HELMHOLTZ
SPITZENFORSCHUNG FÜR GROSSE HERAUSFORDERUNGEN

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Shear viscosity and resonance lifetimes in the hadron gas

Jean-Bernard Rose

with D. Oliinychenko, J. Torres-Rincon, A. Schäfer, J. Hammelmann, H. Elfner (Petersen)

based on Phys Rev C 97.055204 (arXiv:1709.03826) and arXiv:1709.00369

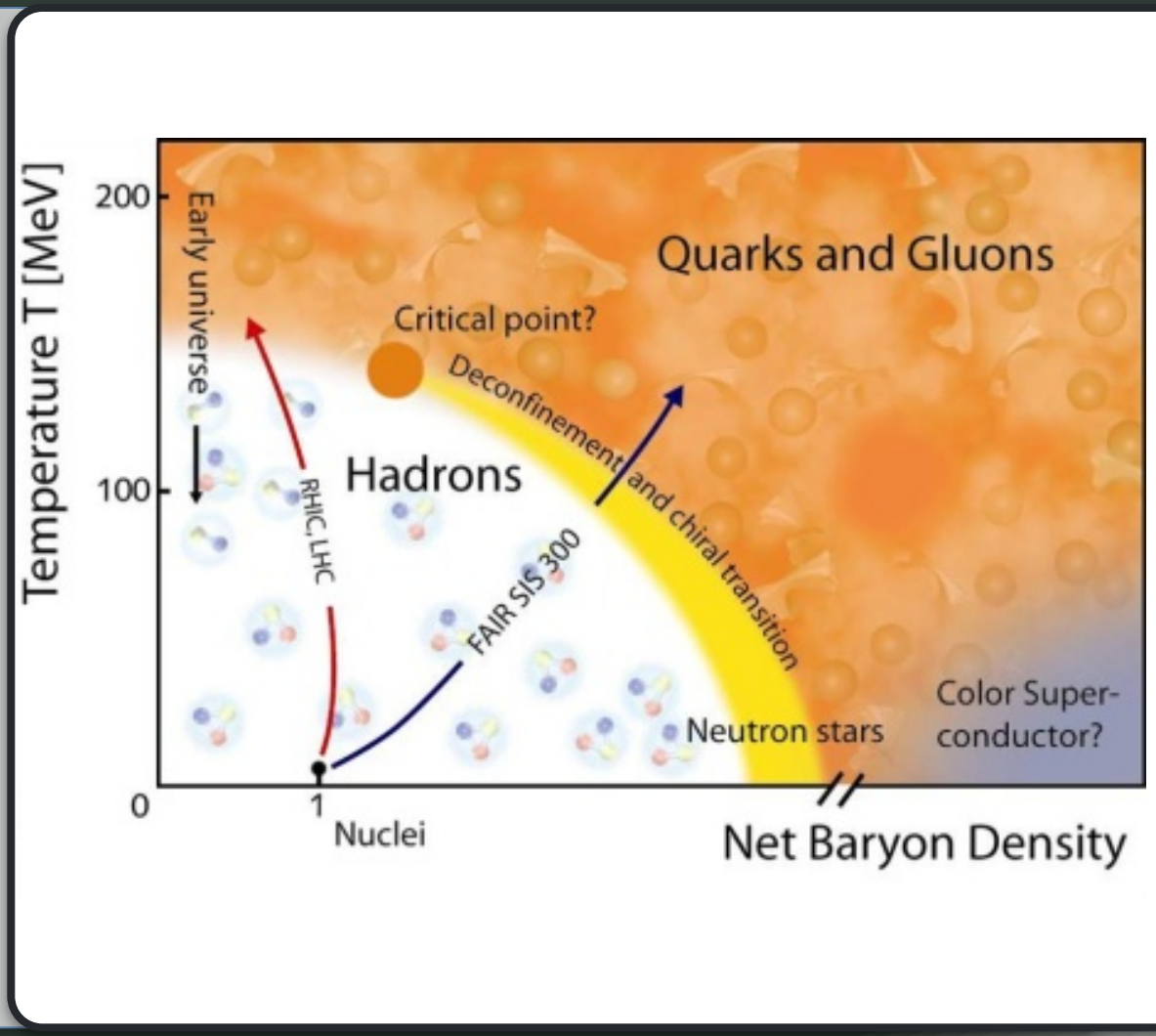
HIC for FAIR
Helmholtz International Center

