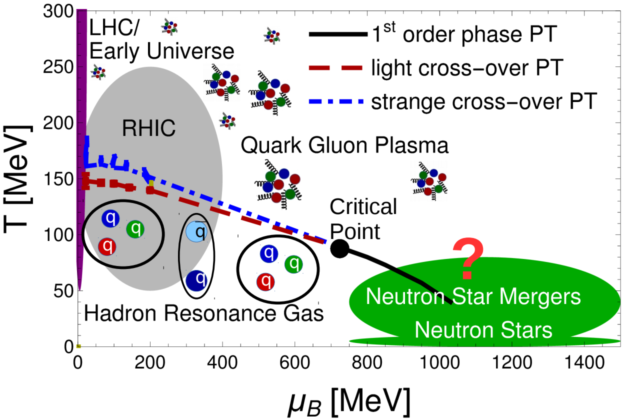
Freeze-out temperature from net-Kaon fluctuations at RHIC

Jacquelyn Noronha-Hostler

Alba, Bellwied, Mantovani Sarti, Parotto, Vazquez, Ratti, Stafford+[WB Collaboration]

CPOD 2018 : Sept 26th, 2018

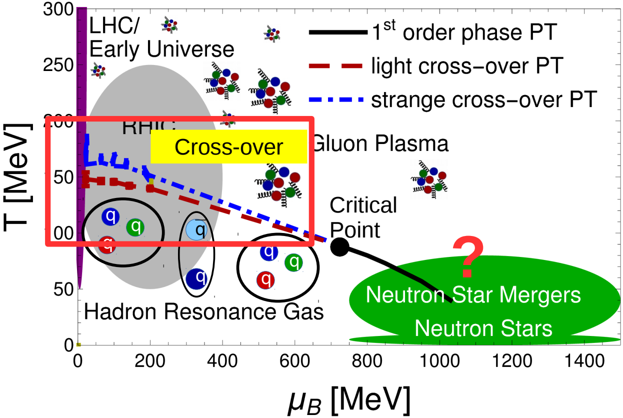
Current Carton of the QCD Phase Diagram



References

Light transition Phys.Lett. B738 (2014) 305-310; **Strange Transition** Bellwied, JNH, Parotto, Vazquez, Ratti, Stafford, arXiv:1805.00088 ; **Neutron Star (mergers)** V. Dexheimer ariXiv:1708.08342; **Holography** Critelli, JNH, Israel Portillo Tues. 19:00 et al, Phys.Rev. D96 (2017) no.9, 096026

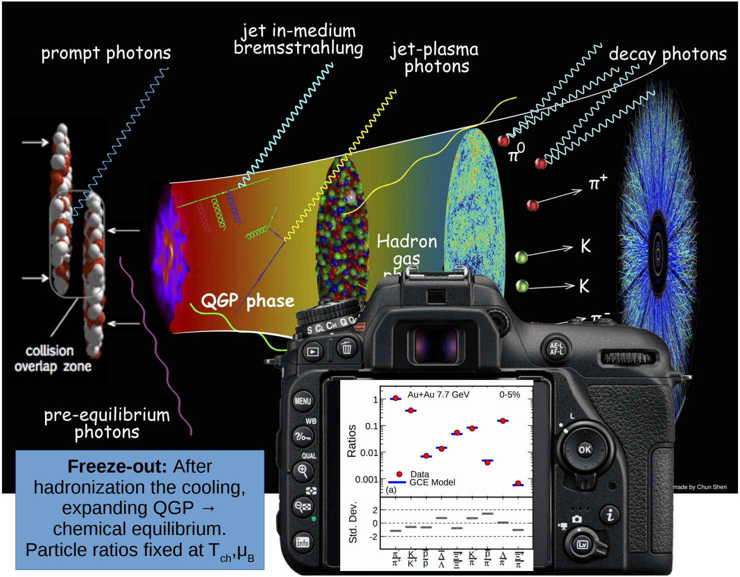
Understanding the cross-over phase transition



References

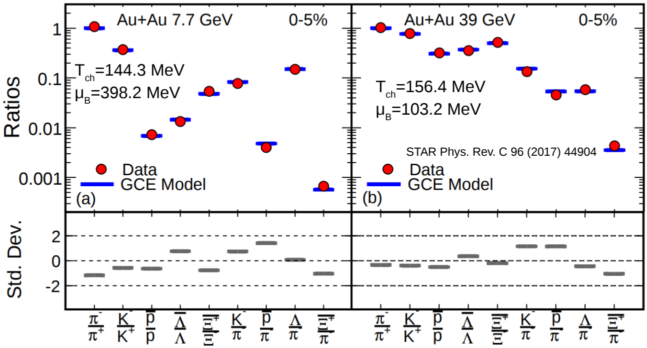
Light transition Phys.Lett. B738 (2014) 305-310; **Strange Transition**Bellwied, JNH, Parotto, Vazquez, Ratti, Stafford, arXiv:1805.00088

Freeze-out: finding the cross-over temperature



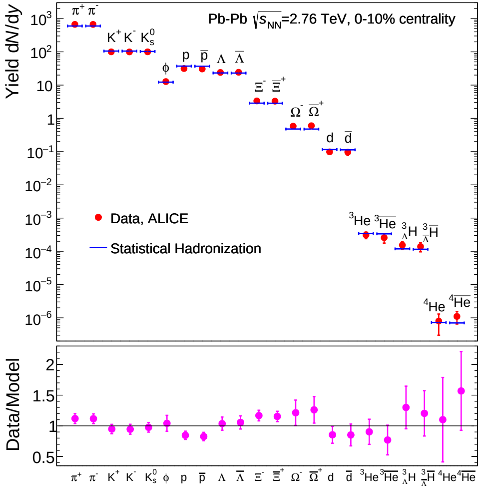
Freeze-out: After hadronization the cooling, expanding QGP → chemical equilibrium. Particle ratios fixed at T_{ch},μ_B

Thermal fits: position on the phase diagram



- Assume particles are in thermal and chemical equilibrium (Grand Canonical Ensemble)
- Calculate ratios of particles in HRG across T and μ_B (volume cancels)
- Extract chemical equilibrium T & μ_B, $\rightarrow$ shortly after hadronization
- Lower Beam Energies=Larger μ_B Beam Energy Scan

