Universität Bielefeld

CRC-TR 211
Strong-interaction matter under extreme conditions

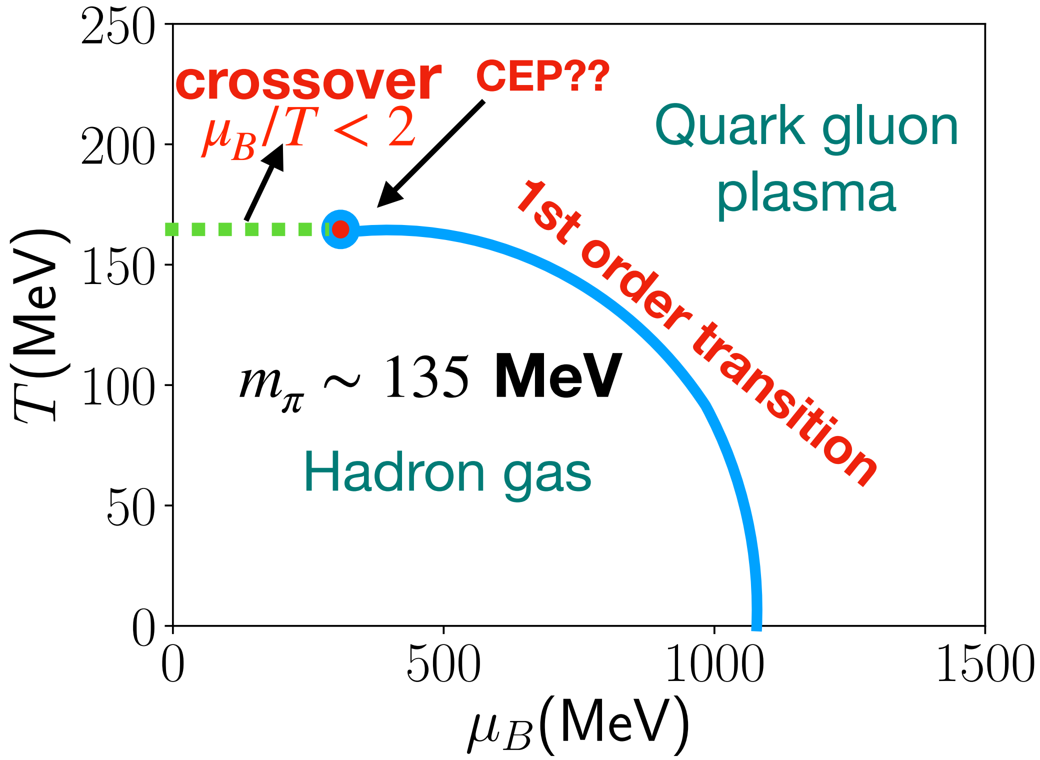
Critical end points in (2+1)-flavor QCD with imaginary chemical potential

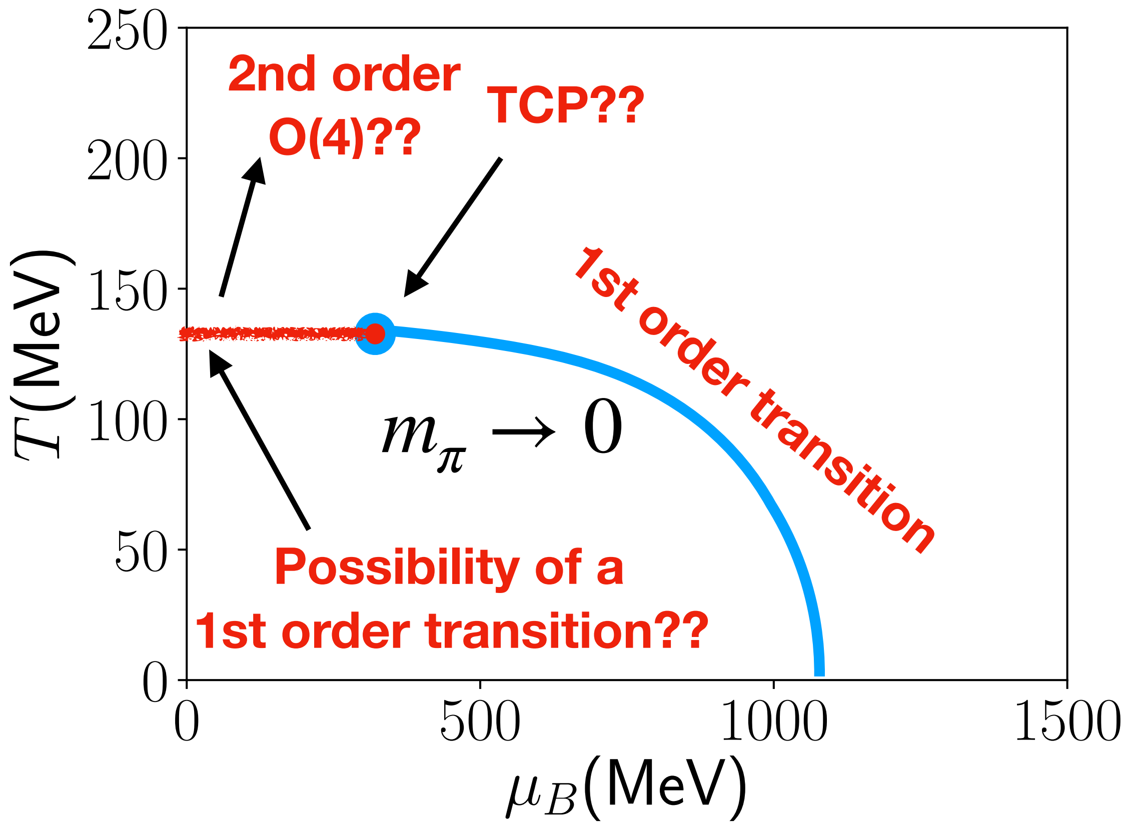
Jishnu Goswami

In collaboration with: Frithjof Karsch, Christian Schmidt, and Anirban Lahiri

The Critical Point and Onset of Deconfinement Conference
"CPOD 2018"

QCD phase diagram in the Chiral limit





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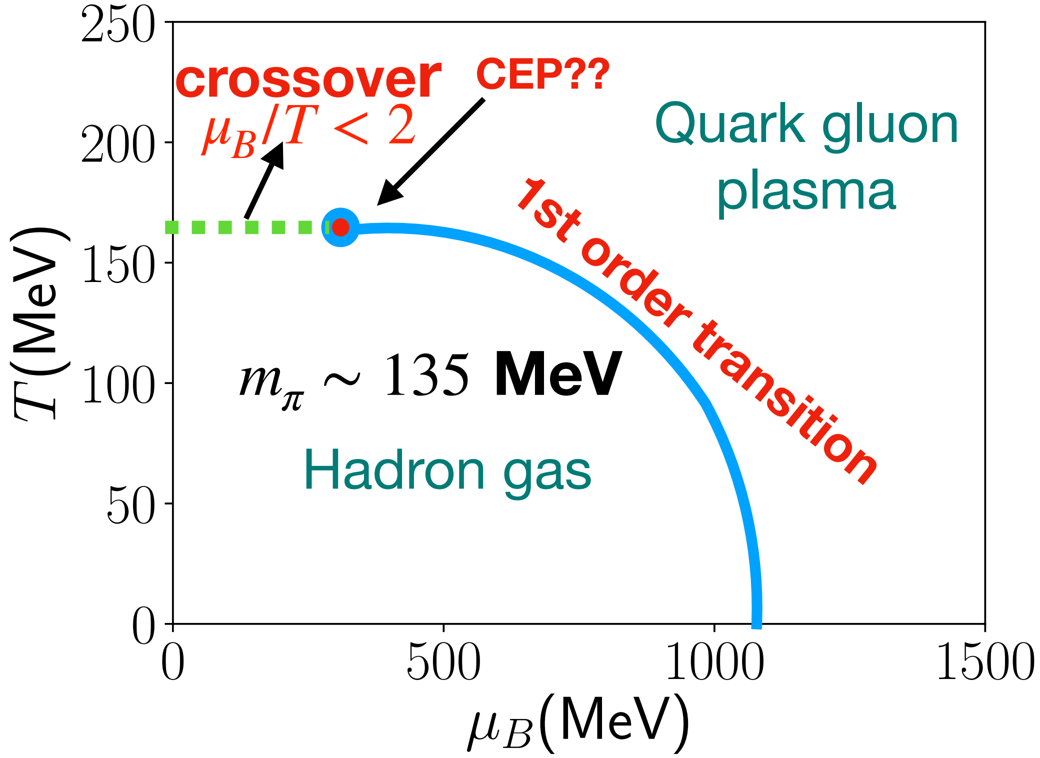

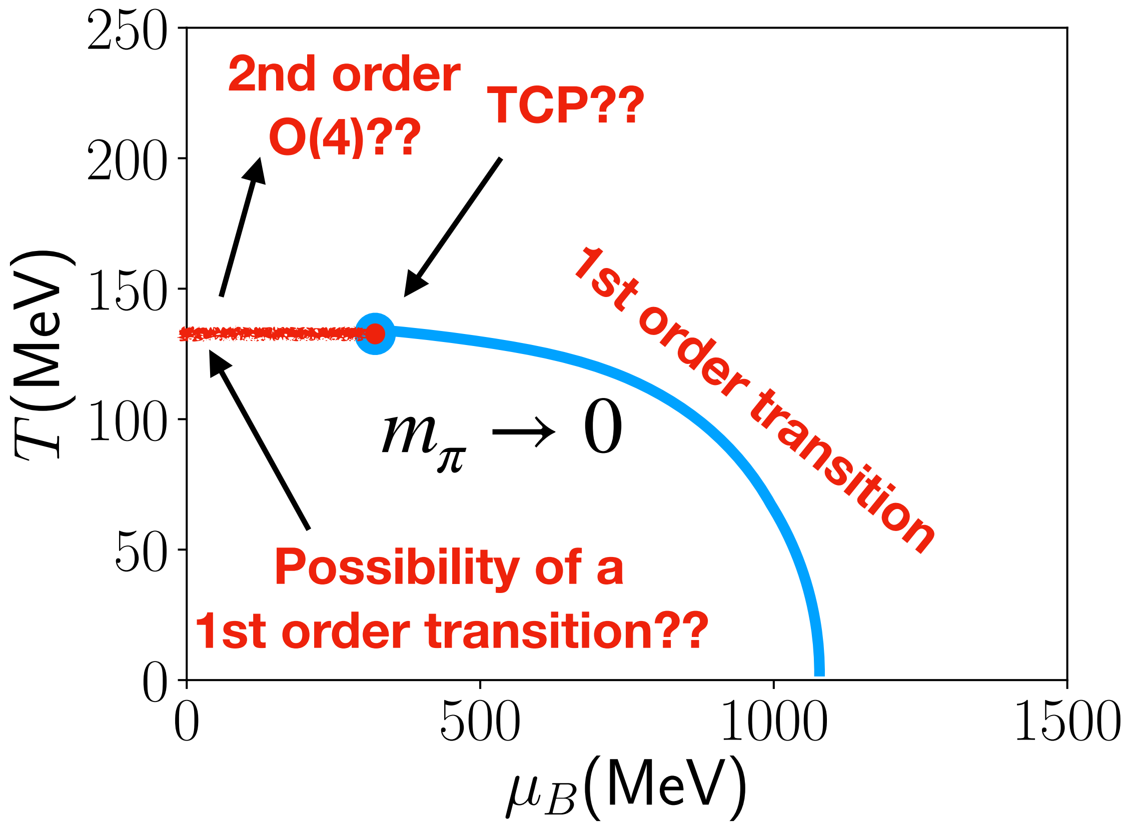

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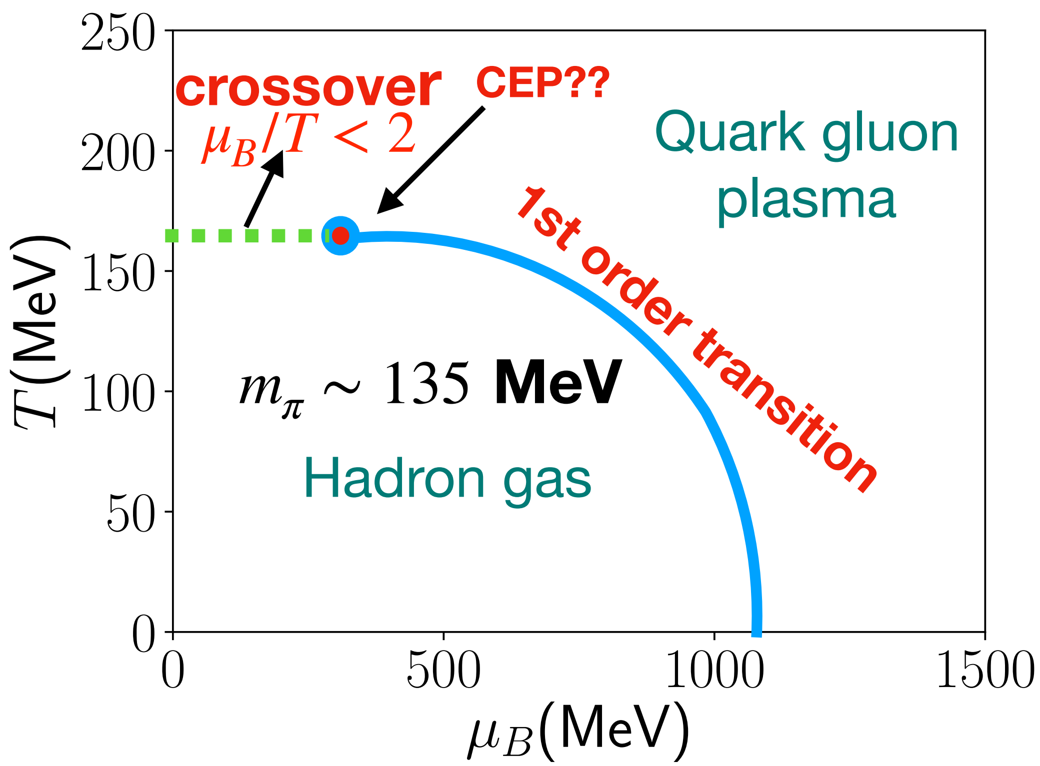

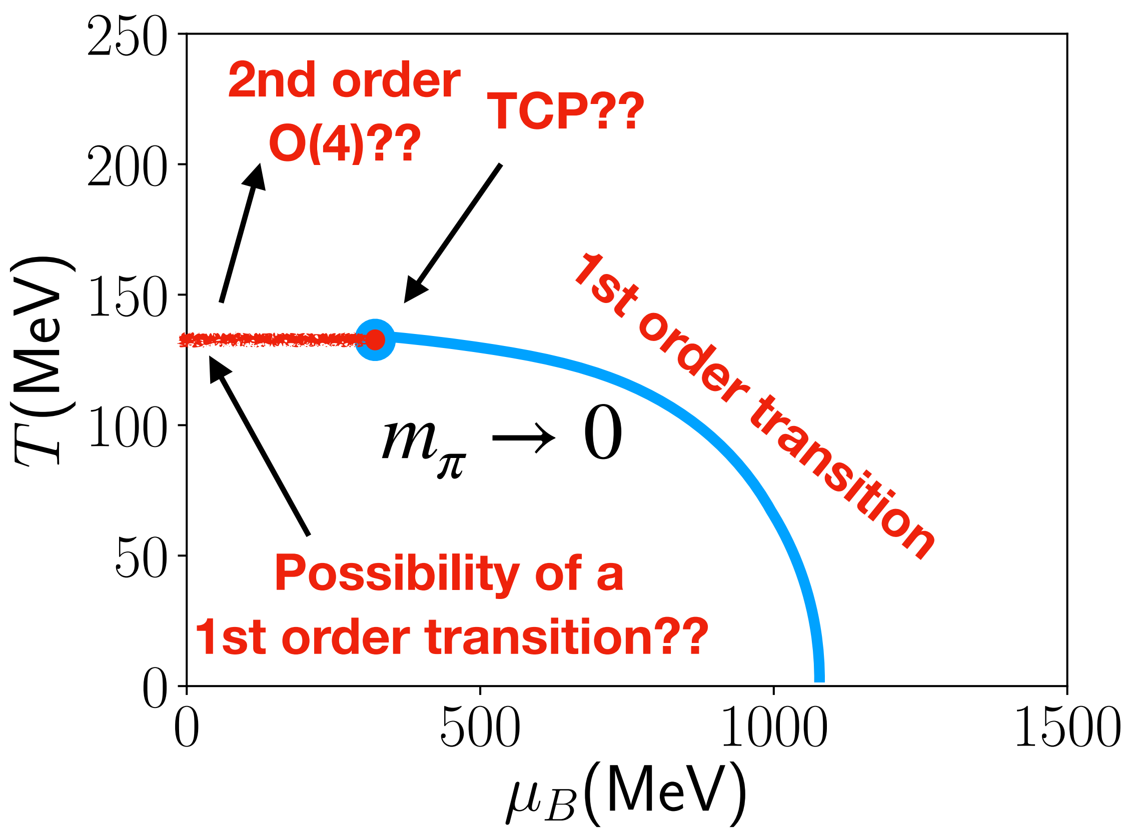

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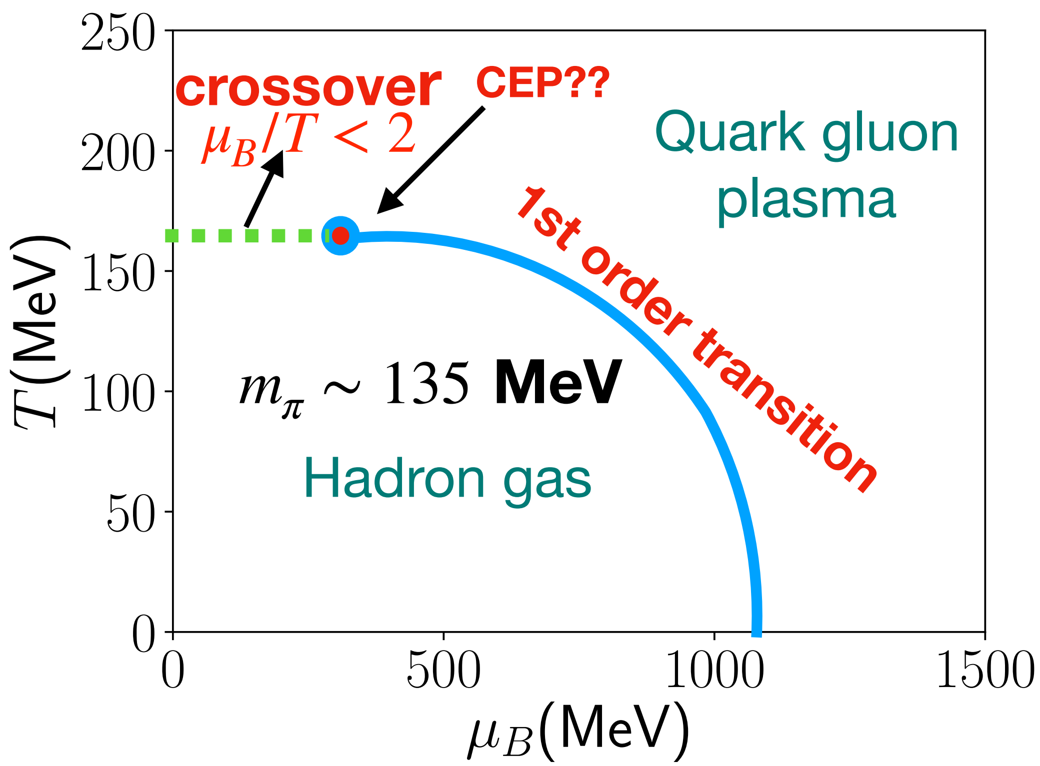

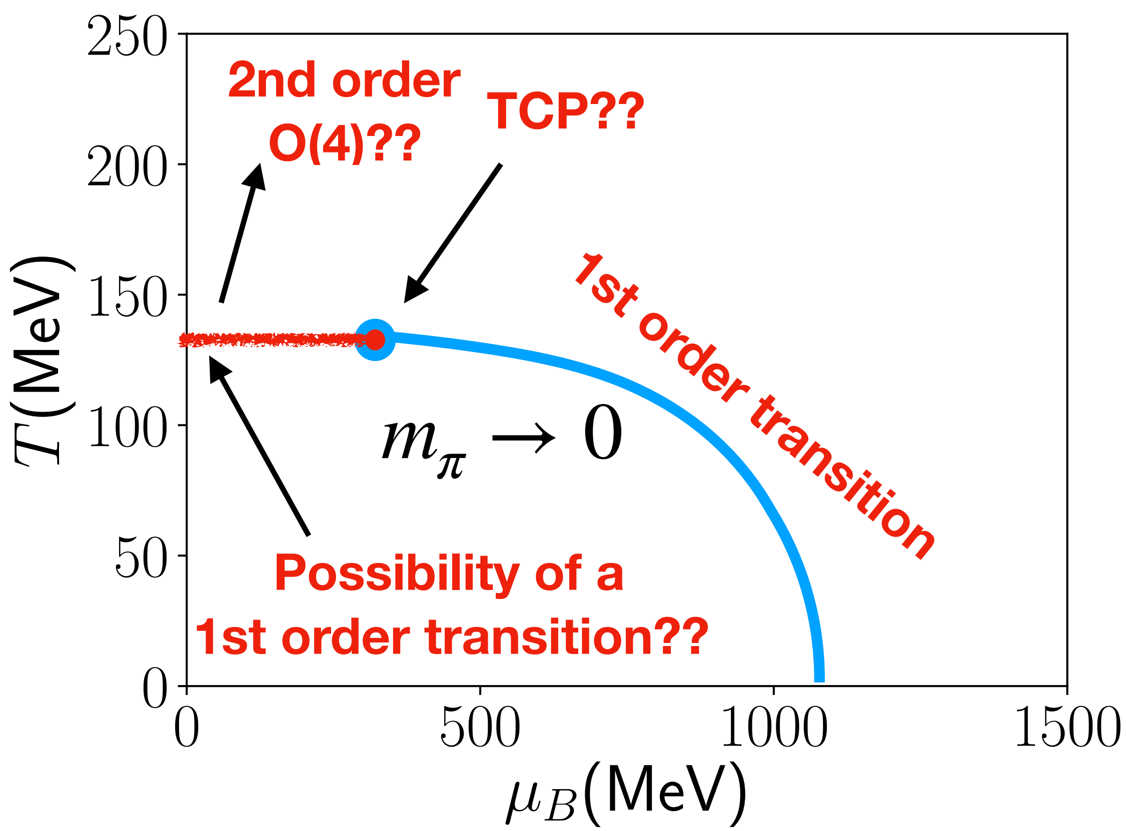

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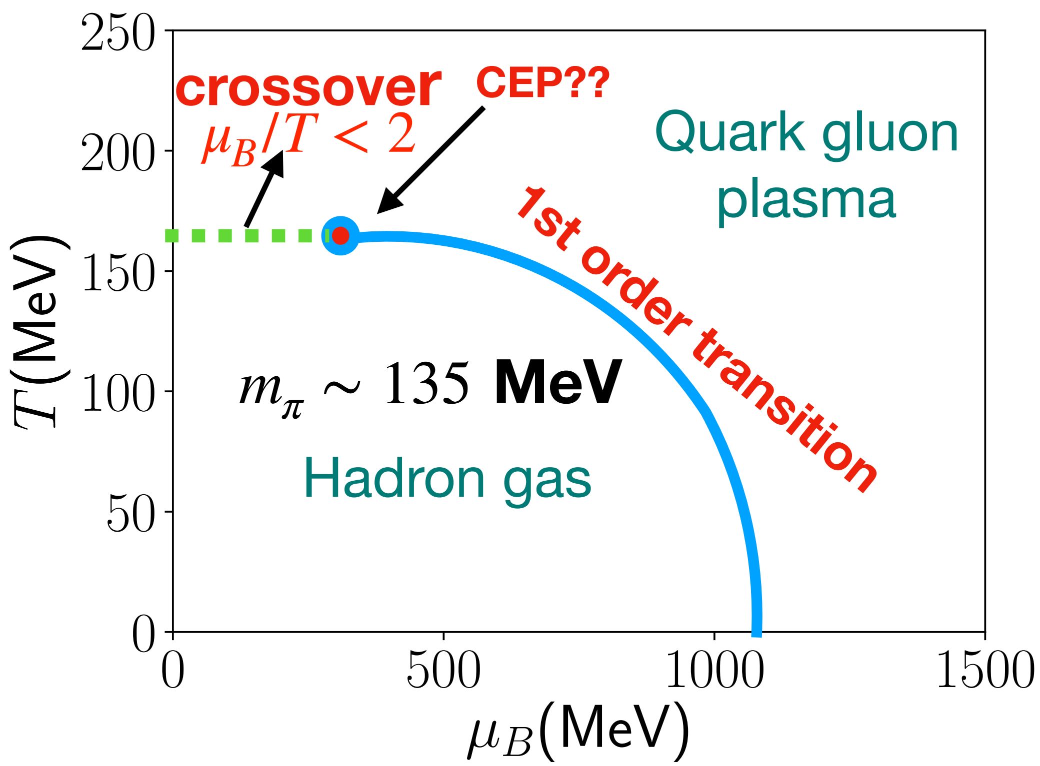

