QCD phase-diagram: searching for criticality

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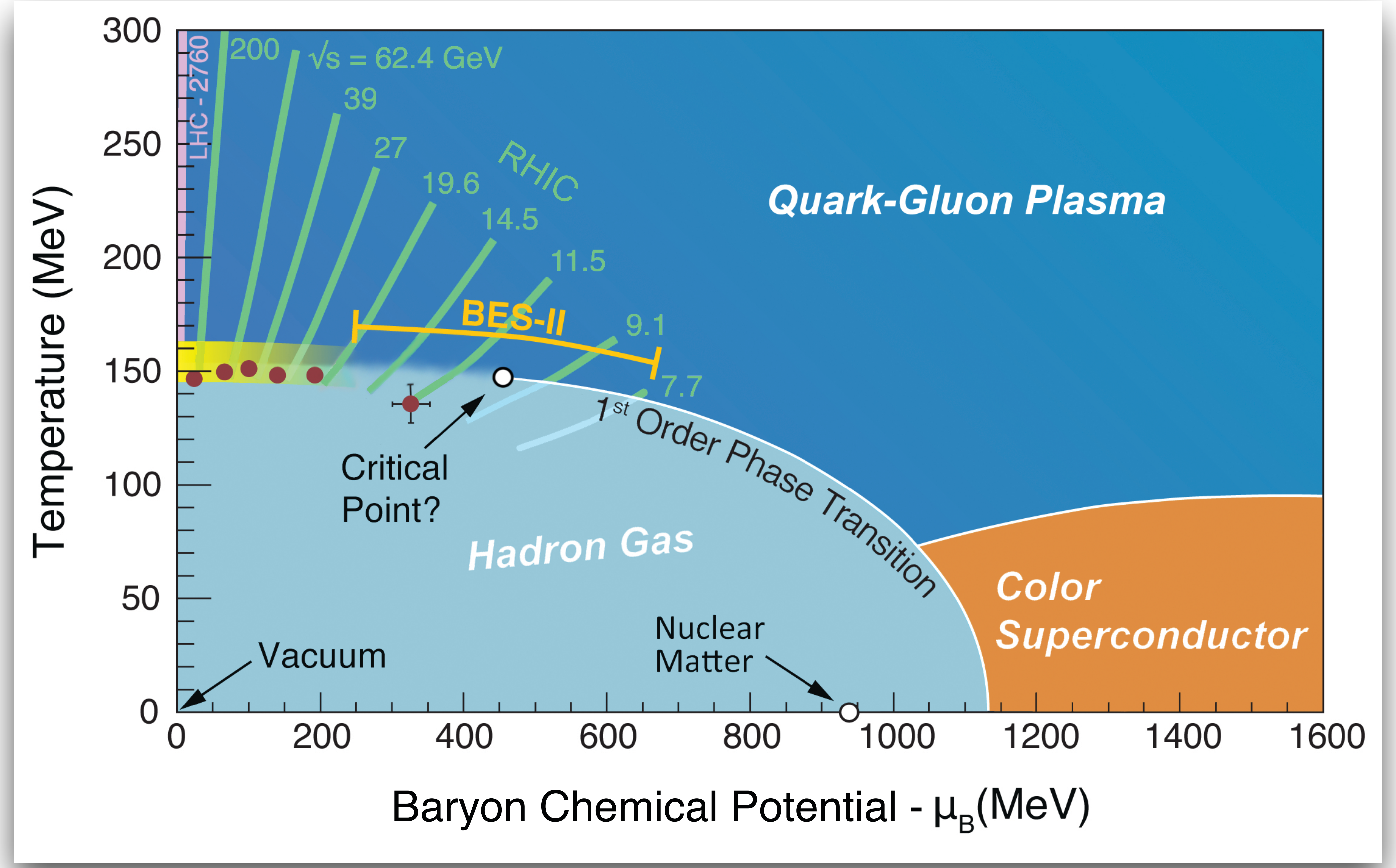
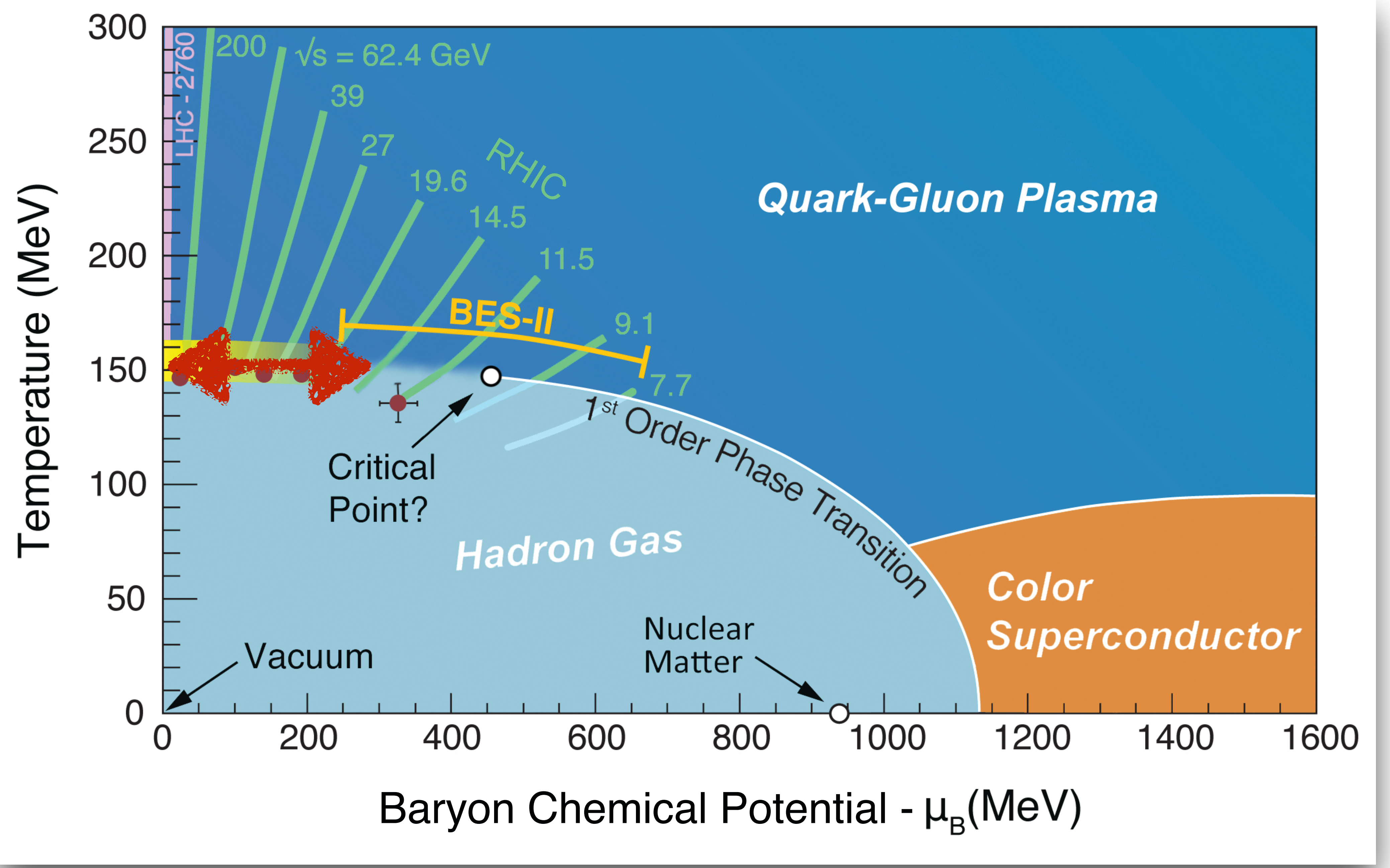
searches for criticality in QCD phase-diagram …



- QCD critical point
  - RHIC, NA61/SHINE, NICA, FAIR, J-PARC-HI
- remnant of the chiral transition @ $\mu_B = 0$
  - LHC

what do we know from (L)QCD ?

physical world @ $\mu_B \geq 0$



chiral crossover temperature: $T_c(\mu_B)$

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2^B \left(\frac{\mu_B}{T_c(0)}\right)^2 - \kappa_4^B \left(\frac{\mu_B}{T_c(0)}\right)^4 + \mathcal{O}\left(\mu_B^6\right)$$

order parameter: $$\Sigma_{\rm sub} \equiv m_s(\Sigma_u + \Sigma_d) - (m_u + m_d)\Sigma_s$$ $$\Sigma_f = \frac{T}{V}\frac{\partial}{\partial m_f} \ln Z$$

susceptibility: $$\chi_{\rm sub} \equiv \frac{T}{V} m_s \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)\Sigma_{\rm sub}$$

and it's quark-line disconnected part: $\chi_{\rm disc}$

Taylor's expansion: $$\frac{\Sigma_{\rm sub}}{f_K^4} = \sum_{n=0}^{\infty} \frac{c_n^{\Sigma}}{n!}\hat{\mu}_B^n$$ $$\frac{\chi_{\rm disc}}{f_K^4} = \sum_{n=0}^{\infty} \frac{c_n^{\chi}}{n!}\hat{\mu}_B^n$$

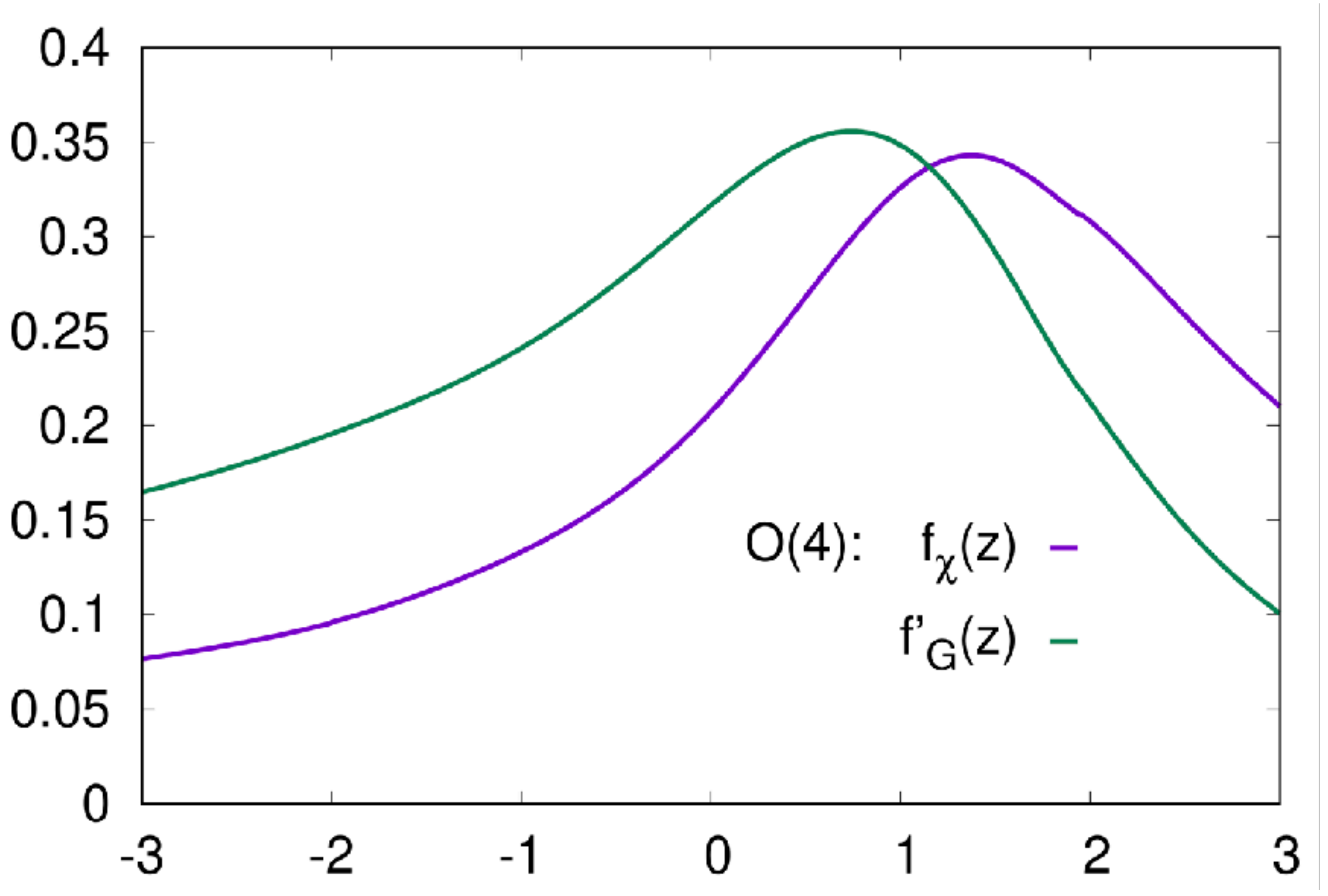
crossover temperature: $$\frac{d^2}{dT^2}\frac{\Sigma_{\rm sub}(T,\mu_B)}{f_K^4} \equiv 0$$ $$\frac{d}{dT}\frac{\chi_{\rm disc}(T,\mu_B)}{f_K^4} \equiv 0$$

$$\chi_t \sim m^{(\beta-1)/\beta\delta} f'_G(z)$$

$$f'_G: \ \partial_T \Sigma_{sub},\ \partial^2_{\mu_B} \Sigma_{sub}$$

$$\chi_m \sim m^{1/\delta-1} f_\chi(z)$$

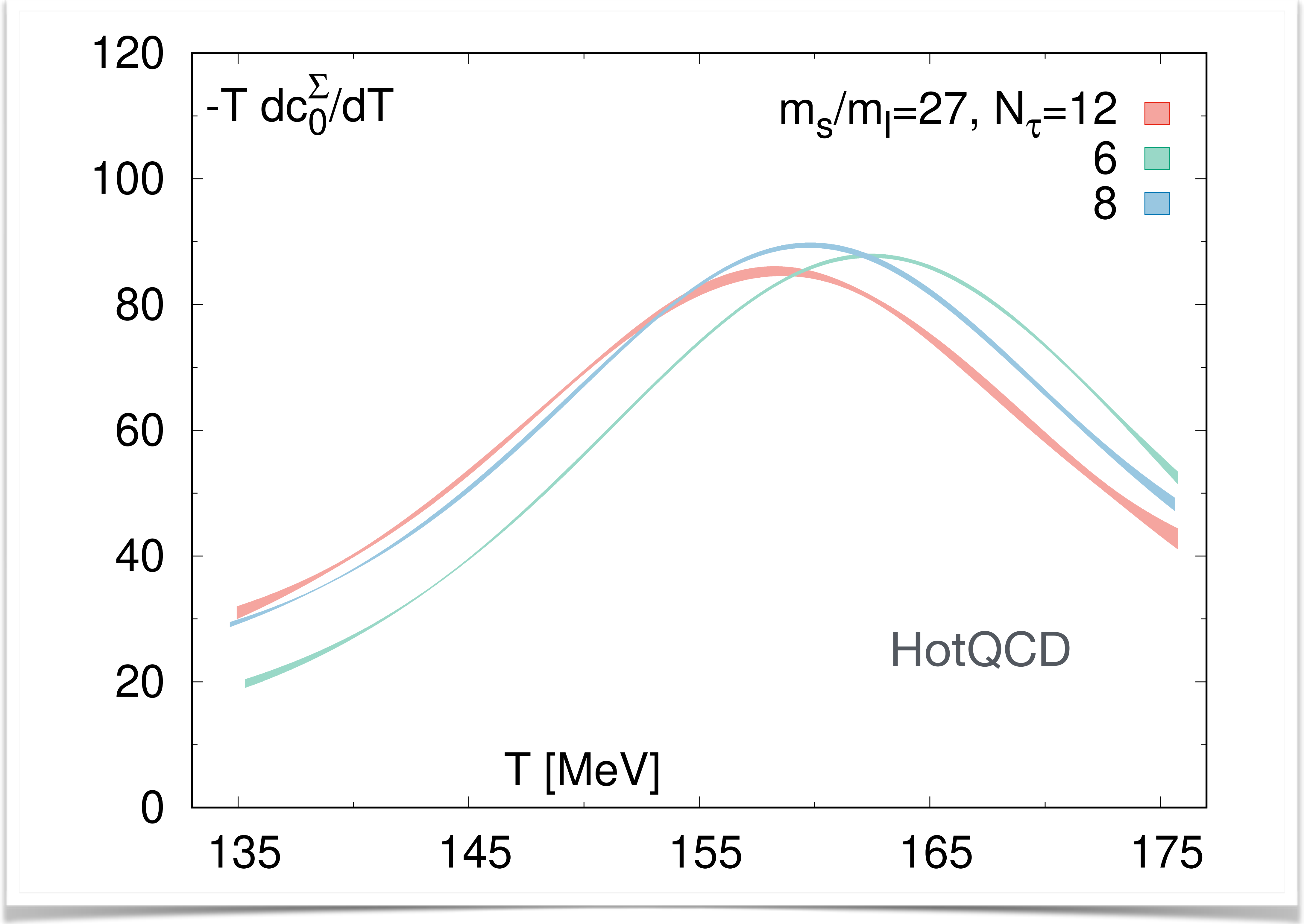
$$f_\chi(z): \ \chi_{sub},\ \chi_{disc},\ \partial^2_{\mu_B} \chi_{disc}$$