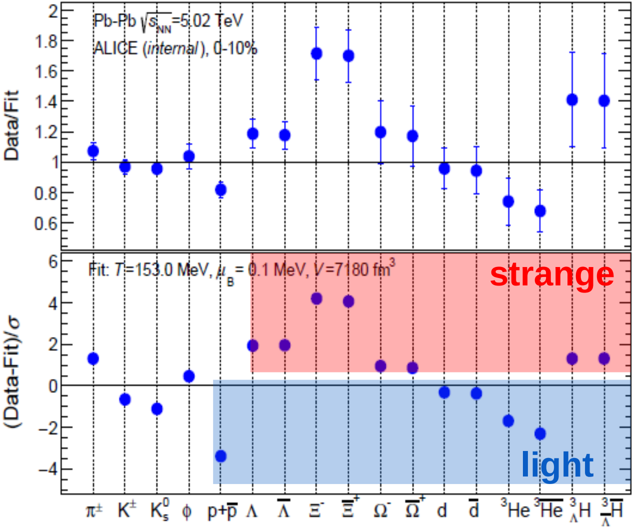
Tension between protons and strange baryons



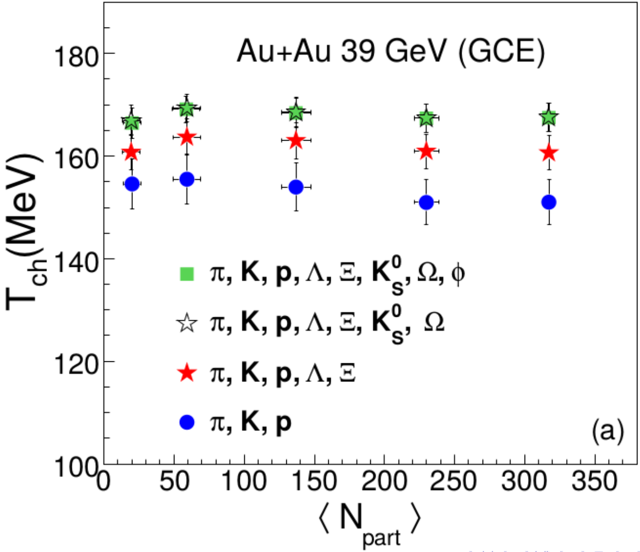
Andronic, Braun-Munzinger, Redlich, Stachel , Nature, 561 (2018) 321-330
Floris Nucl.Phys. A931 (2014) 103-112

protons still overpredicted (prefer lower T_{FO}) and strange baryons underpredicted (prefer higher T_{FO})

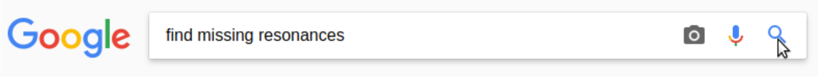
Consistent deviations for light vs. strange baryons



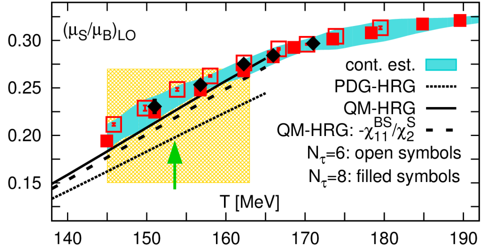
Strange baryons ⇑ T_{ch} by ∼ 10 MeV

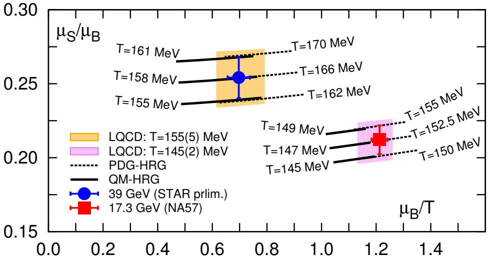


[STAR] Phys. Rev. C 96, 044904 (2017)


find missing resonances

Determining the Number of Hadronic States





Quark Model states (predicted, not yet measured)

μ_S/μ_B from Hadron Resonance Gas matches Lattice QCD

⇓ the freeze-out line**

[HotQCD] Phys.Rev.Lett. 113 (2014) no.7, 072001
*Quark Model States: Phys. Rev. D 34, 2809 (1986),Phys. Rev. D 79, 114029 (2009)

**No decays included

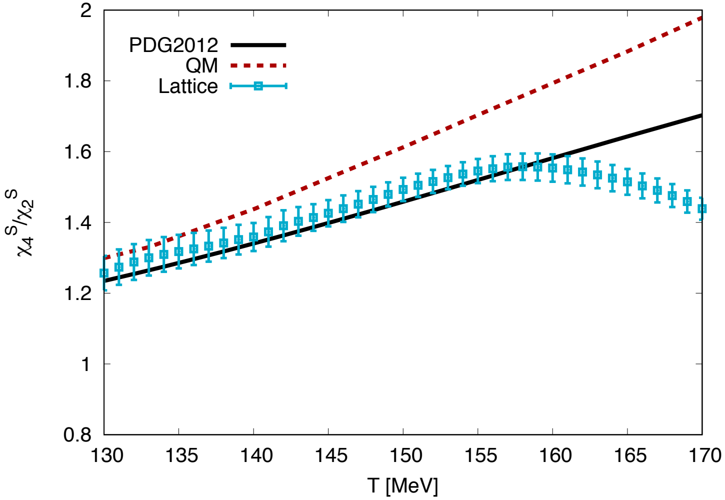
Susceptibilities

Taking derivatives of the pressure gives further information

$$\chi_{lmn}^{BSQ} = \frac{\delta^{l+m+n} p/T^4}{\delta\left(\mu_B/T\right)^l \delta\left(\mu_S/T\right)^m \delta\left(\mu_Q/T\right)^n}$$

- For instance, taking partial derivatives respect to Strangeness selects on only strange hadrons.
- The chemical potentials are constrained by experiments $\langle\rho_S\rangle = 0$ and $\langle\rho_Q\rangle = 0.4\langle\rho_B\rangle$
- Higher-order susceptibilities more sensitive to critical behavior

Quark Model states overpredict χ_4^S/χ_2^S

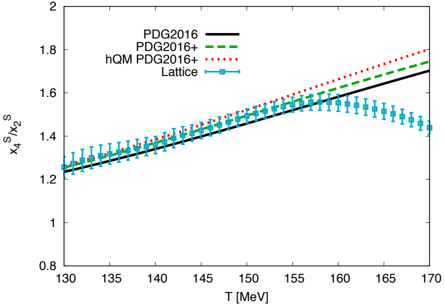


[WB collaboration] Alba, JNH et al Phys.Rev. D96 (2017) no.3, 034517

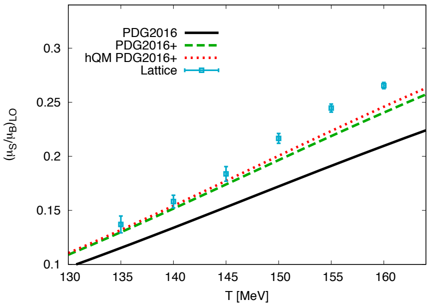
PDG16+ all measured states

PDG16 includes * and ** states, measured but little information known about these states