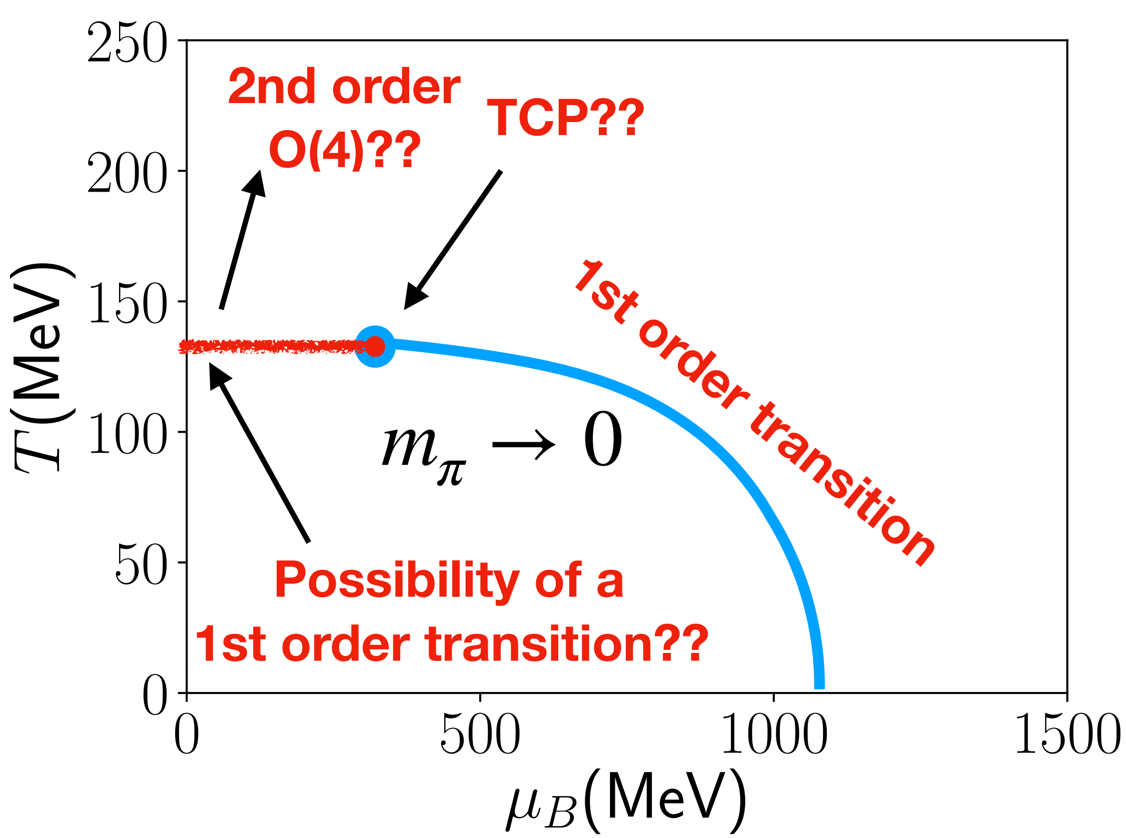

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- Although, if $U(1)_A$ restoration happens alongside the flavour chiral symmetry restoration then there is a possibility of a fluctuation induced 1st order transition.
- Order of the phase transition in the chiral limit is also of relevance for studies of fluctuations at the LHC.[talk by A. Rustamov]

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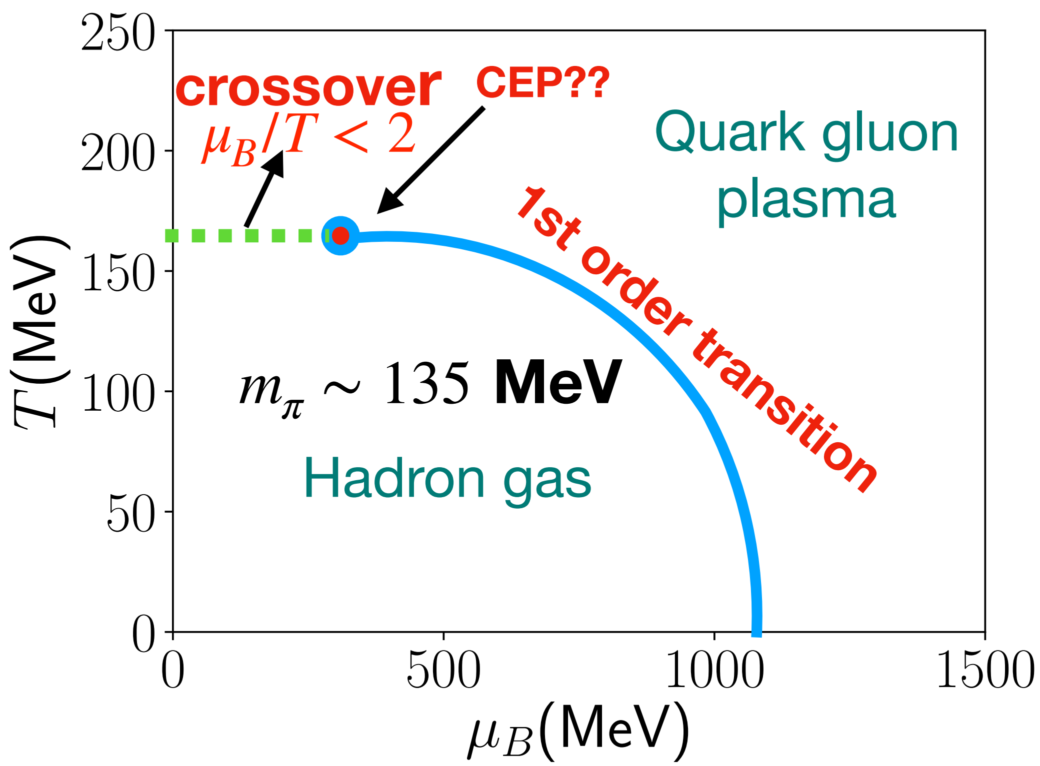
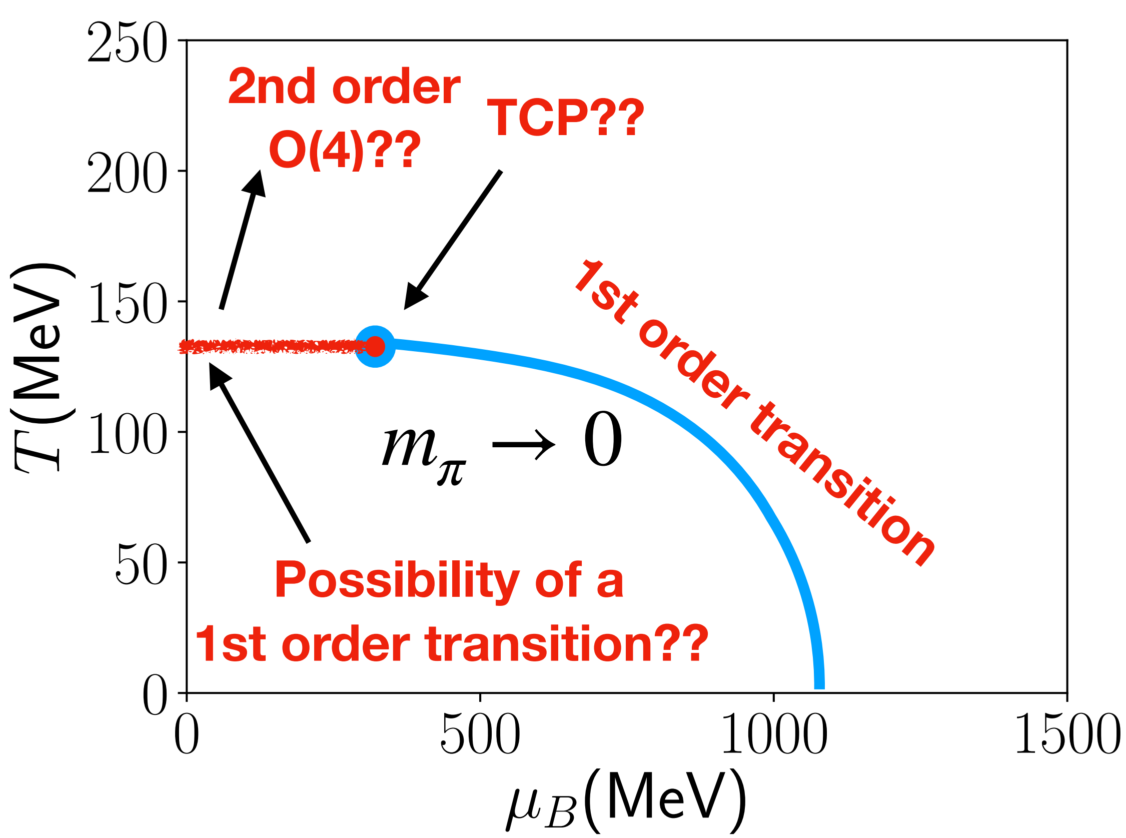
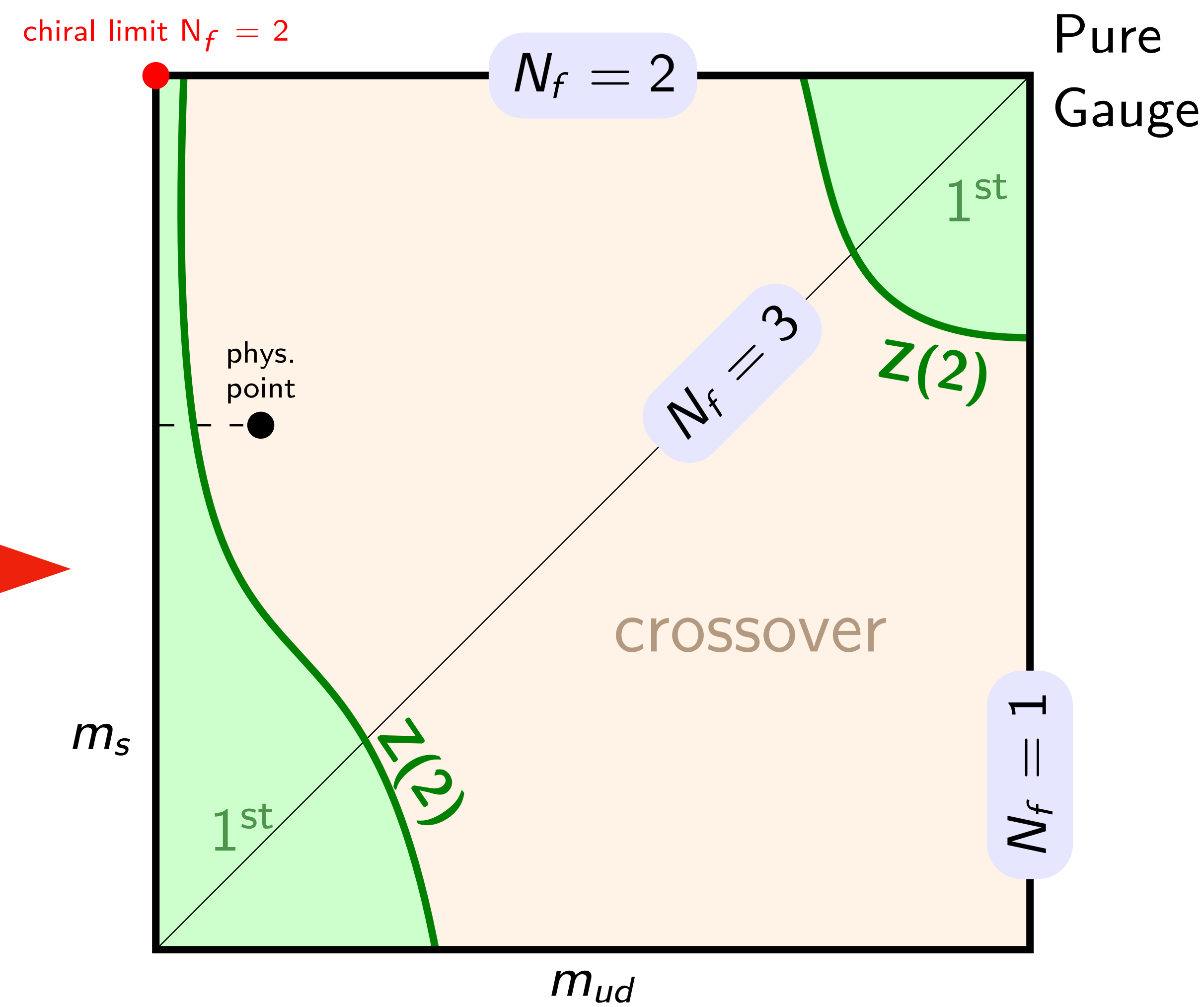




Pisarski, Wilczek, PRD 29 (1984)

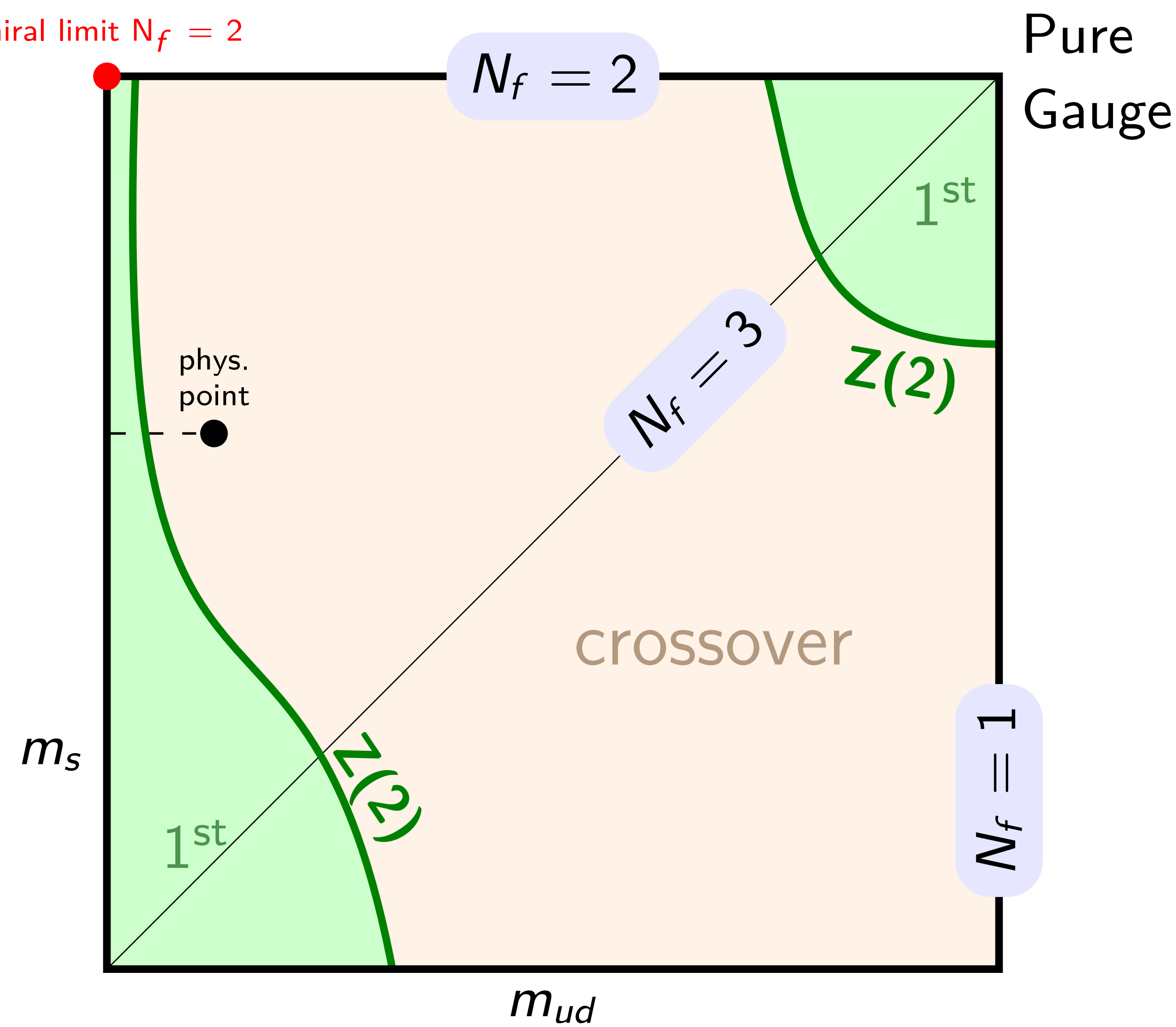
Based on studies with unimproved Lattice action



2-d Columbia plot
$\mu = 0$

Philipsen, Pinke, PRD 93 (2016)

Chiral transition for zero chemical potential with HISQ



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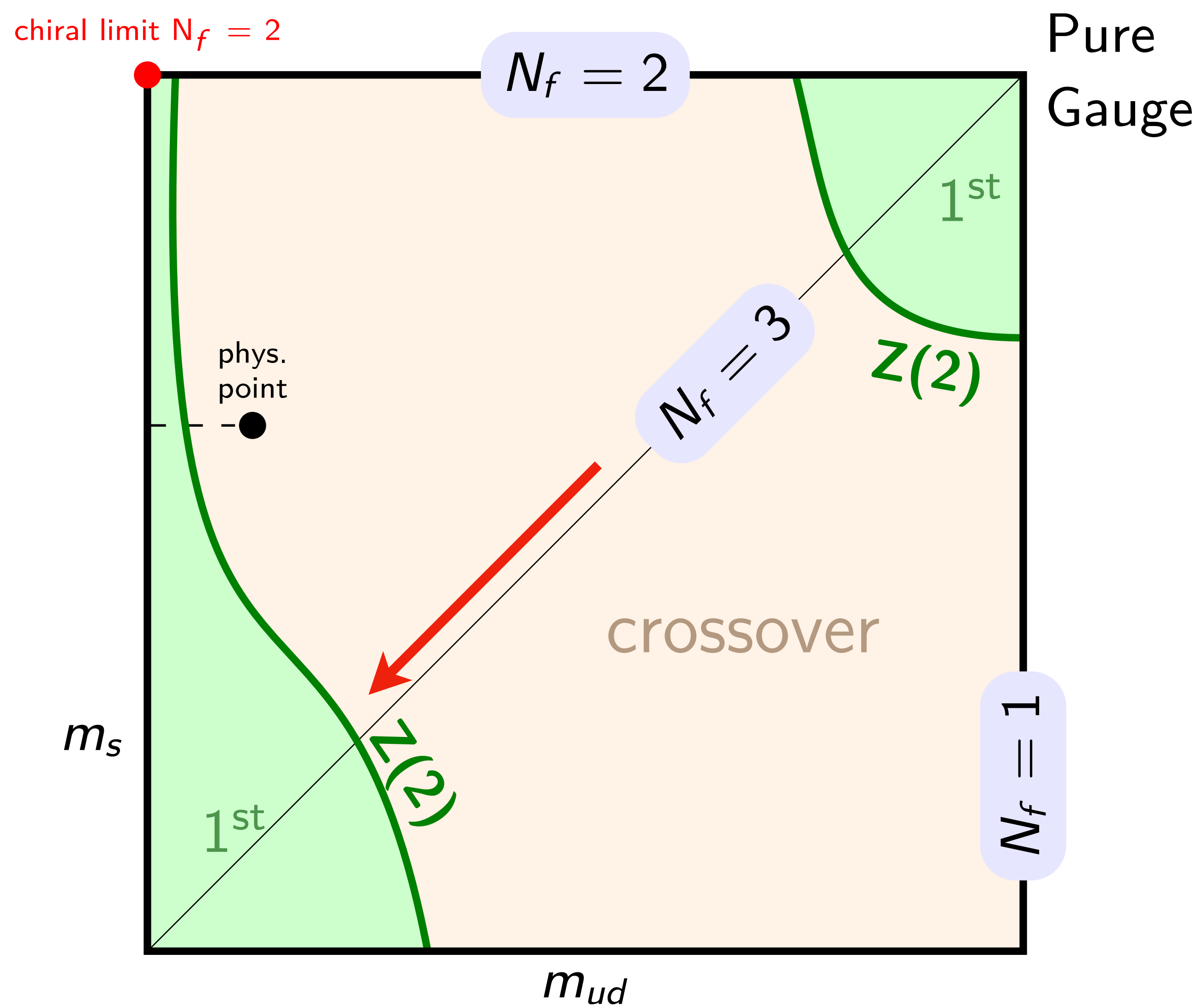

N_f=3 : 1st order phase transition ruled out for 230 MeV > m_π >80 MeV. Bound on critical pion mass is given as, $m_\pi^{cr} \lesssim 50$ MeV from the scaling analysis.

Bazavov et. al. PRD 95, 074505 (2017)

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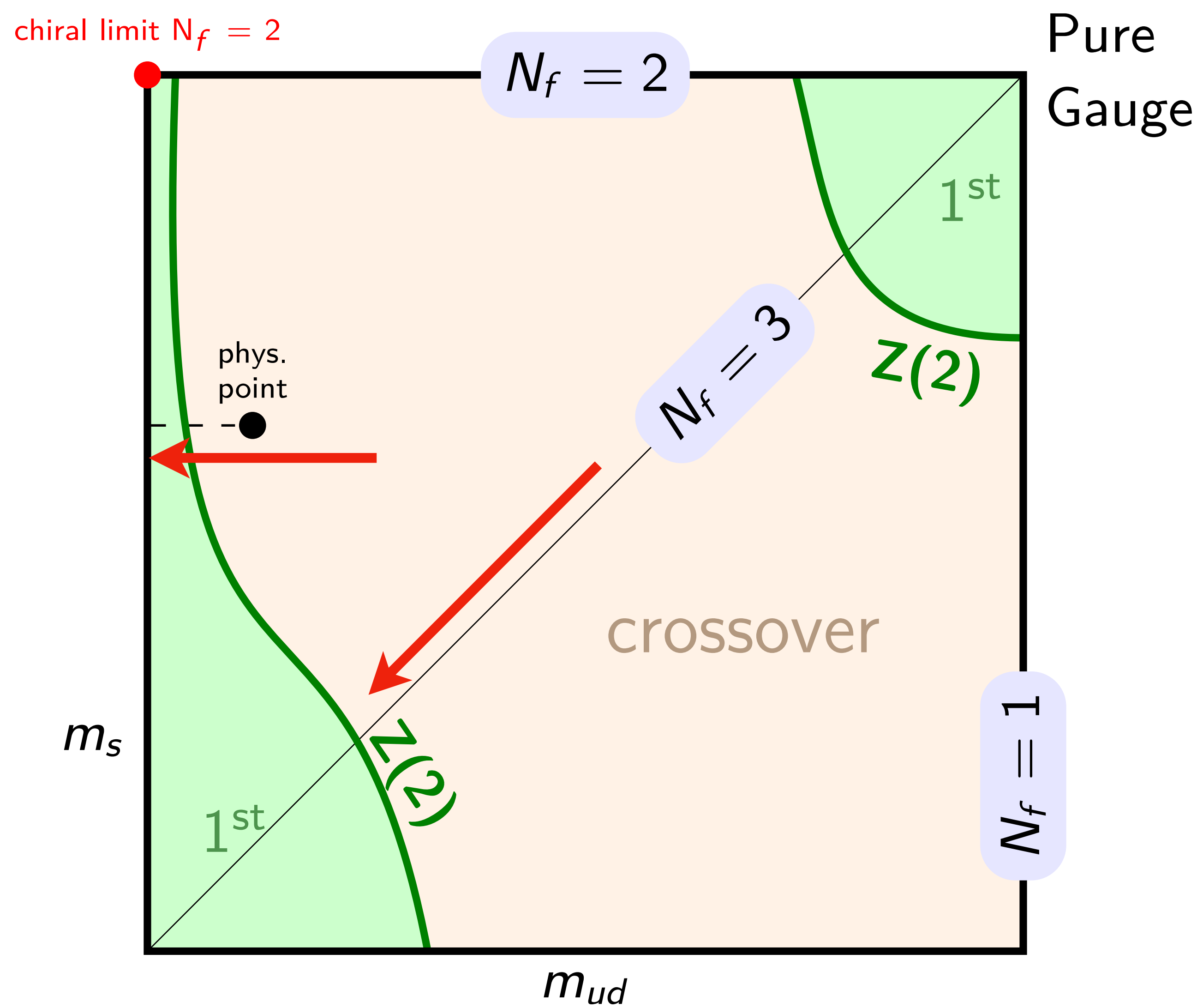

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N_f=2+1 : No hint of 1st order phase transition for $m_\pi > 55$ MeV. chiral transition is most likely 2nd order O(N) rather than 1st order.

