Production of Entropy at the Chiral Phase Transition from Dissipation and Noise

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CPOD, September 2018, Corfu
The QCD Phase Diagram
Finding the Critical Point - Theory

1. First principle calculations
   - Solve partition function $\mathcal{Z}$ on a lattice (sign problem for finite $\mu$)
   - Solve Dyson-Schwinger equations

(Fischer, Luecker, Welzbacher, Phys. Rev. D 90 (2014))
Finding the Critical Point - Theory

2. Effective models

- Extension with Polyakov loop, baryonic degrees of freedom
- Existence/location of CP not universal!


Finding the Critical Point - Theory

3. Susceptibilities

(Skokov, Friman, Redlich, Phys. Rev. C. 83 (2011))
4. Susceptibilities and cumulants

- Generalized susceptibilities:

\[ c_2 = \frac{\partial^2 (p/T^4)}{\partial(\mu/T)^2} = \frac{1}{VT^3} \langle \delta N^2 \rangle \]

\[ c_4 = \frac{\partial^4 (p/T^4)}{\partial(\mu/T)^4} = \frac{1}{VT^3} \left[ \langle \delta N^4 \rangle - 3 \langle \delta N^2 \rangle^2 \right] \]

- Independent of volume and temperature

\[ \kappa \sigma^2 = c_4 / c_2 = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3 \langle \delta N^2 \rangle \]
Finding the Critical Point - Experiment

1. Higher order cumulants: beam energy scan (BES) at STAR

- Higher order cumulants

\[ \sigma^2 = \langle \delta N^2 \rangle \sim \xi^2 \]

\[ S\sigma = \frac{\langle \delta N^3 \rangle}{\langle \delta N^2 \rangle} \sim \xi^{2.5} \]

\[ \kappa \sigma^2 = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3\langle \delta N^2 \rangle \sim \xi^5 \]

(Stephanov, Phys. Rev. Lett. 102 (2009))

(STAR collaboration, Phys. Rev. Lett. 112 (2014))
2. $\xi$-sensitive observables: caveats

- Finite size effects
- Finite time effects
- Critical slowing down

Will influence potential signals

Phenomenologically

$$\frac{d}{dt} m_\sigma(t) = \Gamma(m_\sigma(t)) \left( m_\sigma(t) - \frac{1}{\xi_{eq}(t)} \right)$$

(Berdnikov, Rajagopal, Phys. Rev. D 61 (2000))
Finding the Critical Point - Experiment

3. A dynamical kurtosis

Cumulants are influenced by:
- Relaxation time
- Homogeneous medium
- Inhomogeneous medium

(Mukherjee, Venugopalan, Yin, Phys. Rev. C 92, (2015))

(CH, Nahrgang, Bleicher et al., EPJ A54, (2018))
Finding the Critical Point - Experiment

- Nonequilibrium evolution
  \[ \frac{\partial^2 \sigma}{\partial t^2} - \nabla^2 \sigma + \eta \frac{\partial \sigma}{\partial t} + \frac{\delta \Omega}{\delta \sigma} = \xi \]

- Net-proton kurtosis follows sigma kurtosis

(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))

- Corresponds with
  \[ \langle \delta N^4 \rangle = \langle N \rangle + \kappa_4 \left( \frac{gd}{T} \int p \frac{n_p}{\gamma_p} \right)^4 + \ldots \]

(Stephanov, Phys. Rev. Lett. 107, (2011))

- Cumulants of sigma determine evolution of experimental observables
Finding a First-order Phase Transition

1. Nonequilibrium enhancement of fluctuations
   - Nonequilibrium fluctuations interesting at first-order transition
   - Spinodal decomposition
   - Amplification of inhomogeneities

Finding a First-order Phase Transition

2. Dynamical model

- Formation of metastable phase
- Dynamical fragmentation
- Droplets
- Non-statistical multiplicity fluctuations
Finding a First-order Phase Transition

3. Directed flow

- **a) antiproton**
- **b) proton**
- **c) net proton**

- $v_1$ sensitive to EoS
- Possible signal for first-order phase transition

(Steinheimer et al., PRC 89 (2014))
Ingredients for Nonequilibrium Chiral Fluid Dynamics $N\chi$FD model

- Nonequilibrium dynamics and Bjorken expansion
- *damping* and *stochastic fluctuations*

$$\ddot{\sigma} + \eta \dot{\sigma} + \frac{\delta \Omega}{\delta \sigma} = \xi$$

$$\dot{\varepsilon} = -\frac{e + P}{\tau} + \left( \frac{\delta \Omega}{\delta \sigma} + \eta \dot{\sigma} \right) \dot{\sigma}, \quad \dot{n} = -\frac{n}{\tau}$$

(Herold, Kittiratpattana, Steinheimer, Nahrgang, in prep. (2018))

Based on $L\sigma M$

$$\mathcal{L} = \bar{q} (i \gamma^\mu \partial_\mu - g \sigma) + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma), \quad \text{possibly extended with } \ell, \chi$$

- Successfully describes: *spinodal dynamics, criticality*
**NχFD - Entropy production**

- **Full Langevin:**
  
  \[
  \ddot{\sigma} + \eta \dot{\sigma} + \frac{\delta \Omega}{\delta \sigma} = \xi
  \]

- **W/o dissipation and noise:**
  
  \[
  \ddot{\sigma} + \frac{\delta \Omega}{\delta \sigma} = 0
  \]

- **W/ dissipation, w/o noise:**
  
  \[
  \ddot{\sigma} + \eta \dot{\sigma} + \frac{\delta \Omega}{\delta \sigma} = 0
  \]
NχFD - Initial Conditions

Impact of expansion rate $1/\tau$:
- Trajectory
- Amount of reheating
- Entropy production
- Entropy production becomes stronger at higher $\mu_B$
- Possible signal for first-order phase transition?
- Search for steps in $\pi$ multiplicities or $\pi/p$ ratio
Summary

- Dissipation and noise produce entropy
- Relevant effect for first-order chiral phase transition
- Possibly observable in $\pi/p$ ratio