PDG16+ matches $\chi_4 \chi_2$



Partial pressure closer to Lattice QCD data



[WB collaboration] Alba, JNH et al Phys.Rev. D96 (2017) no.3, 034517

Pressure by Baryon Number, Strangeness, Charge

Can separate pressure by quantum numbers e.g. for strange hadrons (can separate by any BSQ, though)

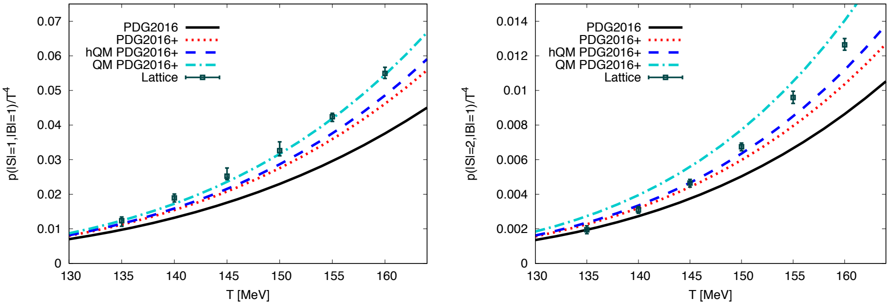
$$
\begin{aligned}
P_S(\hat{\mu}_B, \hat{\mu}_S) &= P_{0|1|}\cosh(\hat{\mu}_S) + P_{1|1|}\cosh(\hat{\mu}_B - \hat{\mu}_S) \\
&+ P_{1|2|}\cosh(\hat{\mu}_B - 2\hat{\mu}_S) + P_{1|3|}\cosh(\hat{\mu}_B - 3\hat{\mu}_S)
\end{aligned}
$$

$$
\begin{aligned}
P_{0|1|} &= \chi_2^S - \chi_{22}^{BS} \\
P_{1|1|} &= \frac{1}{2}\left(\chi_4^S - \chi_2^S + 5\chi_{13}^{BS} + 7\chi_{22}^{BS}\right) \\
P_{1|2|} &= -\frac{1}{4}\left(\chi_4^S - \chi_2^S + 4\chi_{13}^{BS} + 4\chi_{22}^{BS}\right) \\
P_{1|3|} &= \frac{1}{18}\left(\chi_4^S - \chi_2^S + 3\chi_{13}^{BS} + 3\chi_{22}^{BS}\right)
\end{aligned}
$$

Note all $P_{B|S|}$ taken at the limit of $\mu_B = 0$

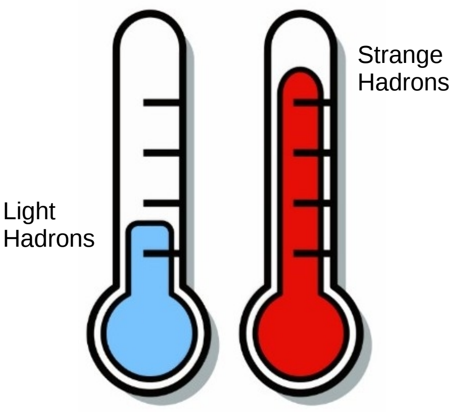
Partial pressure closer to Lattice QCD data

PDG16+ and hQM+PDG16+ closest to Lattce QCD data. Use PDG16+ since the most information is known about these states.



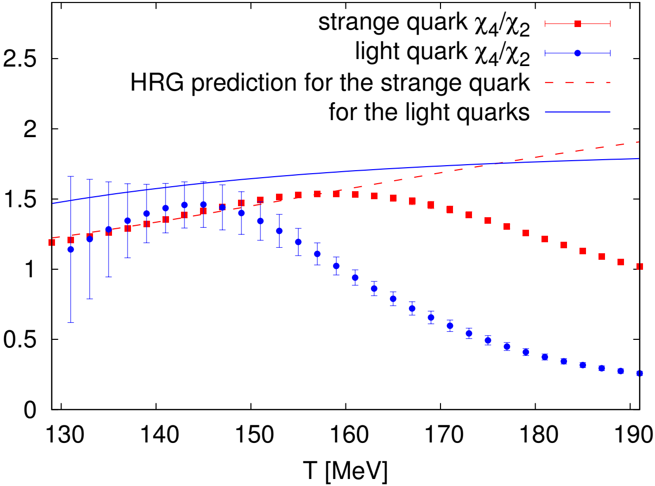
[WB collaboration] Alba, JNH et al Phys.Rev. D96 (2017) no.3, 034517

Should light freeze-out = strange hadrons freeze-out?



Look to fluctuations of conserved charges

Flavor heirarchy in χ_4^S/χ_2^S



Bellwied et al, Phys.Rev.Lett. 111 (2013) 202302

Connecting the Beam Energy Scan to Lattice QCD

RHIC measures [STAR] Phys.Rev.Lett. 112 (2014) 032302

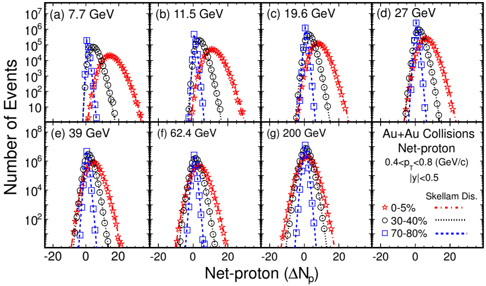
$$\textit{mean}: M = \chi_1 \qquad \textit{variance}: \sigma^2 = \chi_2$$

$$\textit{skewness}: S = \chi_3/\chi_2^{3/2} \qquad \textit{kurtosis}: \kappa = \chi_4/\chi_2^2$$

Lattice QCD calculates Karsch Central Eur.J.Phys. 10 (2012) 1234-1237

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$\sigma^2/M = \chi_2/\chi_1 \qquad S\sigma^3/M = \chi_3/\chi_1$$