A. Lahiri et. al. , QM 2018, arXiv:1807.05727

Chiral transition for zero chemical potential with HISQ

- HotQCD results on chiral phase transition, [$\mu = 0$]

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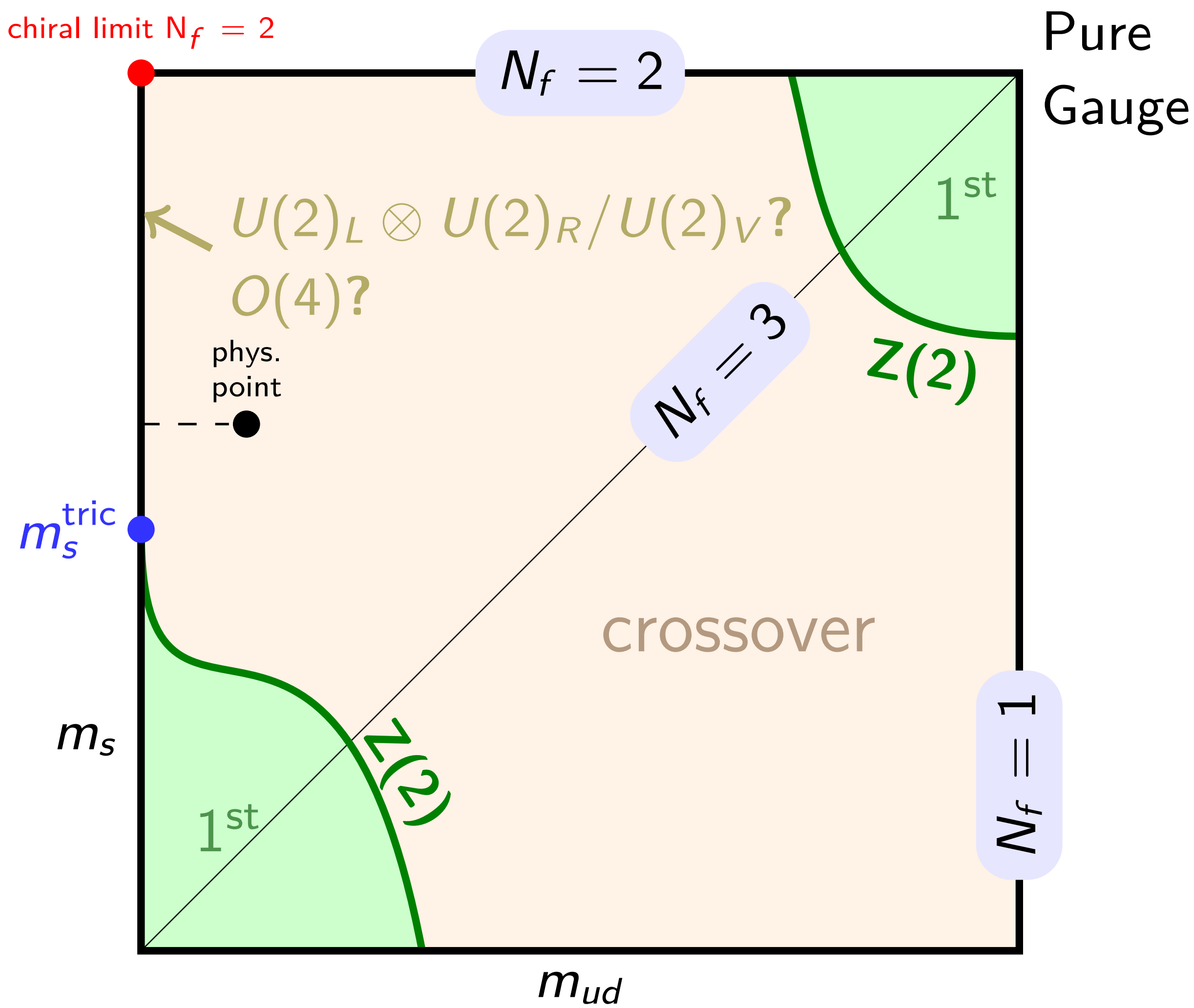

HotQCD: Based on studies with HISQ action

Chiral transition for zero chemical potential with HISQ

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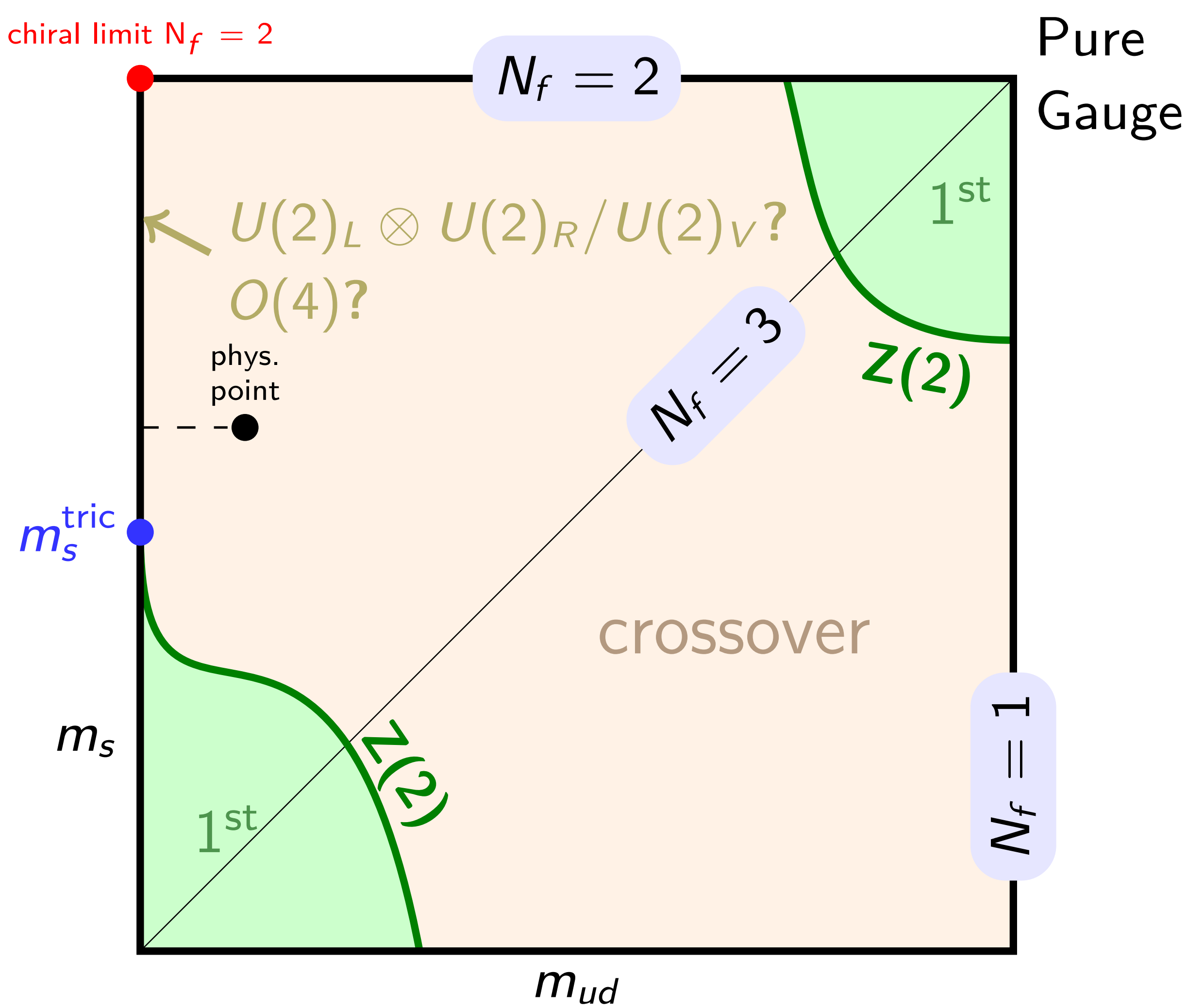

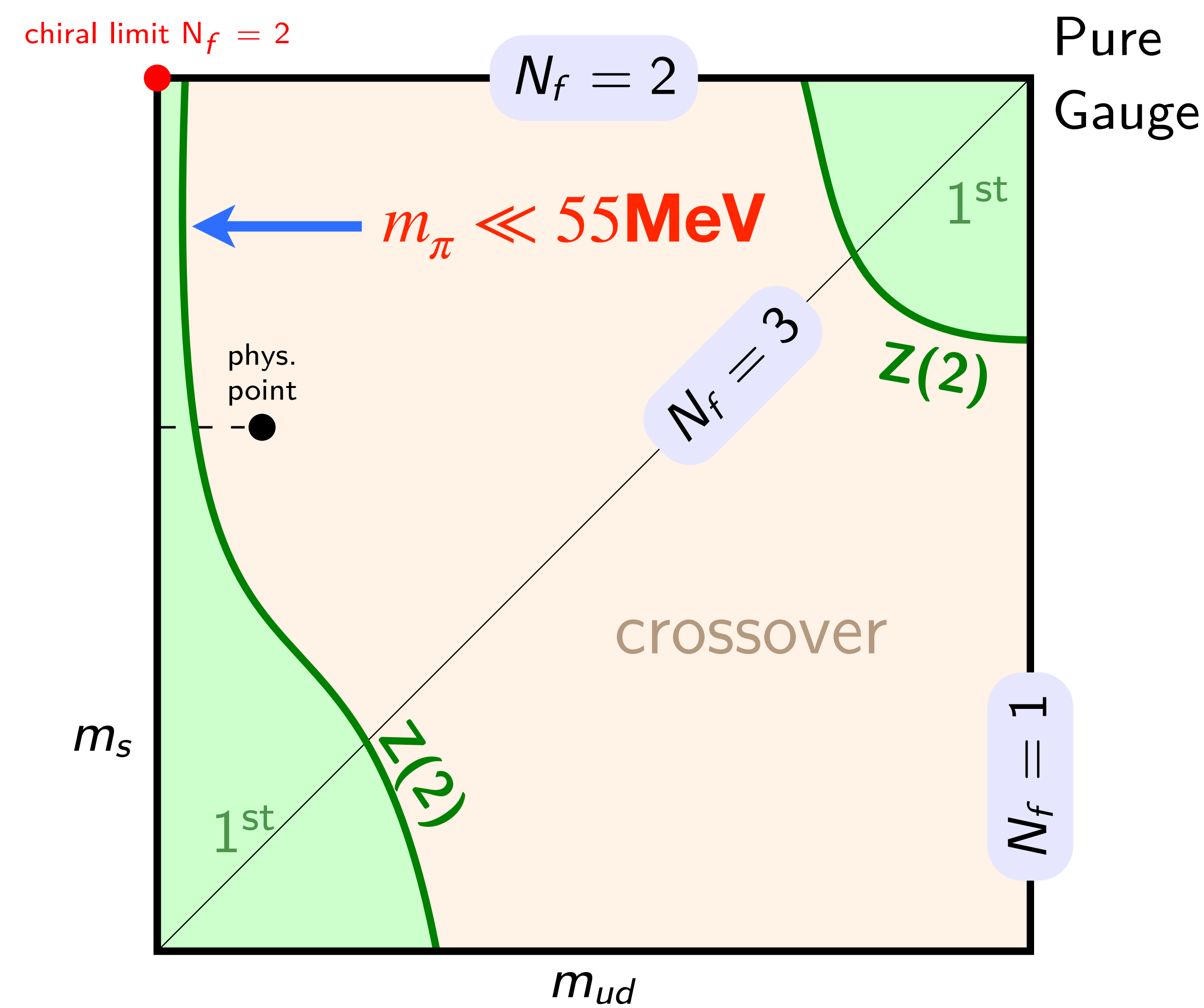

HotQCD: Based on studies with HISQ action

However, still one can argue that 1st order transition can be possible for,

$m_\pi \ll 55$ **MeV**

Central Question:
Nature of the chiral symmetry restoring transition at $\mu=0$ at the chiral limit??

Does a 1st order chiral symmetry restoring transition exist at $\mu=0$ below a certain critical quark mass (m_{cri}) ??

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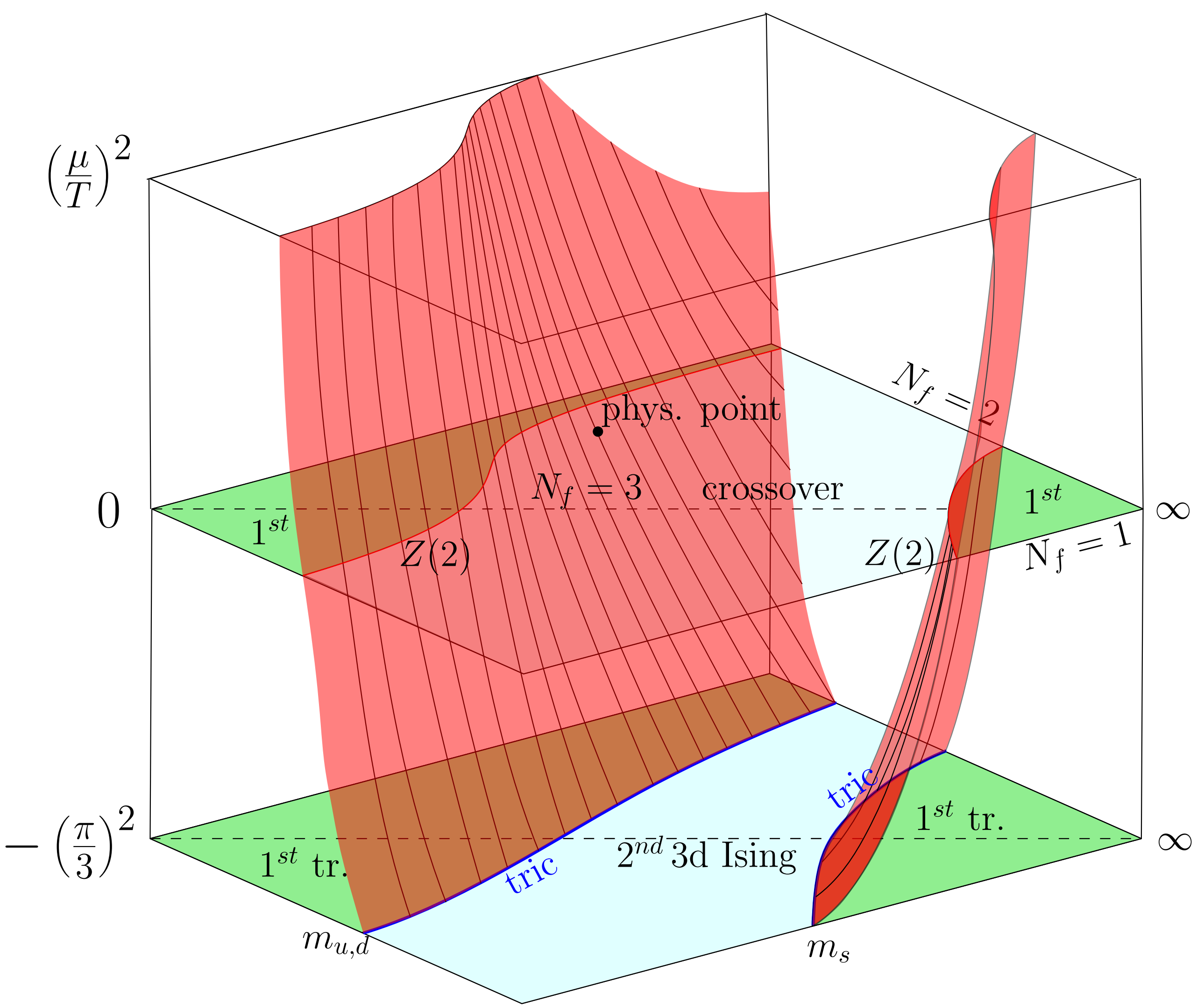

Possible scenario of extended 3d Columbia plot

Another possible way to examine the 1st order nature of chiral transition(μ=0) is to study the phase diagram in the Roberge-Weiss (RW) plane($i\mu/T=i\pi/3$). In the RW plane the first order region(chiral) expected to become largest.

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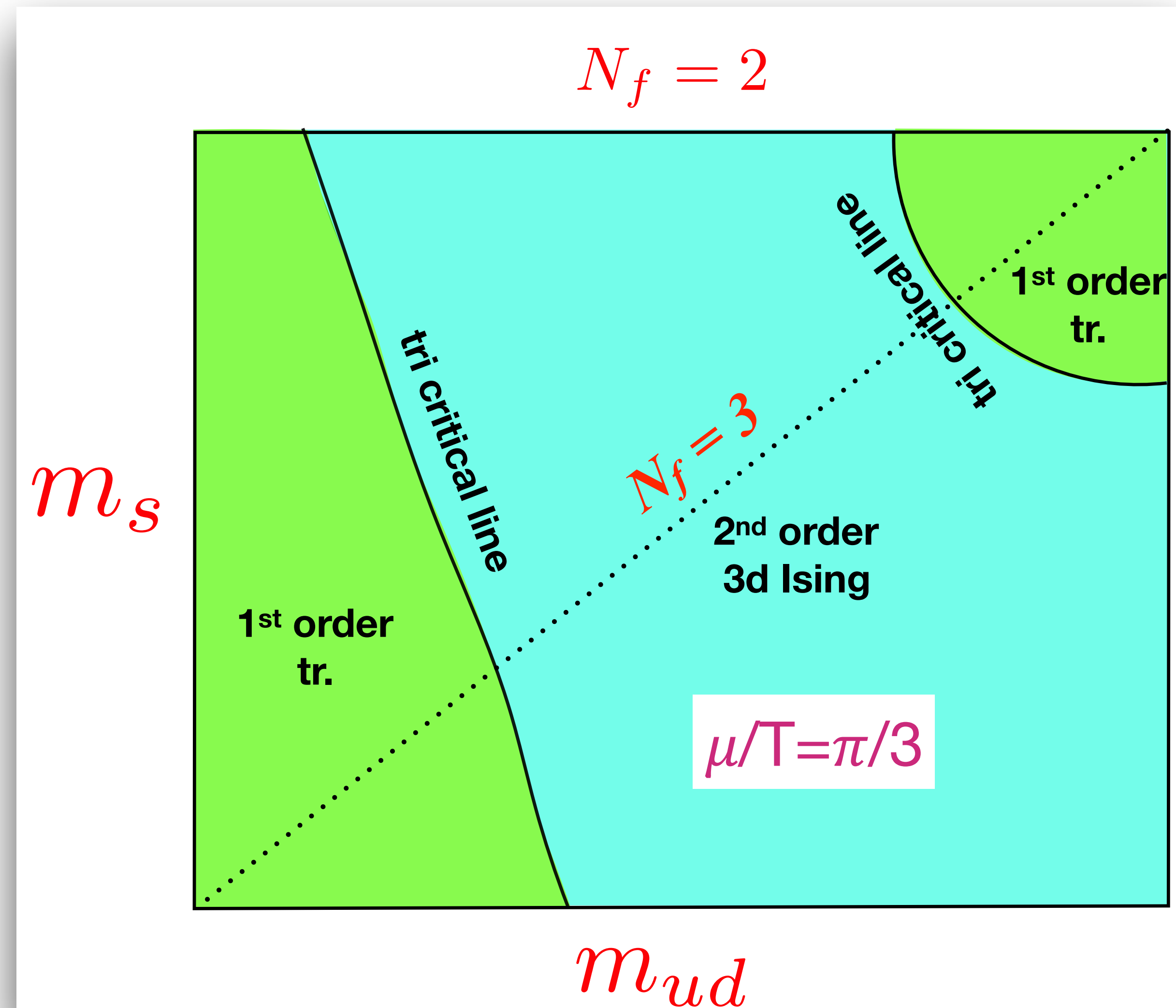

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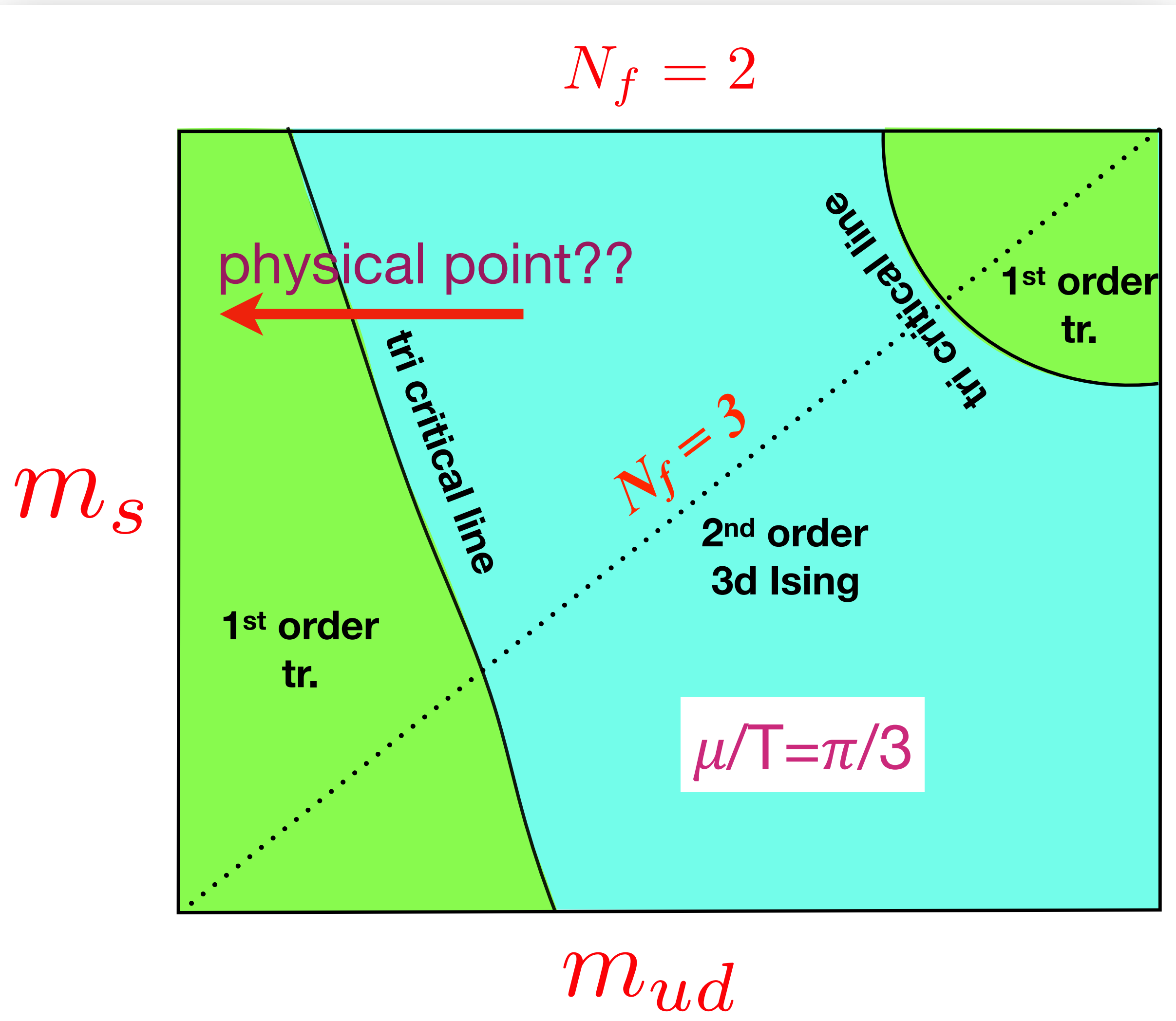

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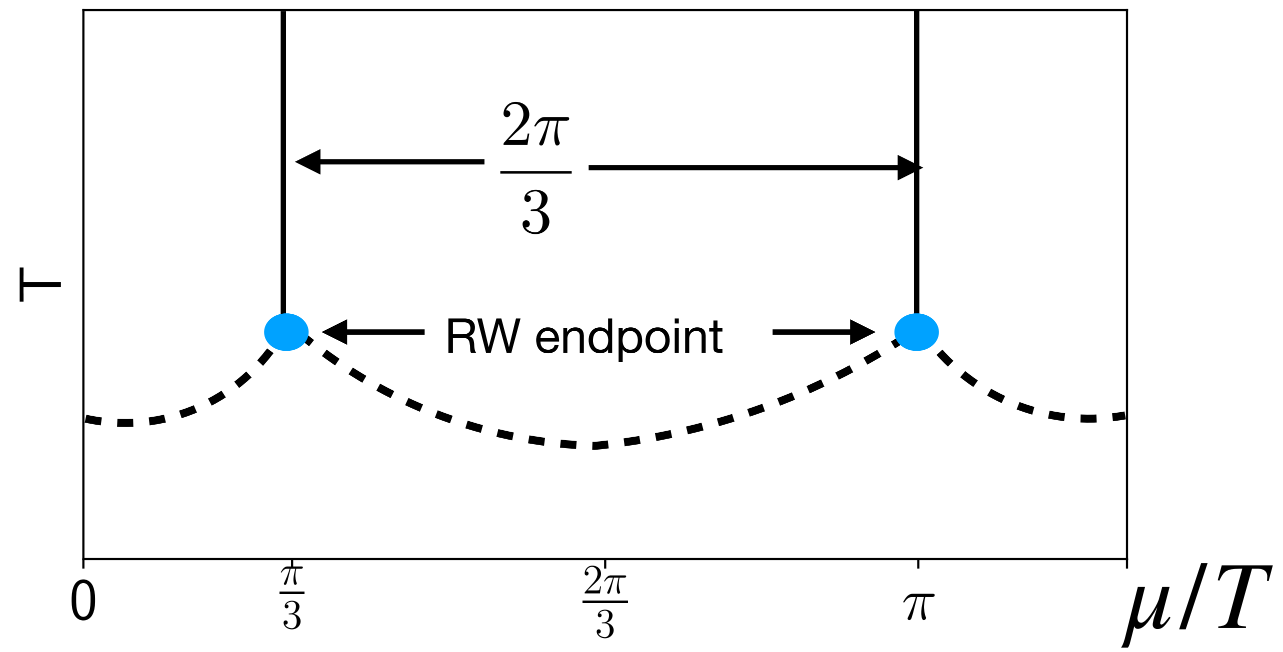
General strategy:Locate the physical point for N_f=2+1, approach the chiral limit while keeping the strange quark mass fix to its physical value. Extrapolate the result to $\mu=0$.