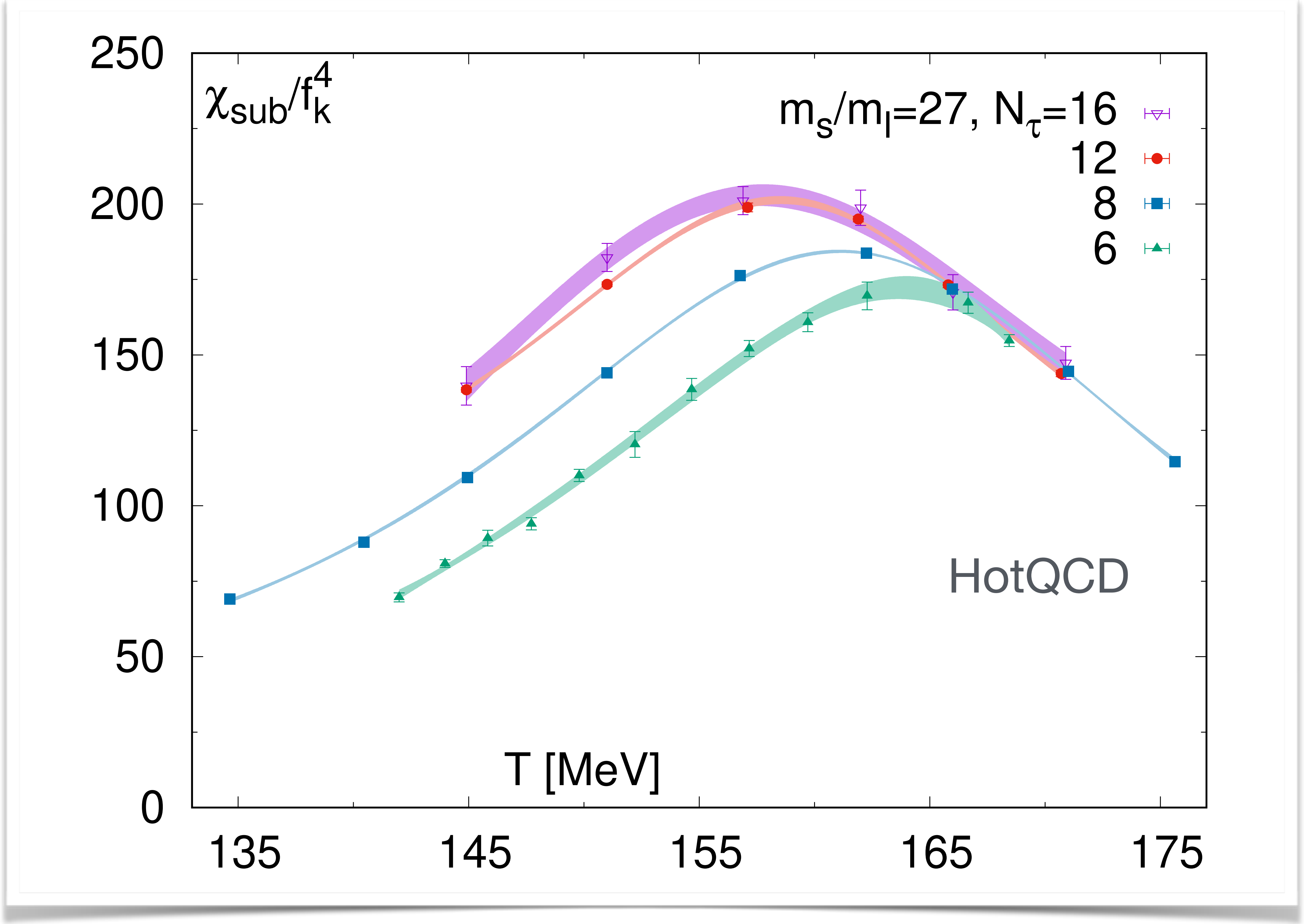
$$z \sim \#[(T_{pc} - Tc) + \#\mu_B^2]/m^{-1/\beta\delta}$$

- m = 0: all these susceptibilities will diverge at a unique transition temperature

- m > 0: crossover, different susceptibilities can lead to different crossover temperatures

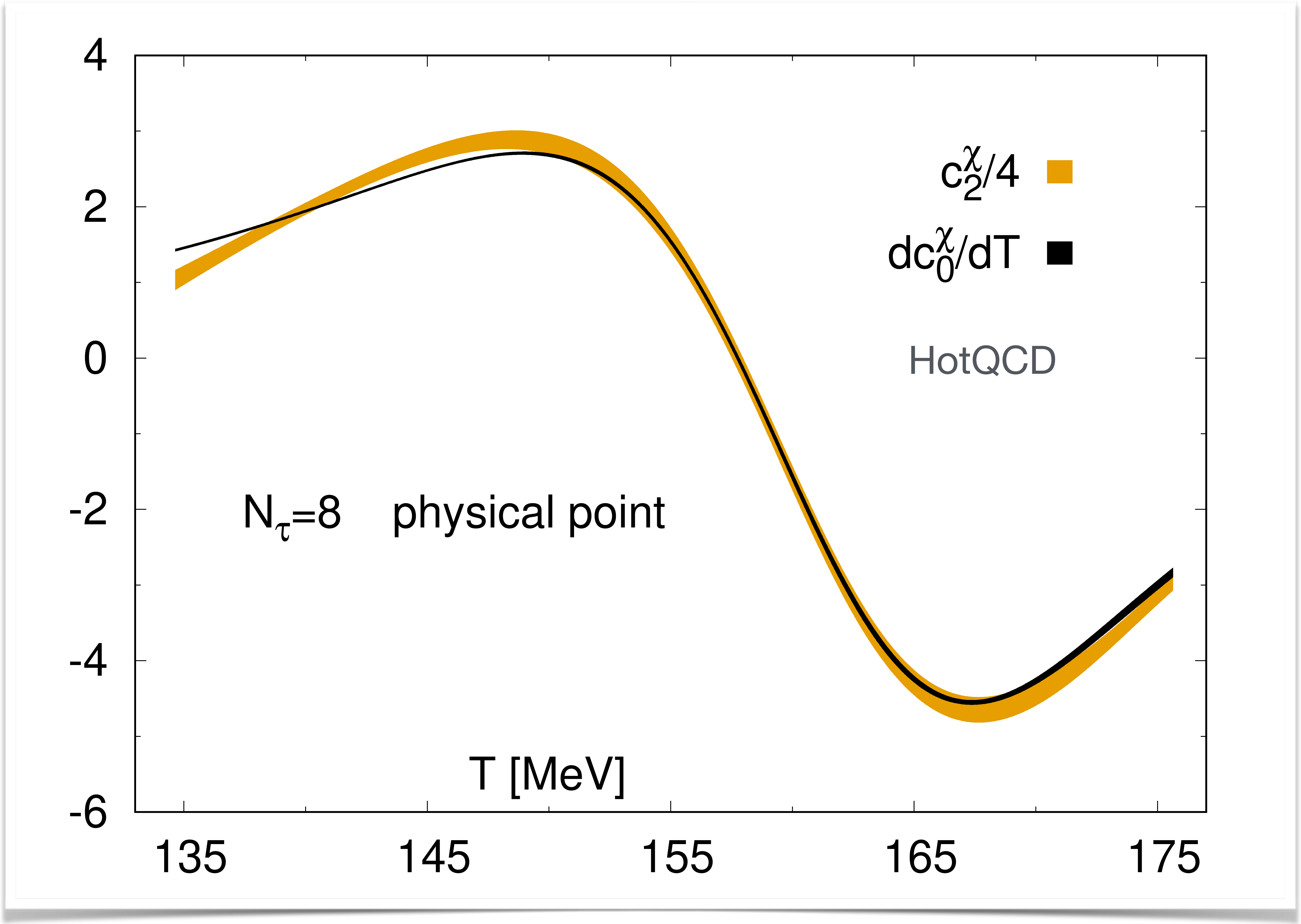


$f'_G : \ \partial_T \Sigma_{sub}$



$f_\chi(z) : \ \chi_{sub}$

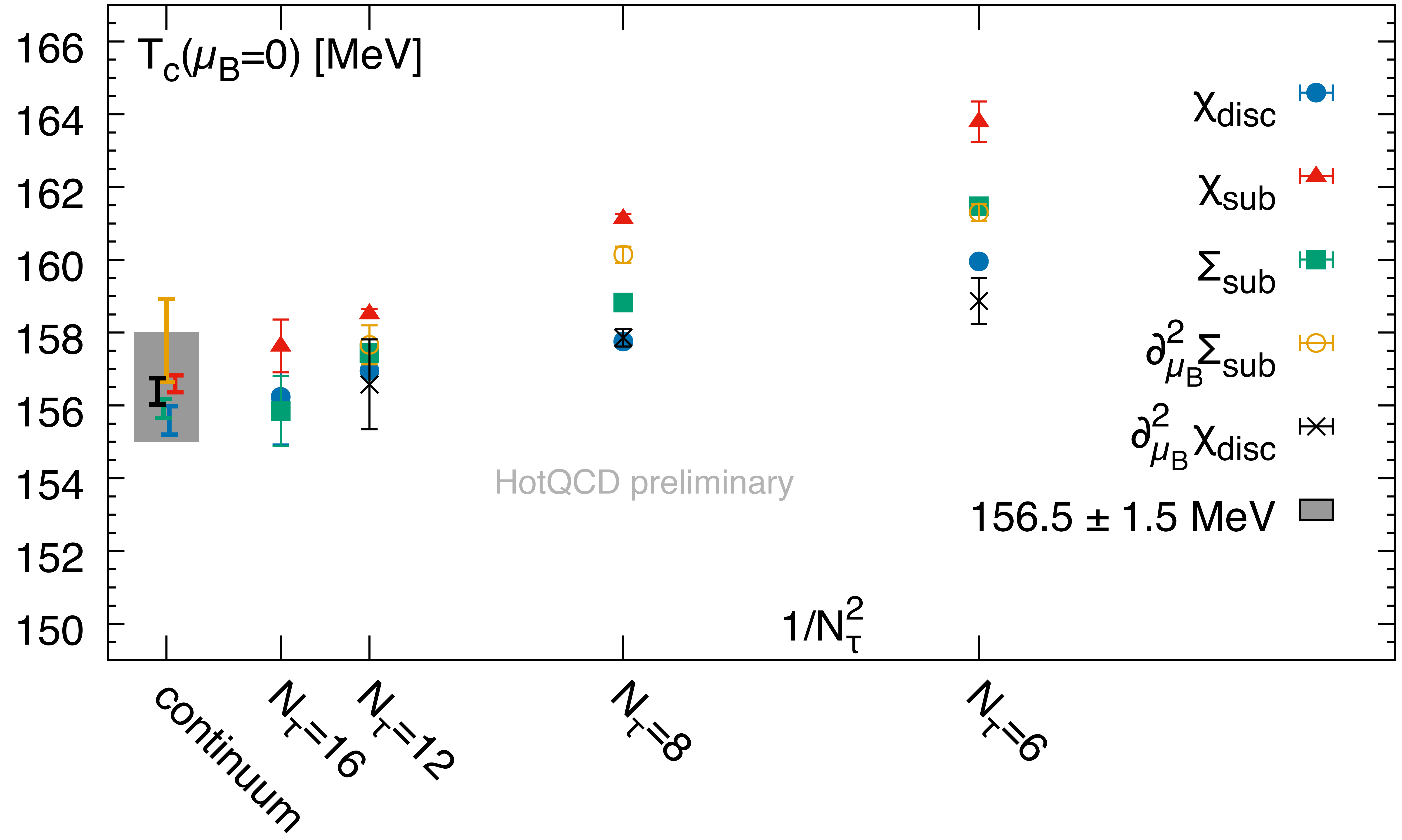
$$\partial_T \chi_{disc} \sim \partial^2_{\mu_B} \chi_{disc}$$



$$\chi_{disc} \sim m^{1/\delta - 1} f_\chi(z)$$

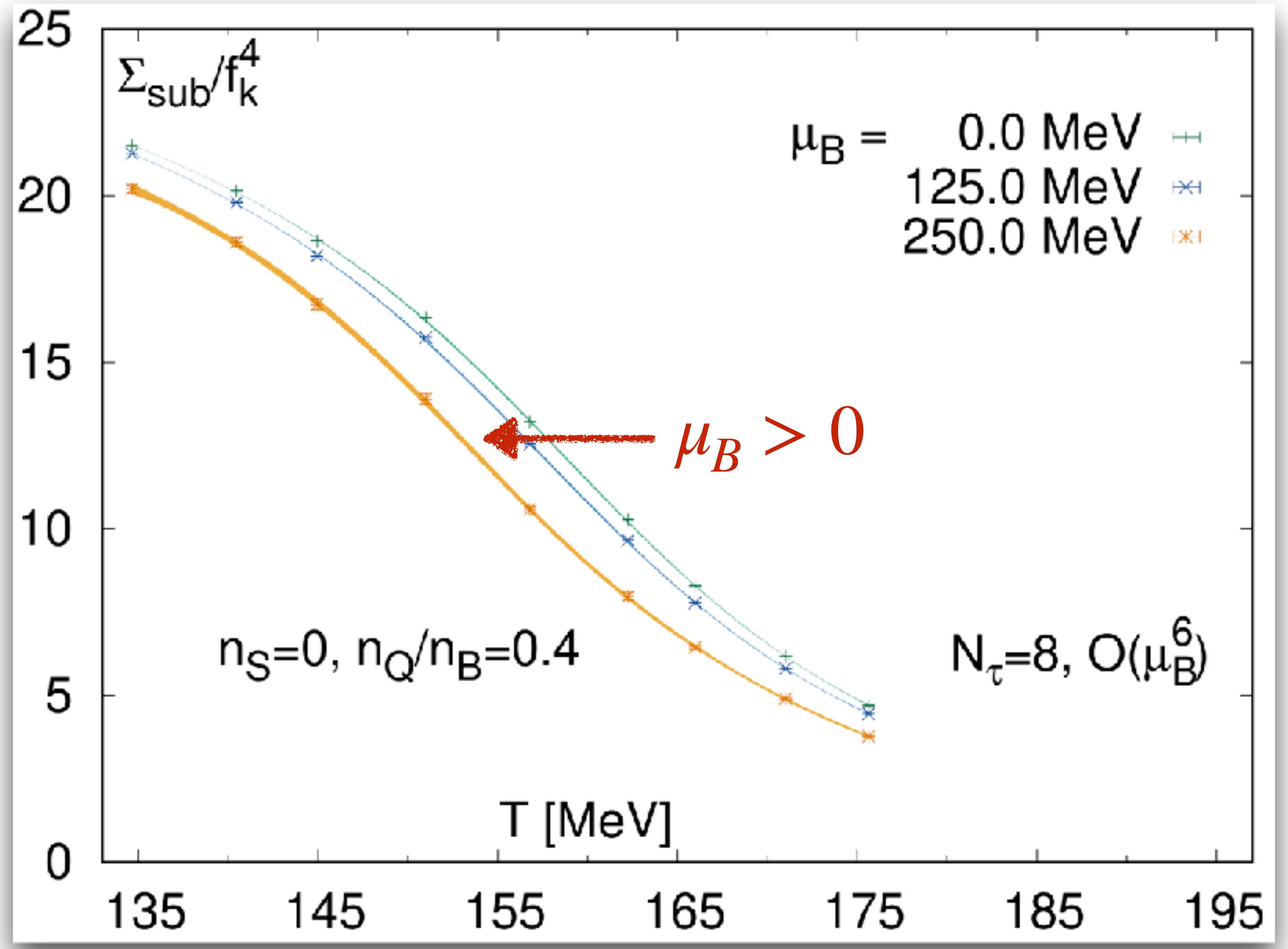
$$z \sim \#[(T_{pc} - Tc) + \#\mu_B^2]/m^{-1/\beta\delta}$$