Acceptance cuts in the Hadron Resonance Gas

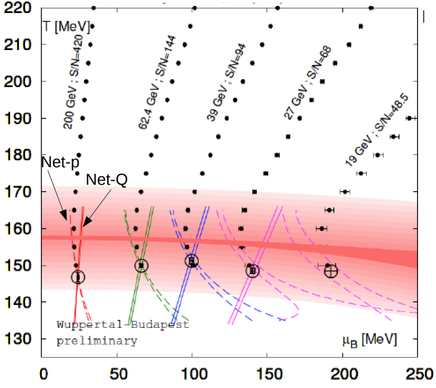
$$\tilde{\chi}_n^{K^\pm} = \sum_i^{N_{HRG}} (Pr_{i \to K^\pm} S_j)^n \frac{d_i}{4\pi^2} \cdot$$

$$\cdot \quad \frac{\partial^{n-1}}{\partial \mu_S^{n-1}} \left\{ \int_{-0.5}^{0.5} \mathrm{d}y \int_{0.2}^{1.6} \mathrm{d}p_T \times \frac{p_T E_T \mathrm{Cosh}[y]}{(-1)^{B_k+1} + \exp((\mathrm{Cosh}[y] E_T - \mu_i/T)} \right\}$$

where $E_T = \sqrt{p_T^2 + m_k^2}$ and $\mu_i = (B_i \mu_b + S_i \mu_S + Q_i \mu_Q)$

Comparison of HRG to Lattice QCD

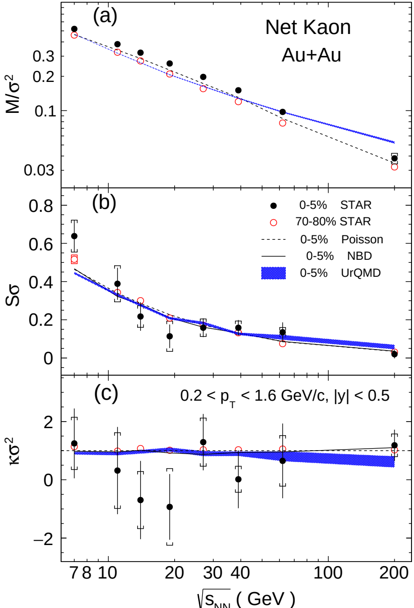
Isentrope lines $S/N_B = const$ generated from Freeze-out point



[WB] Phys.Rev.Lett. 113 (2014) 052301

Caveats: effects from acceptance cuts, decays, finite size effects etc

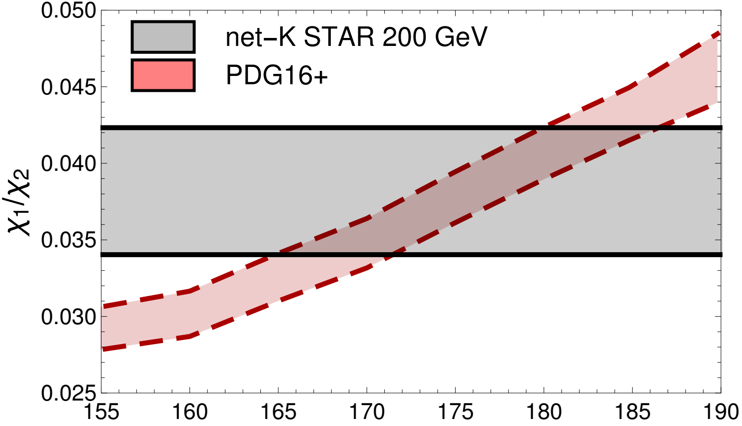
Fluctuations of net-Kaons



UrQMD fails to capture M/σ^2 in $0 - 5\%$

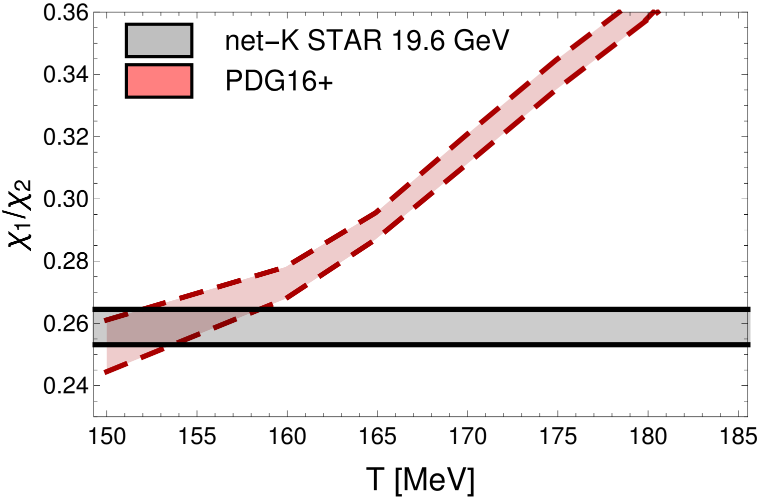
[STAR] Phys. Lett. B 785, 551 (2018)

Isentropes vs. STAR data at $s_{NN} = 200$ GeV



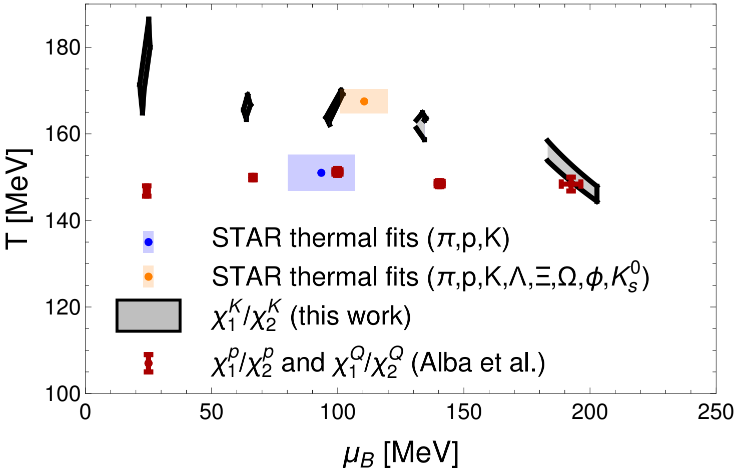
Bellwied, JNH, Parotto, Vazquez, Ratti, Stafford, arXiv:1805.00088

Isentropes vs. STAR data at $\sqrt{s_{NN}} = 19.6$ GeV



Bellwied, JNH, Parotto, Vazquez, Ratti, Stafford, arXiv:1805.00088

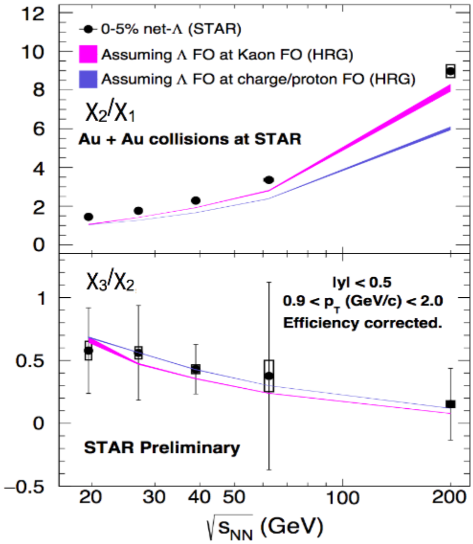
Flavor hierarchy seen compared to STAR data at RHIC