Few details about the RW plane

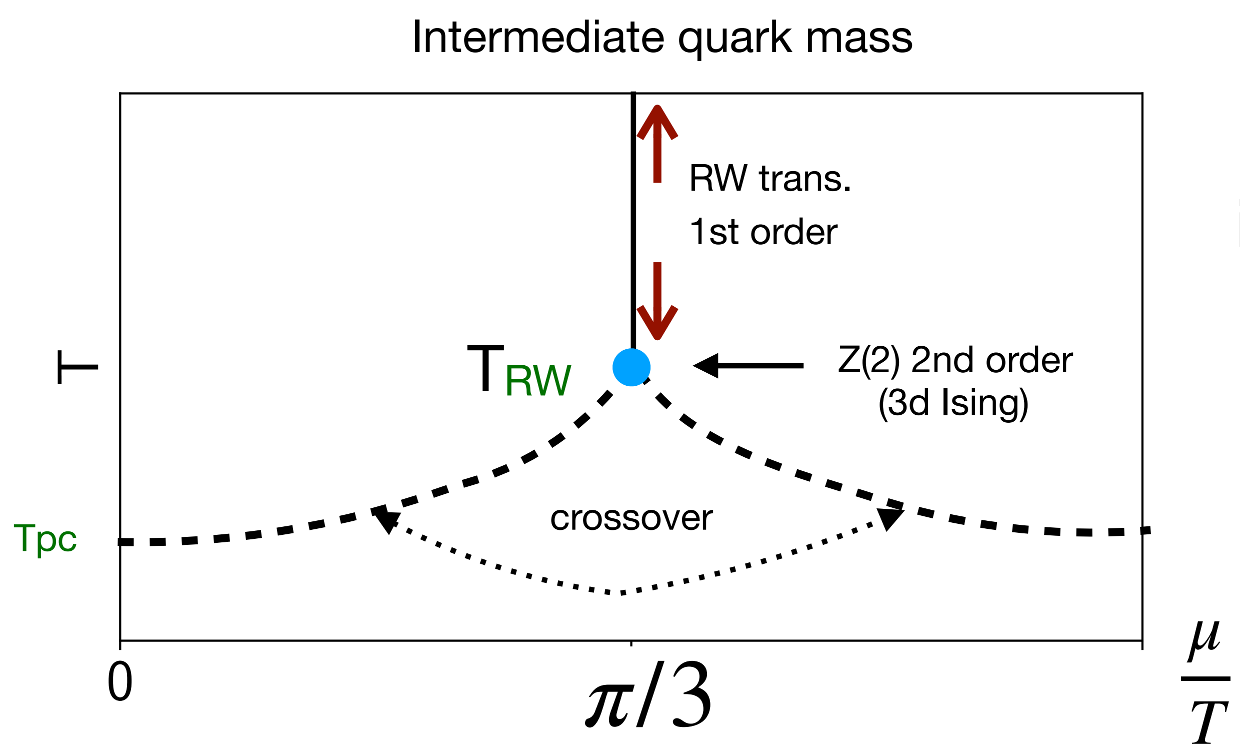


- If *(iμ) is purely imaginary then we can compare the results with the real chemical potential as*, [Taylor expansion method]

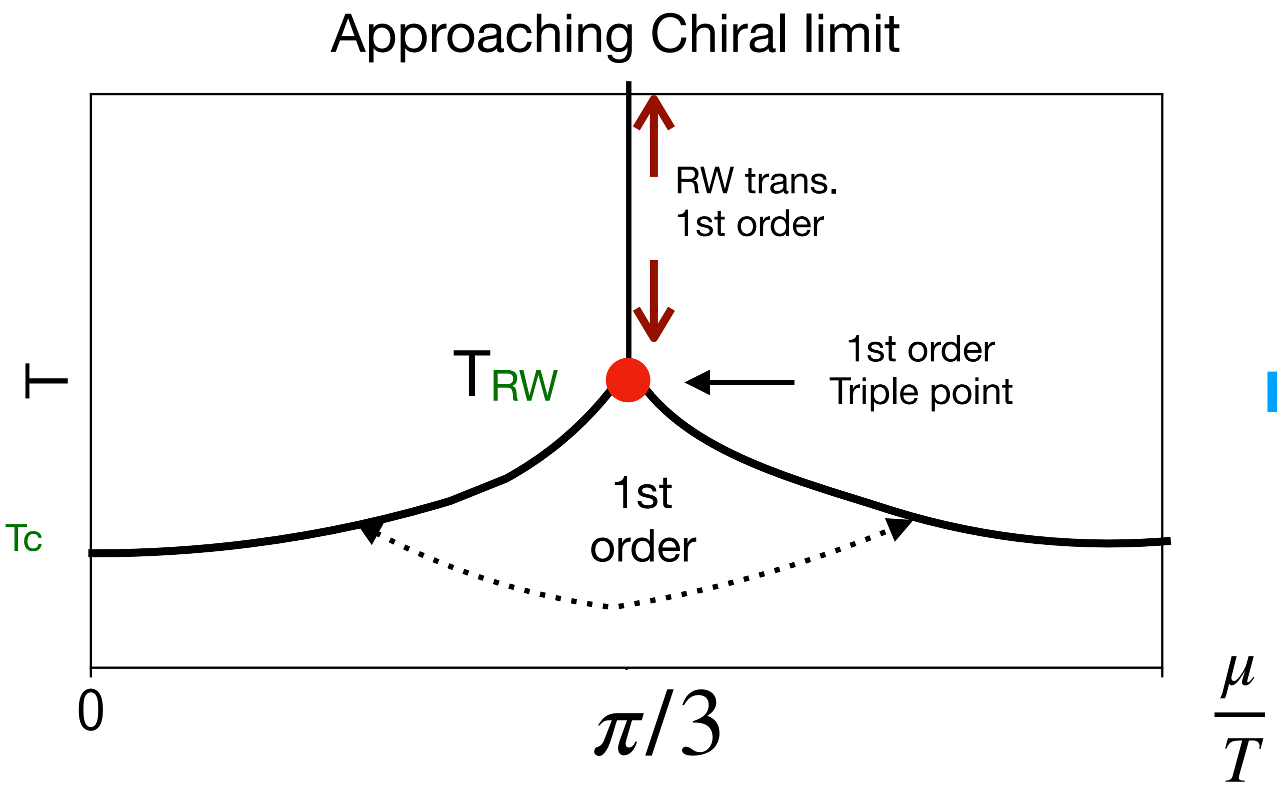
$$\langle O \rangle = \sum_n c_n \left(\frac{i\mu}{T} \right)^n, n = 2, 4, 6.....$$

- The partition function is periodic in μ/T as, Z(μ/T)=Z(μ/T+2kπ/3), known as , Roberge-Weiss(RW) periodicity. And μ/T=(2k+1)π/3, is known as RW plane.

Possible fate of the RW end point



For, intermediate pion mass RW end point is of Z(2) 2nd order.



For, smaller pion mass RW end point may become a 1st order triple point.

This could indicate that the chiral transition at μ=0 is 1st order.

Studies in the RW plane

Mostly with unimproved actions

$N_f=2$: 1st order triple point (at the end of the line of 1st order RW transitions) exist for $\mu/T=\pi/3$ and $m_{cri} > m_{phy}$.

Standard staggered action:

$m_\pi \sim 400$ MeV($N_\tau=4$)

Standard Wilson action:

$m_\pi \sim 930$ MeV($N_\tau=4$)

$m_\pi \sim 680$ MeV ($N_\tau=6$)

The results are strongly fermion discretization scheme and cut-off(N_τ) dependent.

P. de Forcrand et. al, PRL 105, 152001(2010), Owe Philipsen et. al, PRD 89, 094504(2014), Christopher Czaban et al, PRD 93, 054507 (2016)

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Very recent studies with improved actions,

Stout improved staggered fermions($N_f=2+1$): For physical quark mass the RW endpoint is belongs to the Z(2) universality class.

C. Bonati et. al,PRD 93, 074504 (2016)

No, 1st order triple point exist at least for $m_\pi > 50$ MeV.

C. Bonati et. al,arXiv:1807.02106 [hep-lat]

HISQ($N_f=2$): Order of the phase transition at physical point is not clear(large cut-off effects) .

L.K.Wu, et al. PRD 97,114514(2018)

Studies with HISQ in the RW plane

Action, $Z(T,\mu) = \int [\mathscr{D}U] det[M_{ud}(\mu_f)]^{1/2} det[M_s(\mu_f)]^{1/4} \exp[-S_G]$

$M_q = D_{HISQ}(\mu_f) + m_q$, μ_f is purely imaginary

Simulation details, $N_f = 2+1, \frac{\mu_f}{T} = \frac{\pi}{3}$

N_σ	N_τ	$\frac{m_l}{m_s}$	m_π (MeV)
8	4	1/27	135
12	4	1/27	135
16	4	1/27,1/40, 1/60	135, 110, 90
24	4	1/27,1/40, 1/160,1/320	135, 110,55,40

T is varied in the range, 176-215 MeV, corresponds to, $T \sim T_c \pm 0.1 T_c$

Generally we generated 20k trajectory per T value away from T_c and 80k trajectory near T_c

Ising endpoint of a first order line

$$H_{eff}(t,\xi) = t\mathscr{E} + h\mathscr{M}$$

→ Effective Ising Hamiltonian which defines the universal critical behaviour of the system

- t: temperature like field
- $\mathscr{E}$: energy like operator
- h: magnetic field like
- $\mathscr{M}$: magnetization like operator (order parameter)

under Z(2) transformation,

$$\mathscr{E} \to \mathscr{E}$$
$$\mathscr{M} \to -\mathscr{M}$$

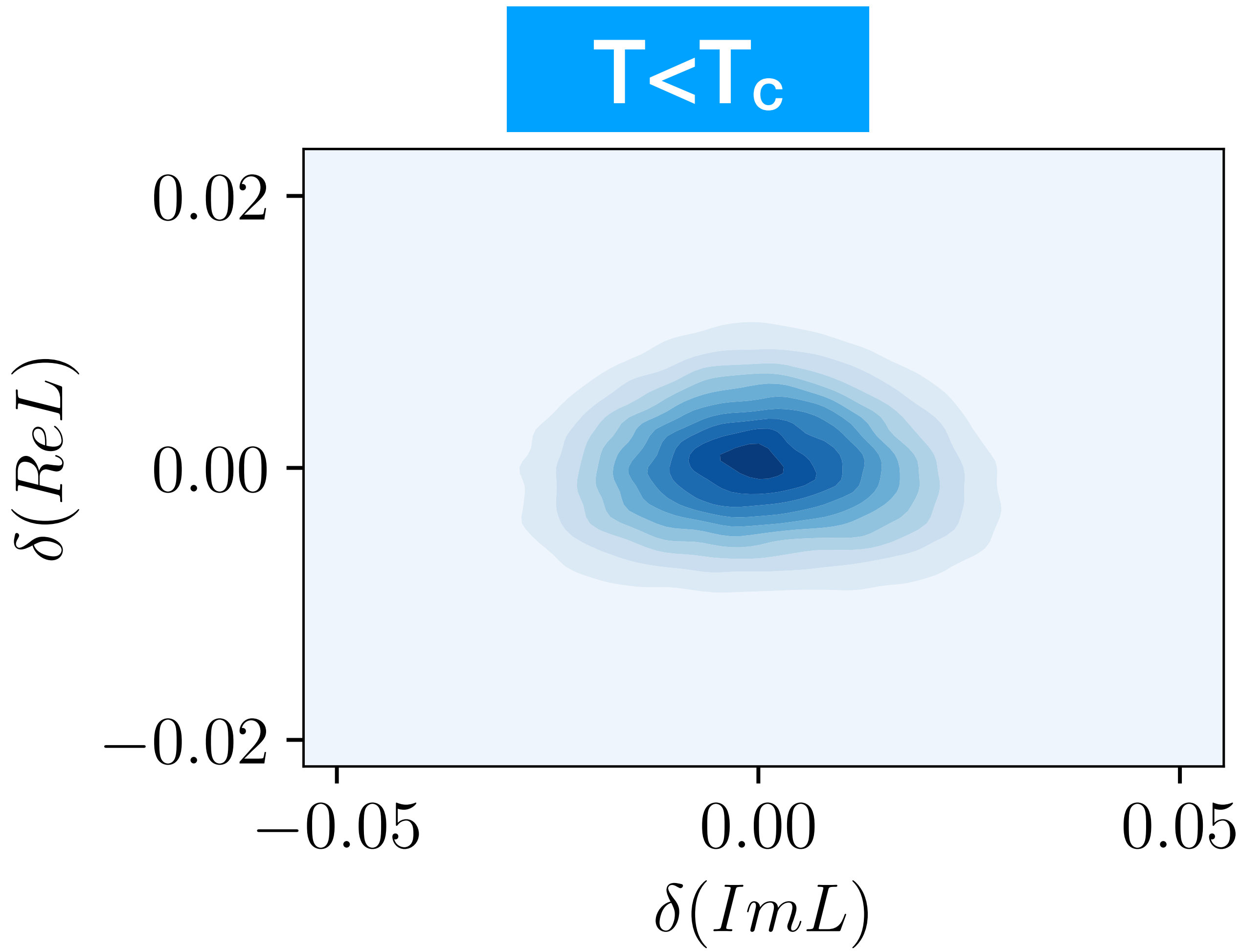
Corresponding critical behaviour of QCD in 2nd RW plane [Z(2) transformation],

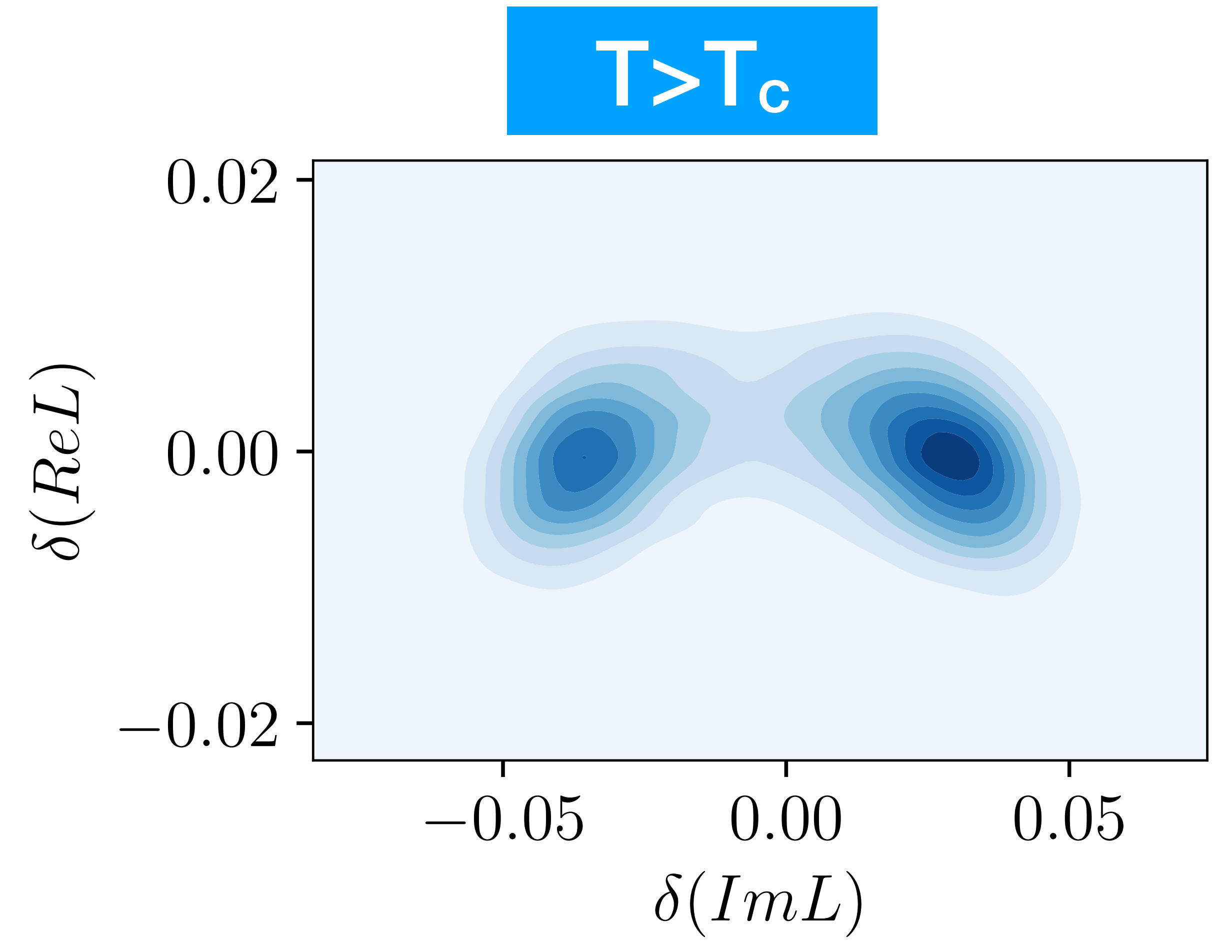
$$Im\ L \to -Im\ L$$ → order parameter

$$Re\ L \to Re\ L$$ → energy like

i.e. at $\mu = \mu_{RW}$

$$\lim_{h\to 0}\lim_{V\to\infty}\langle Im\ L\rangle \equiv \lim_{V\to\infty}\langle|Im\ L|\rangle_{h=0} = \begin{cases} 0, & \text{if } \beta < \beta_c \\ \text{non-zero}, & \text{if } \beta > \beta_c \end{cases}$$





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Finite size scaling and Z(2) universality class

