

Strange matter and kaon to pion ratio in SU(3) PNJL model

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The 'horn': experiments

Firstly, the "horn" was described by the NA49 Collaboration (NA49 Collaboration, PRC 66, 054902,2002) and then it was shown that data can be placed on the same curve (STAR, AGS)

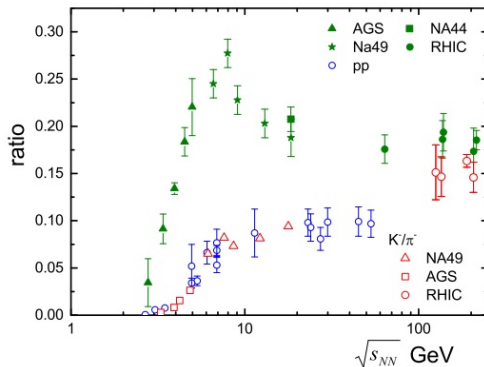


Figure 1: The K^+/π^+ and K^-/π^- ratio as the $\sqrt{s_{NN}}$

The 'horn': theoretical overview

- **the statistical model**: hadron resonances + σ - meson (implies the existence of the critical temperature for hadrons, the hadron phase transition) \Rightarrow the qualitative reproduction of the peak (Andronic PLB 673, 142 (2009)).
- **the SMES (the Statistical Model of Early Stages)**: a slow increase and the following jump in the ratio is a result of the deconfinement transition; the strangeness yield becomes independent of energy in the QGP when deconfinement transition occurs ($m_s \rightarrow m_{s0}$) (M. Gazdzicki, M.I. Gorenstein, Acta Phys. Pol. B 30, 2705 (1999)).
- **the microscopic transport model**: only the hadron phase w/o the quark-gluon phase can not reproduce experimental data (W. Echehalt and W. Cassing, NPA602, 449 (1996); W. Cassing, E. L. Bratkovskaya, PR 308, 65 (1999); H. Petersen et al arXiv:0805.0567 [hep-ph]....)
- **the microscopic transport model + the partial restoration of chiral symmetry** (A. Palmese, et al. PRC 94, 044912 (2016): the quick increase in the K^+/π^+ is due the partial chiral symmetry restoration; the decrease is a result of chiral condensate destruction.

Propose of the work: use the SU(3) PNJL model

The Lagrangian (P. Costa et al. PRD79, 116003 (2009); E. Blanquier J. Phys. G: NPP 38, 105003 (2011)):

$$\begin{aligned} \mathcal{L} = & \bar{q} (i \gamma^\mu D_\mu - \hat{m} - \gamma_0 \mu) q + \frac{1}{2} G_s \sum_{a=0}^8 [(\bar{q} \lambda^a q)^2 + (\bar{q} i \gamma_5 \lambda^a q)^2] \\ & + K \{ \det [\bar{q} (1 + \gamma_5) q] + \det [\bar{q} (1 - \gamma_5) q] \} - \mathcal{U}(\Phi, \bar{\Phi}; T) \end{aligned}$$

$D_\mu = \partial^\mu - iA^\mu$, where A^μ is the gauge field with $A^0 = -iA_4$ and $A^\mu(x) = G_s A_a^\mu \frac{\lambda_a}{2}$.
 The effective potential has to reproduce the Lattice calculation in the pure gauge sector:

$$\begin{aligned} \frac{\mathcal{U}(\Phi, \bar{\Phi}; T)}{T^4} &= -\frac{b_2(T)}{2} \bar{\Phi} \Phi - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\bar{\Phi} \Phi)^2, \\ b_2(T) &= a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3. \end{aligned}$$

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- explain and describe spontaneous chiral symmetry broken as $m_q = m_0 + \langle \bar{q} q \rangle$;
- build the phase diagram with crossover at low chemical potential and 1st order transition at high chemical potential ($m_0 \neq 0$),
- simulate the confinement/deconfinement transition

Meson masses

The meson masses are defined by the Bethe-Salpeter equation at $P = 0$

$$1 - P_{ij} \Pi_{ij}^P(P_0 = M, P = 0) = 0 ,$$

whith

$$P_\pi = G_s + K \langle \bar{q}_s q_s \rangle , \quad P_K = G_s + K \langle \bar{q}_u q_u \rangle$$

and the polarization operator:

$$\Pi_{ij}^P(P_0) = 4 \left((I_1^i + I_1^j) - [P_0^2 - (m_i - m_j)^2] I_2^{ij}(P_0) \right) ,$$

When the $T > T_{\text{Mott}}$ and $P_0 > m_i + m_j$, the meson turns into the resonance state. In this case, the complex properties of the polarization operator have to be taken into account and the solution has to be defined in the form $P_0 = M_M - \frac{1}{2}i\Gamma_M$.

