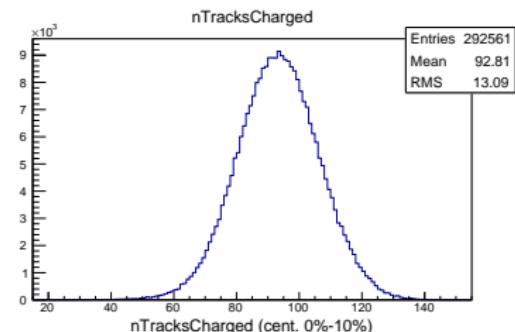
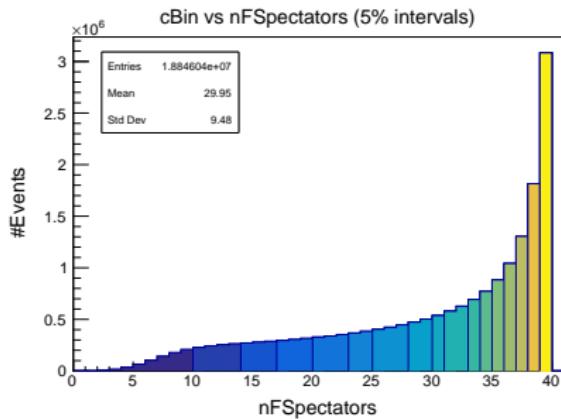
- Target IN/OUT,
- BPD status,
- Vertex track fitted to the main vertex,
- Vertex fit quality = ePerfect,
- Fitted vertex position -580 ± 10 cm,
- Centrality 10% (based on nFSpec)

Track cuts

- Track status,
- Charge ± 1 ,
- Impact point $[\pm 4\text{cm}; \pm 2\text{cm}]$,
- Total number of clusters ≥ 30 ,
- VTPCs clusters ≥ 15 ,
- **NO GTPC clusters,**
- TTD cut $> 2\text{ cm}$,
- proton selection – **matching closest simTrack**,
- $3.98\text{ GeV}/c \leq p_{tot} \leq 126\text{ GeV}/c$
(to match effect of dE/dx p_{tot} cut),
- acceptance cut

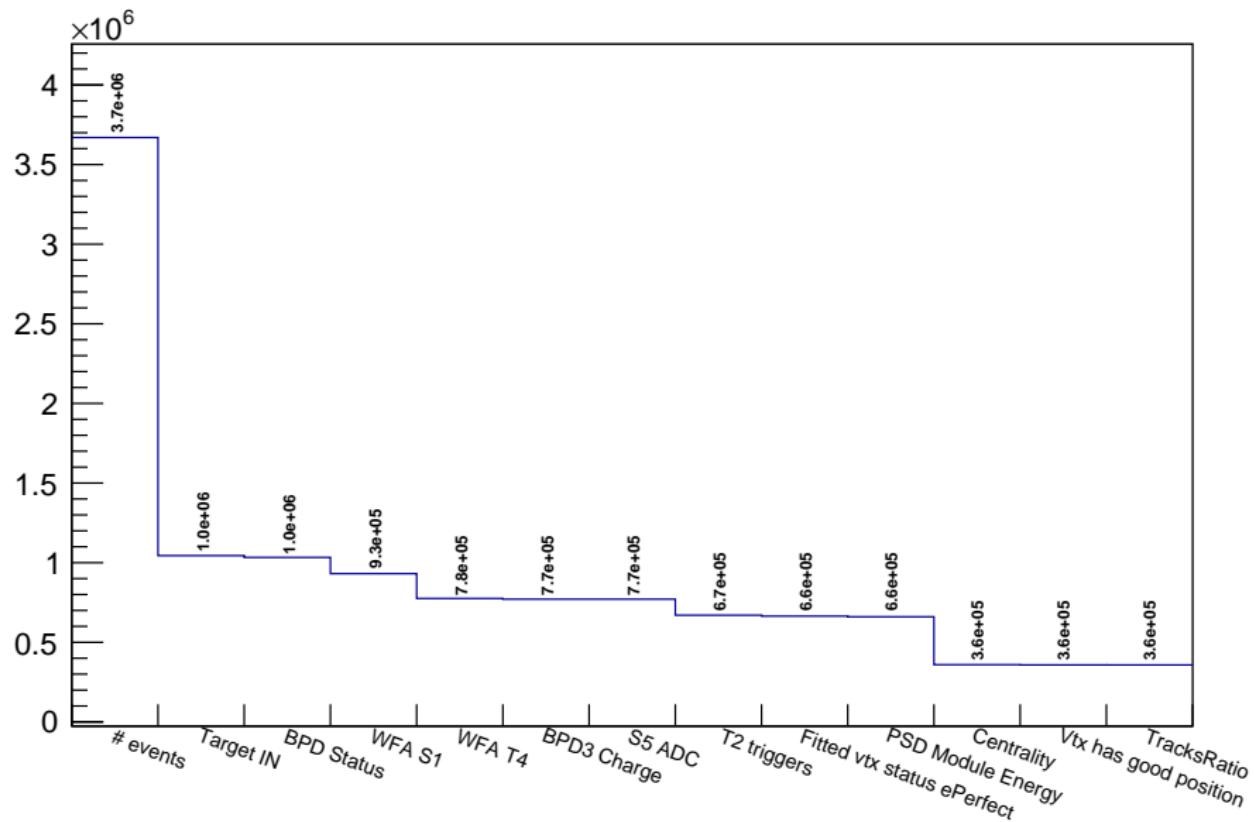
Centrality selection via # forward spectators