Free energy and the universal functions for second order transition near the critical point can be written as,[t=(T-T$_c$)/T$_c$]

$$f = b^{-d} f_s(b^{y_t} u_t, b^{y_h} u_h, b^{-1} N_\sigma) + f_{ns} \ , \quad u_t \sim c_t t, u_t \sim c_h h$$

$$\langle |Im\ L| \rangle = \left.\frac{\partial f}{\partial h}\right|_{h\to 0} \sim N_\sigma^{-\beta/\nu} f_h(z_0 t N_\sigma^{1/\nu})$$

universal scaling function of order parameter

$$\chi_h = \left.\frac{\partial^2 f}{\partial h^2}\right|_{h\to 0} \sim N_\sigma^{\gamma/\nu}\ f_\chi(z_0 t N_\sigma^{1/\nu})$$

universal scaling function of order parameter susceptibility

$$B_4 - (1/N_\sigma^d) B_{ns} \sim f_B(z_0 t N_\sigma^{1/\nu})$$

universal scaling function of Binder cumulant

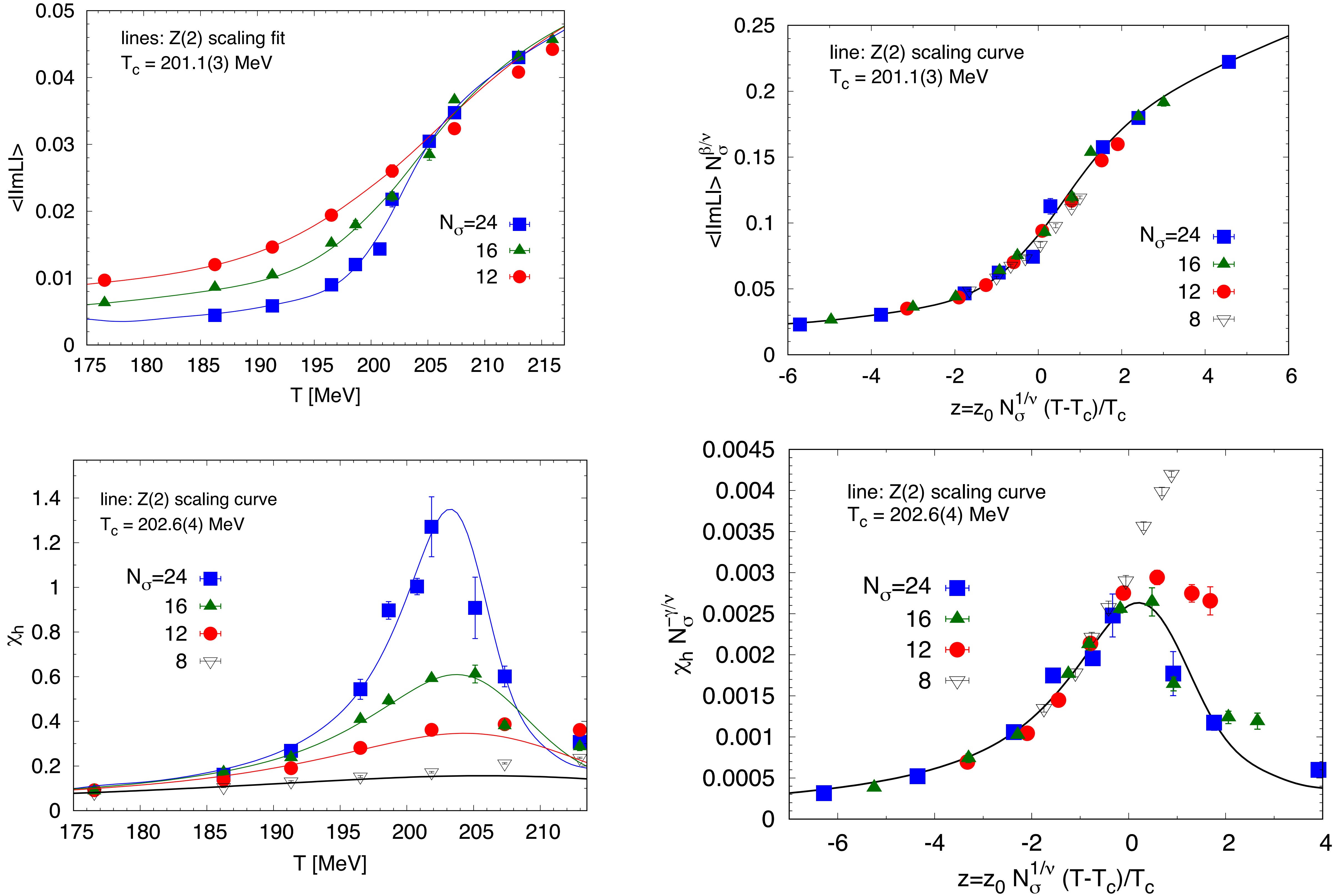
$$\chi_t = \left.\frac{\partial^2 f}{\partial t^2}\right|_{h\to 0} \sim N_\sigma^{\alpha/\nu}\ f_c(z_0 t N_\sigma^{1/\nu})$$

universal scaling function of specific heat

In our case β, ν, α and γ *are Z(2) critical exponents*

Finite size scaling of order parameter and its susceptibility

$m_\pi \sim 135$ **MeV**



Finite size scaling of specific heat and Binder cumulant

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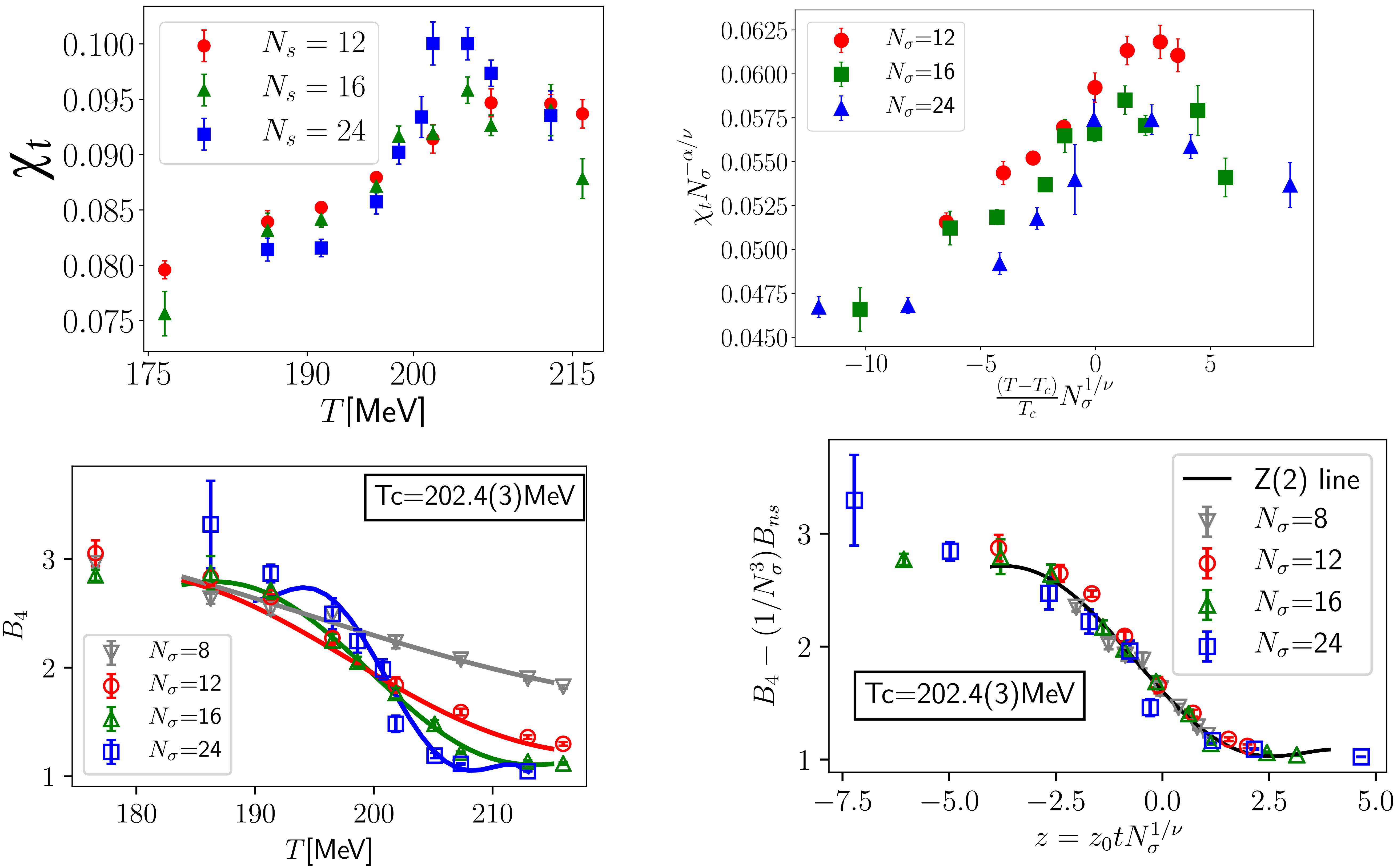
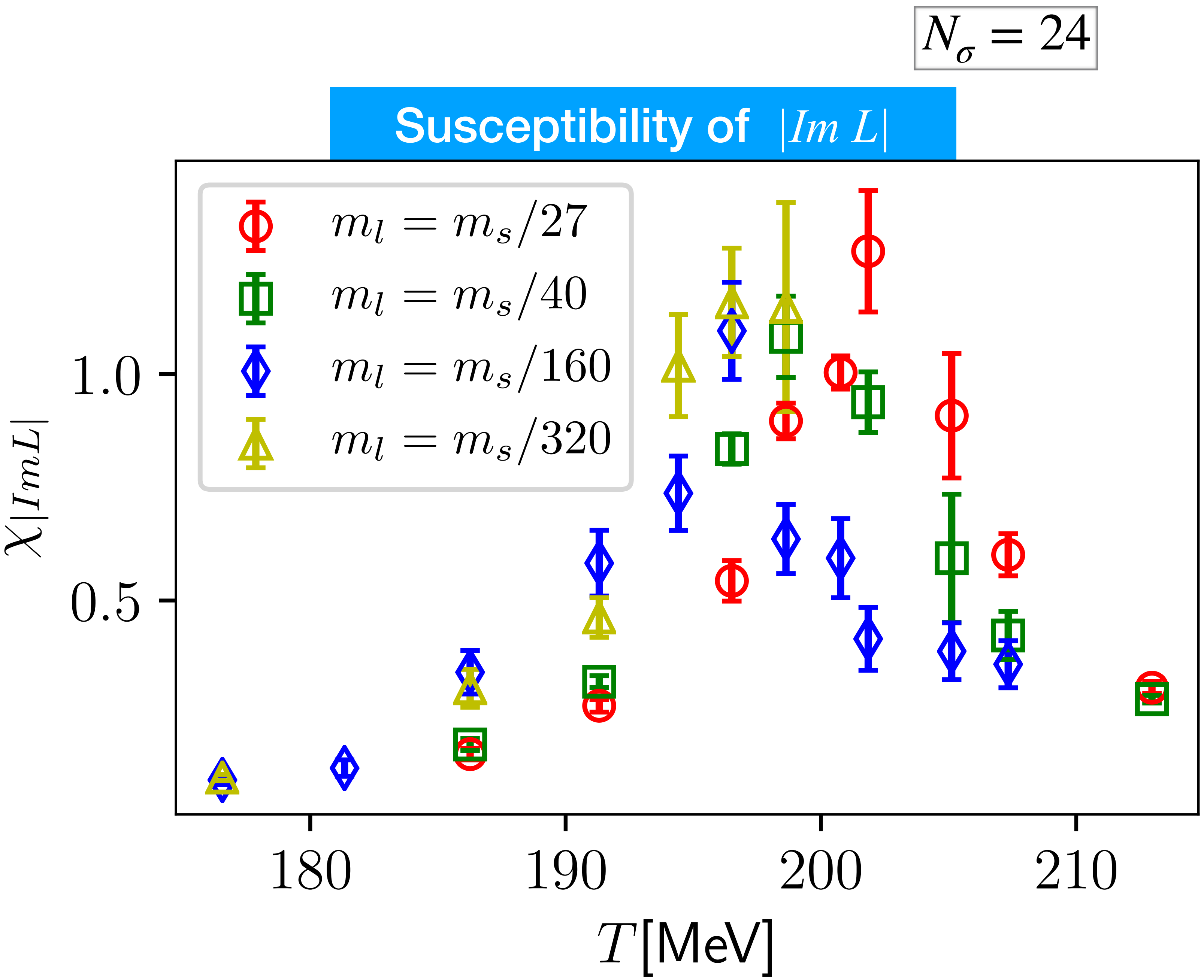


Quark mass dependence of RW transition

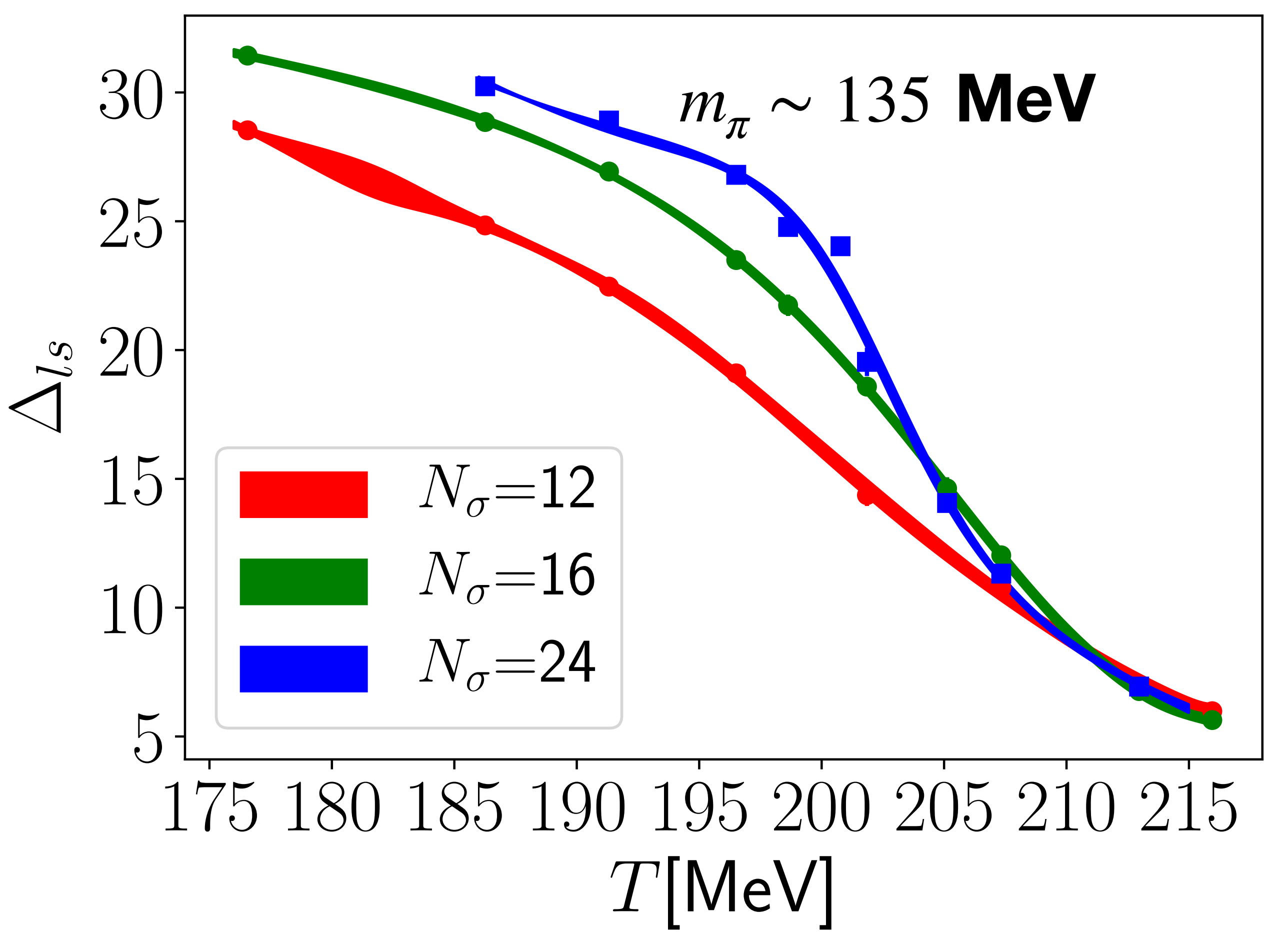


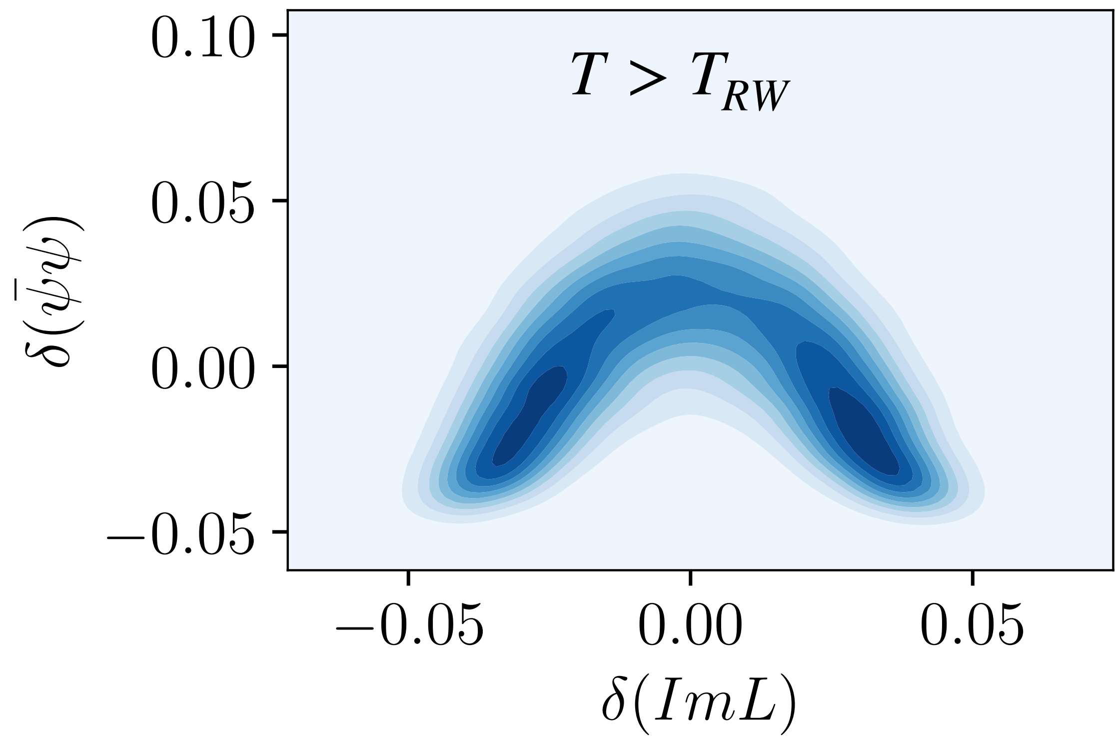
m_l	m_π(MeV)
$m_s/27$	135
$m_s/40$	110
$m_s/160$	55
$m_s/320$	40

No significant rise in the peak of the susceptibility of the order parameter with respect to pion mass upto, $m_\pi \sim 40$ MeV

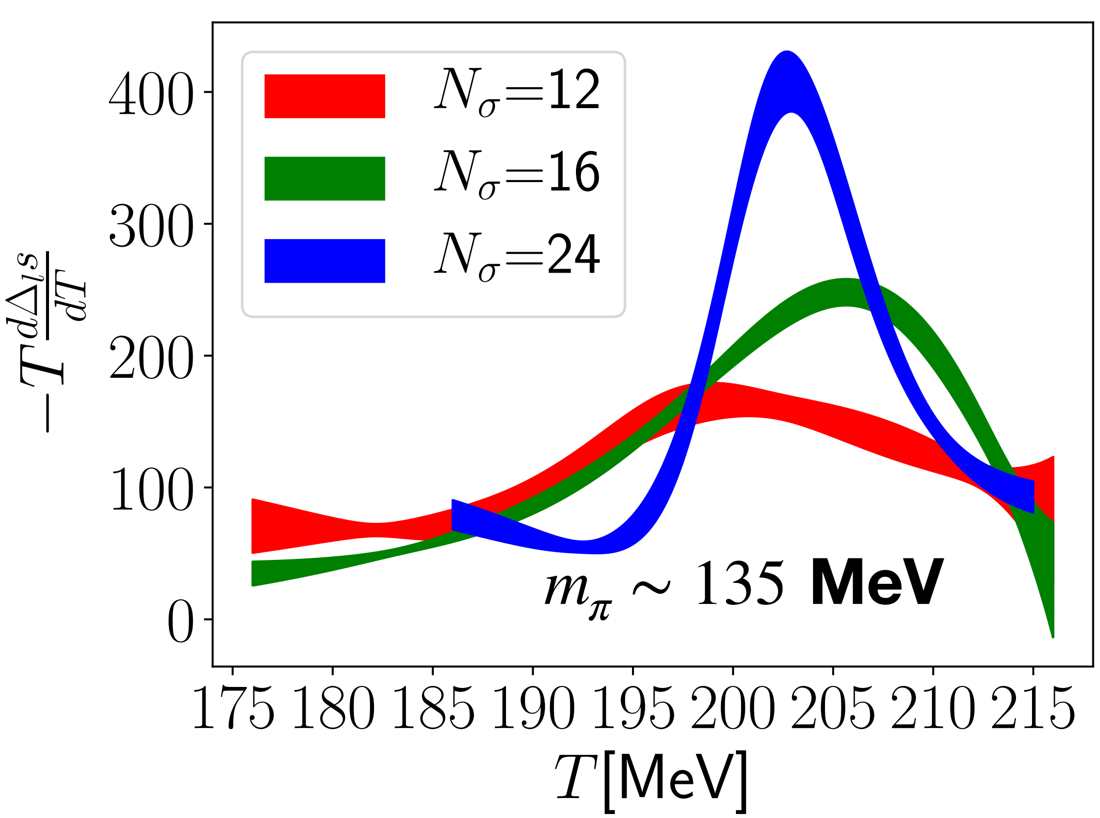
Order of the RW transition seems to be unchanged ??

Finite size effects on chiral observables





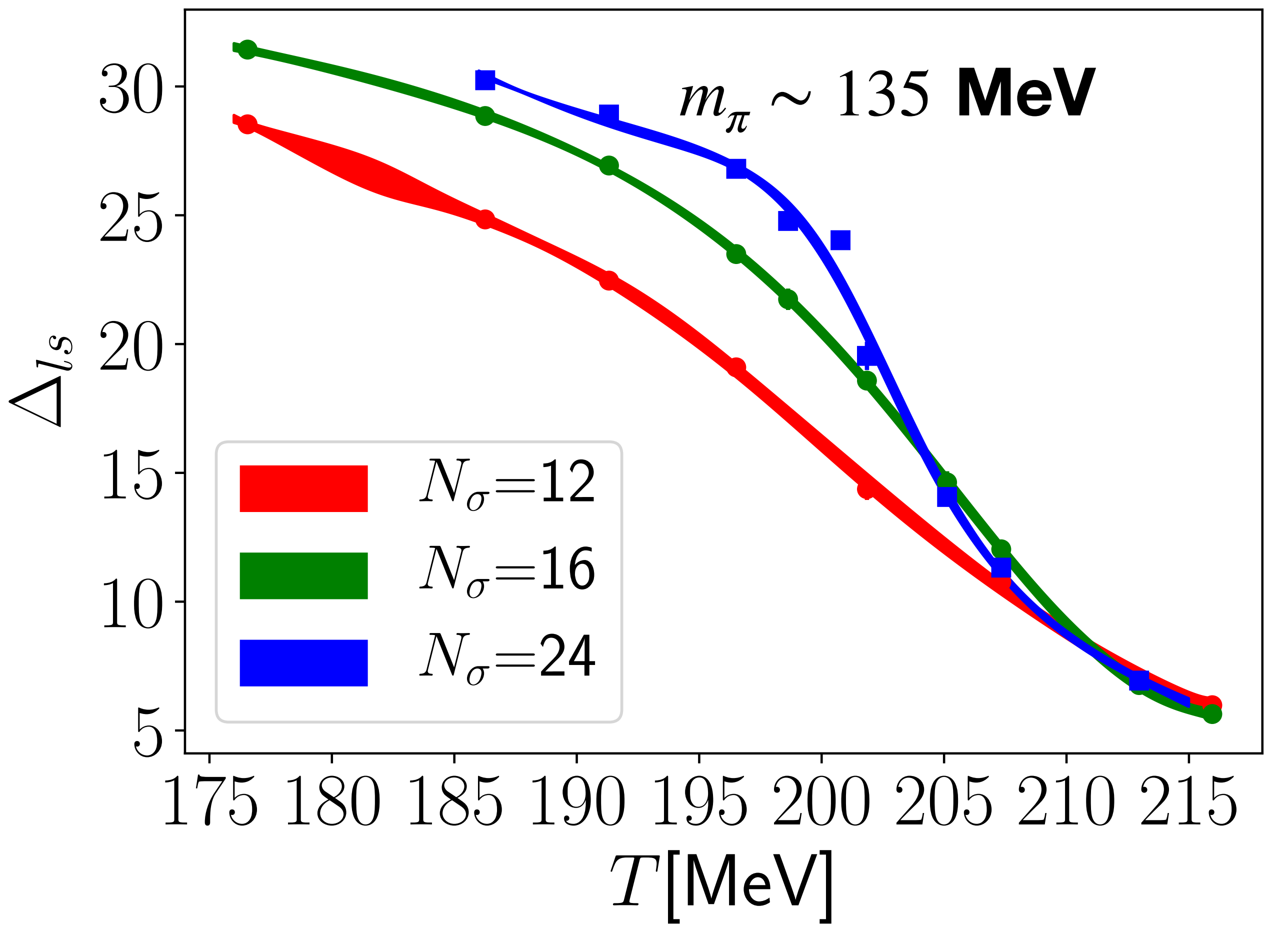
$$\Delta_{ls} = (m_s/f_k^4)\big(\langle \bar{l}l \rangle - (m_l/m_s)\langle \bar{s}s \rangle\big)$$

