Gribov horizon, Polyakov loop and finite temperature

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The Faddeev–Popov procedure

- Starting with a Yang-Mills Lagrangian in the Euclidean space

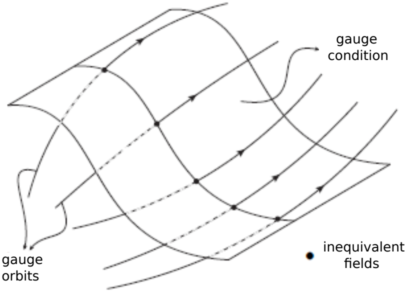
$$\mathcal{L}_{YM} = \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

- Which is invariant under local $SU(N)$ gauge transformations

$$A_\mu \to A'_\mu = U^\dagger A_\mu U + U^\dagger \partial_\mu U$$

where $U \in SU(N)$ is a finite group element

The Faddeev–Popov procedure



To compute the propagators using path integral, we must read off the gauge redundance

The Faddeev–Popov procedure

Introduce the unity into the path integral

$$1 = \int \mathcal{D}\alpha\,\delta(G(A^\alpha))\,\mathrm{Det}\left(\frac{\partial A^\alpha}{\partial\alpha}\right)$$

and the fact that

$$\mathrm{Det}\mathcal{M} = \int \mathcal{D}\bar{c}\mathcal{D}c\,\exp\left\{\int d^4x\,\partial^\mu\bar{c}^a\left(\partial_\mu\delta^{ab} + gf^{abc}A^c_\mu\right)c^b\right\} ,$$

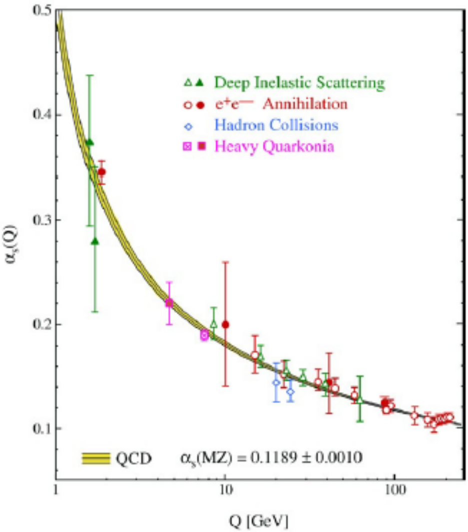
$$\mathcal{M}^{ab} = -\partial^\mu\left(\partial_\mu\delta^{ab} - gf^{abc}A^a_\mu\right) .$$

The Faddeev–Popov procedure

Finally, adding a gauge fixing term we have the celebrated FP Lagrangian

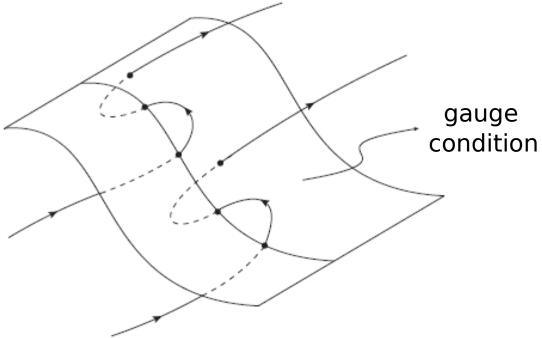
$$\mathcal{L}_{FP} = \mathcal{L}_{YM} + \partial^\mu \bar{c}^a D_\mu c^a + b^a \partial^\mu A^a_\mu \,.$$

The asymptotic freedom



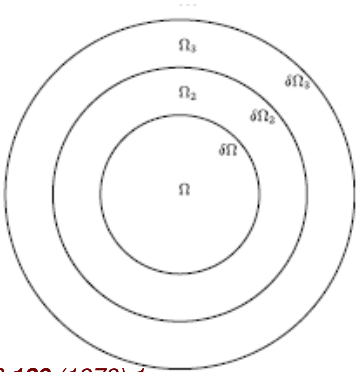
S. Bethke, Prog. Part. Nucl. Phys. ***58****, 351 (2007).*