- A probabilistic selection based on nFSpec percentiles used to select centrality bin.

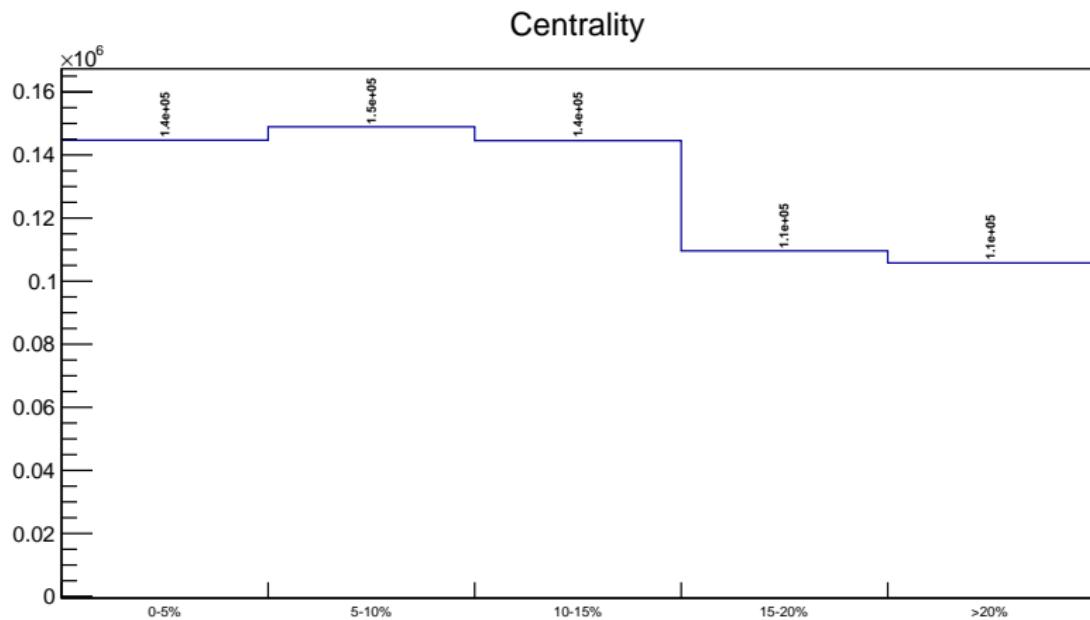


- A discrepancy observed in multiplicity distribution between data (above) & EPOS (below).
- Acceptance cut fixes the problem.