Bellwied, JNH, Parotto, Vazquez, Ratti, Stafford, arXiv:1805.00088

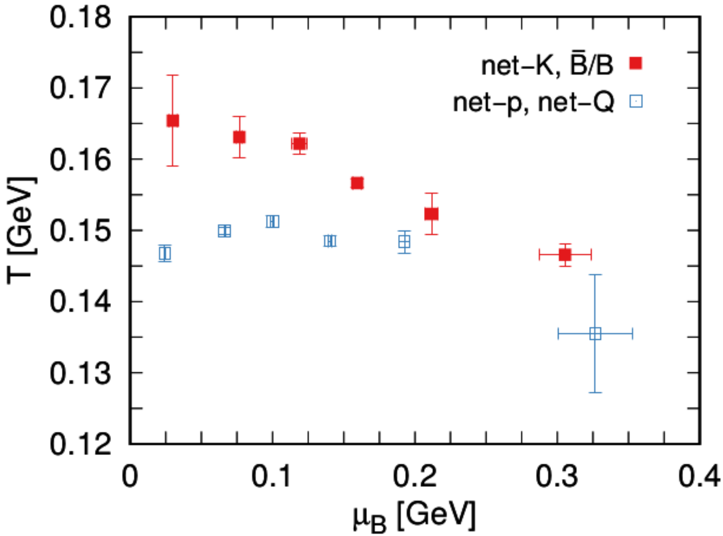
Thermal fits also see flavor hierarchy [STAR] Phys. Rev. C 96 (2017) 44904

Predictions from net-Λ's



Bellwied, JNH, Parotto, Vazquez, Ratti, Stafford, arXiv:1805.00088

Independent confirmation- net-K fluctions and Hyperon yields



Probablistic decays

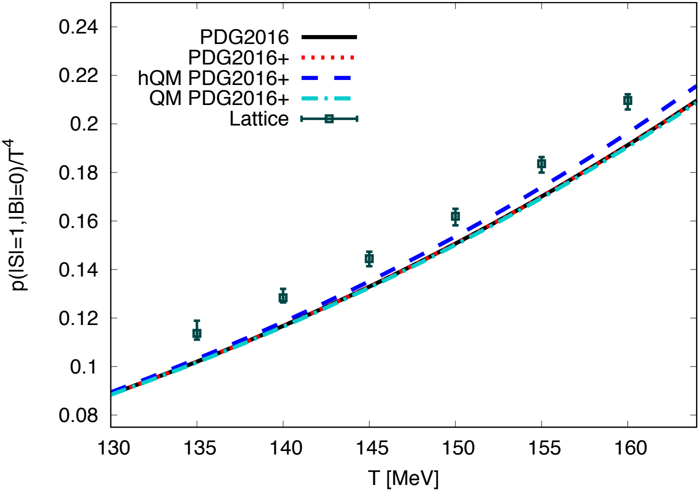
$$\sigma_K^2 = \langle(\Delta N^*_{K+})^2\rangle_T + \langle(\Delta N^*_{K-})^2\rangle_T$$
$$+ \sum_R \langle(\Delta N^*_R)^2\rangle_T \left(\langle n_{K+}\rangle_R^2 + \langle n_{K-}\rangle_R^2\right)$$
$$- 2\sum_R \langle(\Delta N^*_R)^2\rangle_T \langle n_{K+}\rangle_R \langle n_{K-}\rangle_R$$
$$+ \sum_R \langle N^*_R\rangle_T \left(\langle(\Delta n_{K+})^2\rangle_R + \langle(\Delta n_{K-})^2\rangle_R\right)$$
$$- 2\sum_R \langle N^*_R\rangle_T \langle \Delta n_{K+} \Delta n_{K-}\rangle_R \,, \quad (3)$$

Probabilistic contributions of decay process, ↑ T_{FO} ~5%

Bluhm and Narhgang, arXiv:1806.04499

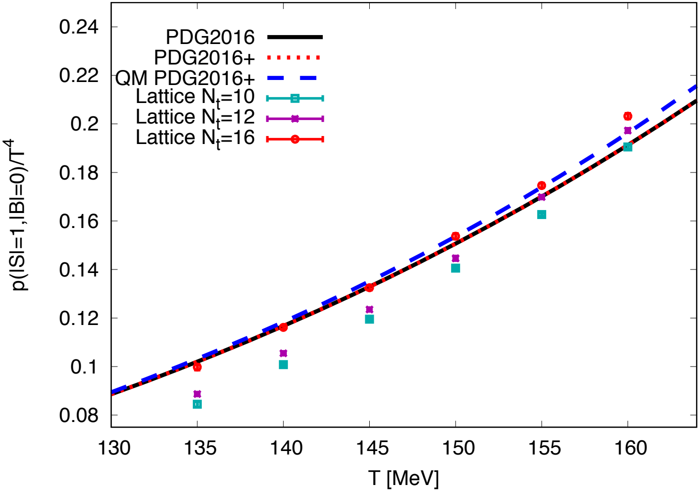

FUTURE

Understand kaon resonances



[WB collaboration] Alba, JNH et al Phys.Rev. D96 (2017) no.3, 034517

Continuum extrapolation difficult



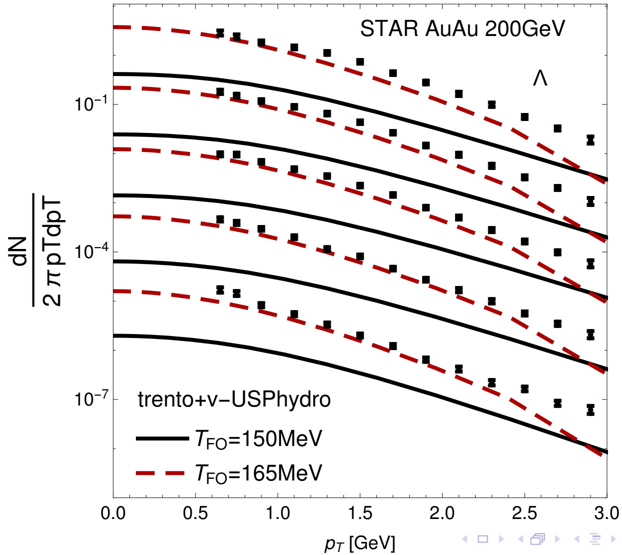
[WB collaboration] Alba, JNH et al Phys.Rev. D96 (2017) no.3, 034517

Hydrodynamics simulations

- New Equation of State with strange particles freezing out at a higher temperature
- Strangeness and baryon number conservation
- Strangeness and baryon number diffusion
- Initial conditions with a strange and baryon number distribution
- Hadronization that considers charge conservation

Would it be worth it?