The Gribov ambiguity



The Gribov horizon(s)

$$\underbrace{\Omega}_{\text{Gribov region}} \equiv \left\{ A^a_\mu \, , \; \partial^\mu A^a_\mu = 0 \, , \mathcal{M}^{ab} = -\partial^\mu \left(\partial_\mu \delta^{ab} - g f^{abc} A^a_\mu \right) > 0 \right\}$$



*V. N. Gribov, Nucl. Phys. B **139** (1978) 1.*

The restriction of the domain of integration in the path integral is achieved by adding two additional terms to S_{FP}, namely[1]

$$
\begin{aligned}
S_0 &= \int d^4x \left(\bar{\varphi}^{ac}_{\mu} (-\partial_\nu D^{ab}_{\nu}) \varphi^{bc}_{\mu} - \bar{\omega}^{ac}_{\mu} (-\partial_\nu D^{ab}_{\nu}) \omega^{bc}_{\mu} \right. \\
&\quad \left. + g f^{amb} (\partial_\nu \bar{\omega}^{ac}_{\mu}) (D^{mp}_{\nu} c^p) \varphi^{bc}_{\mu} \right) \\
S_\gamma &= \gamma^2 \int d^4x \left(g f^{abc} A^a_\mu (\varphi^{bc}_{\mu} + \bar{\varphi}^{bc}_{\mu}) \right) - 4\gamma^4 V (N^2 - 1) \, .
\end{aligned}
$$

[1] *N. Vandersickel, D. Zwanziger Phys. Rept.* ***520**, 175 (2012).*

The Gribov-Zwanziger action

Therefore,

$$S_{GZ} = S_{YM} + S_{gf} + S_0 + S_\gamma \ ,$$

The GZ partition function Z_{GZ} and the vacuum energy $\mathcal{E}_v(\gamma)$ are defined by

$$Z_{GZ} = \int \mathcal{D}\Phi \ e^{-S_{GZ}[\Phi]} \ , e^{-V\mathcal{E}_v} = Z_{GZ} \ .$$

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Leading to a tree-level gluon propagator

$$\Delta^{ab}_{\mu\nu}(p) = \delta^{ab} \frac{p^2}{p^4 + \lambda^4} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \, ,$$

where $\lambda^4 = 2Ng^2\gamma^4$.

The Polyakov loop

$$\mathcal{P} = \frac{1}{N}\mathrm{Tr}\langle P e^{ig\int_0^\beta dt A_0(t,x)}\rangle \ .$$

In the presence two static sources of (heavy) quarks, it is standard to use it as an order parameter for the confinement/deconfinement phase transition

$$\mathcal{P} \propto e^{-FT} \ ,$$

with T the temperature and F the free energy of a heavy quark.

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$$\mathcal{P} \begin{cases} = 0, & \text{confined phase} \\ \neq 0, & \text{deconfined phase.} \end{cases}$$

The Polyakov loop

It turns out that in the Polyakov gauge (A_0 time-independent belonging to the $SU(2)$ Cartan subalgebra)

$$\mathcal{P} = \cos\left(\frac{1}{2} g\beta A_0\right) .$$

Therefore, from the Jensen inequality[1],

$$g\beta\langle A_0\rangle \begin{cases} = \pi & \text{confined phase} \\ \neq \pi, & \text{deconfined phase.} \end{cases}$$

[1] *F. Marhauser and J. M. Pawlowski, arXiv:0812.1144 [hep-ph]*

$$S_{GZ+PLoop} = S_{BFG} + S_0 + S_\gamma \ ,$$

where

$$S_{\mathrm{BFG}} = \int d^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \frac{(\bar{D}A)^2}{2\xi} + \bar{c}^a \bar{D}^{ab}_\mu D^{bd}_\mu (a) c^d \right\} \ ,$$

$$\bar{D}^{ab}_\mu = \delta^{ab}\partial_\mu - g f^{abc} \bar{A}^c_\mu$$

and

$$a_\mu = \bar{A}_\mu + A_\mu \ ,$$

$$\bar{D}_\mu a_\mu = 0 \hookrightarrow \text{LDW gauge fixing condition} \ .$$

The GZ + Polyakov loop action

At one-loop order, the vacuum energy is given by

$$\mathcal{E}_v(r,\lambda^2) = -\frac{d(N^2-1)}{2Ng^2}\lambda^4 + \frac{T}{2V}(d-1)\,\mathrm{Tr}\ln\frac{D^4+\lambda^4}{\lambda^4} - d\,\frac{T}{2V}\,\mathrm{Tr}\ln\frac{-D^2}{\Lambda^2}\ ,$$

where V is now just the spacial (three-dimensional) volume and $r \equiv g\beta\bar{A}_0$.