- improved: $T_c(\mu_B = 0) = 156.5 \pm 1.5\ MeV$

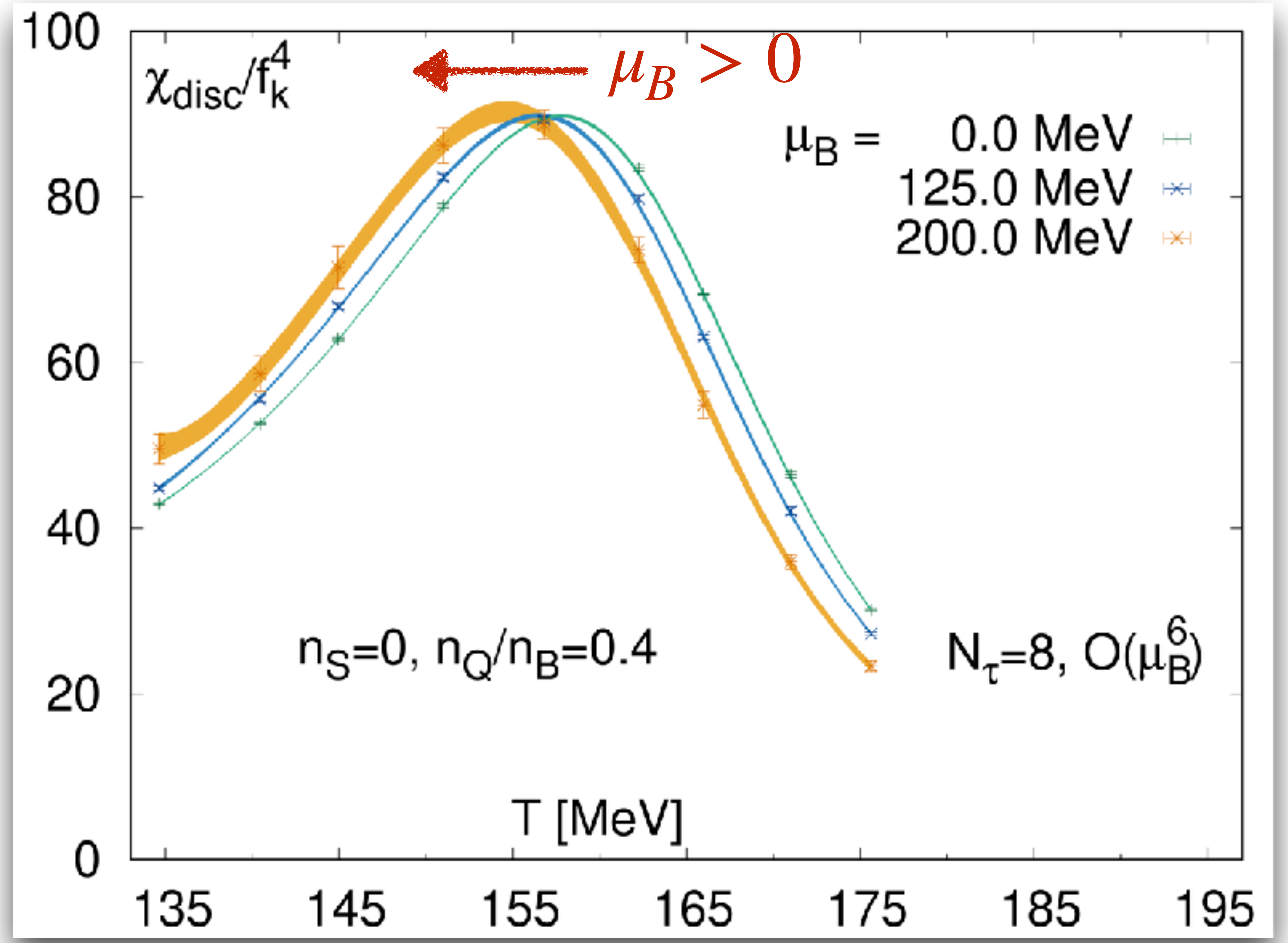


previously: $T_c(\mu_B = 0) = 154(9)\ MeV$

chiral condensate



chiral susceptibility

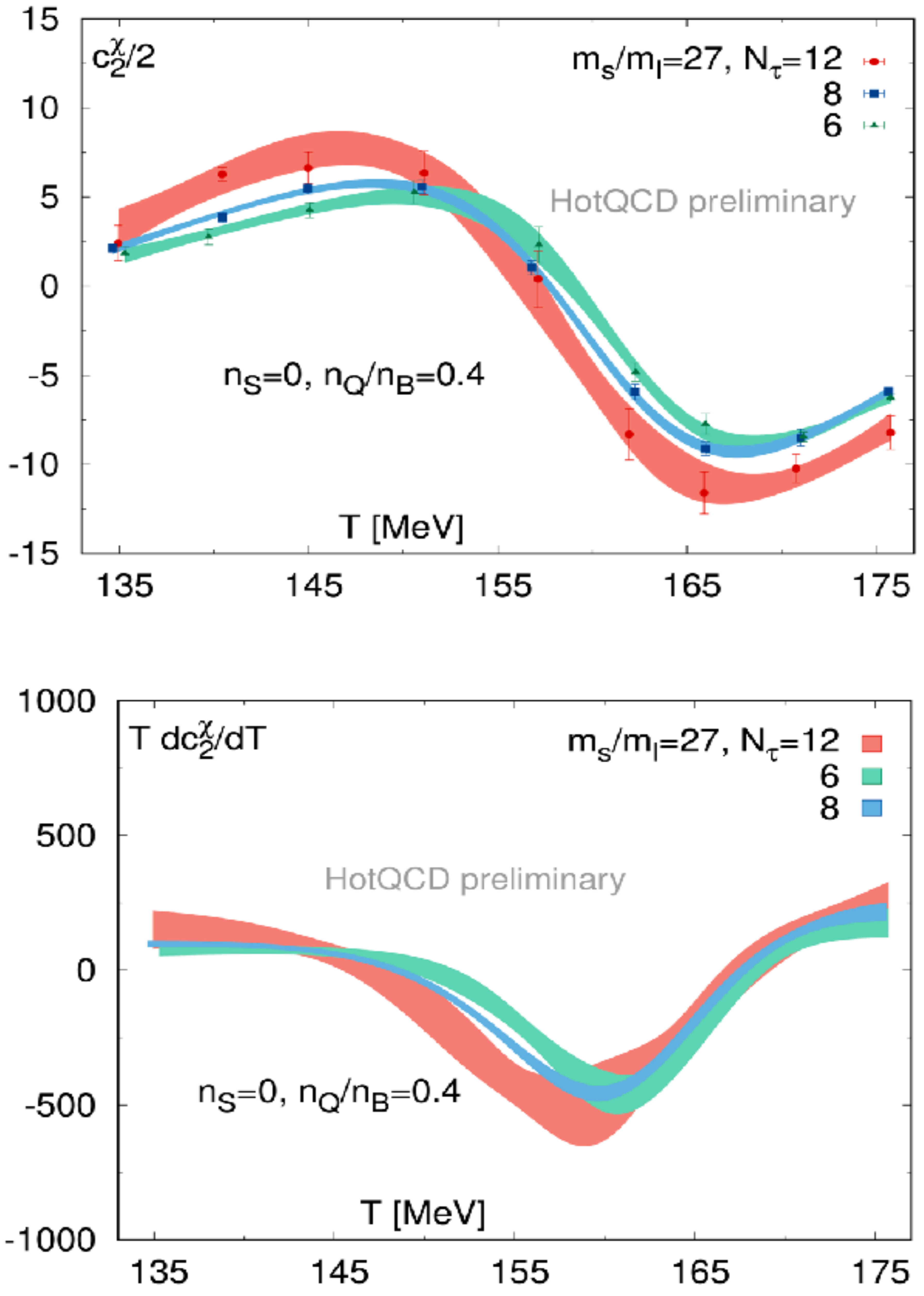


$n_S = 0,\ n_Q/n_B = 0.4$: heavy-ion collision like strangeness neutrality & charge-to-baryon ratio

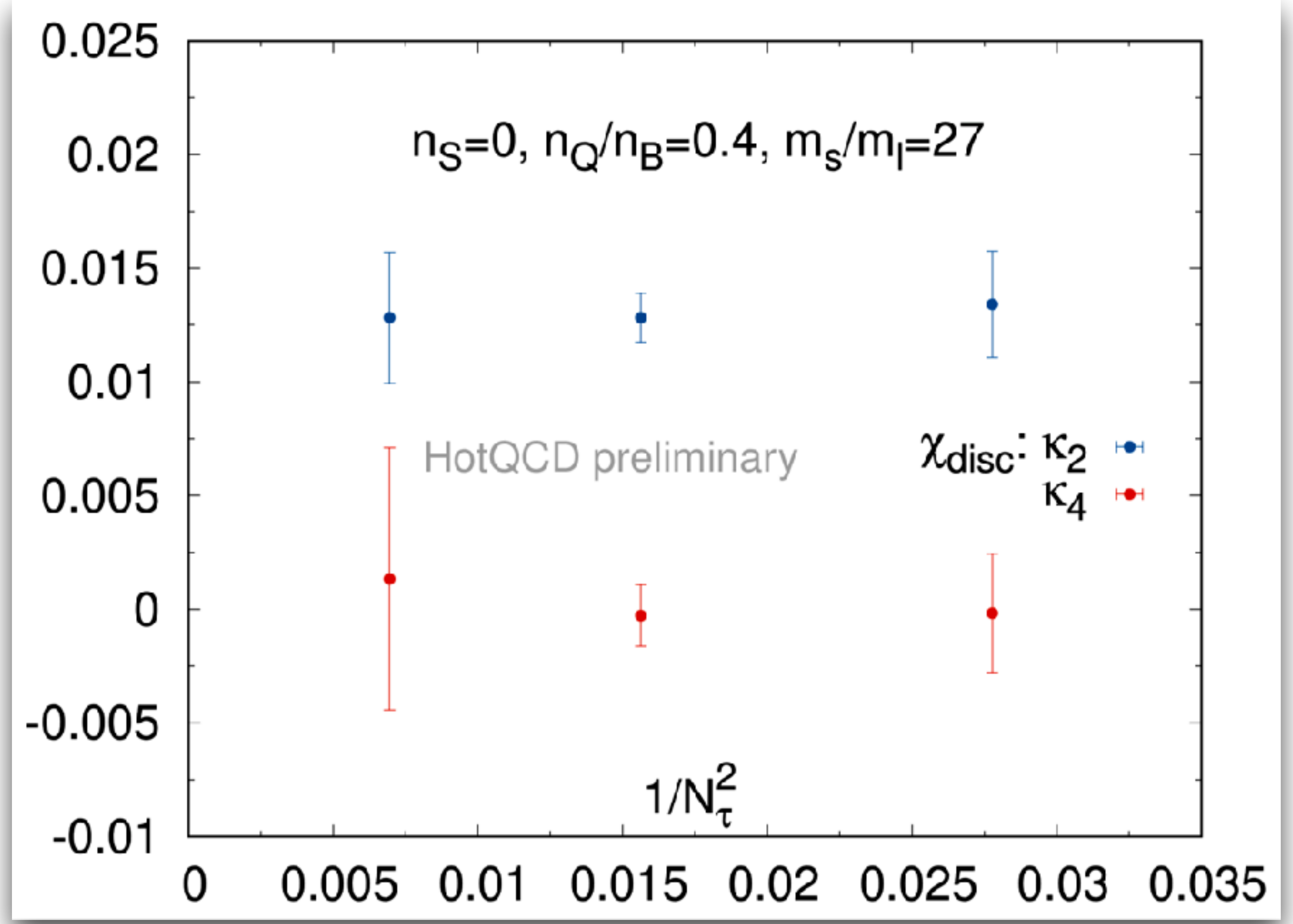
$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)$$