FIAS Frankfurt Institute for Advanced Studies

DAAD

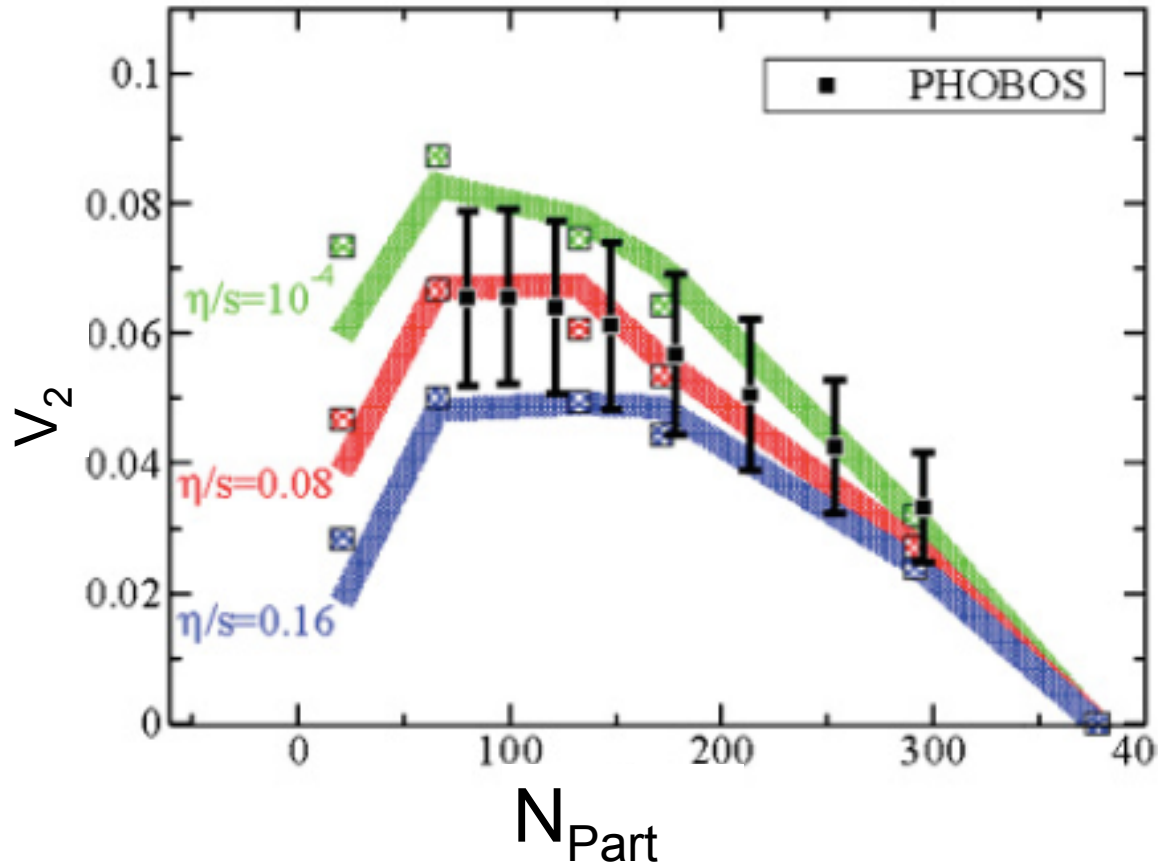
Viscosity in heavy ion collisions

- Investigating deconfinement requires a good knowledge of transport coefficients