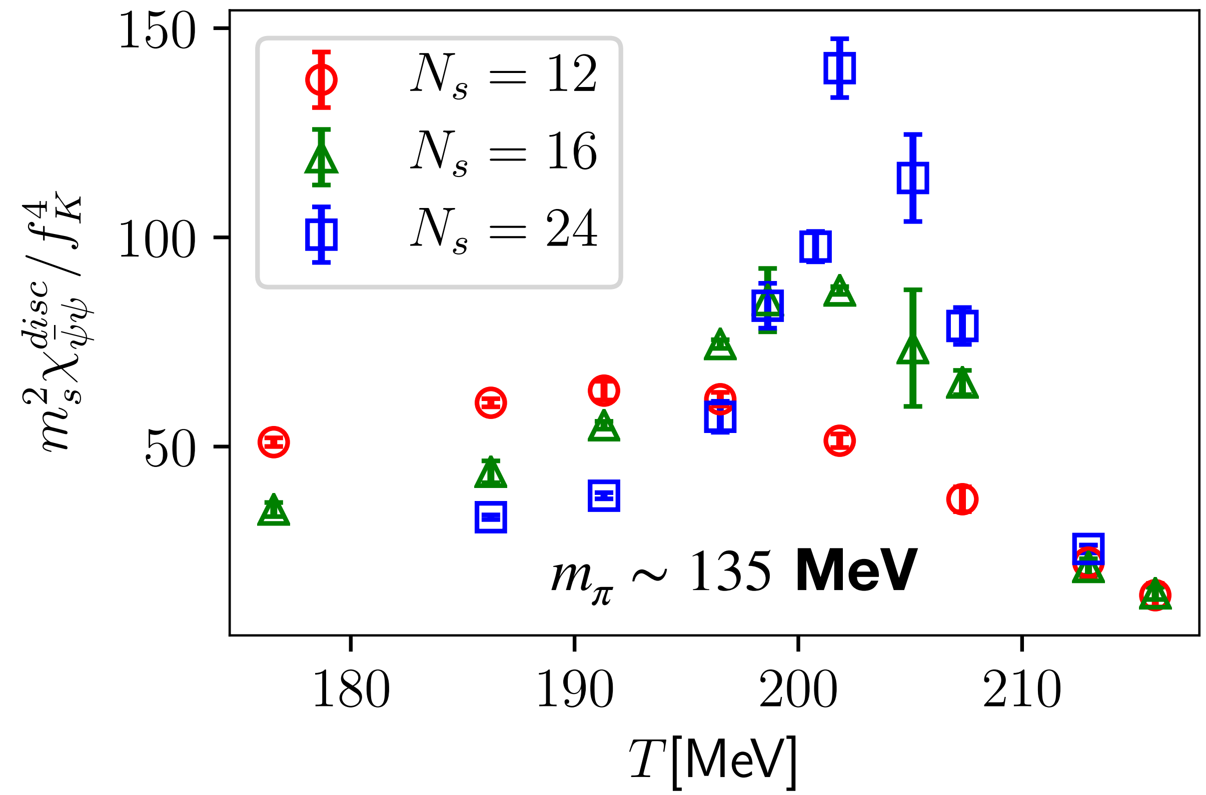


Strong volume dependence of "subtracted chiral condensate" at fixed m_l/m_s below T_{RW}

mixed chiral susceptibility sensitive to transition at the RW endpoint

Finite size effects on chiral observables





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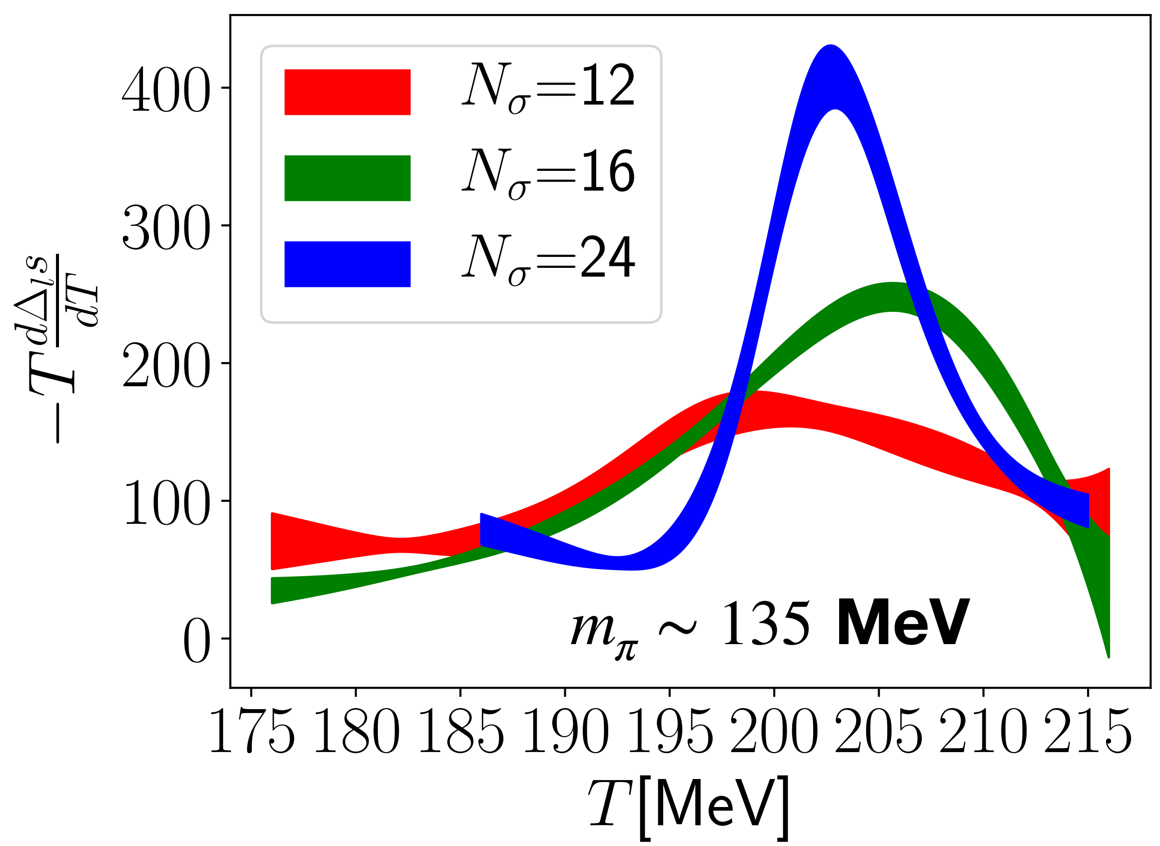
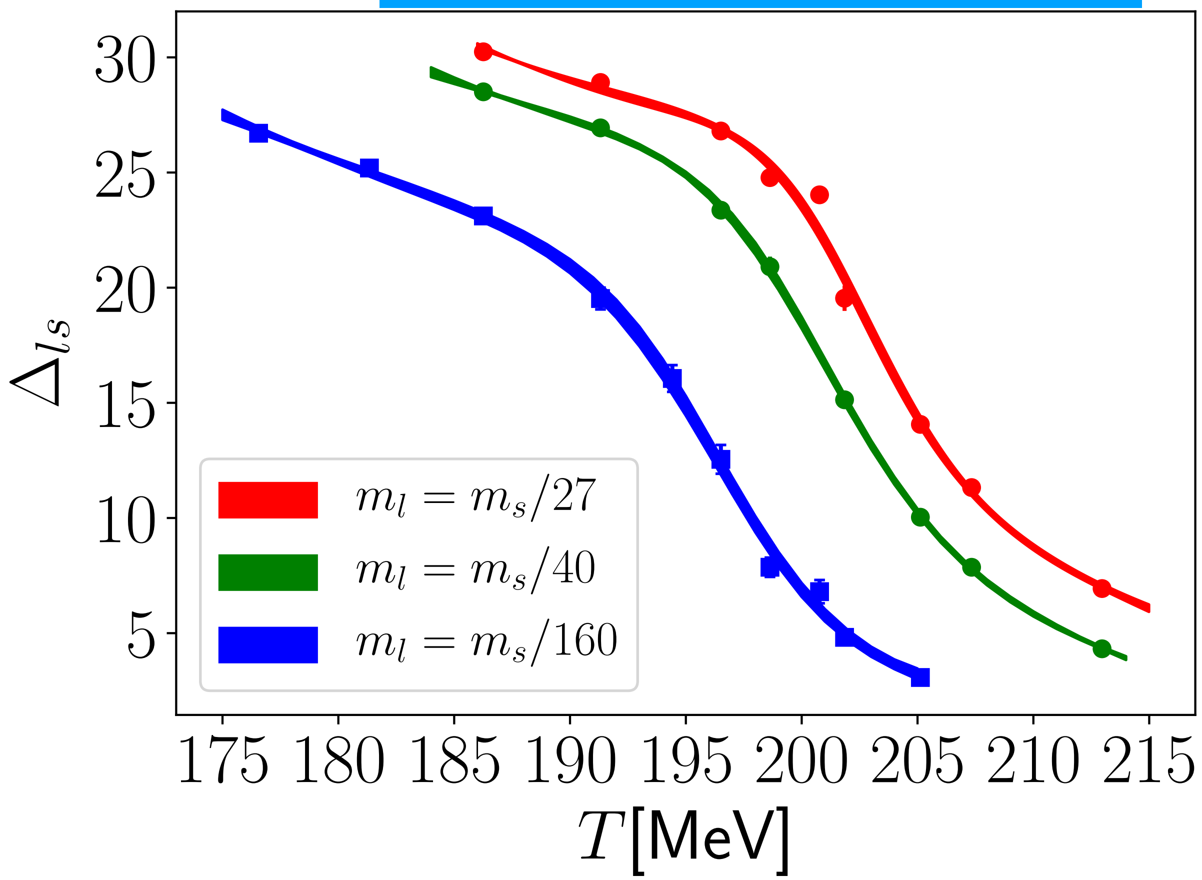


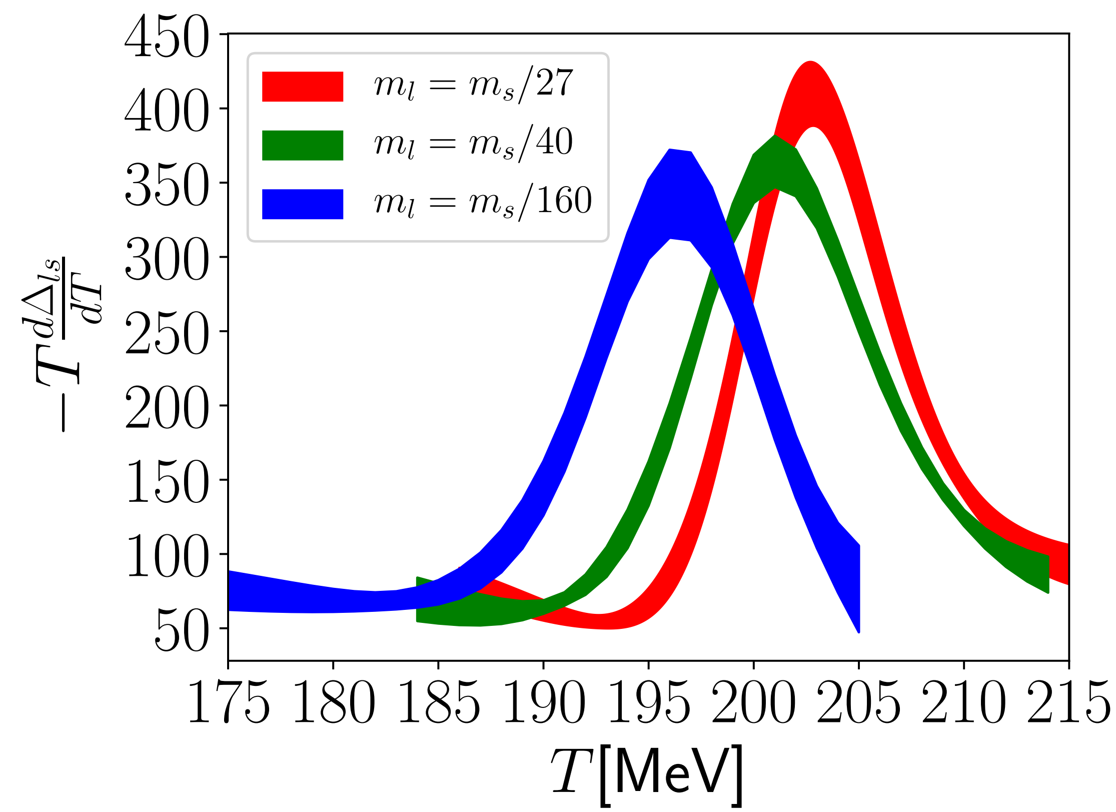
mixed chiral susceptibility sensitive to transition at the RW endpoint

Chiral limit and RW transition

$N_\sigma = 24$

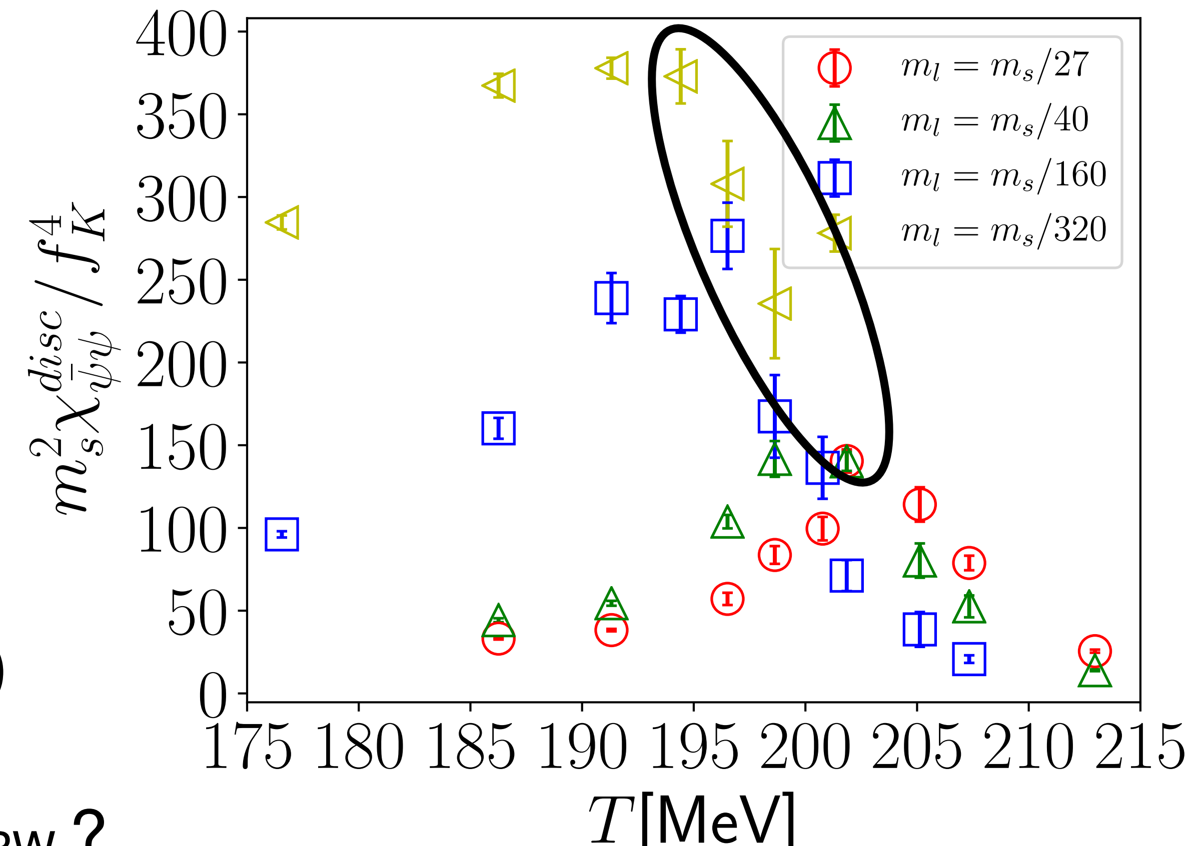
Sub. Chiral condensate





$$\langle\bar{\psi}\psi\rangle \sim const + m^{1/2}$$

$$\chi^{disc}_{\bar{\psi}\psi} \sim m^{-1/2}$$ For, $T < T_c$

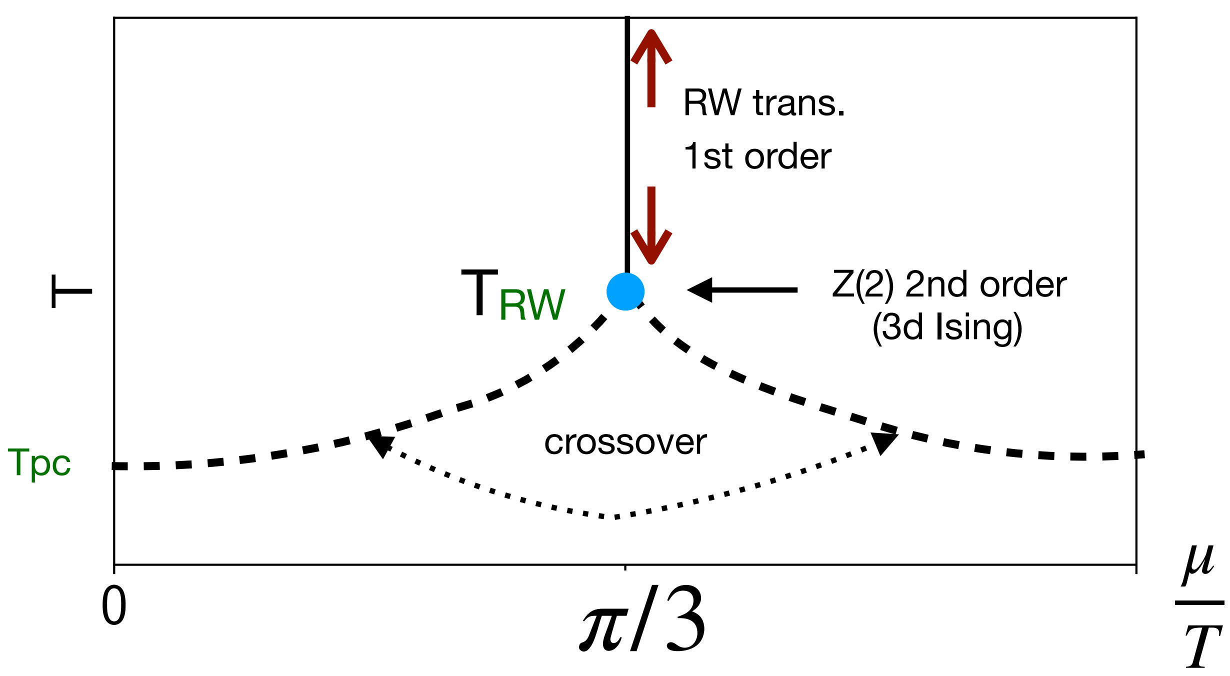


Goldstone effect (square root singularity) in $\chi^{disc}_{\bar{l}l}$ below T_{RW} is evident.

→ chiral symmetry restoration at T_{RW} ?

Outlook on the fate of the RW end point

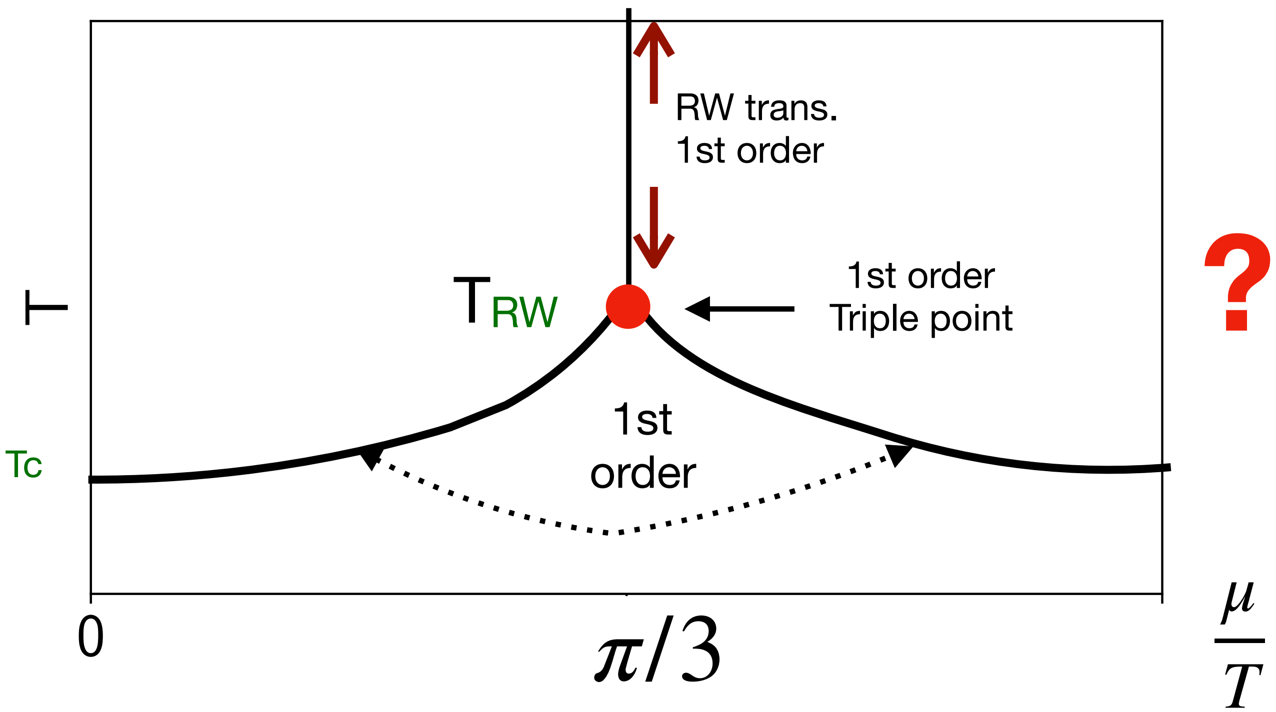
Intermediate quark mass



40 MeV $\le m_\pi \le 135$ MeV

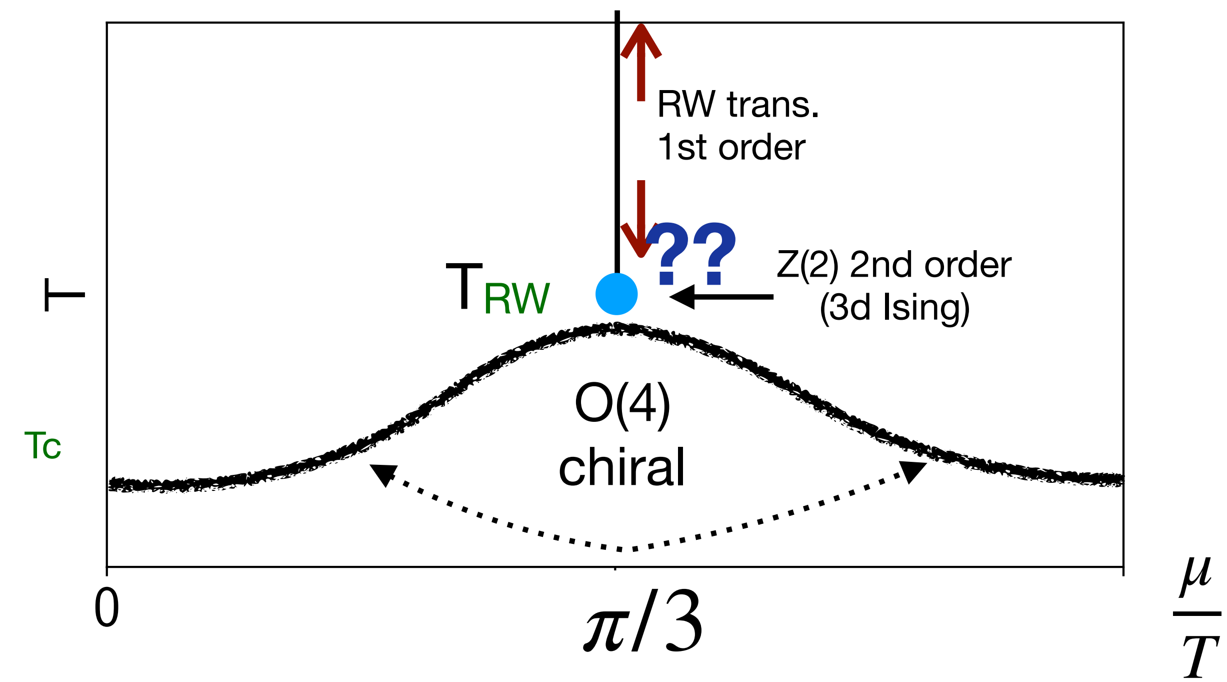
For, ~~intermediate~~ pion mass RW end point most probably stays as Z(2) 2nd order.

Approaching Chiral limit



??

Approaching Chiral limit



Conclusions

- Our studies with HISQ action with physical pion mass suggests that the RW end point is 2nd order and belongs to the Z(2) universality class. This is consistent with the earlier result found with the stout-improved staggered action.
- Preliminary trends at $m_\pi \sim$ 40 MeV suggest that the RW end point remains as Z(2) second order.
- Nature of the chiral transition in the RW plane needs to be examined further. Favours 2nd order(O(4)) transition at μ=0.
- RW transition and chiral phase transition may coincide in the chiral limit.

Calculations on larger lattices are ongoing.