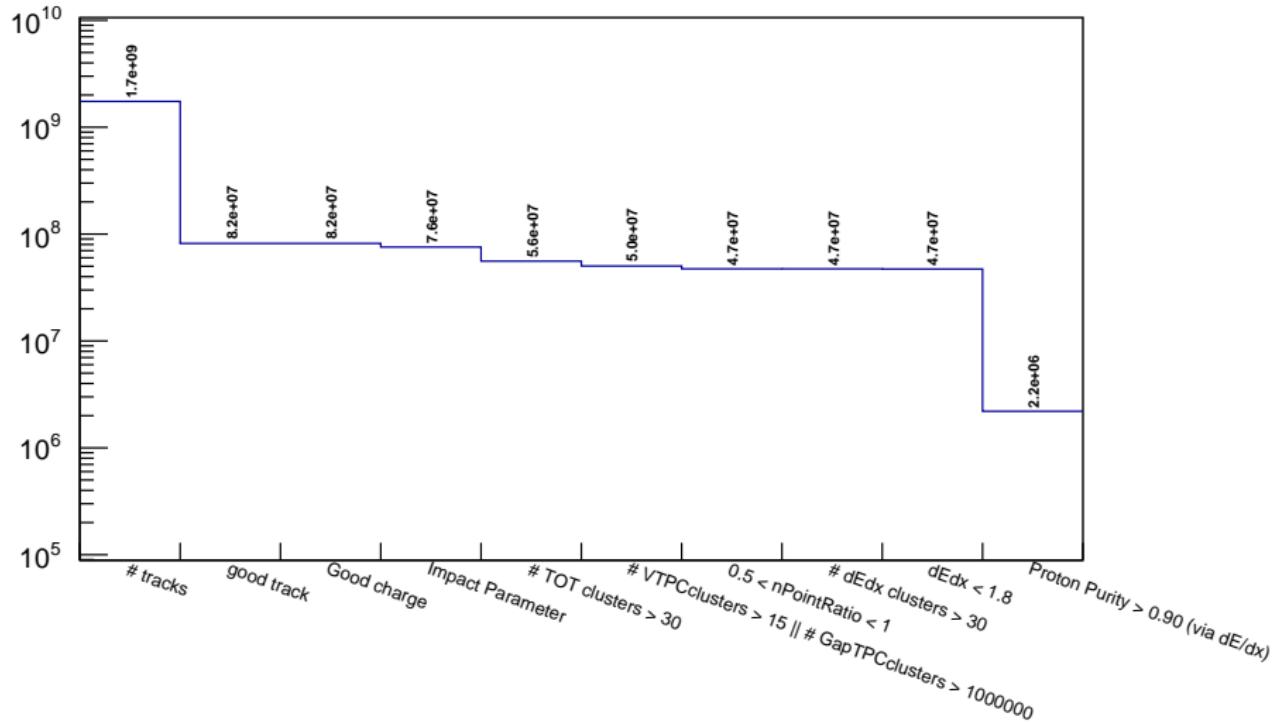
All Event cuts – statistics



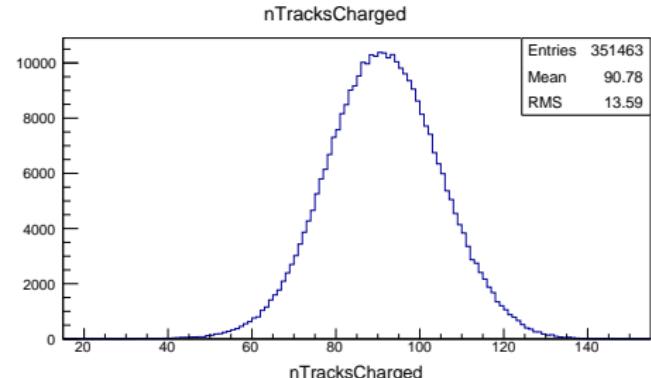
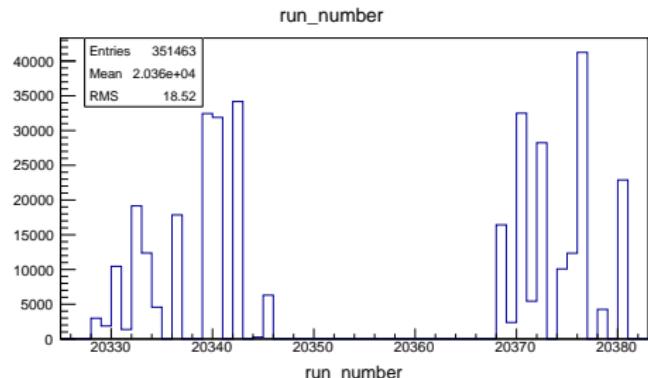
Centrality – statistics