Viscosity in heavy ion collisions

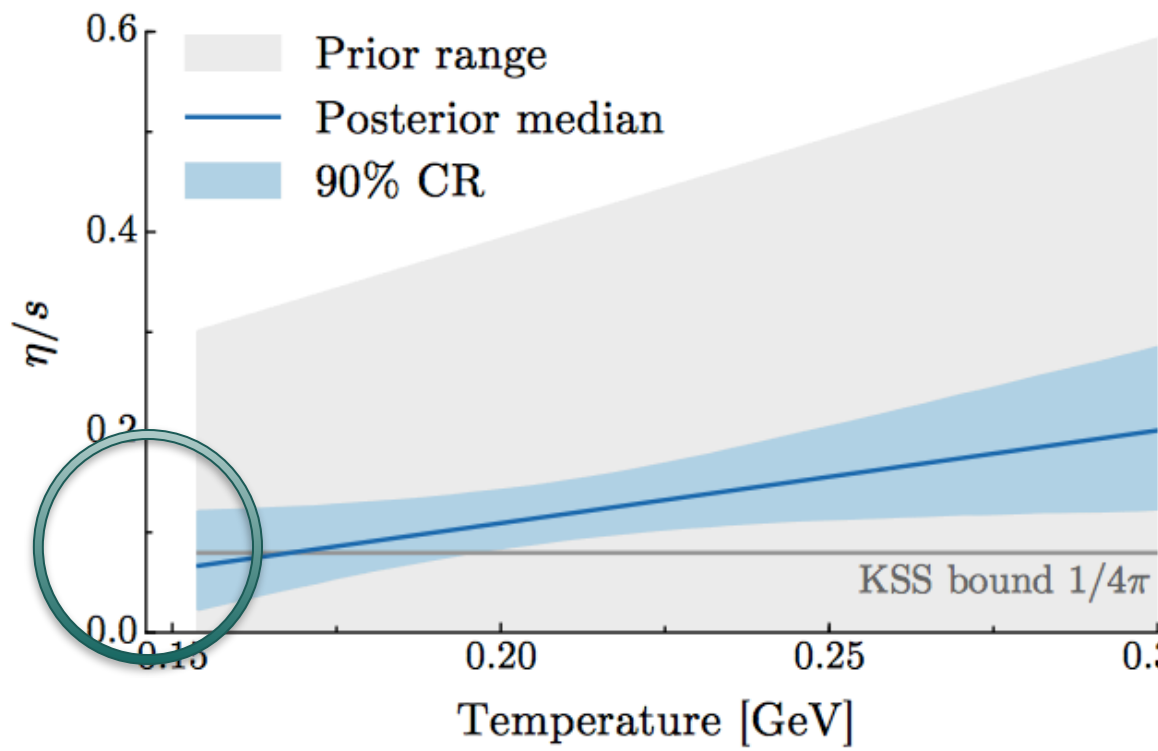
- Investigating deconfinement requires a good knowledge of transport coefficients
- Hydrodynamics relatively successful at explaining this with small *η/s* above the transition