Check in oversimplified hydro



Conclusions and Outlook

- PDG16+ best compromise with current Lattice QCD data
- $\{T_{ch}, \mu_B\}$ extracted from net-K's incompatible with net-p and net-Q
- net-Λ's also appear to favor a higher $\{T_{ch}, \mu_B\}$
- Long way off from conclusive dynamical simulations

Backup Slides

Selecting only charged kaons in Lattice QCD

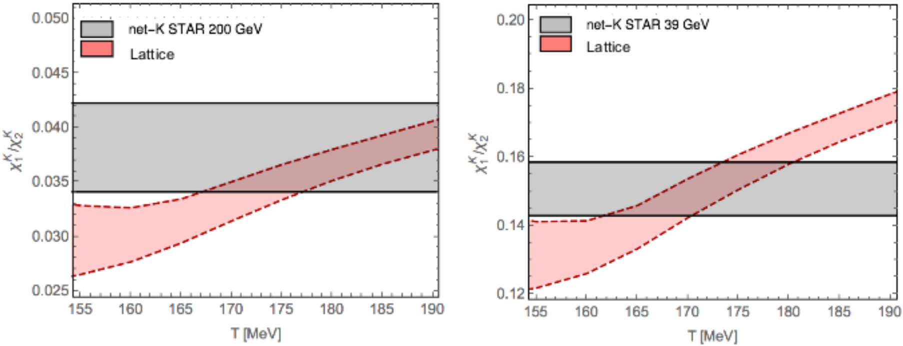
Experiments measure $K^{+/-}$, Lattice QCD includes K^0, Λ, Ξ, Ω

Partial pressure of $K^{+/-}$:
$P_{K^{+/-}} = P_{0|1||1|} \cosh(\hat{\mu}_S + \hat{\mu}_Q)$ where $P_{0|1||1|} = \chi_2^S - \chi_{22}^{BS}$

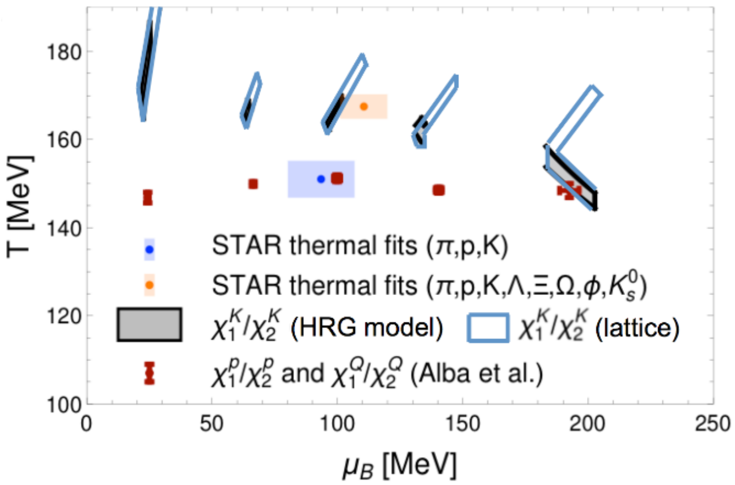
Taking derivatives:

$$\frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}$$
$$\frac{\chi_3^K}{\chi_2^K} = \frac{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}$$
$$\frac{\chi_4^K}{\chi_2^K} = \frac{\chi_e^K}{\chi_e^K} = 1$$

Isentropes and net-Kaons constrain T_{FO}^S



Flavor hierarchy seen compared to STAR data at RHIC

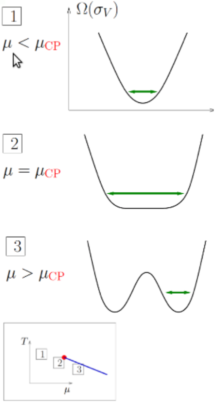


Bellwied, JNH, Parotto, Vazquez, Ratti, Stafford, arXiv:1805.00088

Thermal fits also see flavor hierarchy [STAR] Phys. Rev. C 96 (2017) 44904

Fluctuations near a critical point

Feature of a critical point: divergence of the correlation length ξ



The probability distribution for the order parameter (chiral condensate)

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}$$

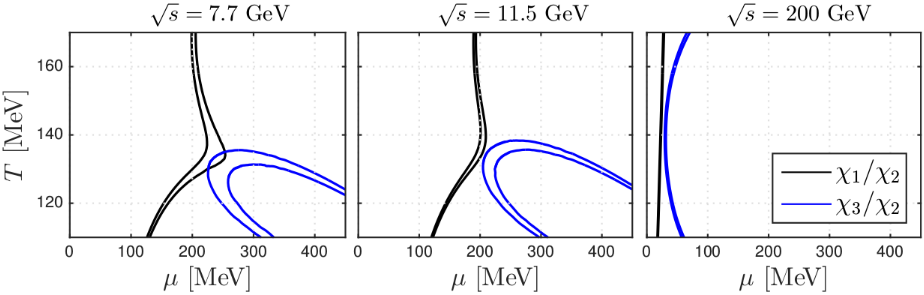
$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma_2 + \frac{\lambda_3}{3}\sigma^3 + \cdots\right]$$

For electric Charge Fluctuation, the correlation length ($\xi = m_\sigma^{-1}$)

$$\xi \sim |T - T_c|^{-\nu} \text{ where } \nu > 0$$

$$\chi_2 = VT\xi^2$$
$$\chi_3 = 2VT^{3/2}\hat{\lambda}_3\xi^{9/2}$$
$$\chi_4 = 6VT^2[2\hat{\lambda}_3^2 - \hat{\lambda}_4]\xi^7$$

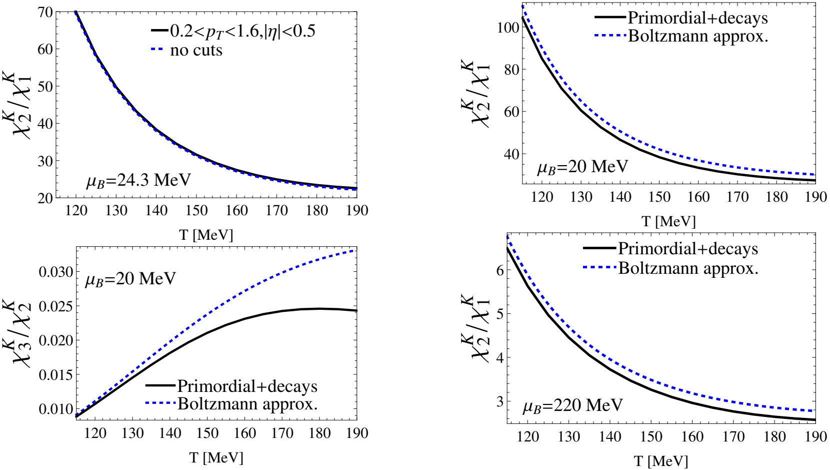
Extraction of the freeze-out line from susceptibilities



- Freeze out points $[T - \mu_B]$ are extracted from the line made by the closer points between χ_1/χ_2 and χ_3/χ_2

Effects of decays, full statistics, acceptance cuts

Acceptance/decays small effect for χ_2/χ_2 but higher order cumulants necessitate decays!