The model properties

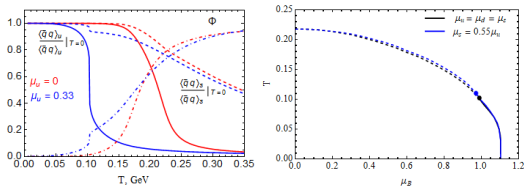


Figure 2: The order parameters and phase diagram of PNJL model

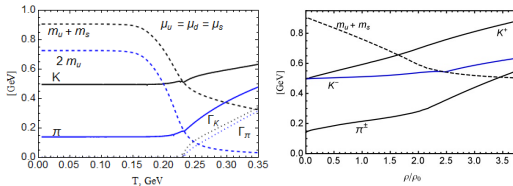


Figure 3: The mass spectra at zero and finite chemical potential (see for discussion P.Costa PRD71 (2005) 116002)

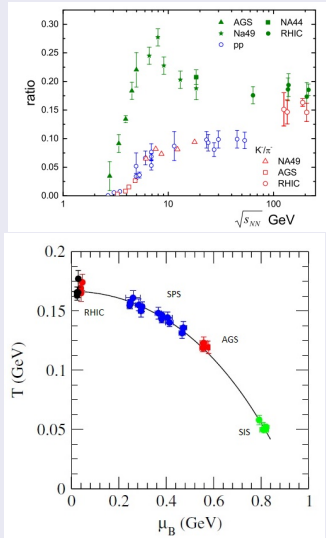


Figure 4: J. Cleymans et al. PRC73, 034905 (2006)

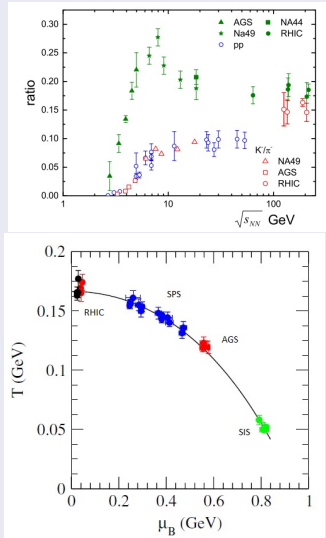


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T- μ variables

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$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \quad \mu_B(\sqrt{s}) = \frac{d}{1 + e\sqrt{s}}$$

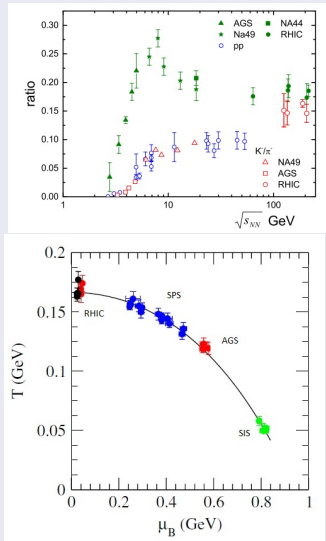
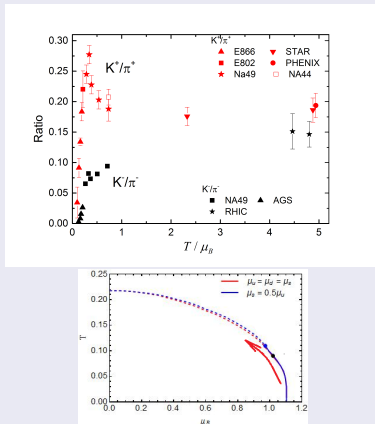


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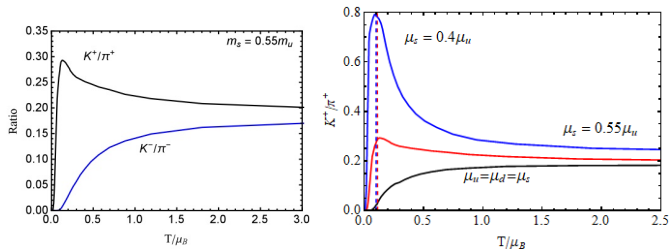