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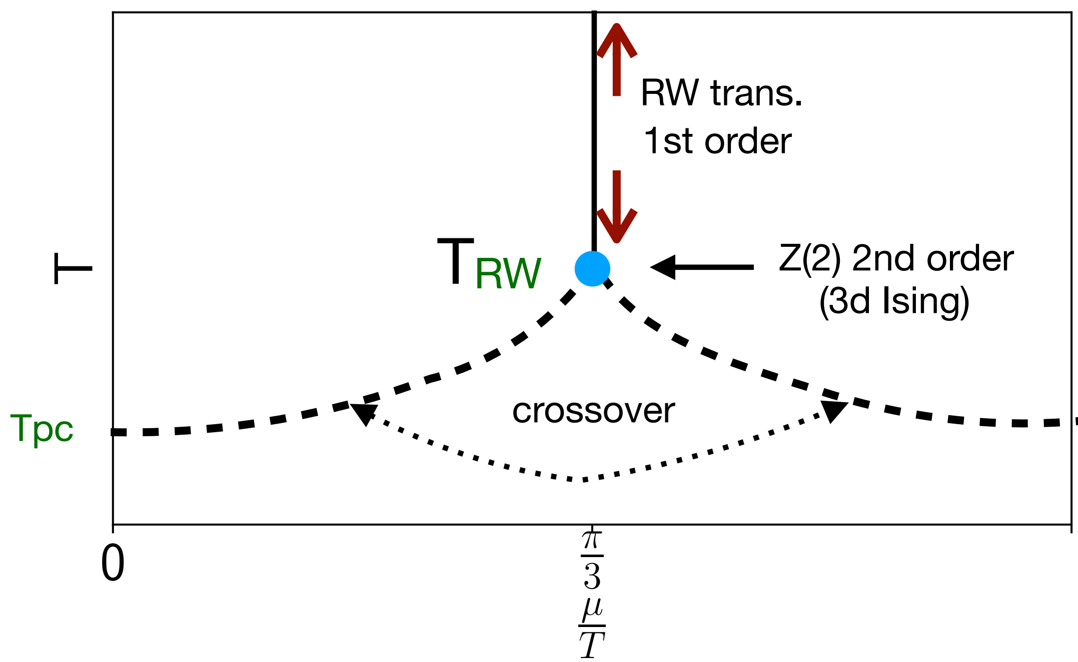
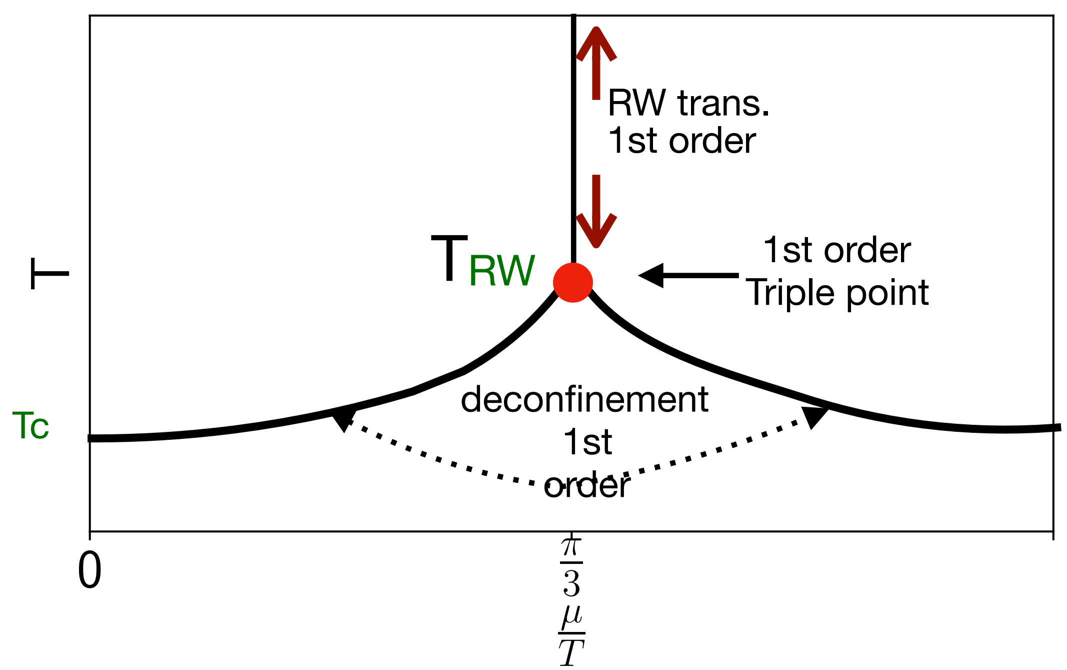
Thank you for your time and attention

Back up slides

Intermediate quark mass

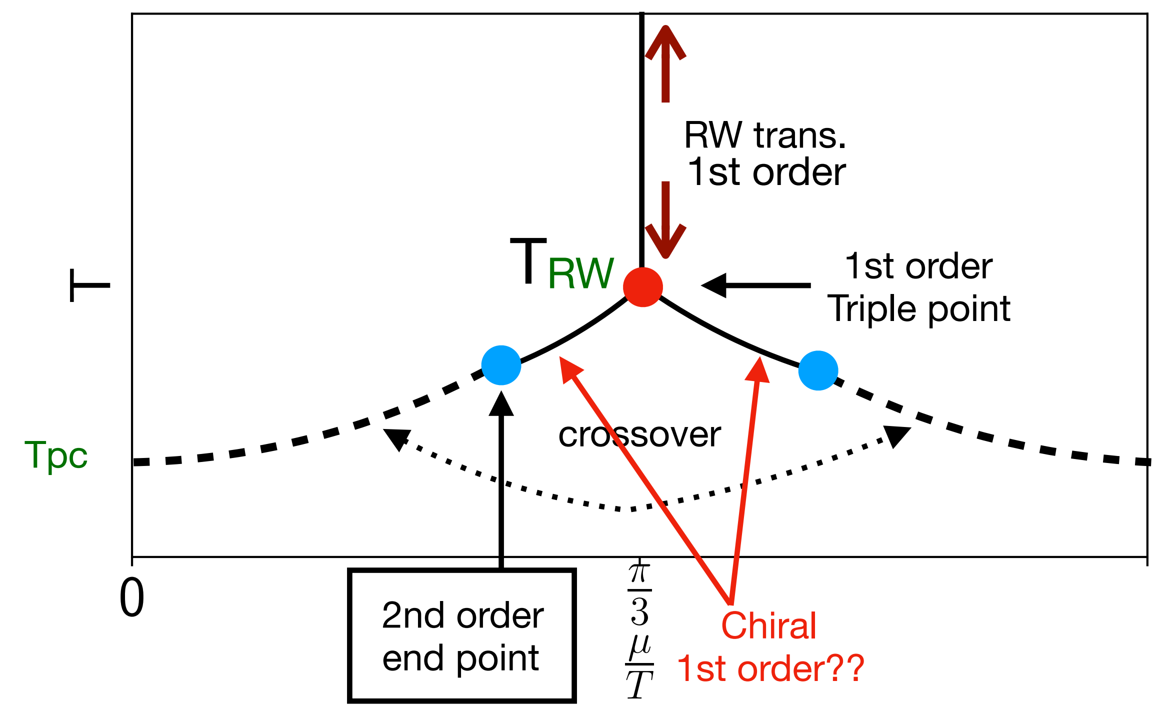


Heavy quark mass



Conjectured phase diagrams in the imaginary chemical potential plane

small quark mass



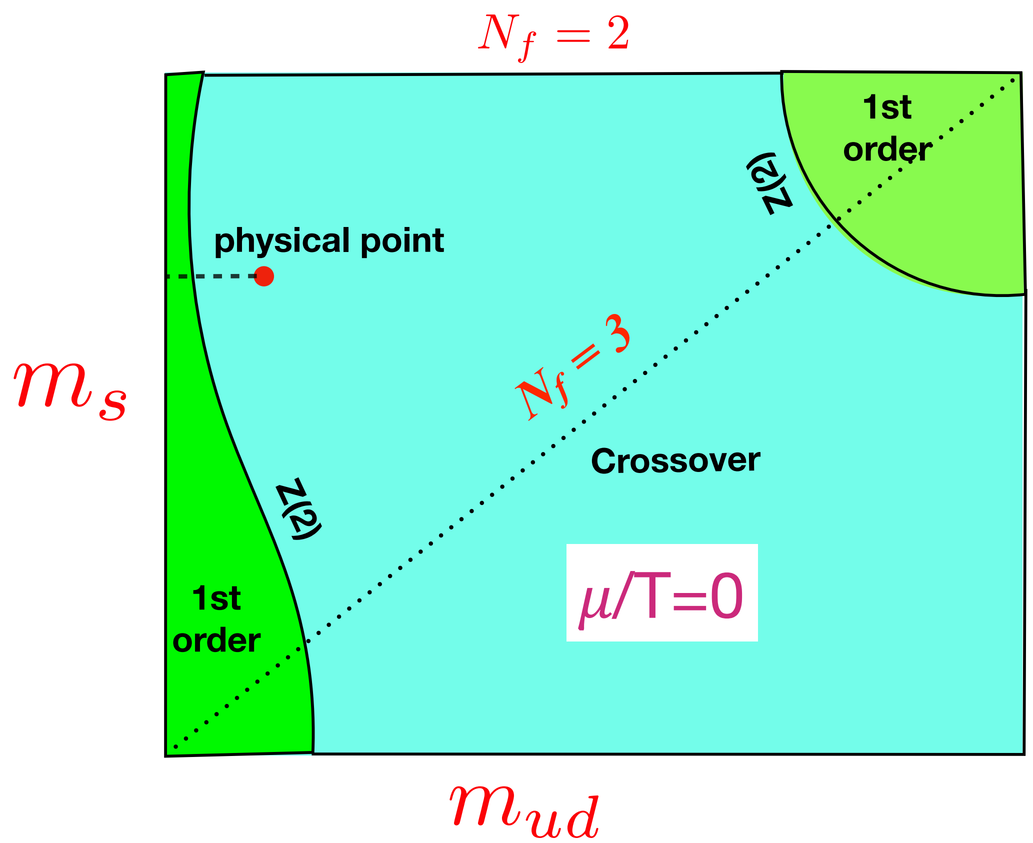
Different scenarios for different quark masses

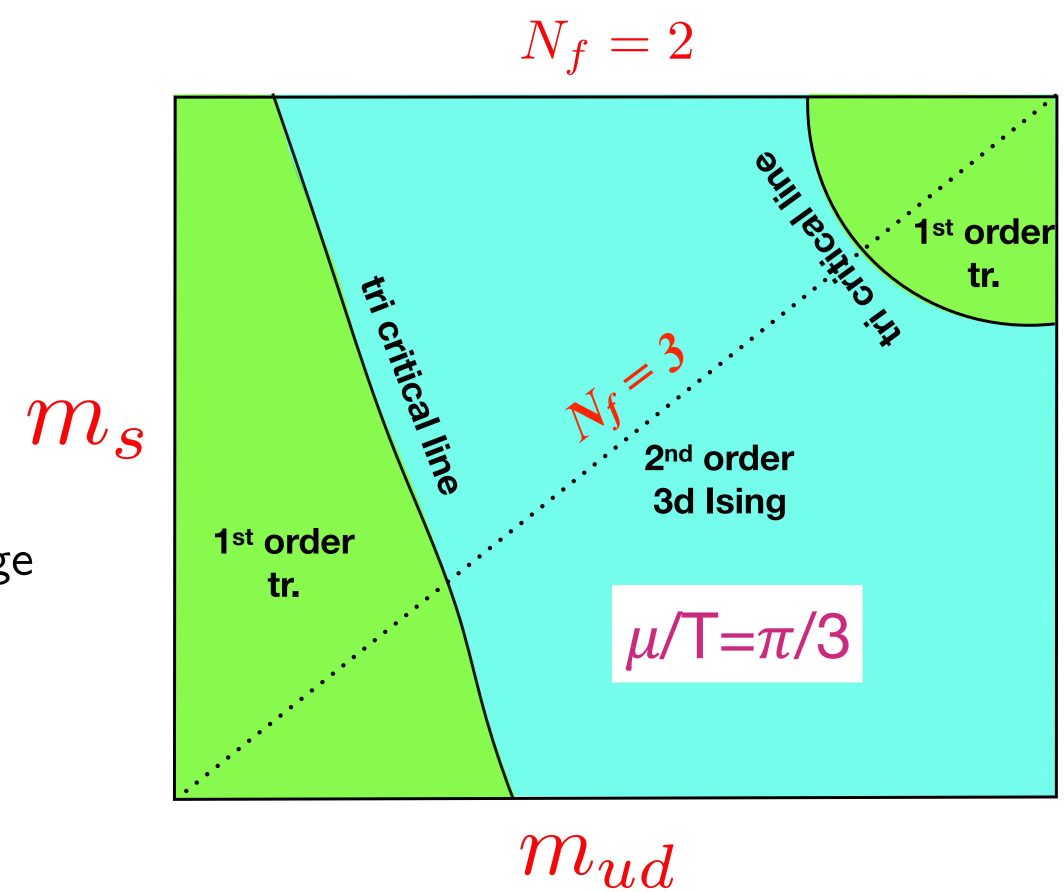
Phases in the RW plane

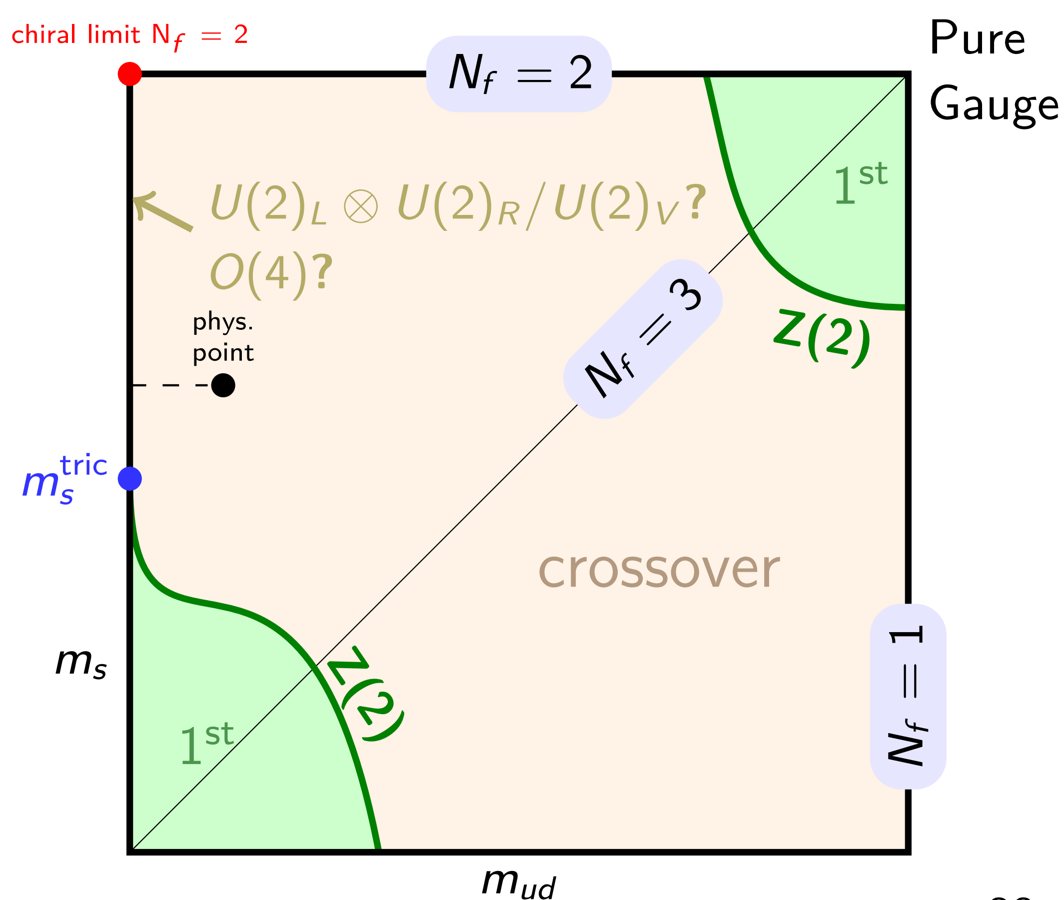
- RW transition happens between two Z(3) sectors of the Polyakov loop. Hence, the order parameter can be the phase or the imaginary part of the Polyakov loop.

- In the RW plane, the 1st order region (for small mass) consists of three 1st order transitions, where high temperature RW transition meets two chiral phase transitions.

- The physical point which is crossover for $\mu=0$ can be 1st or 2nd order in the RW plane. So, our first goal is to confirm this issue and then going to the chiral limit to "search for a 1st order" transition.







Columbia plot in μ=0
and RW plane

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Finite size scaling and Z(2) universality class

Free energy,

$$f = f_{ns} + b^{-d} f_s(b^{y_t} u_t, b^{y_h} u_h, b^{-1} N_\sigma)$$

Responsible for the universal critical behaviour

universal functions →

$$< O > = (\ldots.)\frac{\partial}{\partial h} f_s(\ldots)|_{h\to o}$$ order parameter

$$\chi_h = (\ldots.)\frac{\partial^2}{\partial h^2} f_s(\ldots)|_{h\to o}$$ susceptibility of op

$$\chi_t = (\ldots.)\frac{\partial^2}{\partial t^2} f_s(\ldots)|_{h\to o}$$ specific heat

$$t = \frac{T - T_c}{T_c} \sim \beta - \beta_c$$

near, $T \to T_c$ $u_t \sim c_t t, u_t \sim c_h h$

$$\chi_t = z_2 N_\sigma^{\alpha/\nu} f_t(z_0 t N_\sigma^{1/\nu})$$

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Susceptibility of $|Im\ L|$

Susceptibility of $|Re\ L|$

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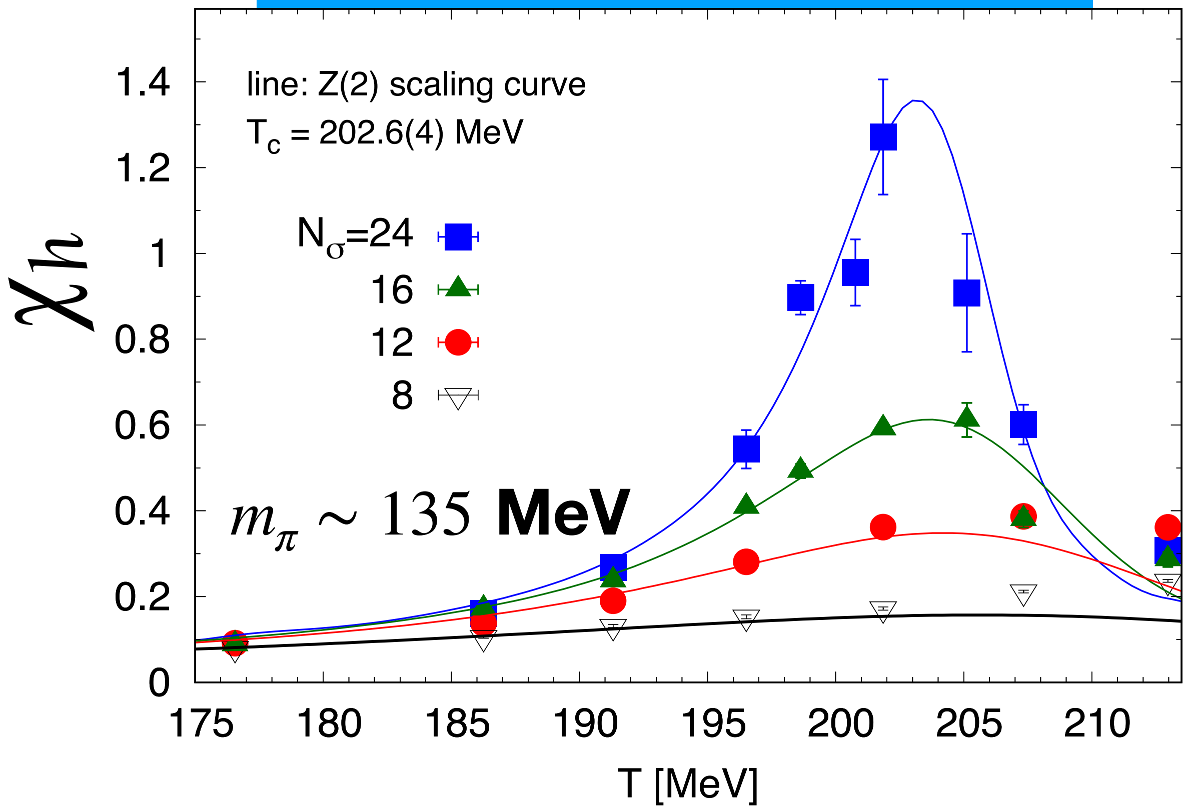

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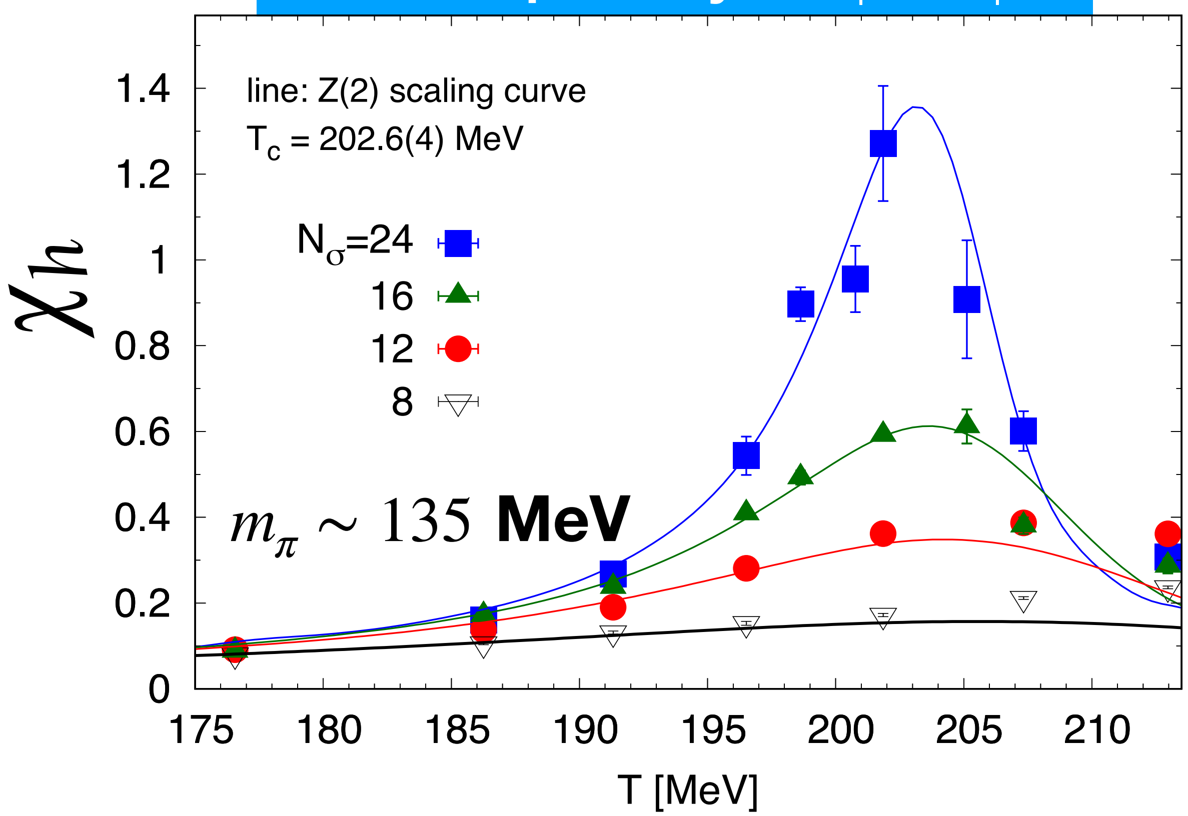