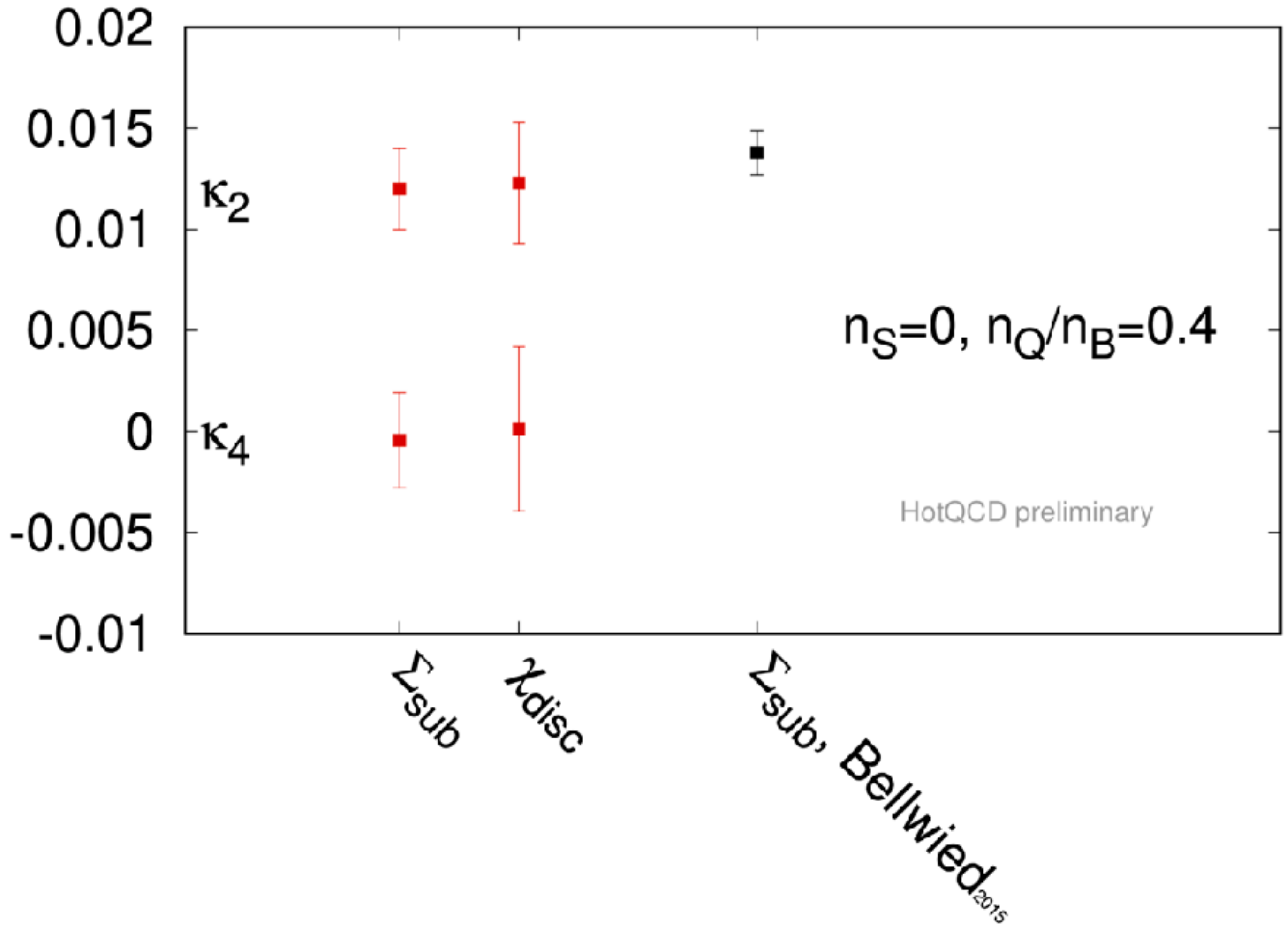
$$\frac{d}{dT} \frac{\chi_{\rm disc}(T,\mu_B)}{f_K^4} = (\ldots)\mu_B^2 + (\ldots)\mu_B^4 + \ldots = \mathbf{0}$$

$$\kappa_2 = \frac{1}{2T_0^2} \frac{T_0 \left.\frac{\partial c_2^\chi}{\partial T}\right|_{(T_0,0)} - 2\, c_2^\chi\big|_{(T_0,0)}}{\left.\frac{\partial^2 c_0^\chi}{\partial T^2}\right|_{(T_0,0)}}$$



$$\frac{T_c(\mu_B)}{T_c(0)} = 1-\kappa_2^B\left(\frac{\mu_B}{T_c(0)}\right)^2-\kappa_4^B\left(\frac{\mu_B}{T_c(0)}\right)^4+\mathcal{O}\left(\mu_B^6\right)$$

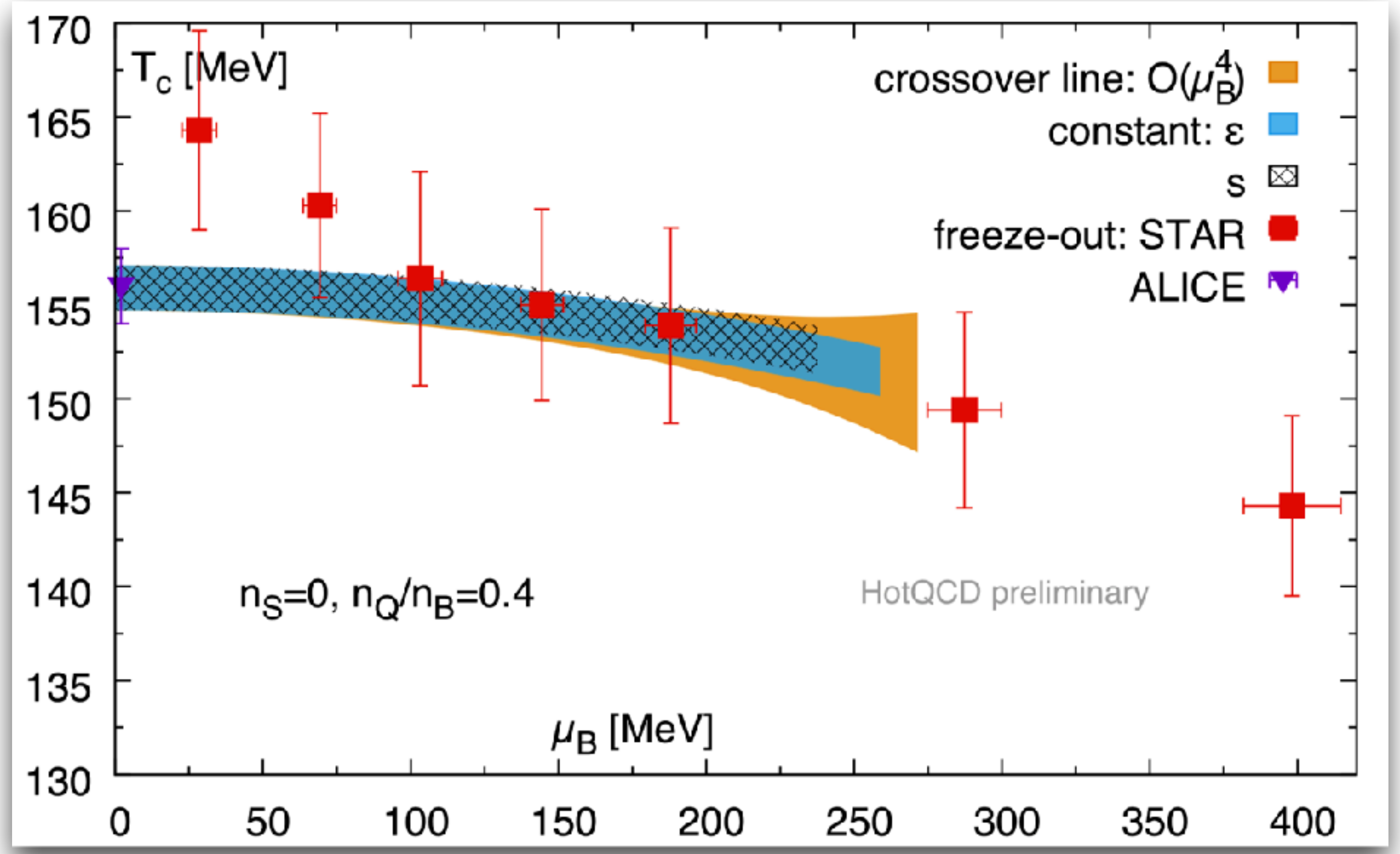




- 4^{th} order corrections order of magnitude smaller than 2^{nd}

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2^B \left(\frac{\mu_B}{T_c(0)}\right)^2 - \kappa_4^B \left(\frac{\mu_B}{T_c(0)}\right)^4 + \mathcal{O}\left(\mu_B^6\right)$$

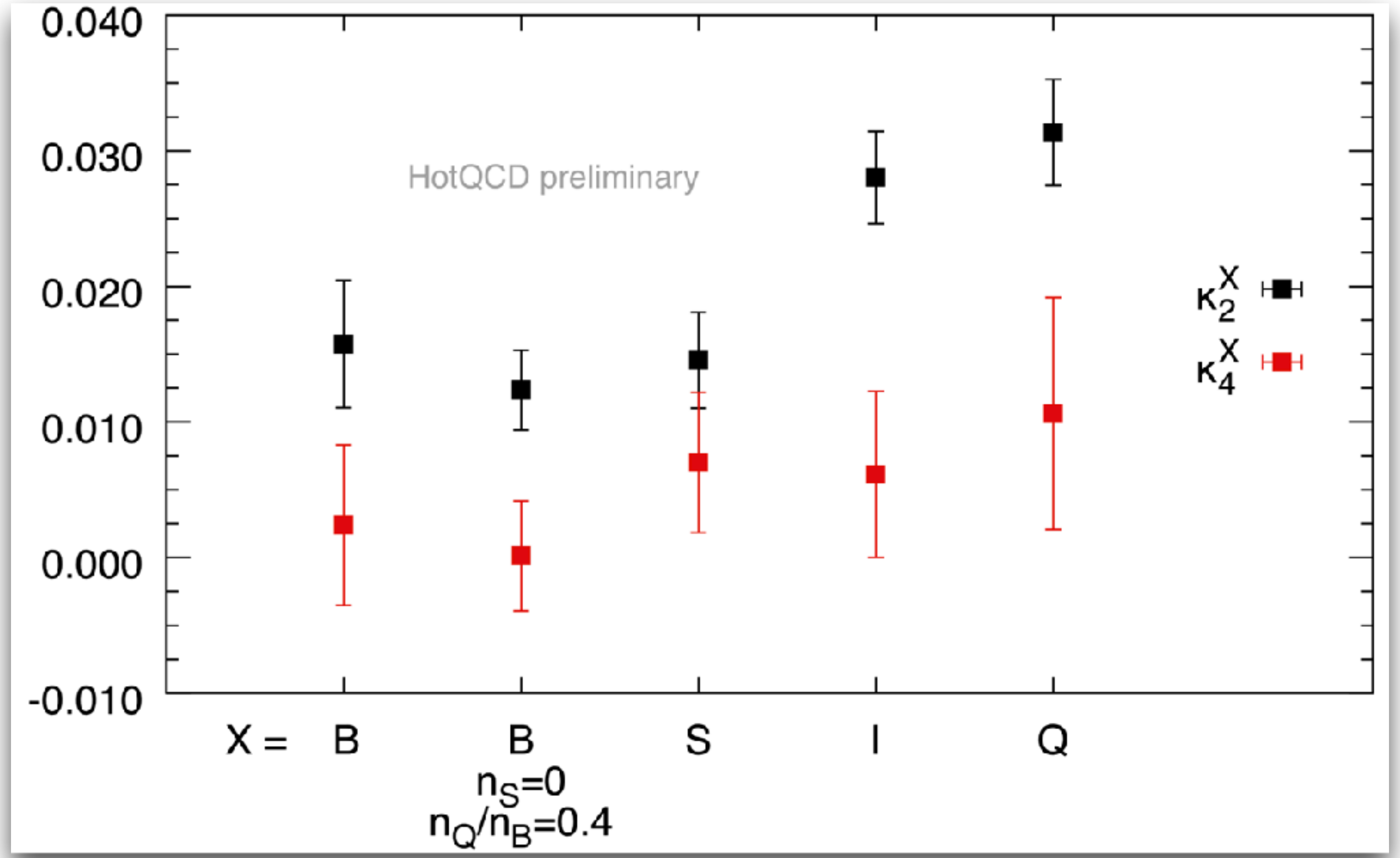
- along the chiral crossover energy density & entropy density remains constant
- freeze-out line coincides with the chiral crossover



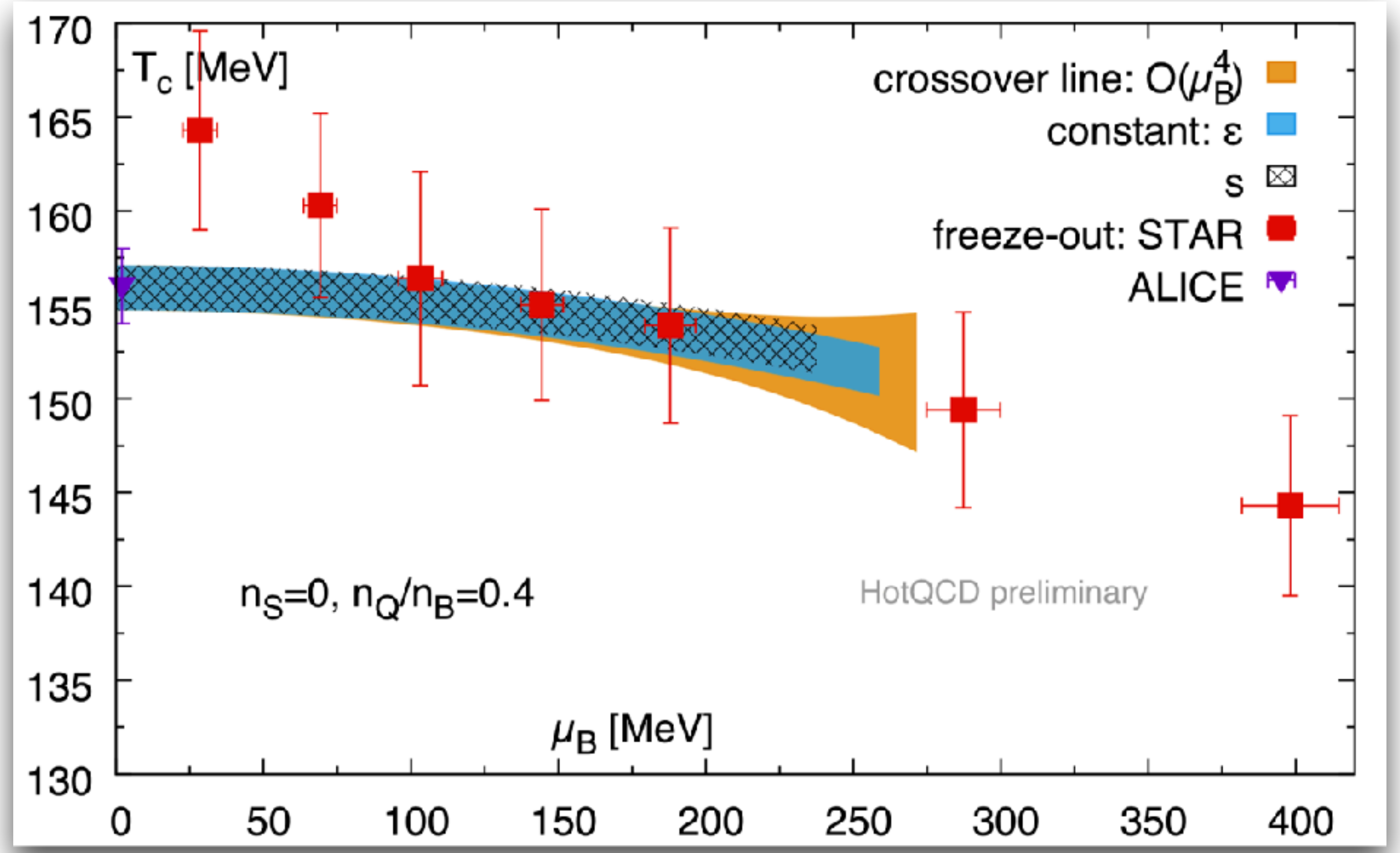
chiral crossover surface: $T_c(\mu_X)$

$$\frac{T_c(\mu_X)}{T_c(0)} = 1 - \kappa_2^B \left(\frac{\mu_X}{T_c(0)}\right)^2 - \kappa_4^B \left(\frac{\mu_X}{T_c(0)}\right)^4 + \mathcal{O}\left(\mu_B^6\right)$$