Kaon to pion ratio

$$n_{K^\pm} = \int_0^\infty p^2 dp \frac{1}{e^{(\sqrt{p^2 + m_{K^\pm}} \mp \mu_{K^\pm})} - 1},$$

$$n_{\pi^\pm} = \int_0^\infty p^2 dp \frac{1}{e^{(\sqrt{p^2 + m_{\pi^\pm}} \mp \mu_{\pi^\pm})} - 1}.$$

with parameter $\mu_\pi = 0.135$ (M. Kataja, P.V. Ruuskanen PLB 243, 181 (1990)) and $\mu_K = \mu_u - \mu_s$ (see for example A. Lavagno and D. Pigato, EPJ Web of Conferences 37, 09022 (2012)).



How we can improve the PNJL model?

- introduce a phenomenological dependence of $G_s(\Phi)$ (Y. Sakai et al PRD 82, 076003 (2010), P. de Forcrand, O. Philipsen NPB 642, 290(2002))

$$\tilde{G}_s(\Phi) = G_s[1 - \alpha_1 \Phi \bar{\Phi} - \alpha_2 (\Phi^3 + \bar{\Phi}^3)]$$

with $\alpha_1 = \alpha_2 = 0.2$.

- introduce a $K = K_0 \exp(-(\rho/\rho_0)^2)$ (see for discussion P. Costa et al AIP Conf.Proc. 775 (2005) 173)

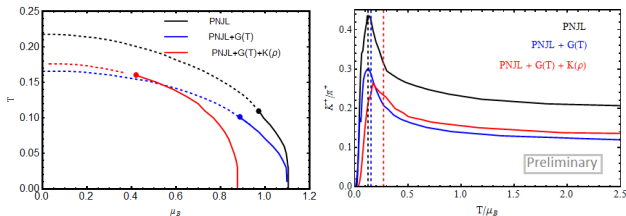
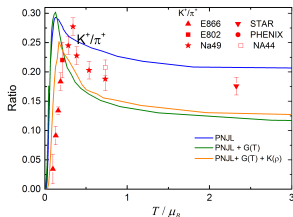


Figure 5: $\mu_s = 0.5\mu_u$

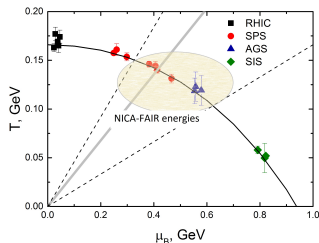
Results and outlooks

Theory + experiment

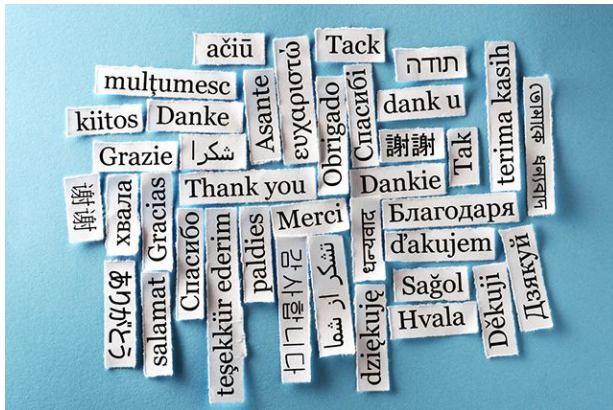


- splitting of kaons masses at high densities \Rightarrow the difference in the behavior of the K/π at low energies.
- the height of the peak in the model depends on the properties of the matter (strange chemical potential, T and μ_B) - it looks like we need more realistic description of the media.

CEP at NICA/FAIR energies?



- the position of the peak pretends to depend on CEP position. This statement can be checked more carefully, for example in the PNJL model with vector interaction, where at a critical value of G_V the first order transition can disappear.



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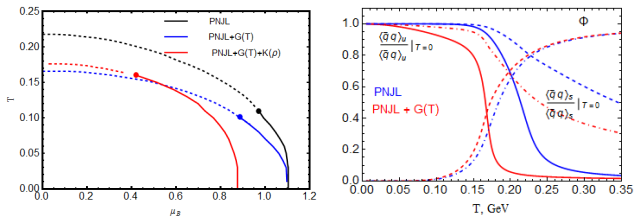


Figure 6: $\mu_s = 0.5\mu_u$