Luzum & Romatschke 10.1103/Phys. Rev. C 78.034915

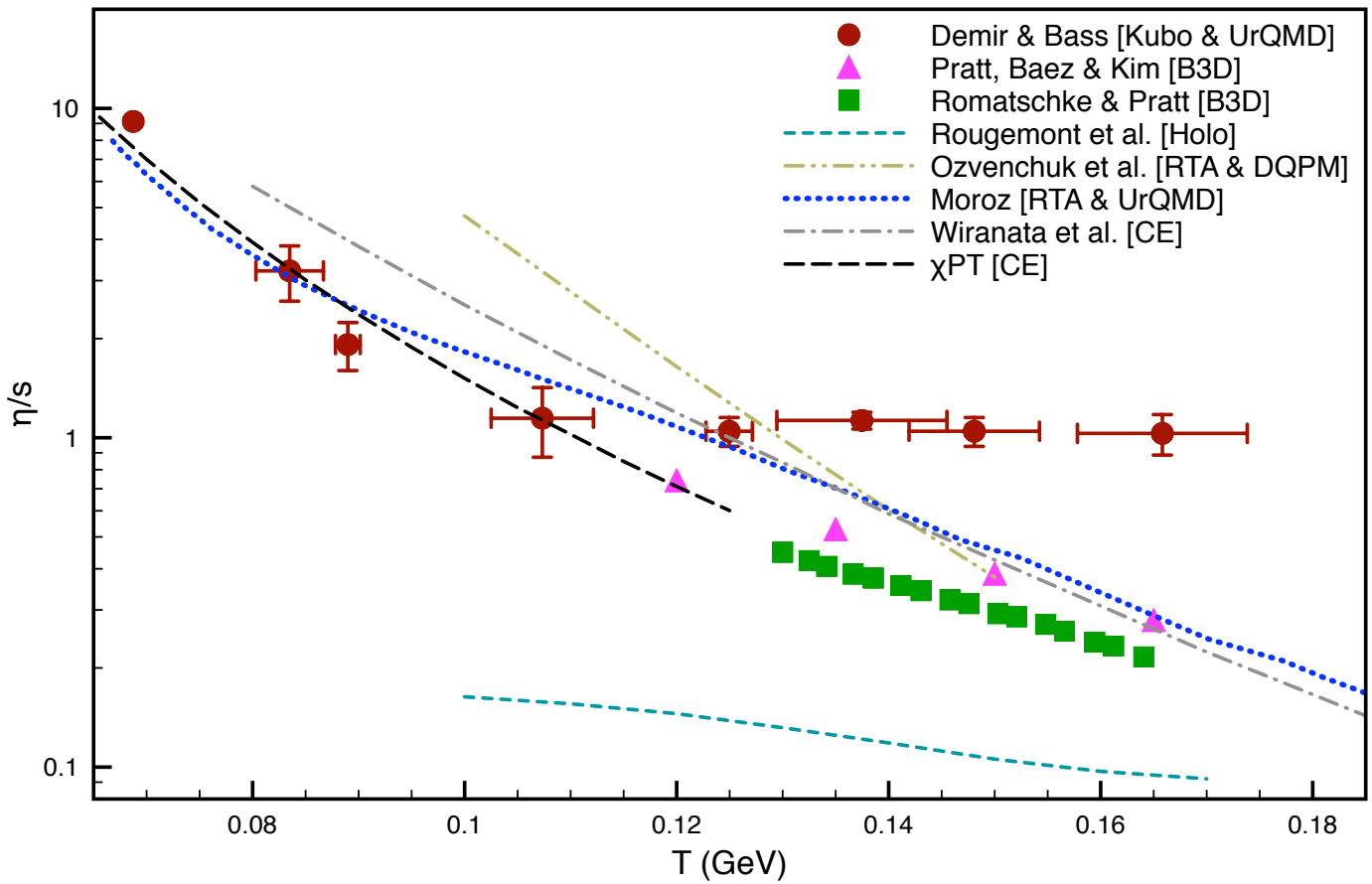
Viscosity in heavy ion collisions

- Investigating deconfinement requires a good knowledge of transport coefficients
- Hydrodynamics relatively successful at explaining this with small *η/s* above the transition
- Still not clear what the behavior of η/s is at low energies (FAIR, late stage RHIC/LHC)