- X=B: baryon
- X=Q: electric charge
- X=S: strangeness
- X=I: isospin



search for criticality: $\mu_B > 0$

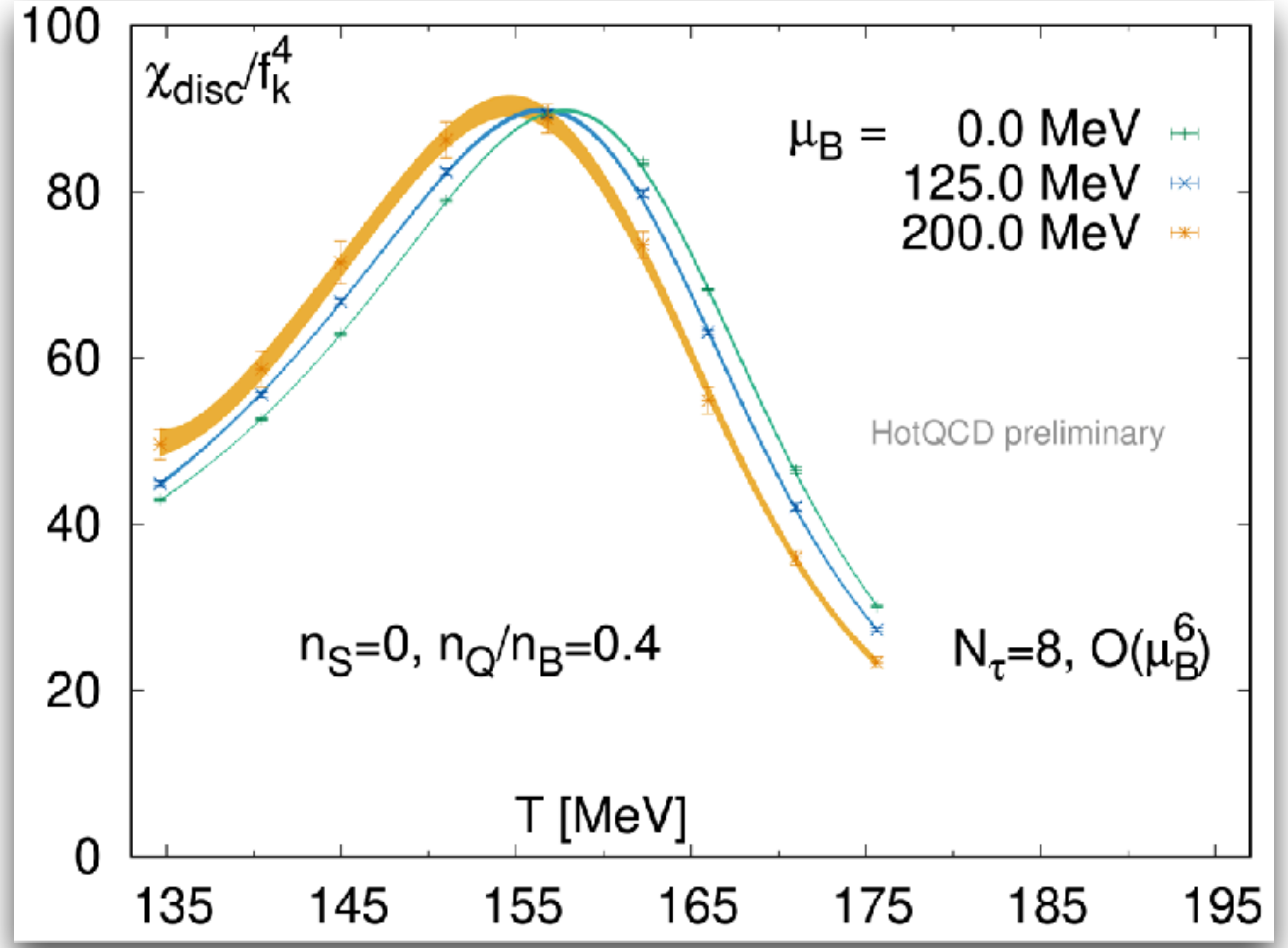


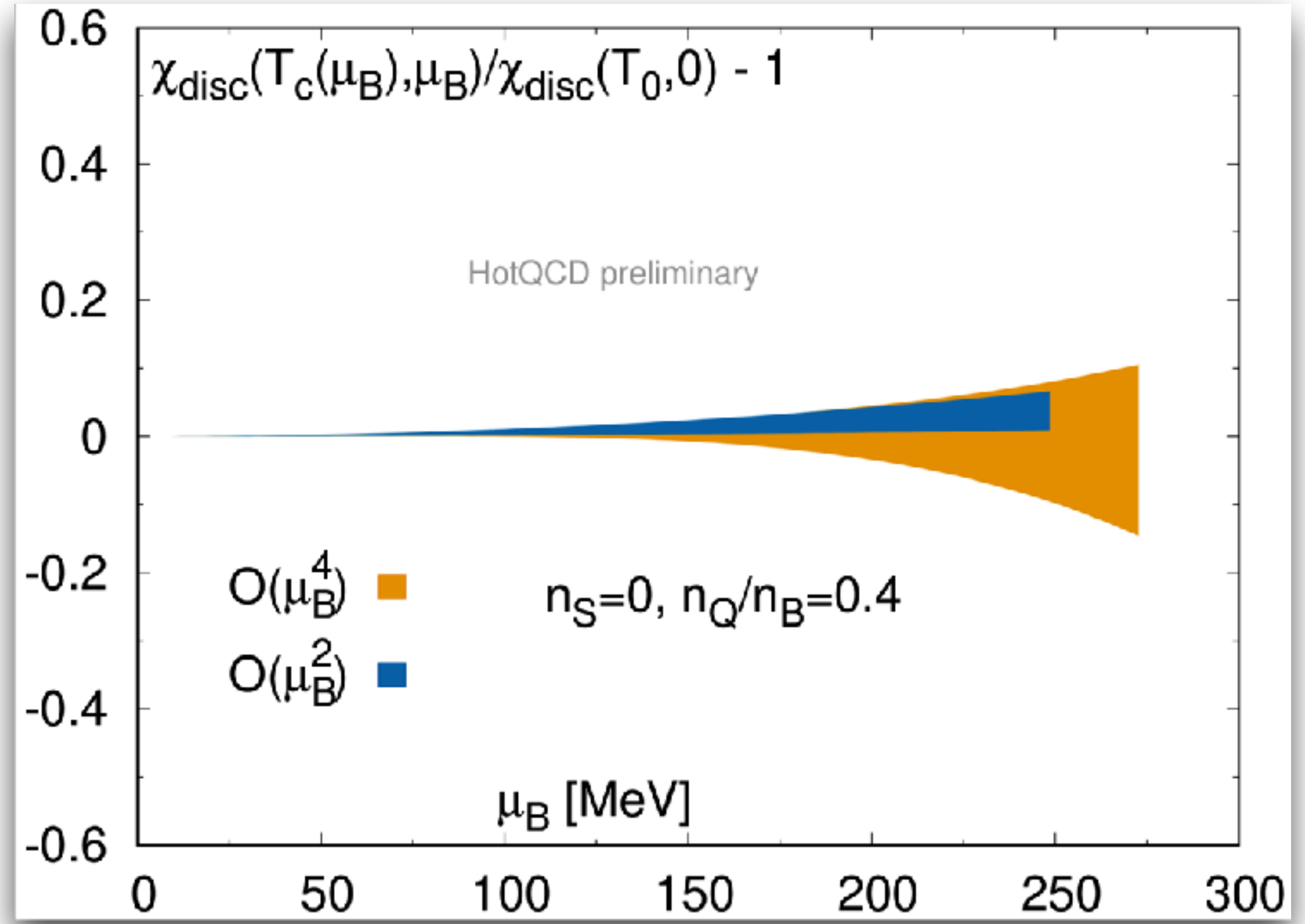
signs of enhanced fluctuations around the phase-boundary ?

chiral susceptibility

$\chi_{disc} \sim \partial^2 \ln \mathcal{Z} / \partial m_q^2$

along $T_c(\mu_B)$





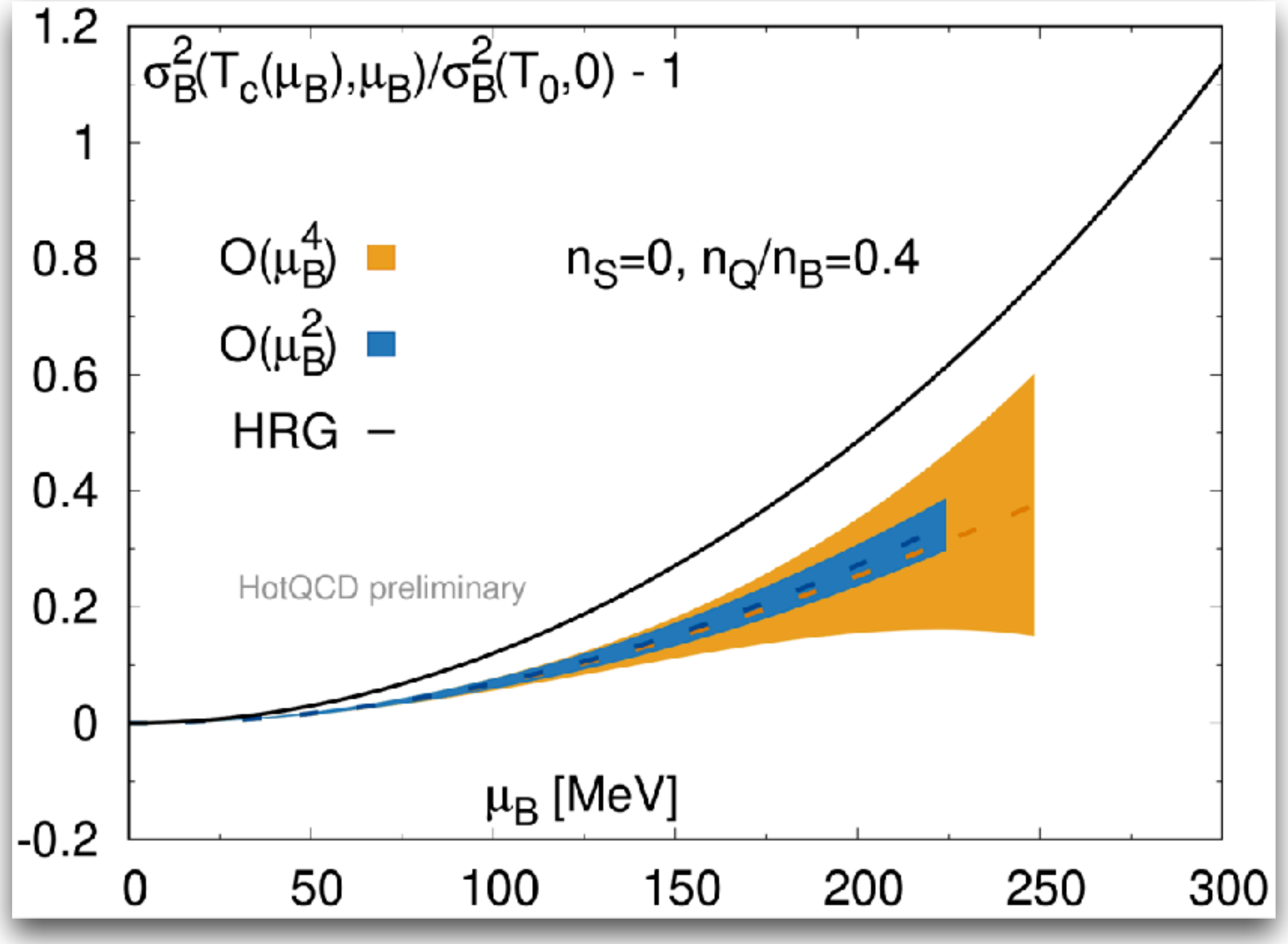
- little change in peak-height & width with increasing baryon chemical potential: no indication of a stronger transition becoming stronger

net-baryon number fluctuations

$$\frac{\sigma_B^2}{Vf_K^3} = \frac{1}{Vf_K^3}\frac{\partial \ln Z}{\partial \hat{\mu}_B^2} = \sum_{n=0}^{\infty} \frac{c_n^B}{n!}\hat{\mu}_B^n$$

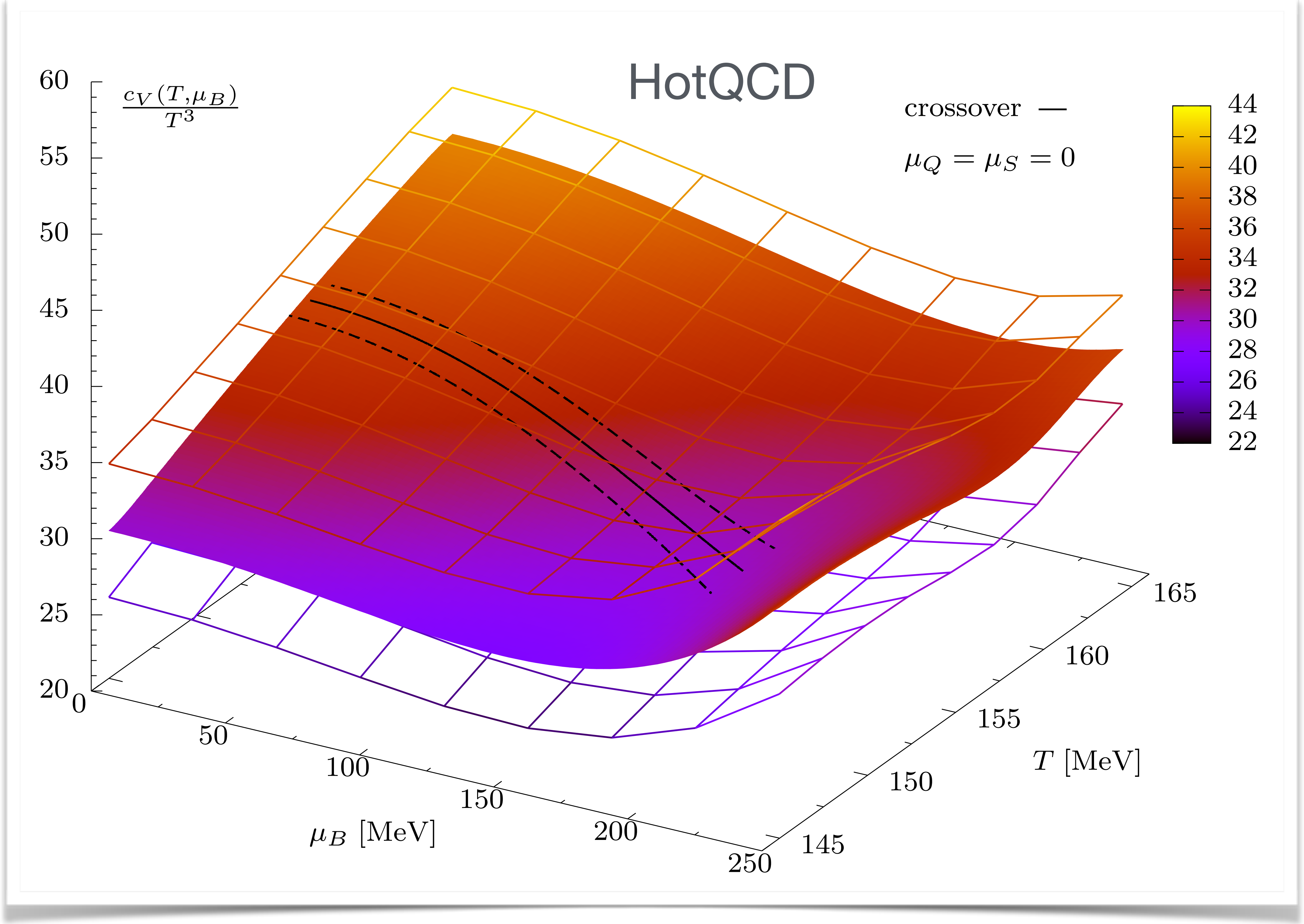
- increase remains less than (ideal) hadron gas resonance gas model (HRG)