Fermi-sign problem

The QCD path integral is computed by Monte Carlo algorithms which samples field configurations with a weight proportional to the exponential of the action

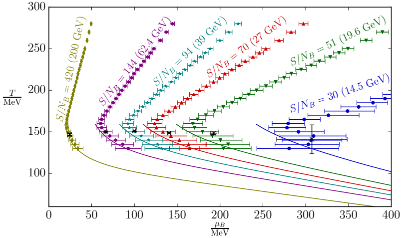
$$Z(\mu_B, T) = \mathrm{Tr}\left(e^{-\frac{H_{QCD}-\mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

where for finite μ_B then $\det M[U, \mu_B] \rightarrow$ complex so Monte Carlo simulations are no longer possible

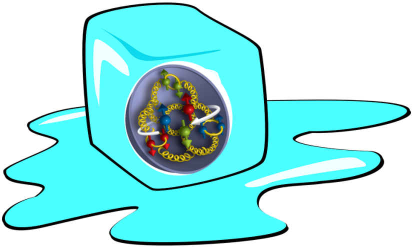
- Taylor expansion around $\mu_B = 0$ (Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003)
- Reweighting (complex phase moved from the measure to observables) (Barbour et al. 1998; Z. Fodor and S, Katz, 2002)
- Simulations at imaginary chemical potentials (plus analytic continuation) (Alford, Kapustin, Wilczek, 1999; de Forcrand, Philipsen, 2002; D'Elia, Lombardo 2003)

Phase Diagram from Lattice QCD

Beam Energy Scan- vary center of mass energy



Trajectories of heavy-ion collisions assuming a constant entropy
[Wuppertal Budapest Collaboration] arXiv:1607.02493



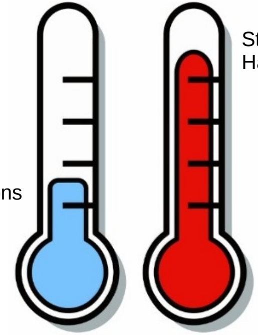
$$\chi^{BSQ}_{lmn} = \frac{\delta^{l+m+n} p/T^4}{\delta\left(\mu_B/T\right)^l \delta\left(\mu_S/T\right)^m \delta\left(\mu_Q/T\right)^n}$$

Melting hadrons

Susceptibilities from Lattice QCD compared to particle numbers of conserved charges used to calculate the freeze-out line

Are the freeze-out T's of light and strange equal?

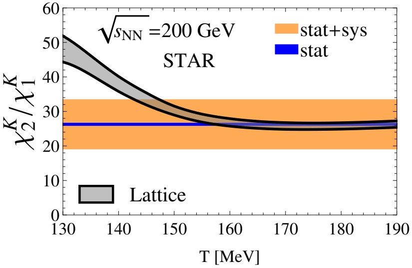
Connecting first principle Lattice QCD calculations to Beam Energy Scan data



Look to experimental data of Baryons vs. Kaons

Only can see a minimum $T^{min}_{strange} > 148$ MeV.

$$M_K/\sigma^2_K = \tanh^{-1}(\hat{\mu}_S + \hat{\mu}_Q)$$



Lattice QCD: JNH, et al, arXiv:1607.02527
STAR data from Ji Xu Strangeness in Quark Matter 2016

Quantum Chromodynamics

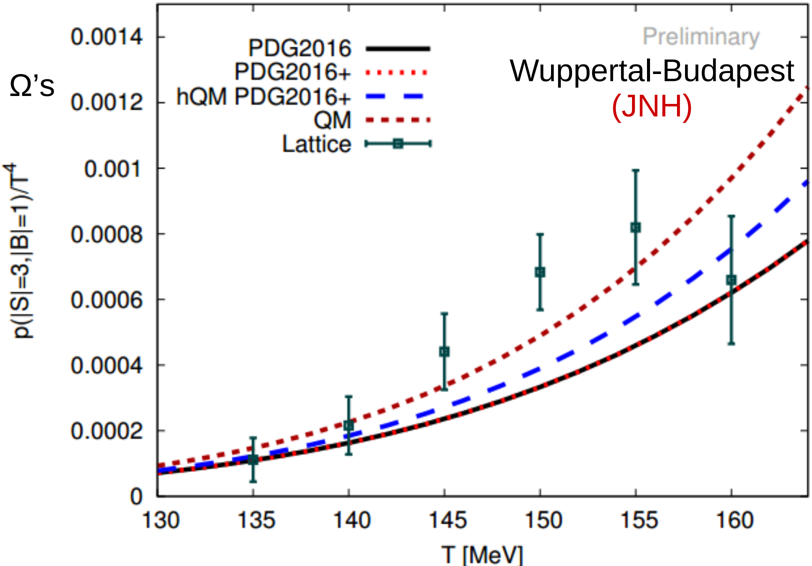
Describing interactions between quarks q and gluons g

- **Quarks** described by Spin$= \frac{1}{2}$ Dirac Fields $\psi_\alpha^{i,q}(x)$
  - α Dirac spinor index
  - $i = (1,2,3)$ $SU(3)$ color index
  - $q = (u,d,s,c,b,t)$ flavor index
- **Gluons** described by Spin$= 1$ Vector Fields $A_\mu^a(x)$
  - μ Lorentz vector index
  - $a = (1,2,..8)$ color index
- Gell-Mann matrices in color space $(\lambda^a)_{ij}$
  - $i,j = 1,2,3$
  - SU(3), naturally generalize the Pauli matrices for SU(2)
  - $\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = i f_{abc} \frac{\lambda_c}{2}$