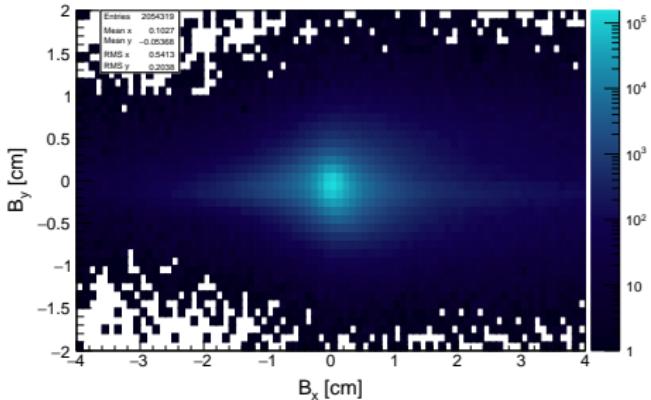
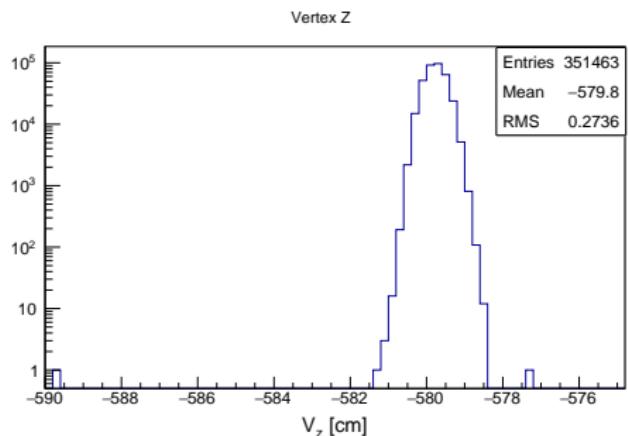
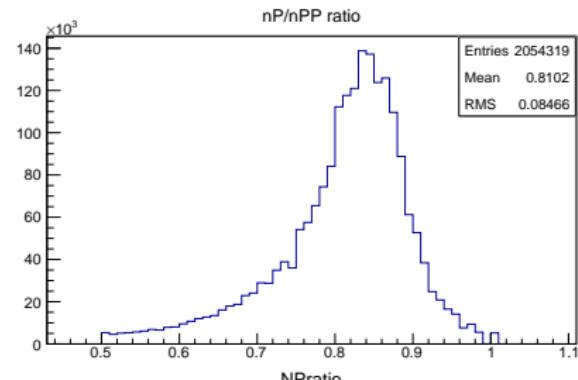
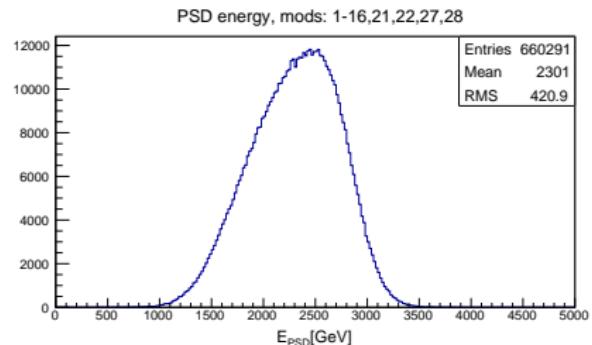
Track cuts – statistics



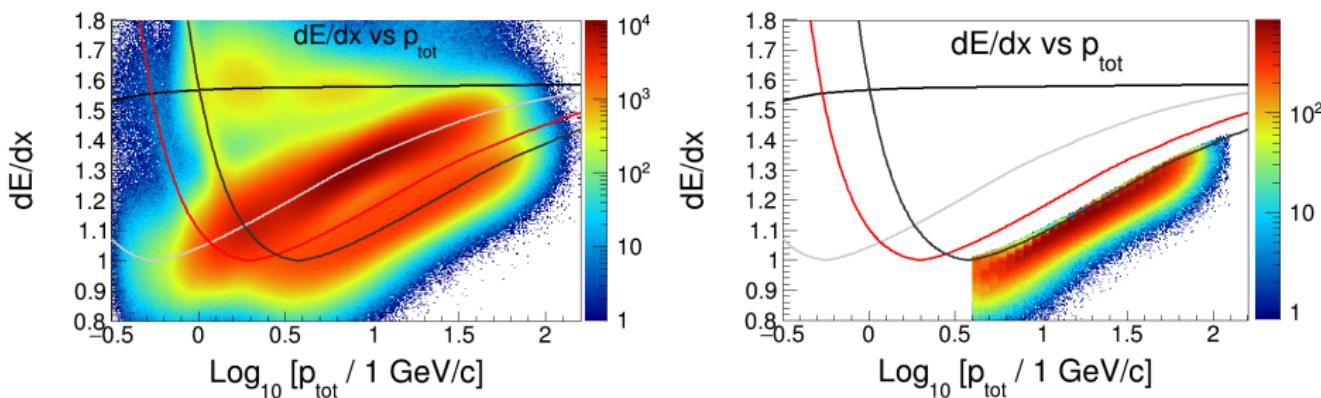
Cuts (plots)



Cuts (plots)