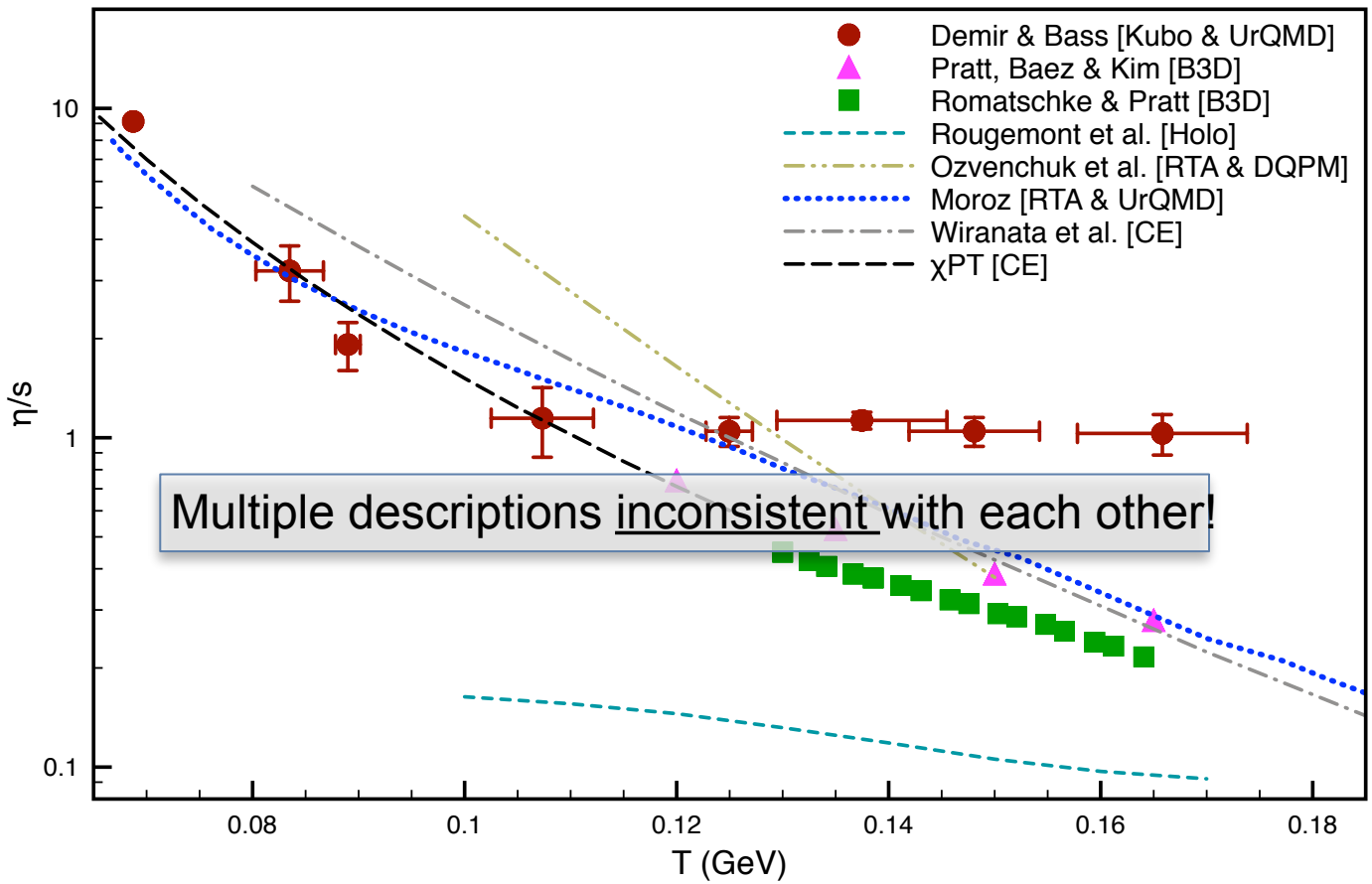
Bernhard, Moreland, Bass, Liu & Heinz Phys. Rev. C94 no. 2 (2016) 024907

Previous HG viscosity calculations



-Demir & Bass, Phys.Rev.Lett. 102 (2009) 172302
-Pratt, Baez & Kim, Phys. Rev. C95 (2017) 024901
-Romatschke & Pratt, arXiv:1409.0010v1
-Rougemont et al., arXiv:1704.05558
-Ozvenchuk et al., Phys. Rev. C87 (2013) 064903
-Moroz, arXiv:1301.6670
-Wiranata et al., Phys. Rev. C88 (2013) 044917
-Torres-Rincon, arXiv:1205.0782