$$\mathcal{D}^2 = (2\pi nT + rsT)^2 + \vec{q}^2\ ,$$

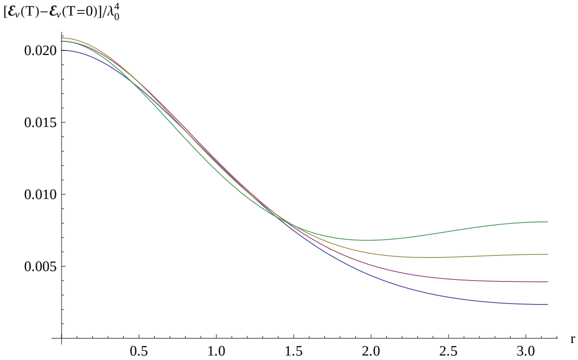
where n is the Matsubara mode.

Therefore, the transition can be identified by the condition

$$\frac{d^2}{dr^2}\mathcal{E}_v\bigg|_{r=\pi} = 0 \ .$$

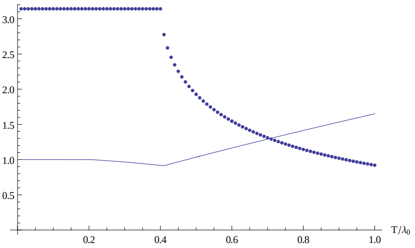
By assuming first that the temperature does not influence the Gribov parameter $\lambda = \lambda_{T=0}$.

T−independent λ



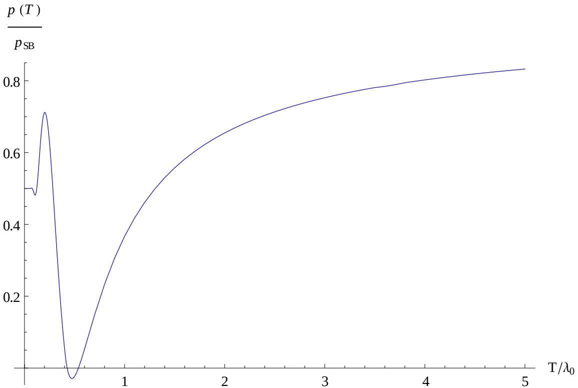
The effective potential $\mathcal{E}_v$ at the temperatures (from below upwards at $r = \pi$) 0.42, 0.44, 0.46, and 0.48 times $\lambda_{T=0}$.

$T-$dependent λ and r



The dotted line curve represents $r(T)$, while the continuous line is $\lambda(T)$. At $T \approx 0{,}40\lambda_0$, both curves clearly have a discontinuous derivative.

Thermodynamic Pressure= $\frac{T}{V}\ln Z$



GZ pressure relative to the Stefan-Boltzmann limit pressure $\sim T^4$.

Conclusions and perspectives

- Implementing the Polyakov loop to the GZ action gives us a deconfinement critical temperature $T_C \simeq 0{,}233\ GeV$, which is not far from lattice value[1] for $SU(2)$ $T_C \simeq 0{,}295\ GeV$.

[1] *A. Cucchieri, A. Maas and T. Mendes, Phys. Rev. D* ***75*** *(2007) 076003.*

[2] *F. Canfora, A. Giacomini, P. Pais, L. Rosa, A. Zerwekh , Eur. Phys. J. C.* **76***, 443 (2016).*

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- The refined GZ action, which takes into account the condensate of the extra fields, improves the result to $T_C \simeq 0{,}250\ GeV$.

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- The refined GZ action, which takes into account the condensate of the extra fields, improves the result to $T_C \simeq 0{,}250\ GeV$.
- The thermodynamic pressure has a negative sector, which is related to the complex poles of the GZ propagator[2].

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Thank you!