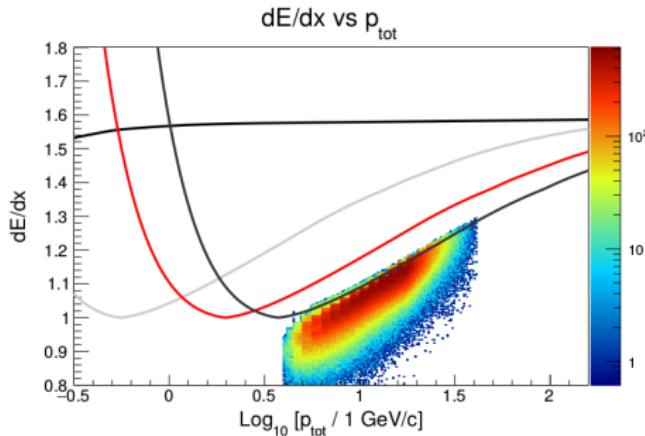
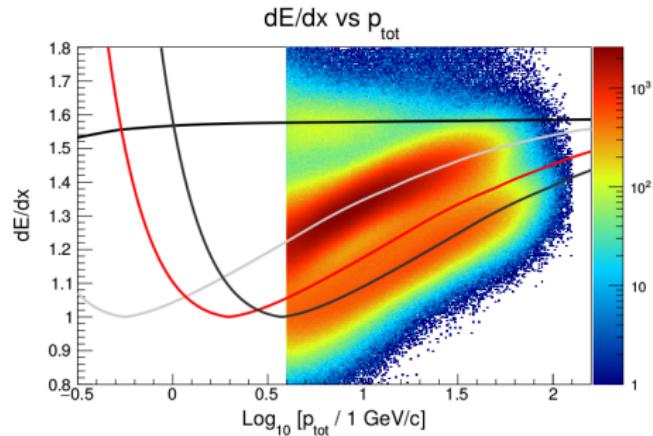


dE/dx vs p_{tot} (proton ID)



- Avoid p_{tot} region where Bethe-Bloch curves overlap ($3.98 \text{ GeV}/c \leq p_{tot} \leq 126 \text{ GeV}/c$)
- Using Hans Dembinski/Raul R Prado's dE/dx fitting software – Bins: p_{tot}, p_T
- Presented in Moscow meeting by Prado, Herve & Unger
- 30 Bins in $\text{Log}_{10}(p_{tot})$: $10^{0.6} \rightarrow 10^{2.1} \text{ GeV}/c$
- 20 Bins in p_T : $0.0 \rightarrow 2.0 \text{ GeV}/c$
- Preliminary p selection: 90% purity removing deuterons from the model
- Cut along Bethe-Blochs: $BB_p + 0.15(BB_K - BB_p)$

dE/dx simulation & proton purity assignment in EPOS

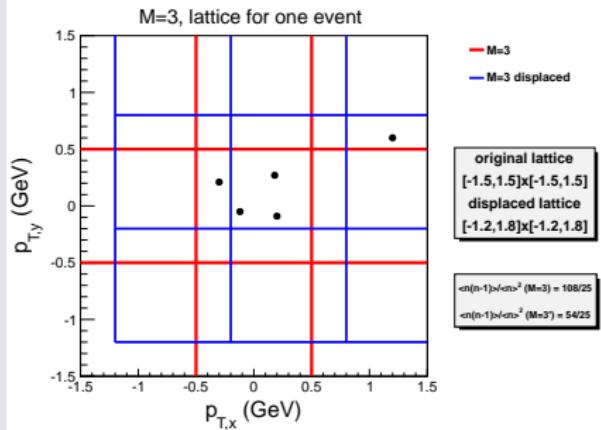


- Used dE/dx spectra from Ar+Sc @150 data in the 6% - 18% centrality interval
- For each track, assign a dE/dx value based on particle species and phase space bin
- Apply dE/dx & purity cuts identical to NA61/SHINE data

Improving calculation of $F_2(M)$ via lattice averaging

- Problem: With low statistics/multiplicity, lattice boundaries may **split pairs** of neighboring points, affecting $F_2(M)$ values (see example below).
- Solution: Calculate moments several times on different, slightly displaced lattices (see example)
- Average corresponding $F_2(M)$ over all lattices. Errors can be estimated by variance over lattice positions.
- Lattice displacement is larger than experimental resolution, yet maximum displacement must be of the order of the **finer binnings**, so as to stay in the correct p_T range.

Displaced lattice — a simple example



Improved confidence intervals for ϕ_2 via resampling

- In order to estimate the statistical errors of $\Delta F_2(M)$, we need to produce variations of the original event sample. This, we can achieve by using the statistical method of resampling (bootstrapping) ⇒
 - Sample original events with replacement, producing new sets of the same statistics (# of events)
 - Calculate $\Delta F_2(M)$ for each bootstrap sample in the same manner as for the original.
 - The variance of sample values provides the statistical error of $\Delta F_2(M)$.

[W.J. Metzger, "Estimating the Uncertainties of Factorial Moments", HEN-455 (2004).]