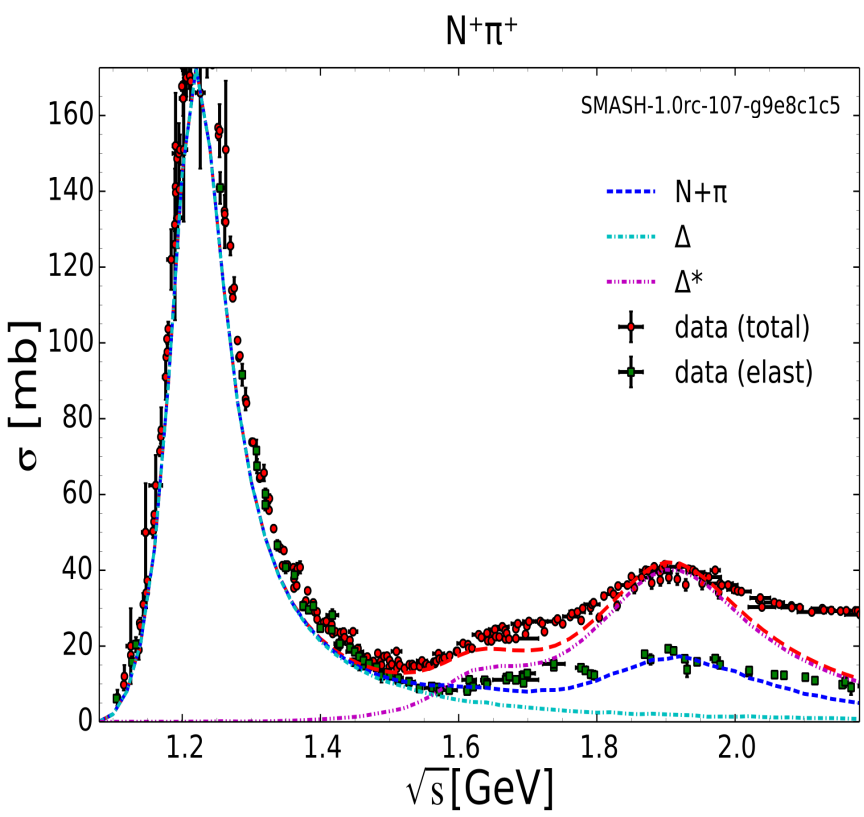
Previous HG viscosity calculations



Multiple descriptions inconsistent with each other!

-Demir & Bass, Phys.Rev.Lett. 102 (2009) 172302
-Pratt, Baez & Kim, Phys. Rev. C95 (2017) 024901
-Romatschke & Pratt, arXiv:1409.0010v1
-Rougemont et al., arXiv:1704.05558
-Ozvenchuk et al., Phys. Rev. C87 (2013) 064903
-Moroz, arXiv:1301.6670
-Wiranata et al., Phys. Rev. C88 (2013) 044917
-Torres-Rincon, arXiv:1205.0782

Modelling the hadron gas: smash



- **SMASH is a new semi-classical transport approach for the hadron gas**
- Geometric collision criterion:

$$d_{trans} < d_{int} = \sqrt{\frac{\sigma_{tot}}{\pi}}$$

- Spectral functions of resonances are described by relativistic Breit-Wigner functions, with resonance lifetime

$$\tau_{res} = \frac{1}{\Gamma(m)}$$

- Elastic scatterings parameterized for NN; all other elastic scatterings assumed to go through resonances
- Inelastic scatterings, currently include
  - NN↔NR, NN↔ ΔR
  - KN↔KN, KN↔πH
  - +antiparticles
- Strings (turned off for detailed balance)