along $T_c(\mu_B)$

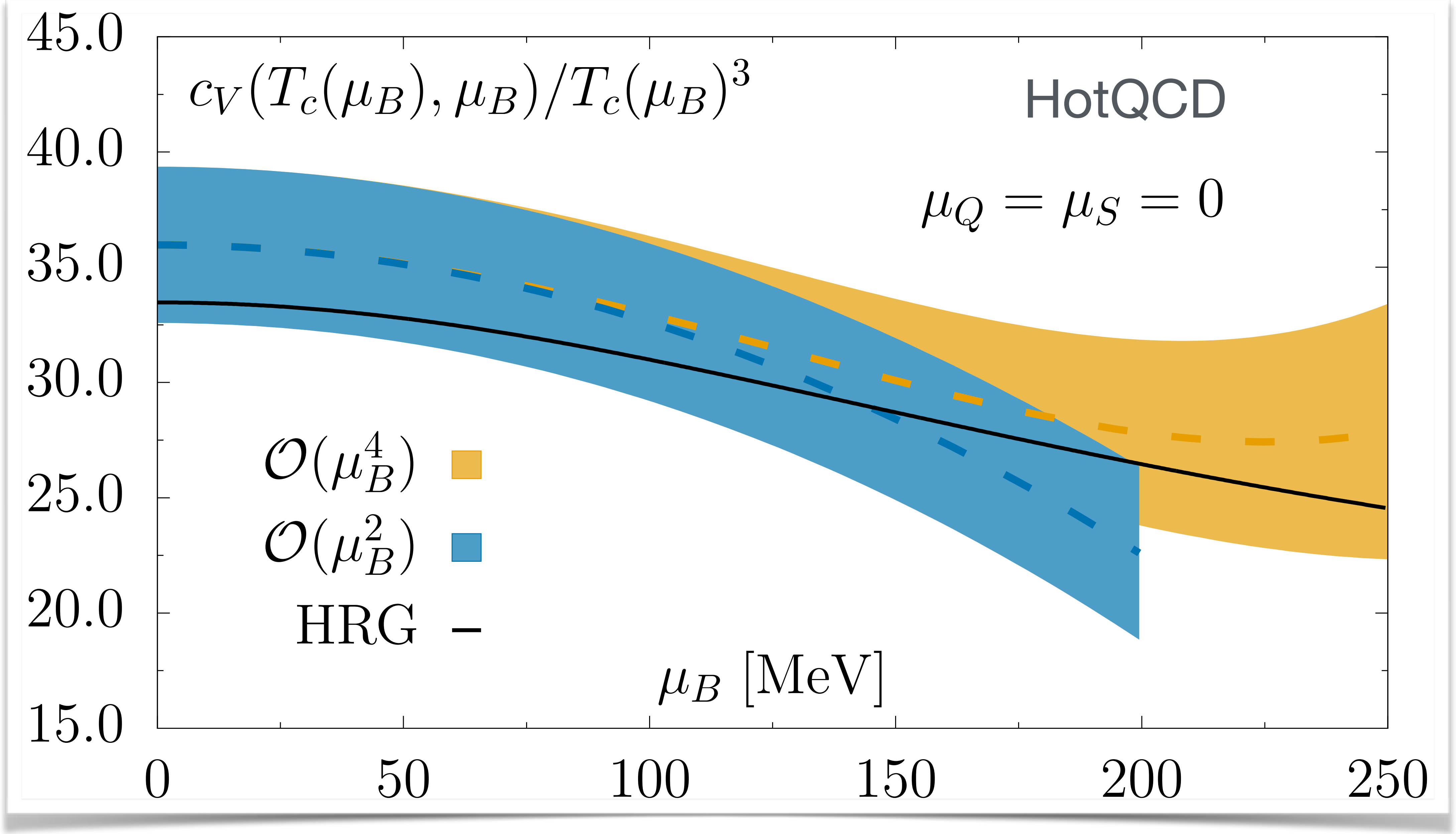


specific heat @ constant volume

$$c_V = T\left(\frac{\partial s}{\partial T}\right)_{n_B}$$



along $T_c(\mu_B)$



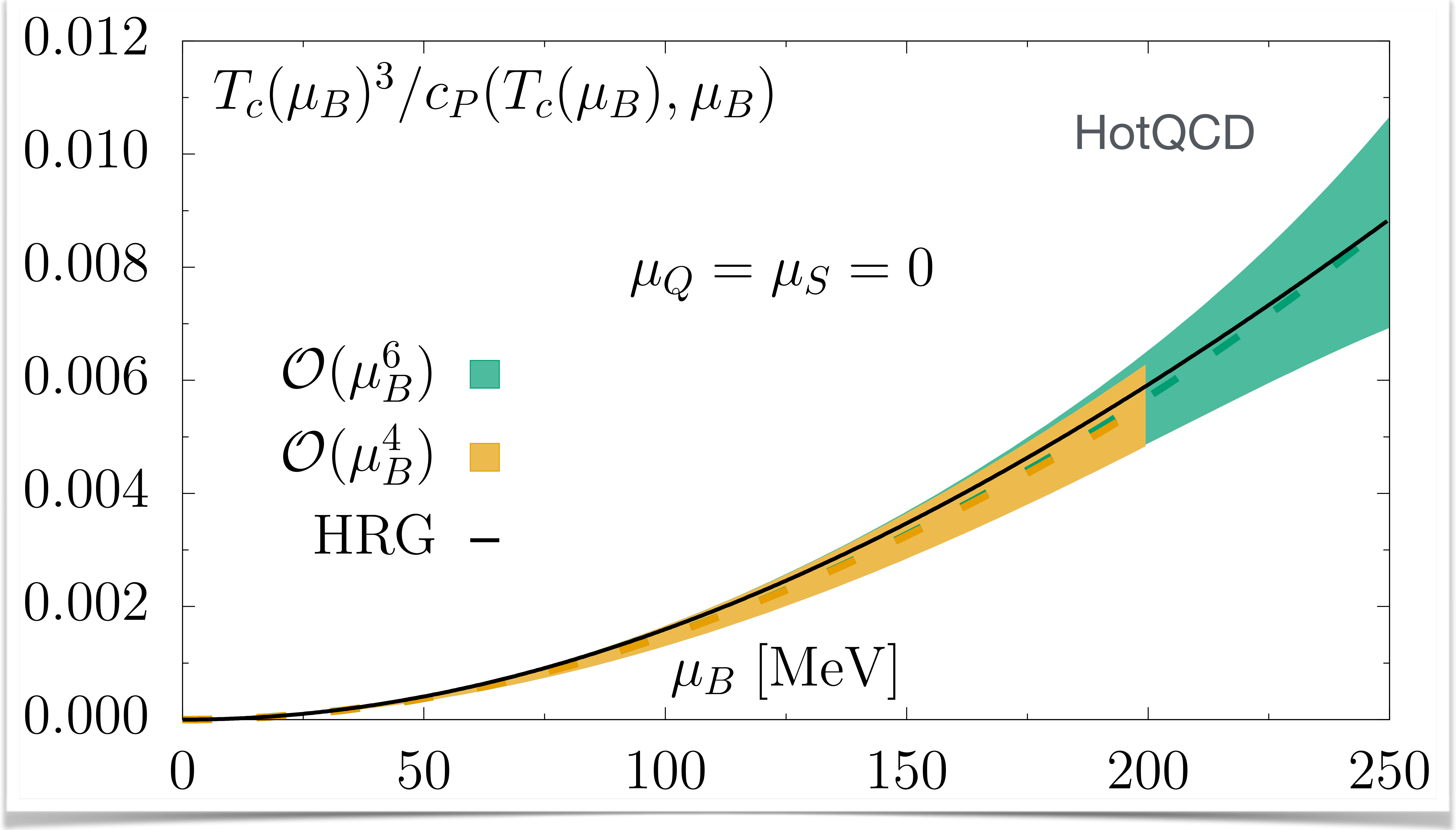
- no increase above HRG

(inverse) specific heat @ constant pressure

$$c_p = \frac{T}{(s/n_B)}\left[\frac{\partial(s/n_B)}{\partial T}\right]_p$$

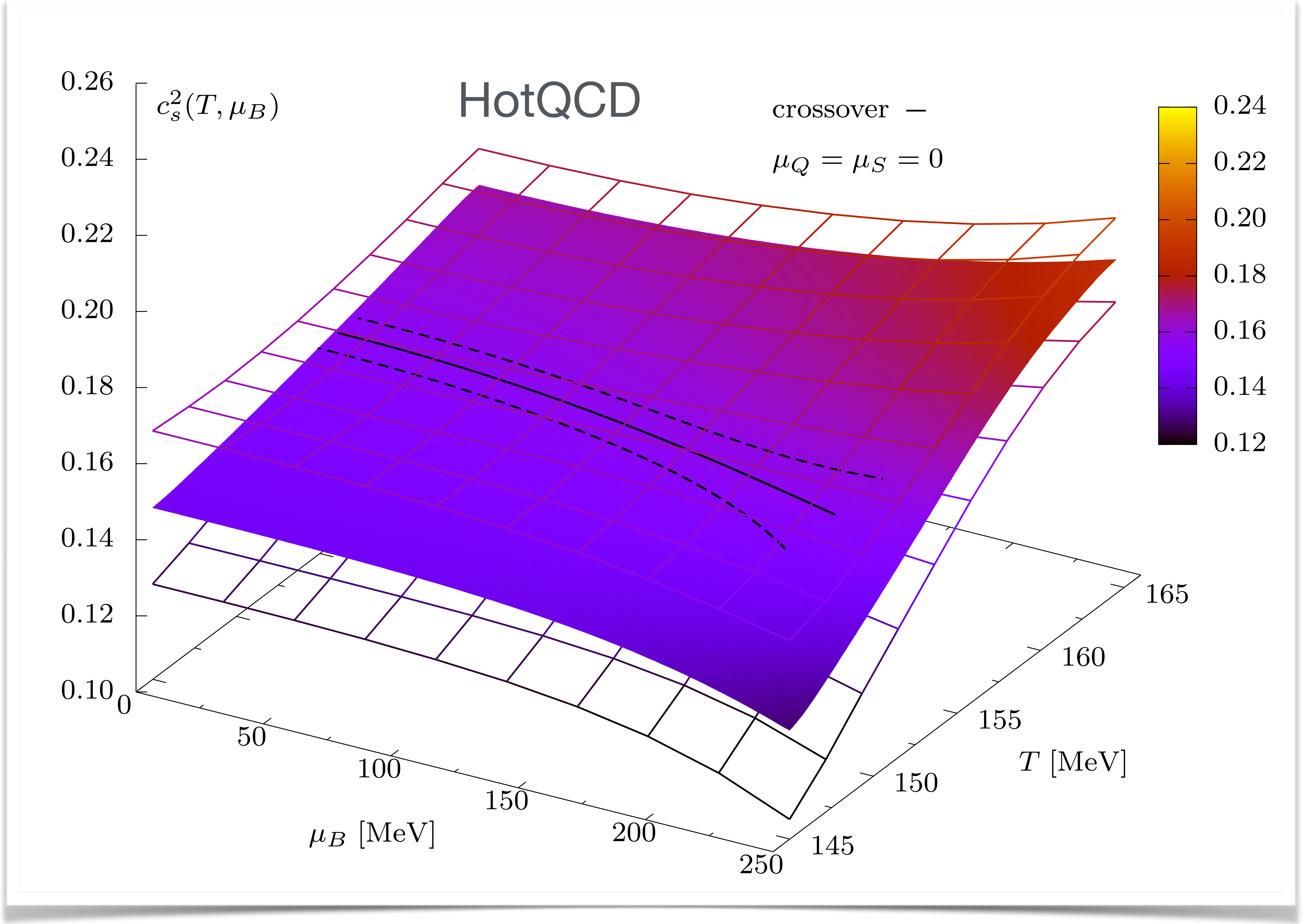
- no increase above HRG

along $T_c(\mu_B)$

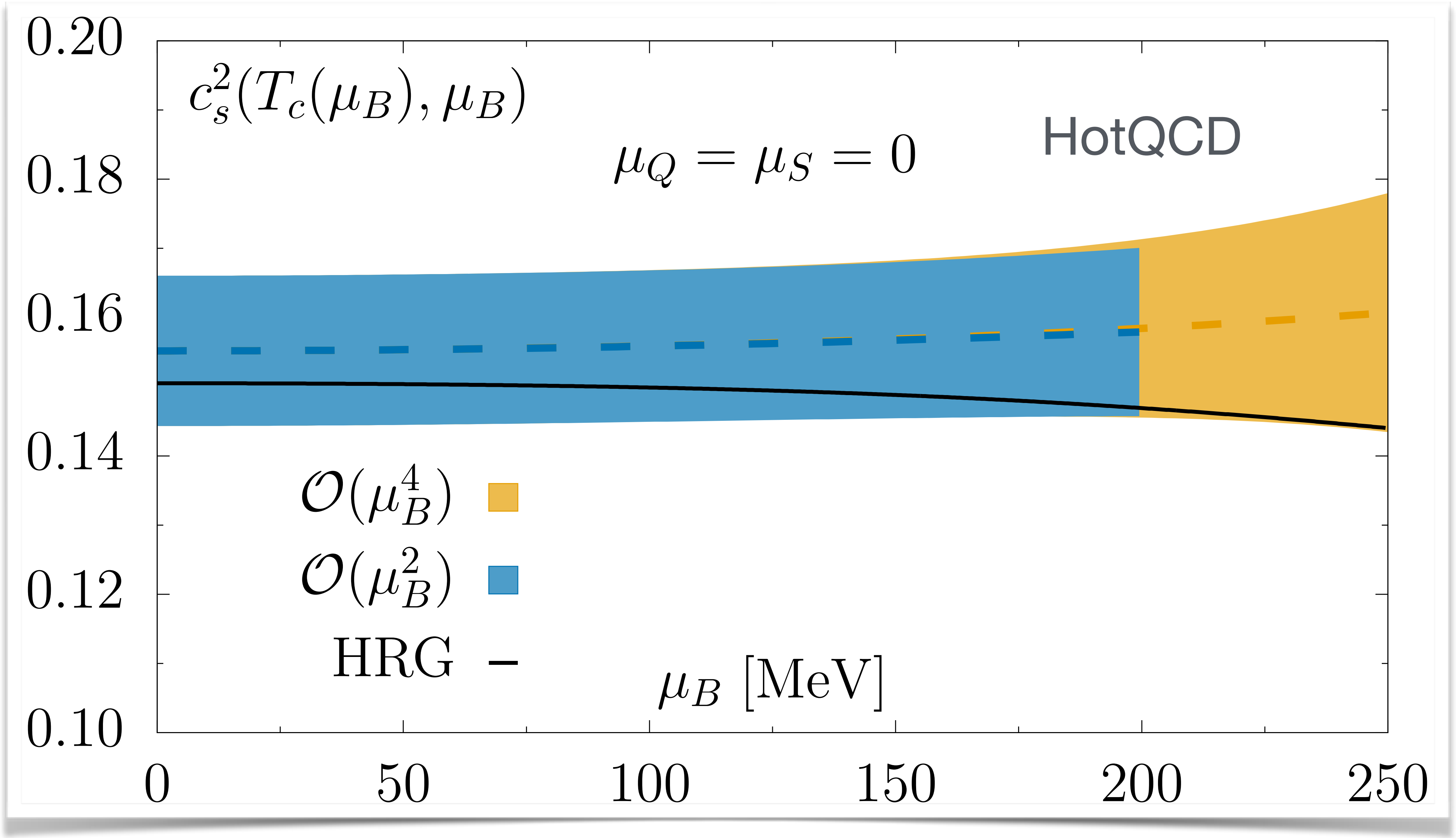


isentropic speed of sound

$$c_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_{s/n_B}$$



along $T_c(\mu_B)$

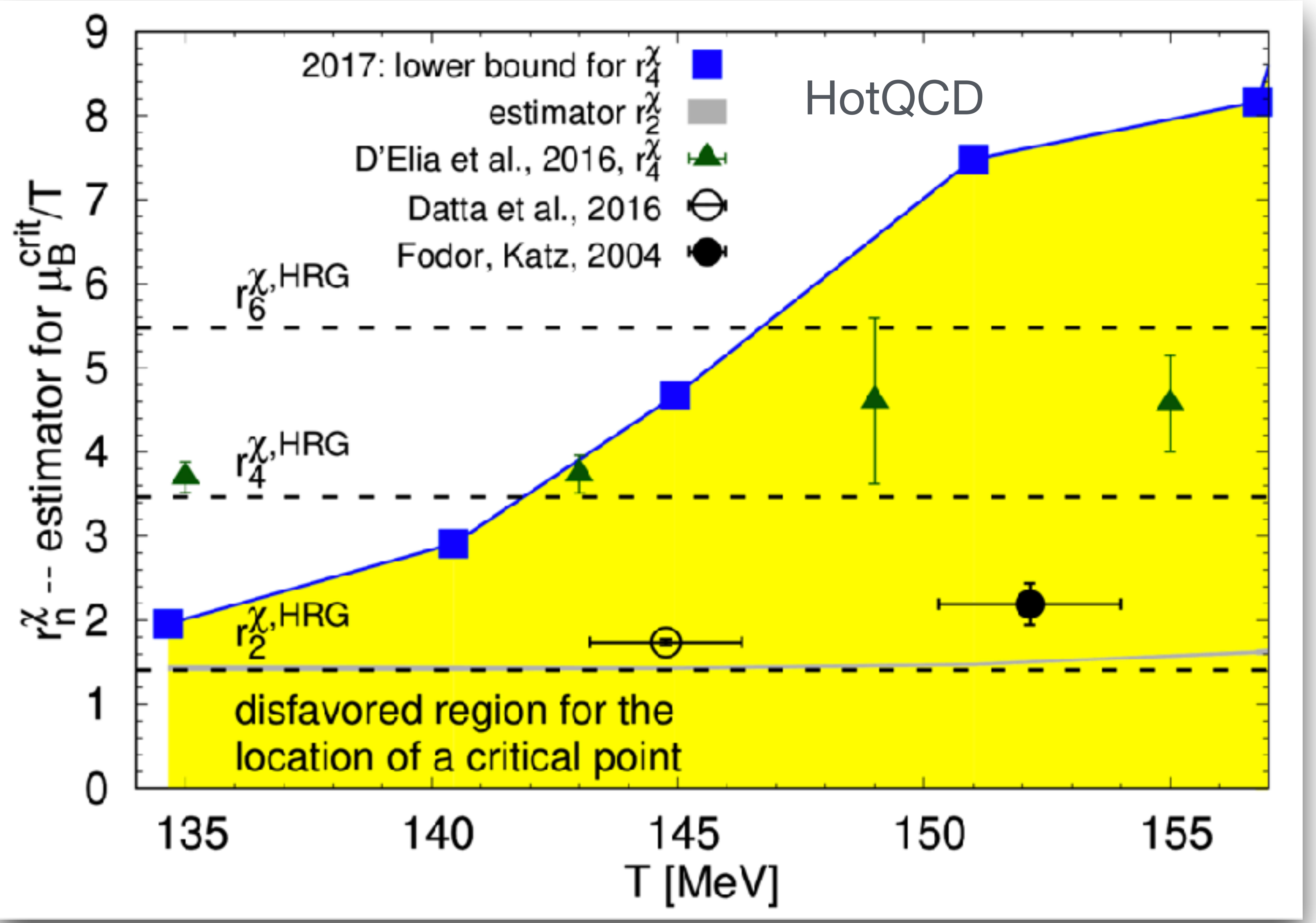


- no decrease below HRG