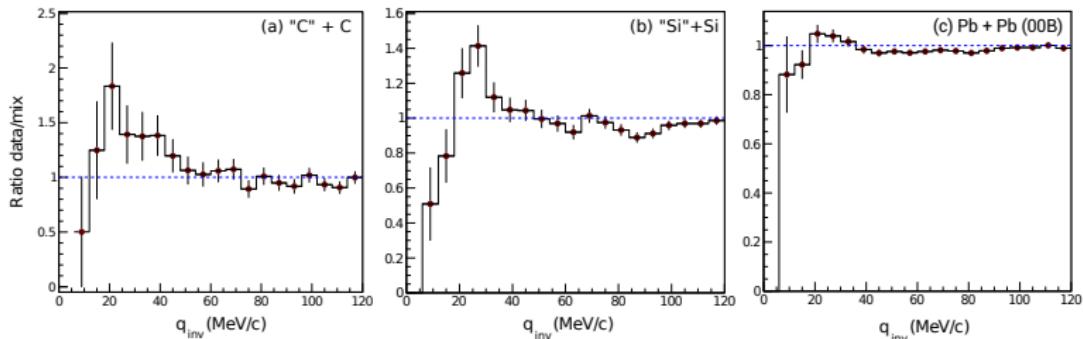
- Furthermore, we can obtain a distribution $P(\phi_2)$ of ϕ_2 values. Each bootstrap sample of $\Delta F_2(M)$ is fit with a power-law:

$$\Delta F_2(M; \mathcal{C}, \phi_2) = e^{\mathcal{C}} \cdot (M^2)^{\phi_2}$$

and we can extract a confidence interval for ϕ_2 from the distribution of values. [B. Efron, *The Annals of Statistics* 7,1 (1979)]

Split tracks; the q_{inv} cut in analysed datasets

- Split tracks can create **false positive** for intermittency \Rightarrow must be **reduced** or **removed**.
- q_{inv} -test – distribution of track pairs: $q_{inv}(p_i, p_j) \equiv \frac{1}{2} \sqrt{-(p_i - p_j)^2}$, p_i : 4-momentum of i^{th} track.
- Calculate ratio $q_{inv}^{data}/q_{inv}^{mixed}$ \Rightarrow **peak** at low q_{inv} (below 20 MeV/c): **possible split track contamination**.

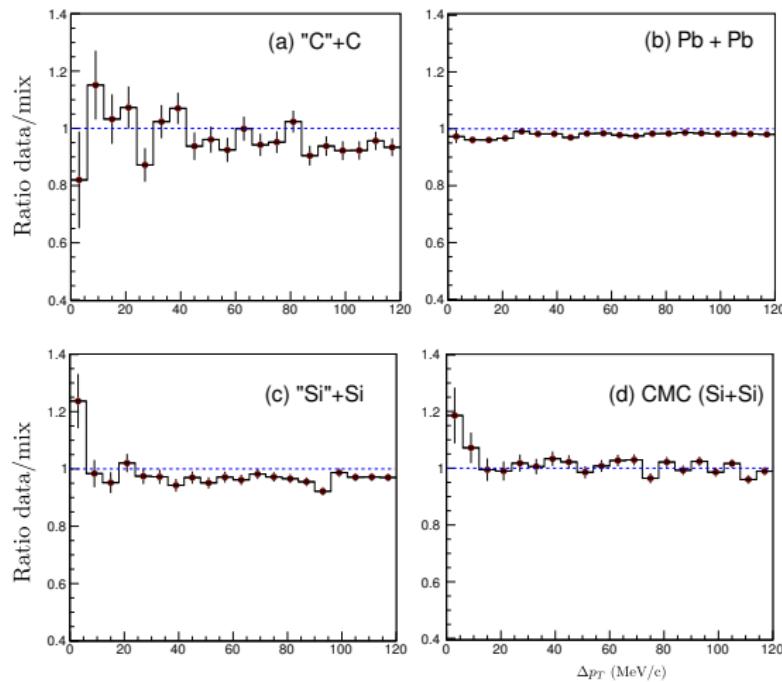


- Anti-correlations due to **F-D effects and Coulomb repulsion** must be removed before intermittency analysis \Rightarrow “dip” in low q_{inv} , peak predicted around 20 MeV/c [Koonin, PLB 70, 43-47 (1977)]
- **Universal cutoff** of $q_{inv} > 25$ MeV/c applied to all sets before analysis.