Weil et al., Phys. Rev. C94 (2016) no.5,

Green-Kubo Formalism

The shear viscosity is calculated from

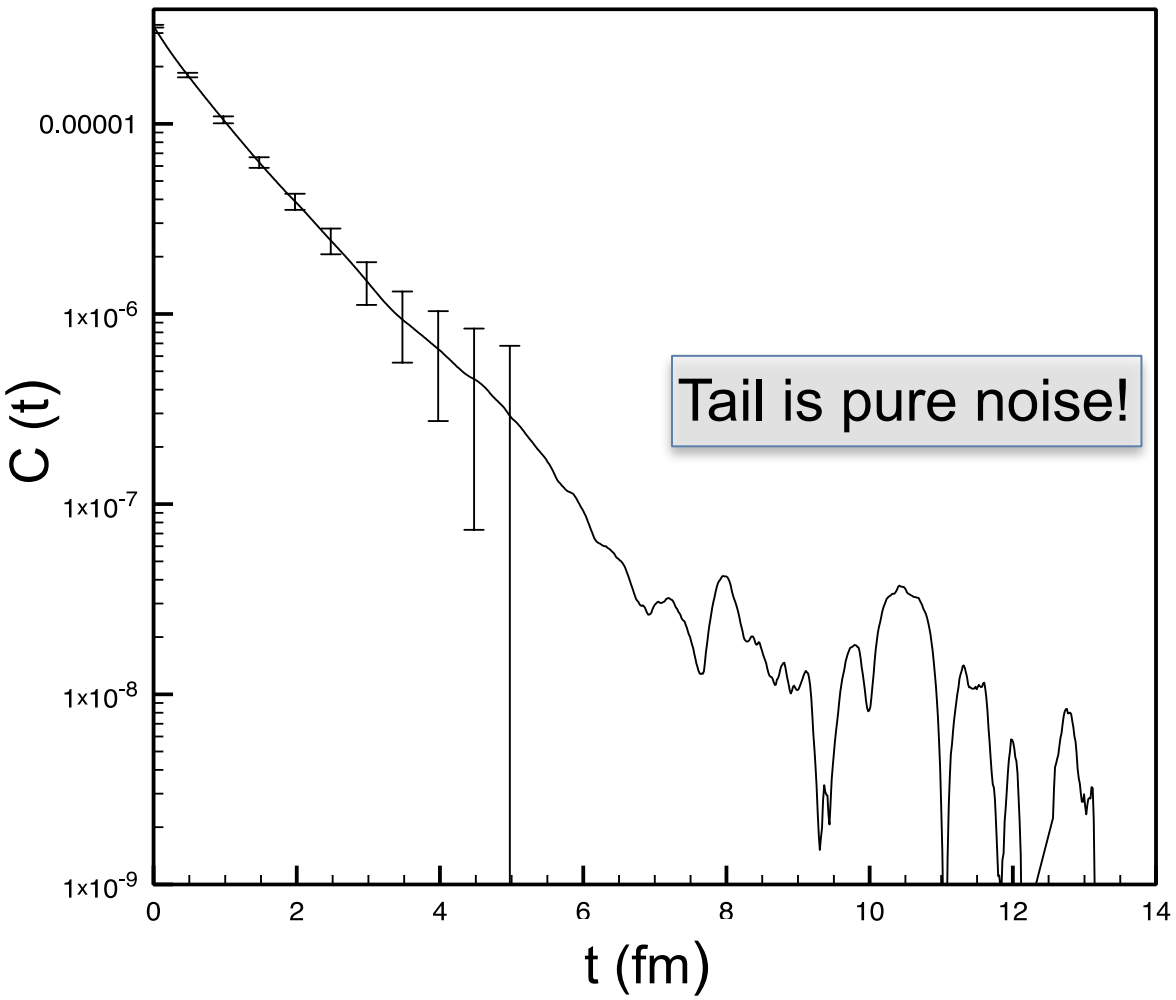
$$\eta = \frac{V}{T}\int_0^\infty C^{xy}(t)\,dt$$