constraining the location of QCD critical point …

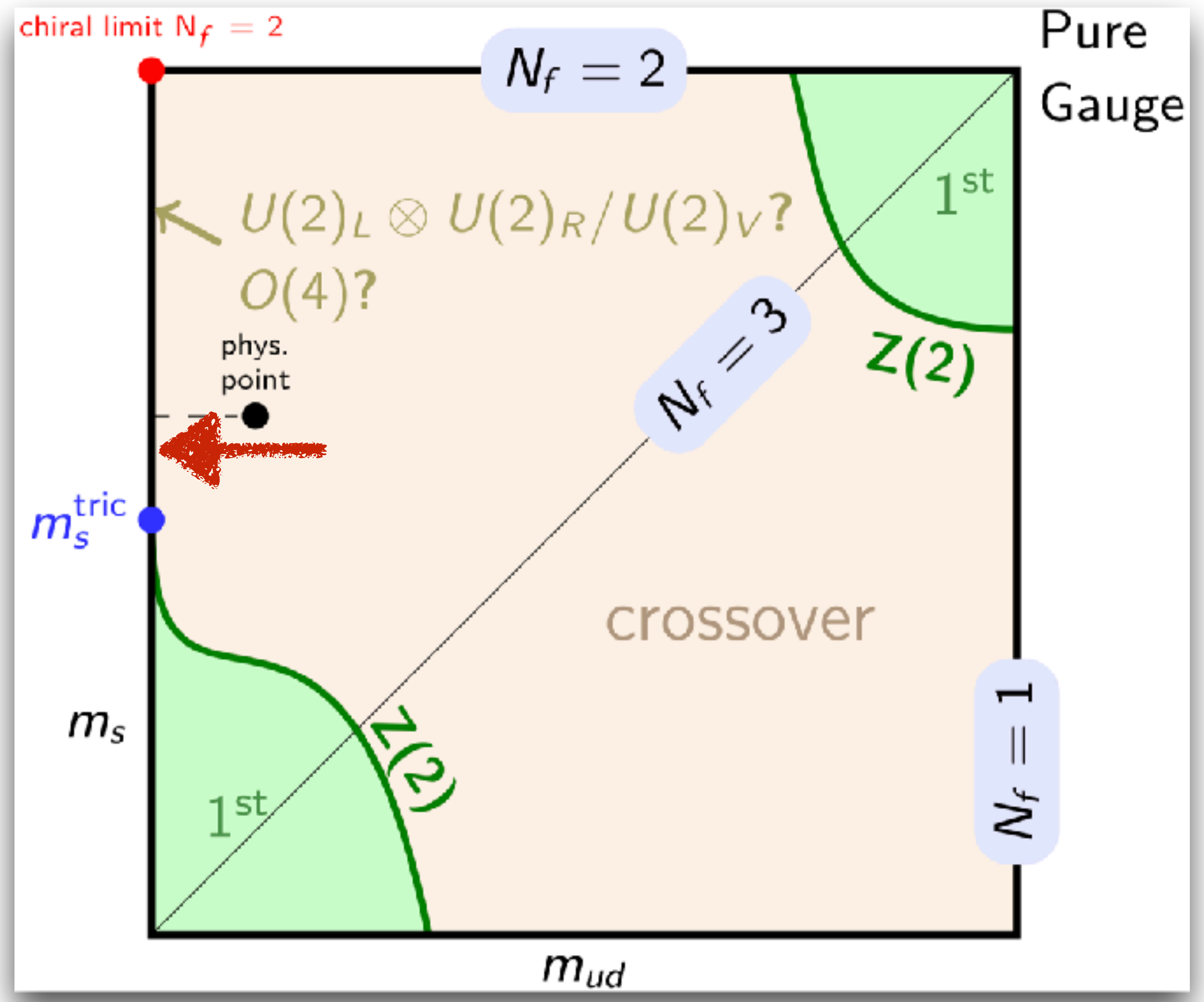
- radius of convergence for baryon number susceptibility:

$$r_{2n}^{\chi} = \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^{B}}{\chi_{2n+2}^{B}}\right|}$$



- disfavored for: $\mu_B \lesssim 300\ MeV$

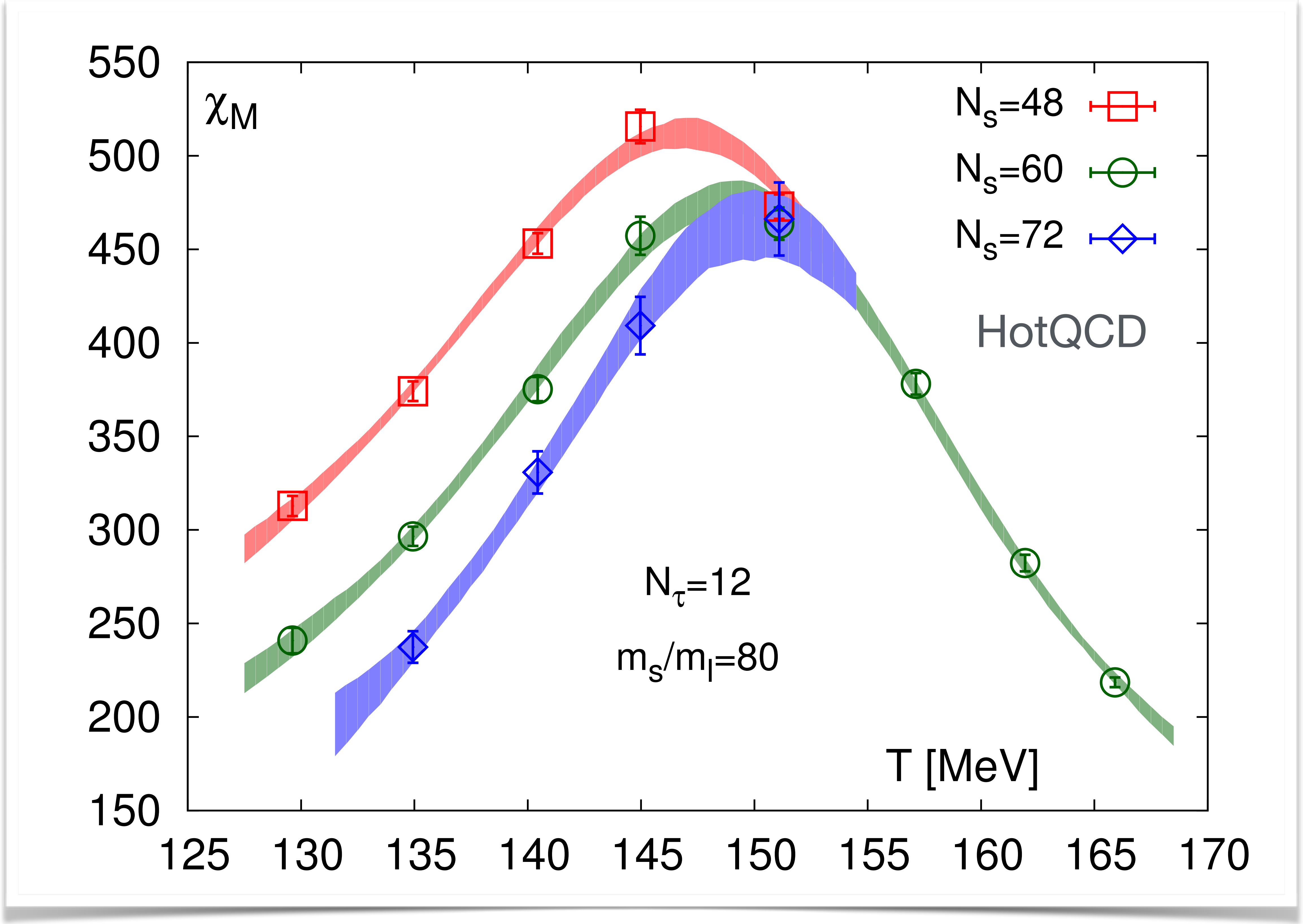
search for criticality: $\mu_B = 0$



towards chiral limit …

- u/d quark masses: $m_u = m_d \equiv m_l \to 0$
- s quark mass: m_s^{phys}

volume dependence of chiral susceptibility



- no direct evidence of 1st order transition for $m_\pi \gtrsim 50\ MeV$

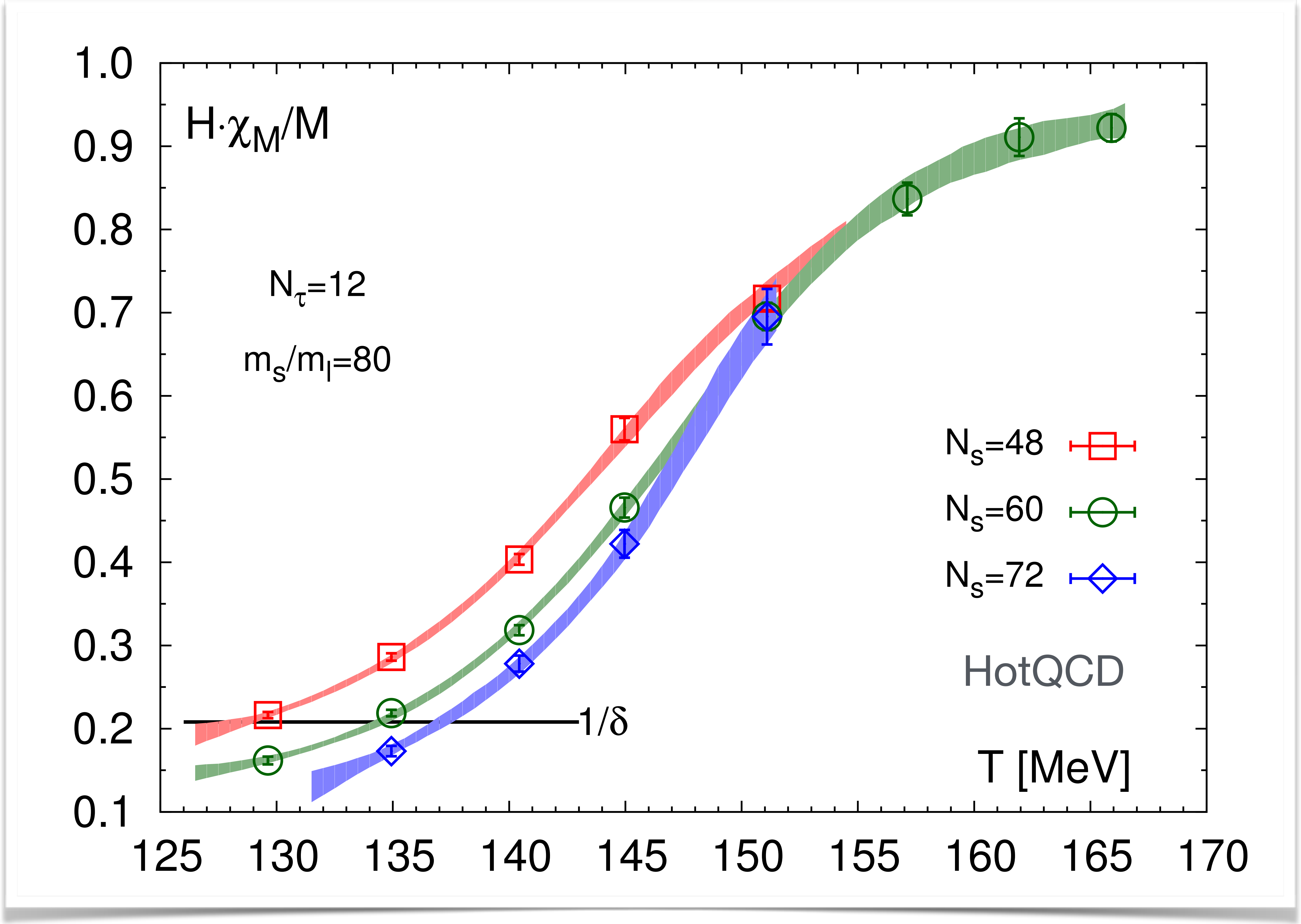
T_c^0 : critical temperature in the chiral limit