NA49 analysis – Δp_T distributions

- We measure correlations in relative p_T of protons via

$$\Delta p_T = 1/2 \sqrt{(p_{X_1} - p_{X_2})^2 + (p_{Y_1} - p_{Y_2})^2}$$



- Strong correlations for $\Delta p_T \rightarrow 0$ indicate **power-law scaling** of the density-density correlation function \Rightarrow intermittency presence
- We find a strong peak in the "Si" + Si dataset
- A similar peak is seen in the Δp_T profile of simulated CMC protons with the characteristics of "Si" + Si.

Split tracks & the q_{inv} cut

- Events may contain **split tracks**: sections of the same track erroneously identified as **a pair of tracks** that are close in momentum space.
- Intermittency analysis is based on pairs distribution \Rightarrow split tracks can create a **false positive**, and so must be **reduced** or **removed**.
- Standard cuts** remove part of split tracks. In order to estimate the residual contamination, we check the q_{inv} distribution of track pairs:

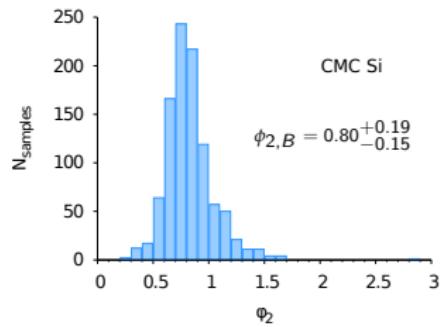
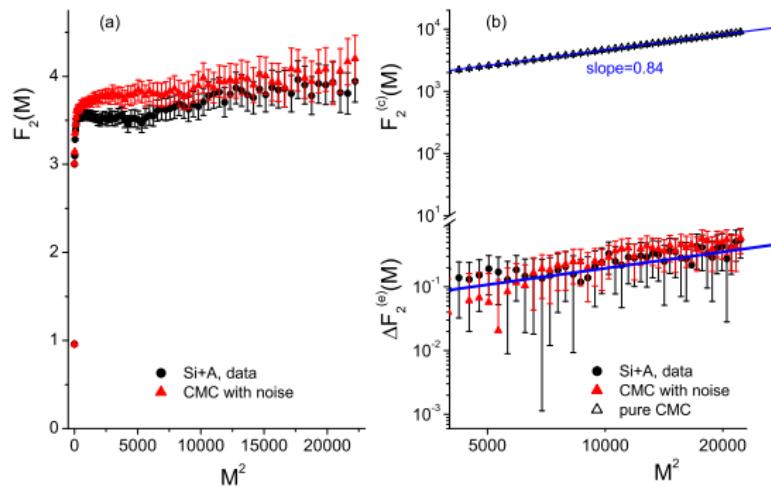
$$q_{inv}(p_i, p_j) \equiv \frac{1}{2} \sqrt{-(p_i - p_j)^2},$$

p_i : 4-momentum of i^{th} track.

- We calculate the ratio of $q_{inv}^{data} / q_{inv}^{mixed}$. A **peak** at low q_{inv} (below 20 MeV/c) indicates a possible split track contamination that must be removed.

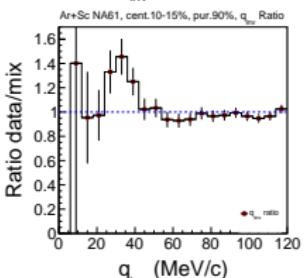
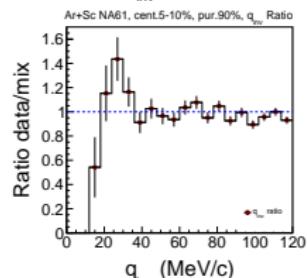
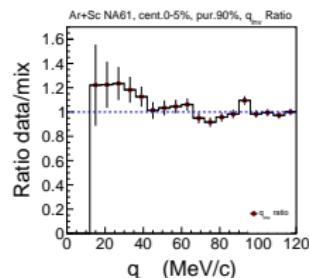
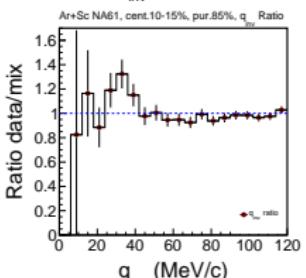
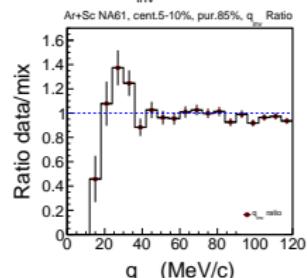
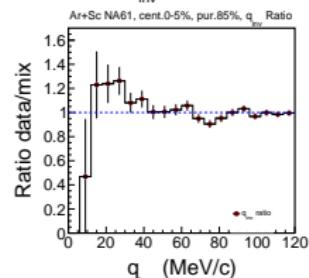
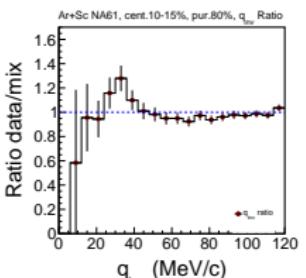
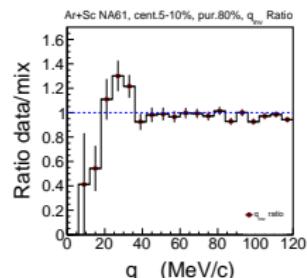
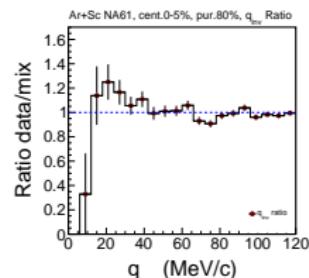
Noisy CMC (baryons) – estimating the level of background