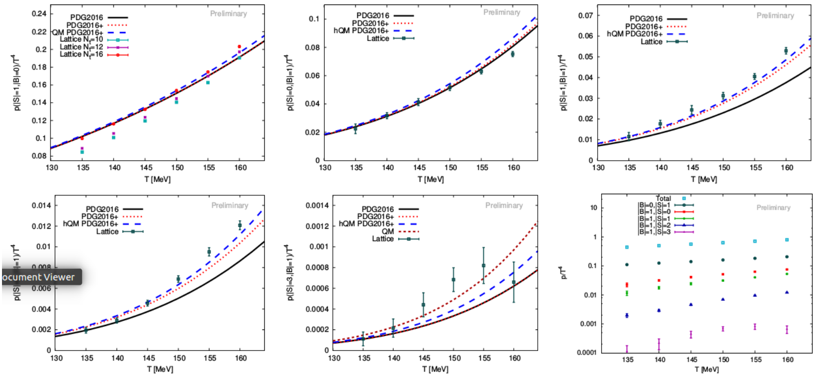
Quantum Chromodynamics Lagrangian

- Gauge Invariance
  $\psi(x) \to \psi'(x) = U(x)\psi(x) = e^{-i\theta_a(x)\lambda_k/2}\psi(x)$
  where $U(x)$ is a 3x3 unitary matrix
- QCD Langrangian
  $$L_{QCD} = \underbrace{-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a}_{\text{Gluon Fields}} + \underbrace{\bar{\psi}^q\left(i\gamma_\mu D^\mu - m_q\right)\psi^q}_{\text{Quark Fields}}$$
  where gluon field $F^a_{\mu\nu} = \underbrace{\partial_\mu A^a_\nu - \partial_\nu A^a_\mu}_{\text{QED}} + \underbrace{g f^{abc} A^b_\mu A^c_\nu}_{\text{gauge invariance}}$
  and $D^\mu = \partial^\mu - igA^\mu(x)$, m_q=quark mass depending on the flavor, and the gauge coupling parameter g is $g^2/(4\pi) = \alpha_s$

Partial Pressure



Partial Pressure



Hadron Resonance Gas model

Pressure:

$$p^{HRG}/T^4 = \frac{1}{VT^3}\sum_i \ln Z_i(T,\mu)$$

Energy Density:

$$\varepsilon^{HRG}/T^4 = -\frac{1}{VT^3}\sum_i \frac{\partial \ln Z_i(T,\mu)}{\partial(1/T)}$$

Number Density:

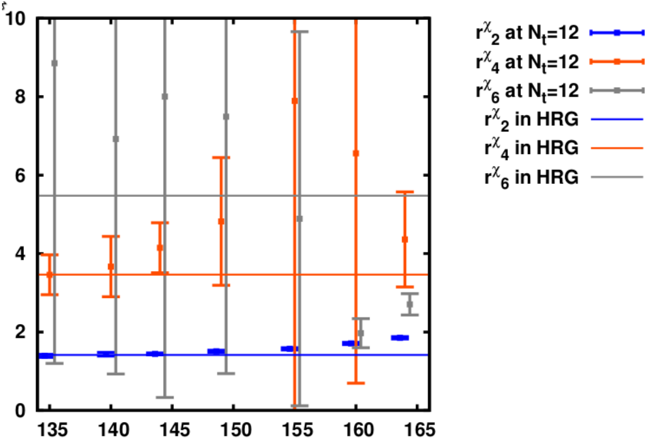
$$n^{HRG}/T^3 = \frac{1}{VT^2}\sum_i \frac{\partial \ln Z_i(T,\mu)}{\partial \mu}$$

Entropy density:

$$s^{HRG}/T^3 = \frac{1}{VT^2}\sum_i \ln \frac{\partial \ln Z_i(T,\mu)}{\partial T} = \frac{\varepsilon + p - \sum_j^{BSQ} \mu_j \rho_j}{T}$$

Einstein-Maxwell-Dirac equations

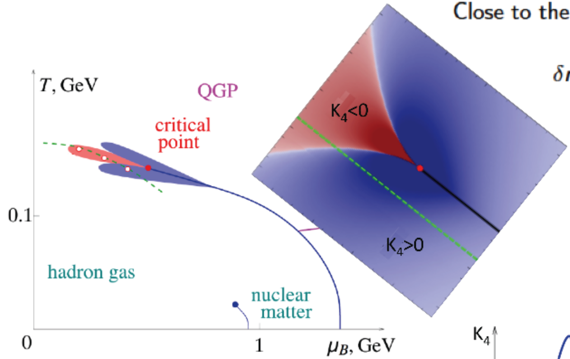
static charged black hole backgrounds that are spatially isotropic and translationally invariant



Kurtosis at CP

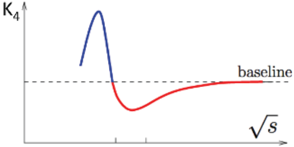
Close to the critical point

$$\delta n_p = \underbrace{\delta n_p^0}_{stat.} + \underbrace{\frac{\partial \bar{n}_p}{\partial m} g \delta\sigma}_{critical}$$



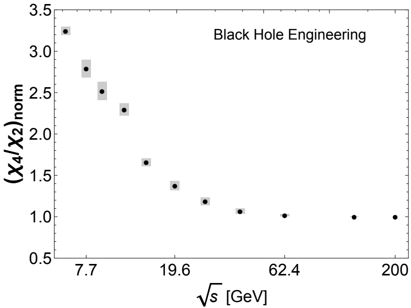
The fluctuation of an observable

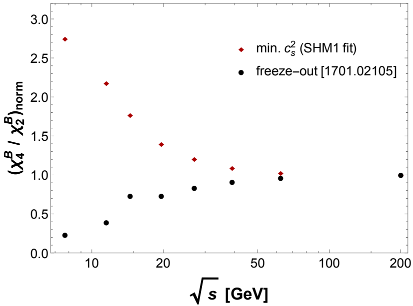
$$\delta N = \sum_p \delta n_p$$



M. A. Stephanov, Phys. Rev. Lett. **107** (2011) 052301

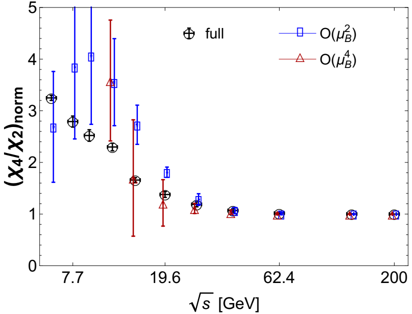
Kurtosis

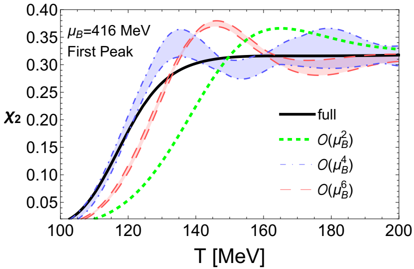




kurtosis strongly depends on the freeze-out line

Reconstructing Kurtosis





Looking for peaks in χ_2 may be easier than kurtosis