$$H \cdot \chi_M / M = 1/\delta \ @ \ T_c^0$$

M: chiral condensate

χ_M : chiral susceptibility

$H = m_l / m_s^{phys}$



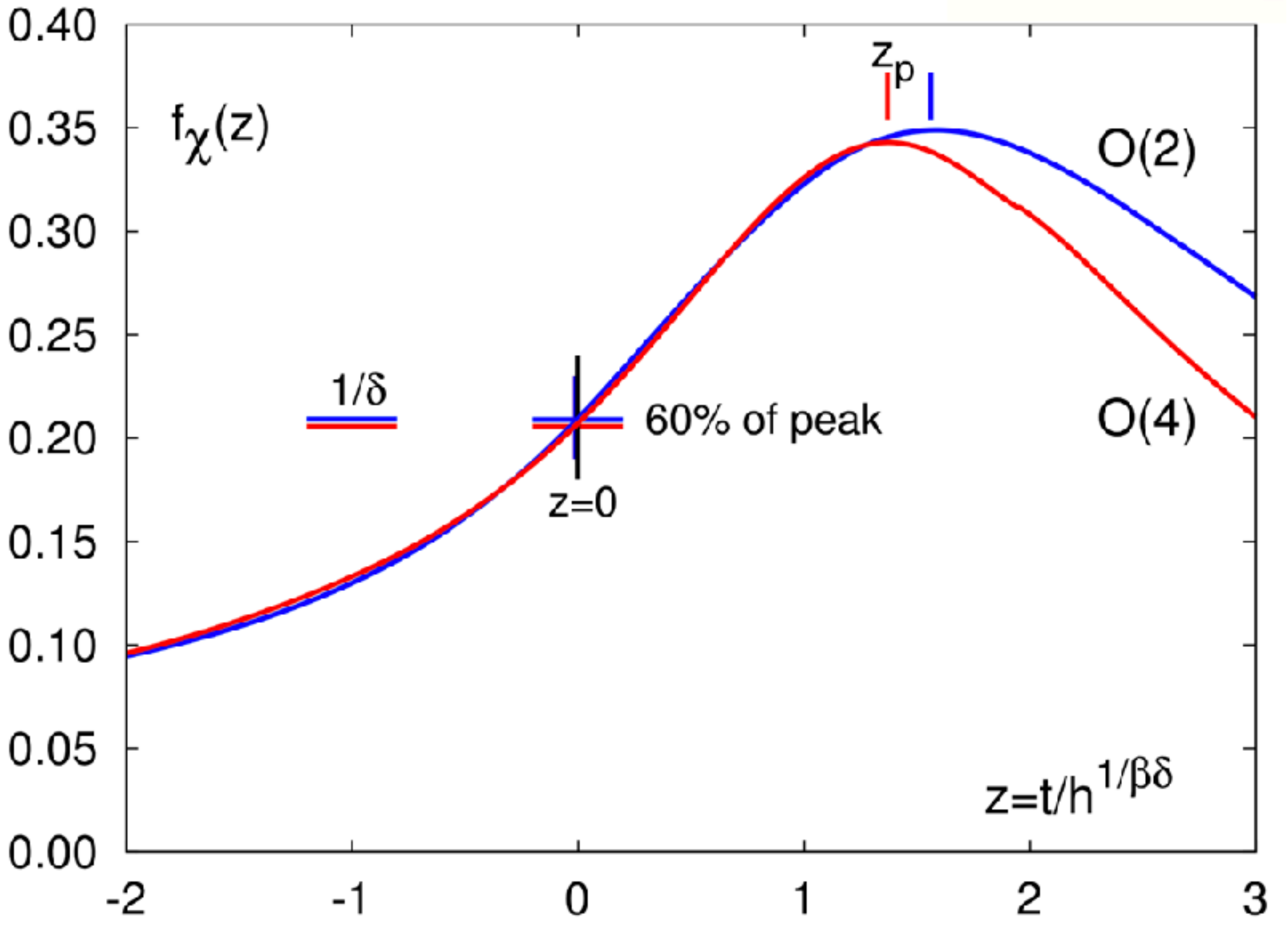
T_c^0 : critical temperature, chiral limit H=0

conventionally: $T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{1/\beta\delta}\right)$

but: $z_{\overline{60\%}} \approx 0$

$$T_{\overline{60\%}}(H) = T_c^0 \left(1 + \frac{z_{\overline{60\%}}}{z_0} H^{1/\beta\delta}\right)$$

- very mild dependence on quark mass
- scaling-violating regular terms are small

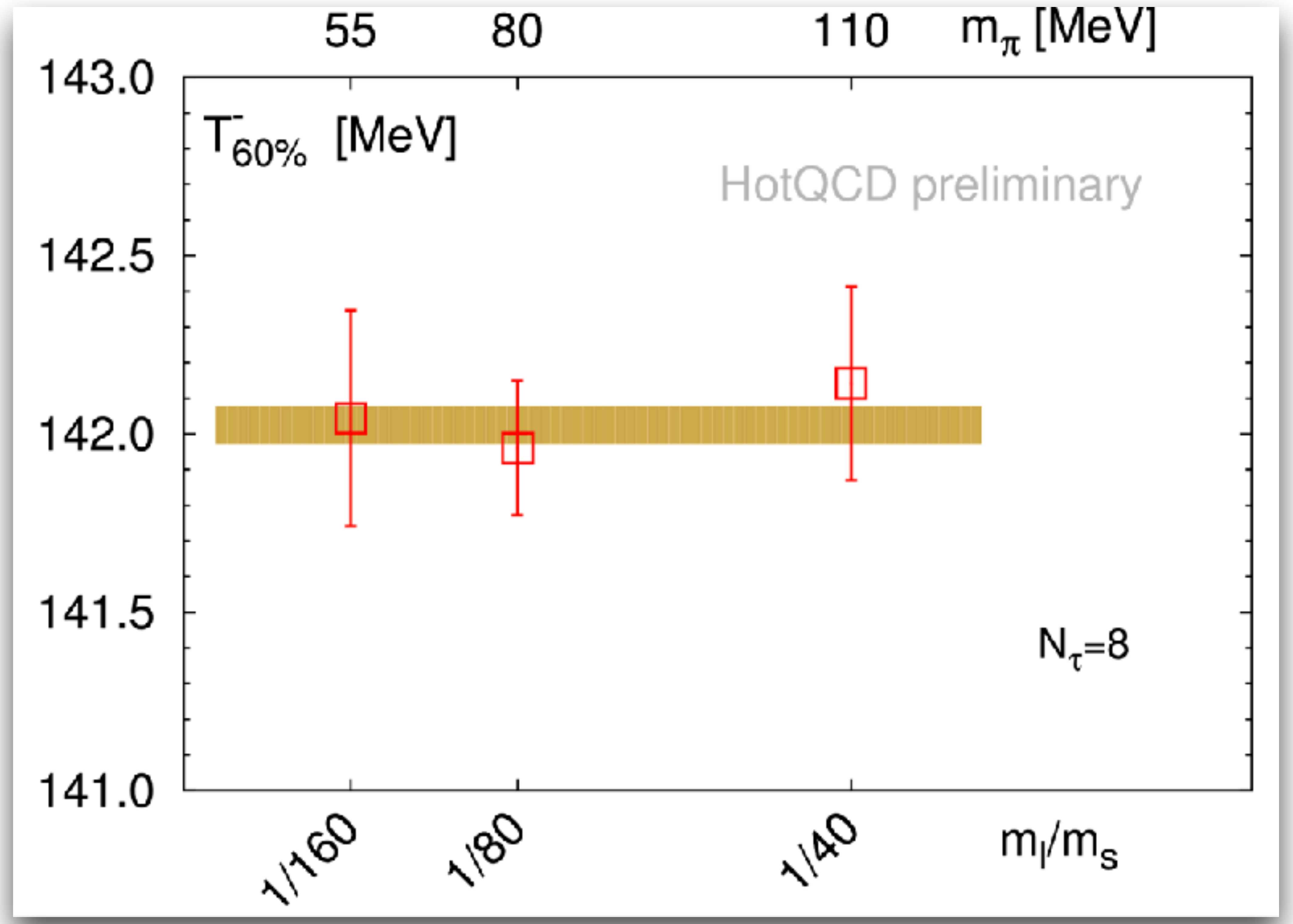


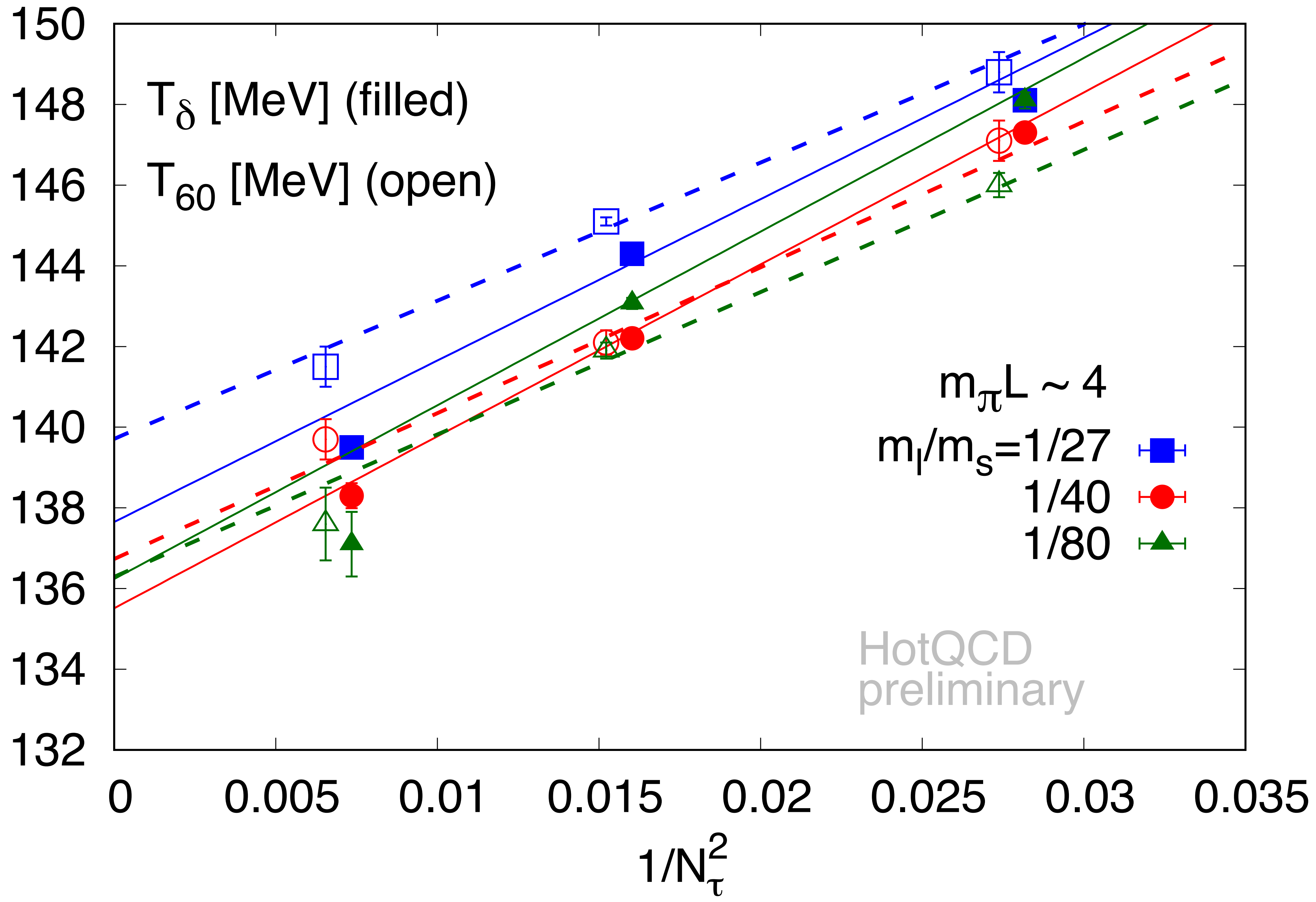
z_p : peak-location of the susceptibility

$z_{\overline{60\%}}$: location of 60% of peak-height

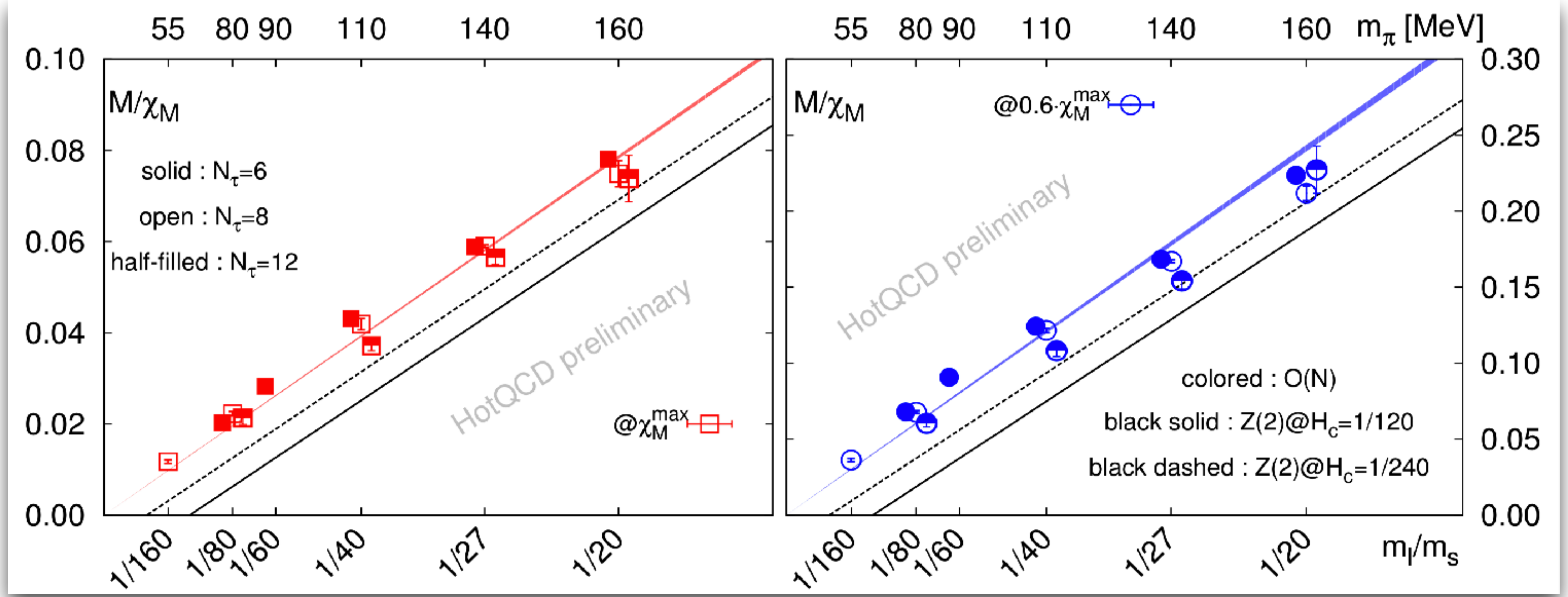
	z_p	$z_{\overline{60\%}}$
$O(2)$	1.56	-0.009
$O(4)$	1.37	-0.01

$T_c^0 \approx 135\ MeV$





- QCD chiral transition consistent with O(N) universality class



$$\frac{M}{\chi_M} \sim \frac{m_l - m_c}{m_s^{phys}} \frac{f_M(z)}{f_\chi(z)}$$

summary:

- precise chiral crossover temperature $T_c(\mu_B = 0) = 156.5 \pm 1.5\ MeV$
- $T - \mu_B$ phase-boundary up to $\mu_B \lesssim 300\ MeV$
- no sing of enhanced fluctuation above HRG
- QCD critical point unlikely to be located within $\mu_B \lesssim 300\ MeV$
- $m_u = m_d \to 0$: chiral transition consistent with O(N) universality class
- chiral critical temperature $T_c^0 \approx 135\ MeV$