- $F_2(M)$ of noisy CMC approximates “Si” + Si for $\lambda \approx 0.99$
- $\Delta F_2^{(e)}(M)$ reproduces critical behaviour of pure CMC, even though their moments differ by orders of magnitude!

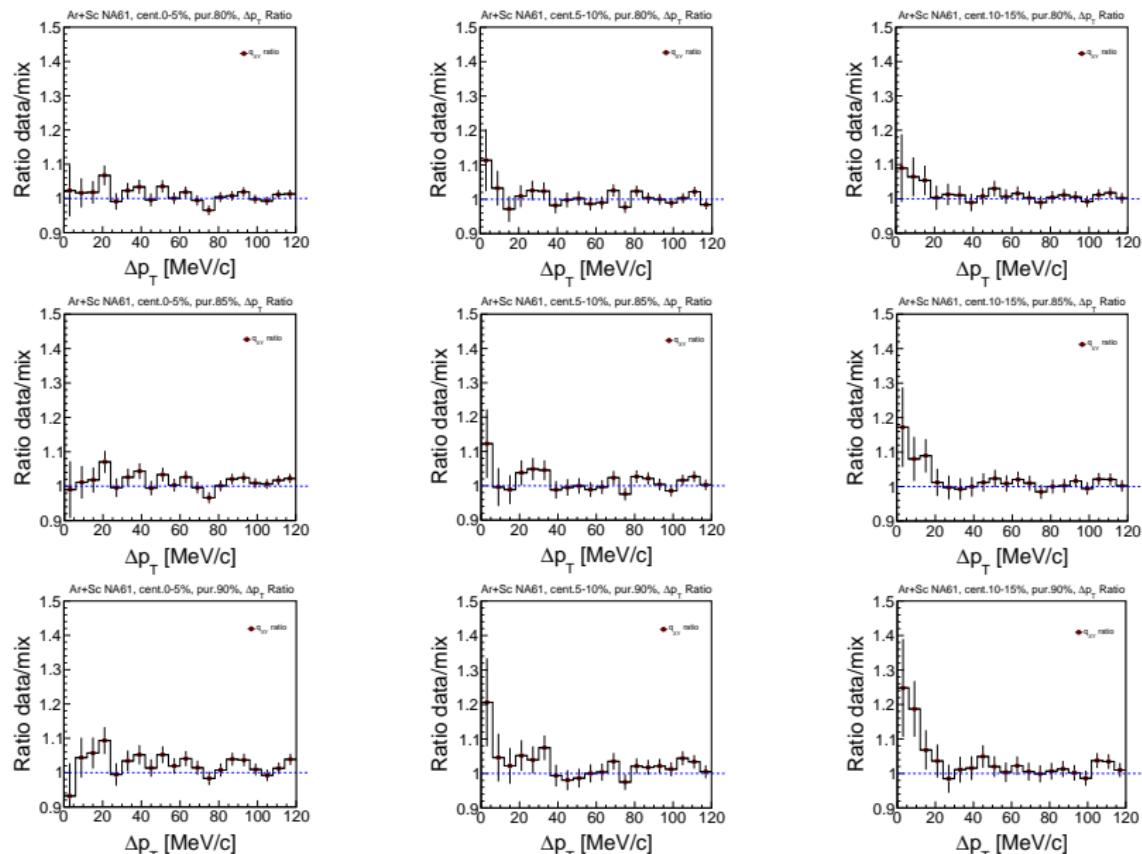


- Noisy CMC results show our approximation is reasonable for dominant background.

q_{inv} proton distributions – NA61/SHINE



Δp_T proton distributions – NA61/SHINE



q_{inv} & Δp_T distributions – EPOS