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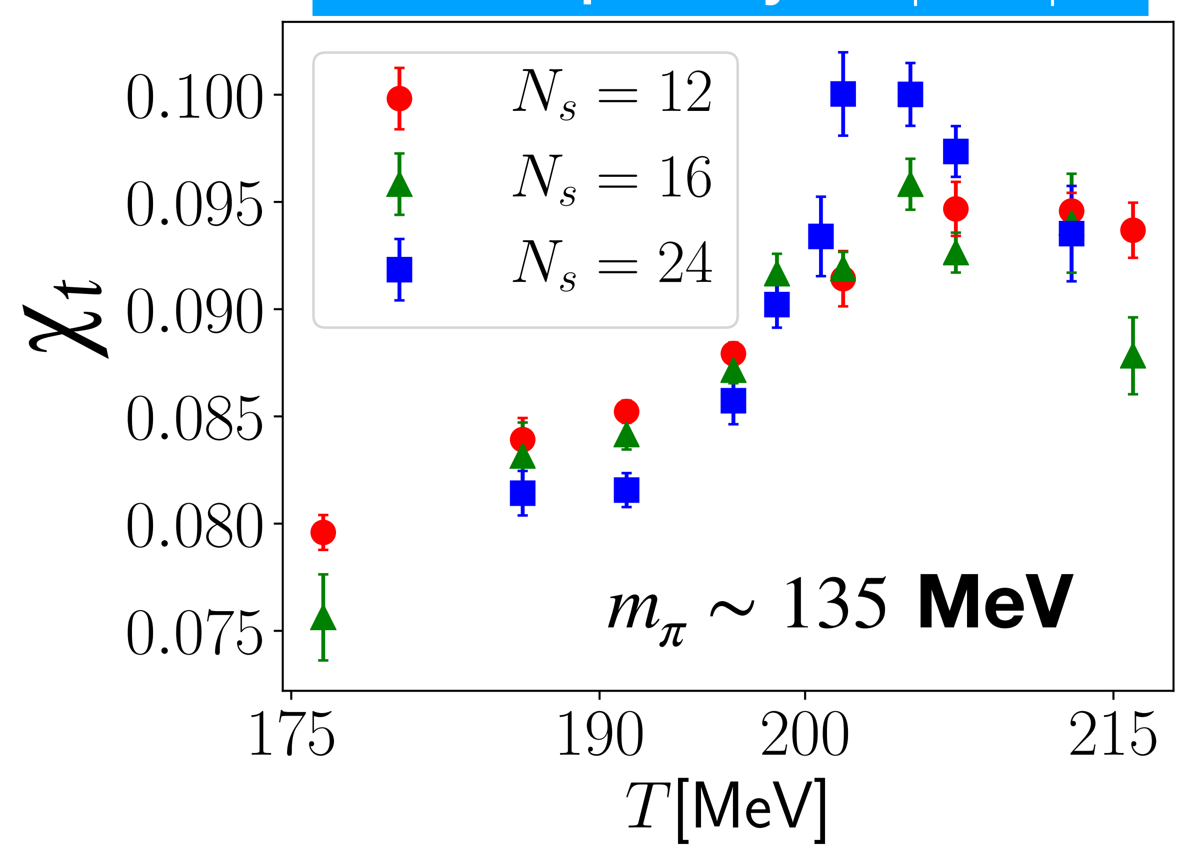

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"Susceptibility" and "specific heat" scale with corresponding Z(2) finite size universal scaling functions

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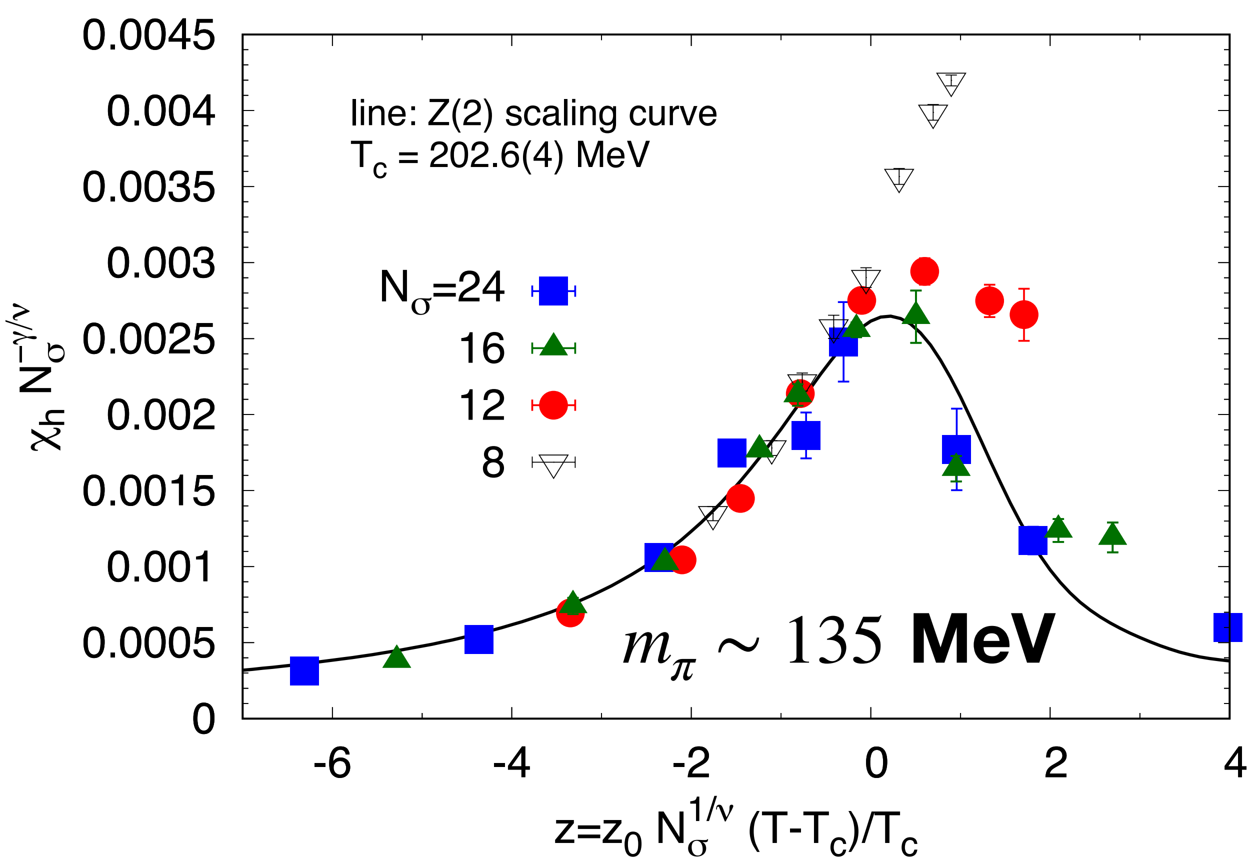

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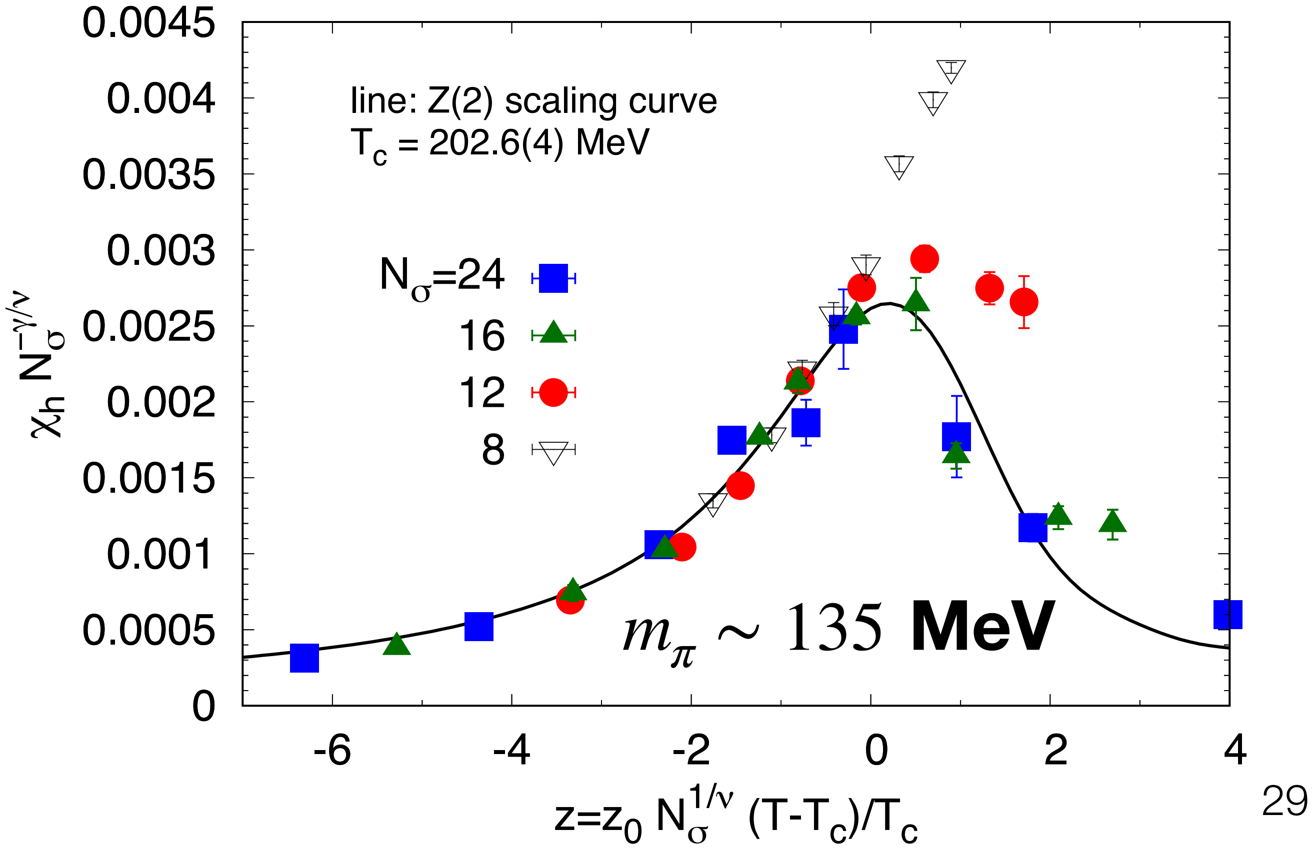
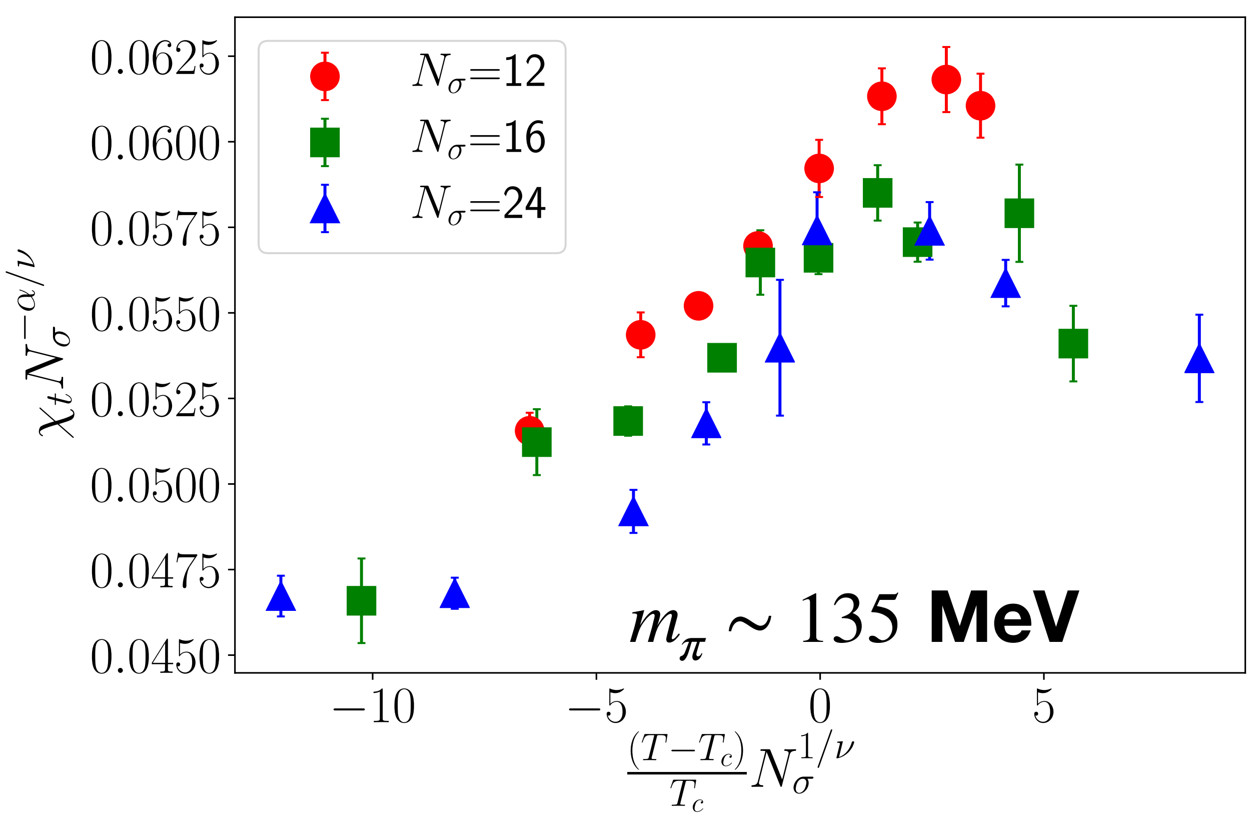
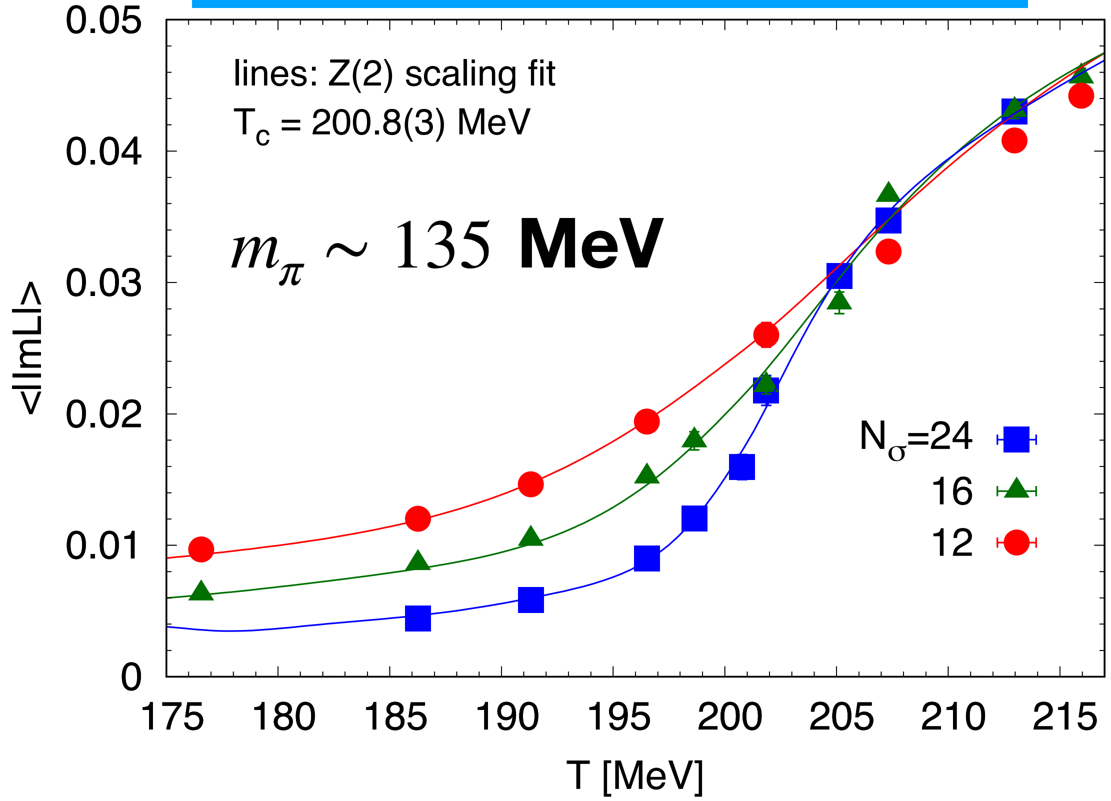
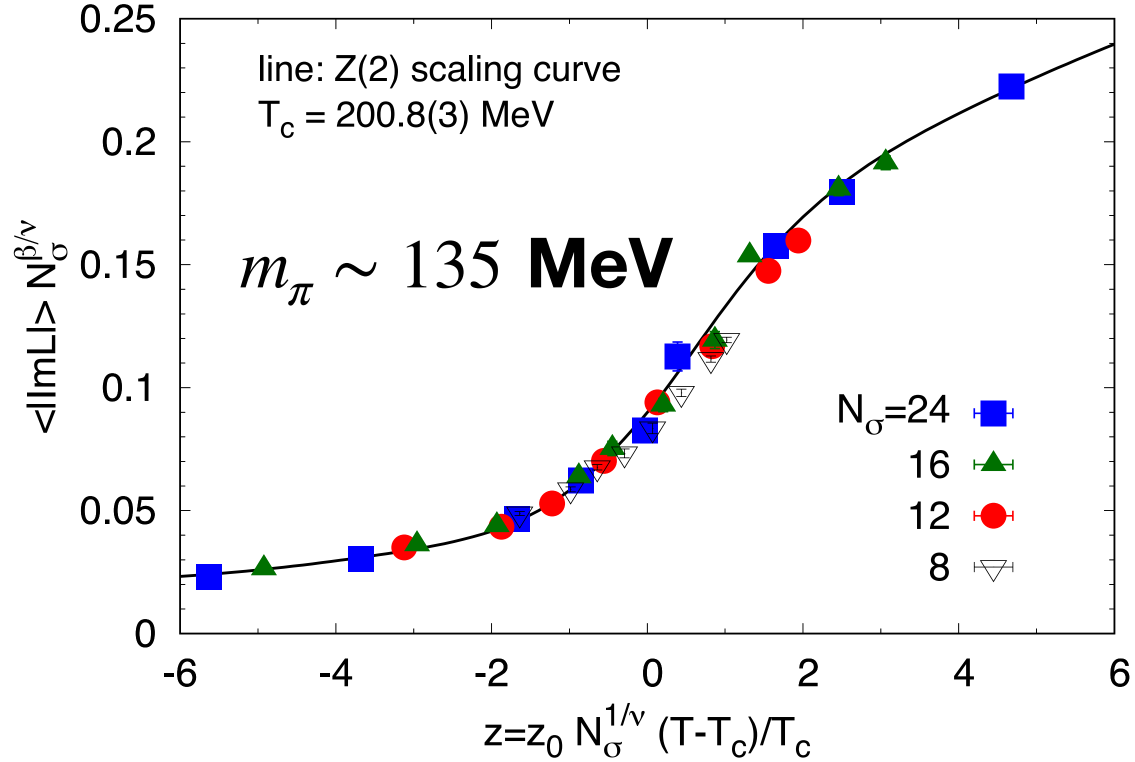




Binder cumulants of order parameter

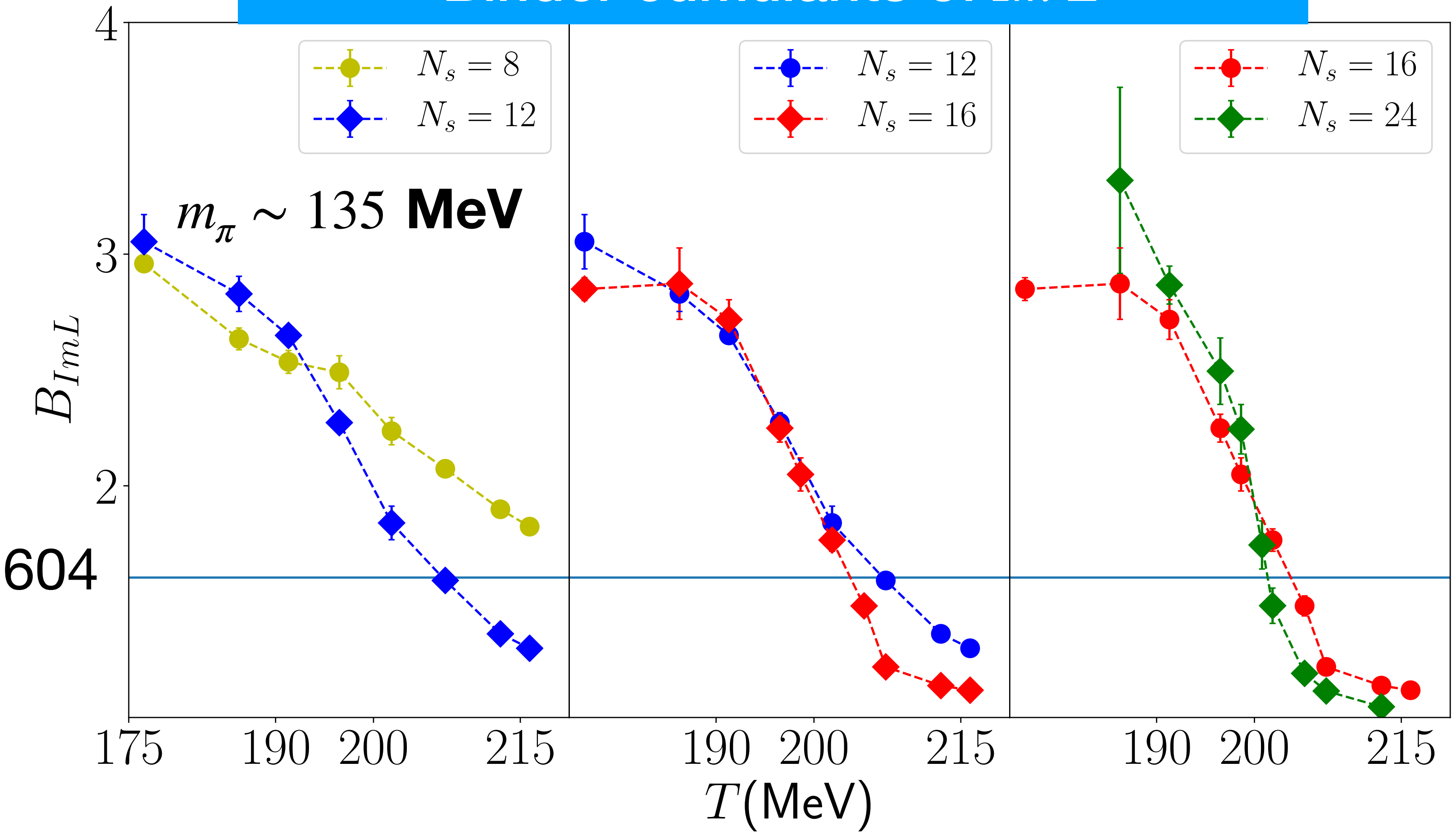
$|Im\ L|$





Universal Z(2) scaling of the order parameter

Binder cumulants of $Im\ L$



Crossing of the Binder cumulants approaches to the universal value in $V \to \infty$

Ising value=1.604

Studies in the RW plane

Mostly with unimproved actions

$m_\pi > 1$ GeV, for the 'heavy quark mass RW transition'

'small quark mass RW transition' $(N_f = 2)$

Standard staggered action: $m_\pi \sim 400$ MeV (N_τ=4)

Standard Wilson action: $m_\pi \sim 930$ MeV(N_τ=4)

$m_\pi \sim 680$ MeV (N_τ=6)

➡ 1st order triple point (at the end of the line of 1st order RW transitions) exist already for $\mu/T=\pi/3$ and $m_{cri} > m_{phy}$.

☆ The results are strongly fermion discretization scheme and cut-off(N_τ) dependent.

P. de Forcrand et. al, PRL 105, 152001(2010), Owe Philipsen et. al, PRD 89, 094504(2014), Christopher Czaban et al, PRD 93, 054507 (2016)

Studies in the RW plane

Very recent studies with improved actions,

- Stout improved staggered fermions(N_f=2+1): At the physical quark mass point($m_\pi \sim 135$ MeV) a 2nd order transition in the 3d-Ising universality class happens instead of a 1st order at the RW endpoint.

C. Bonati et. al,PRD 93, 074504 (2016)

No 1st order end point (of the line of 1st order RW transitions) for $m_\pi > 50$ MeV.

C. Bonati et. al,arXiv:1807.02106 [hep-lat]

- HISQ(N_f=2): Order of the phase transition at physical point is not clear(large cut-off effects) .

L.K.Wu, et al. PRD 97,114514(2018)