where

$$C^{xy}(t) = \frac{1}{N}\sum_s^N T^{xy}(s)T^{xy}(s+t)$$

and

$$T^{\mu\nu} = \frac{1}{V}\sum_i^{N_{part}} \frac{p_i^\mu p_i^\nu}{p_i^0}$$



N is the number of time steps, and N_{part} the number of particles

Green-Kubo Formalism

It has been shown that the correlation function in

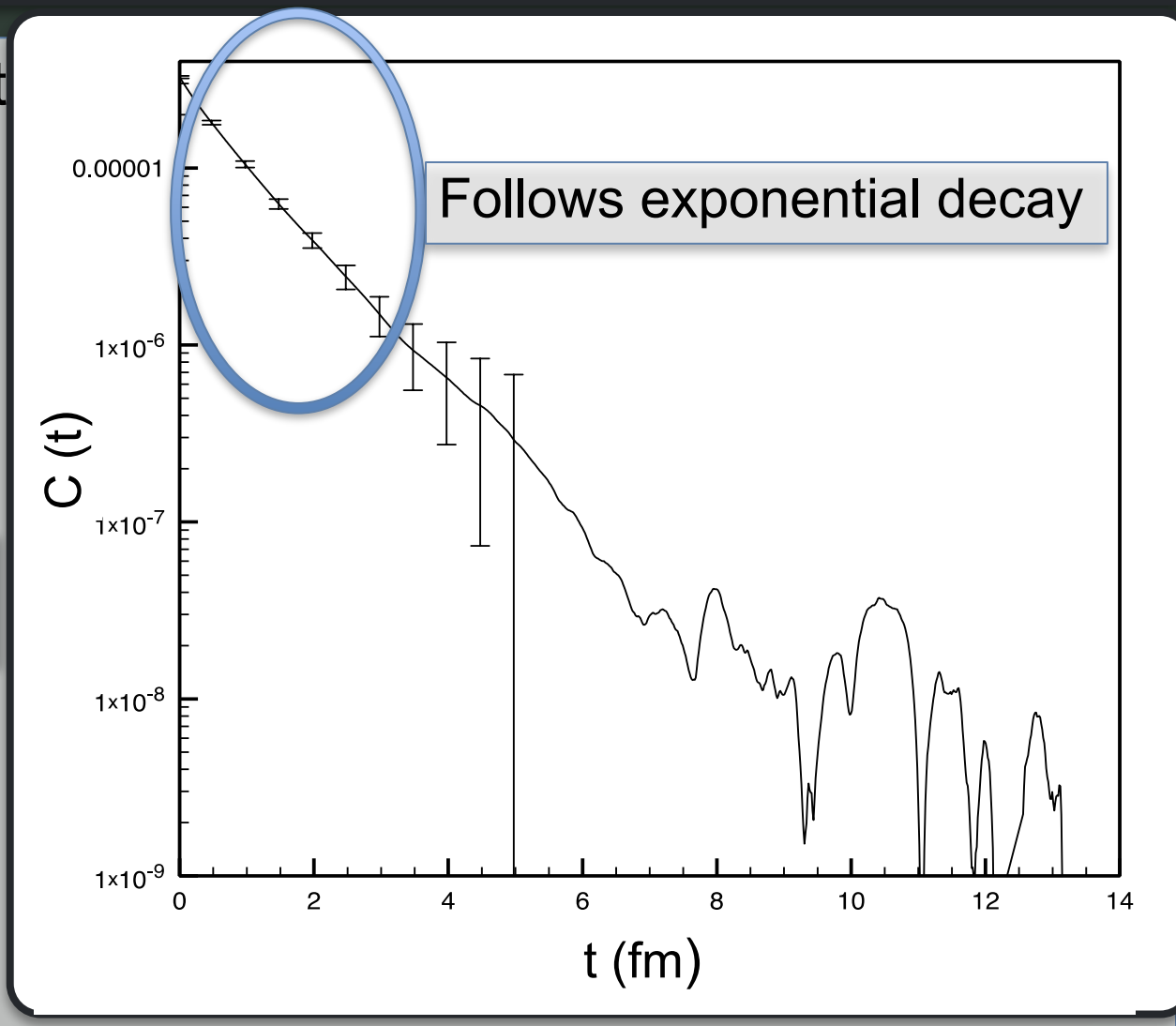
$$\eta = \frac{V}{T}\int_0^\infty C^{xy}(t)\,dt$$

Follows

$$C^{xy}(t) = C^{xy}(0)\exp\left(-\frac{t}{\tau}\right)$$

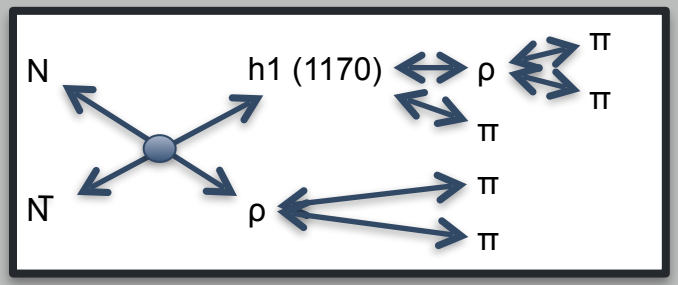
So that

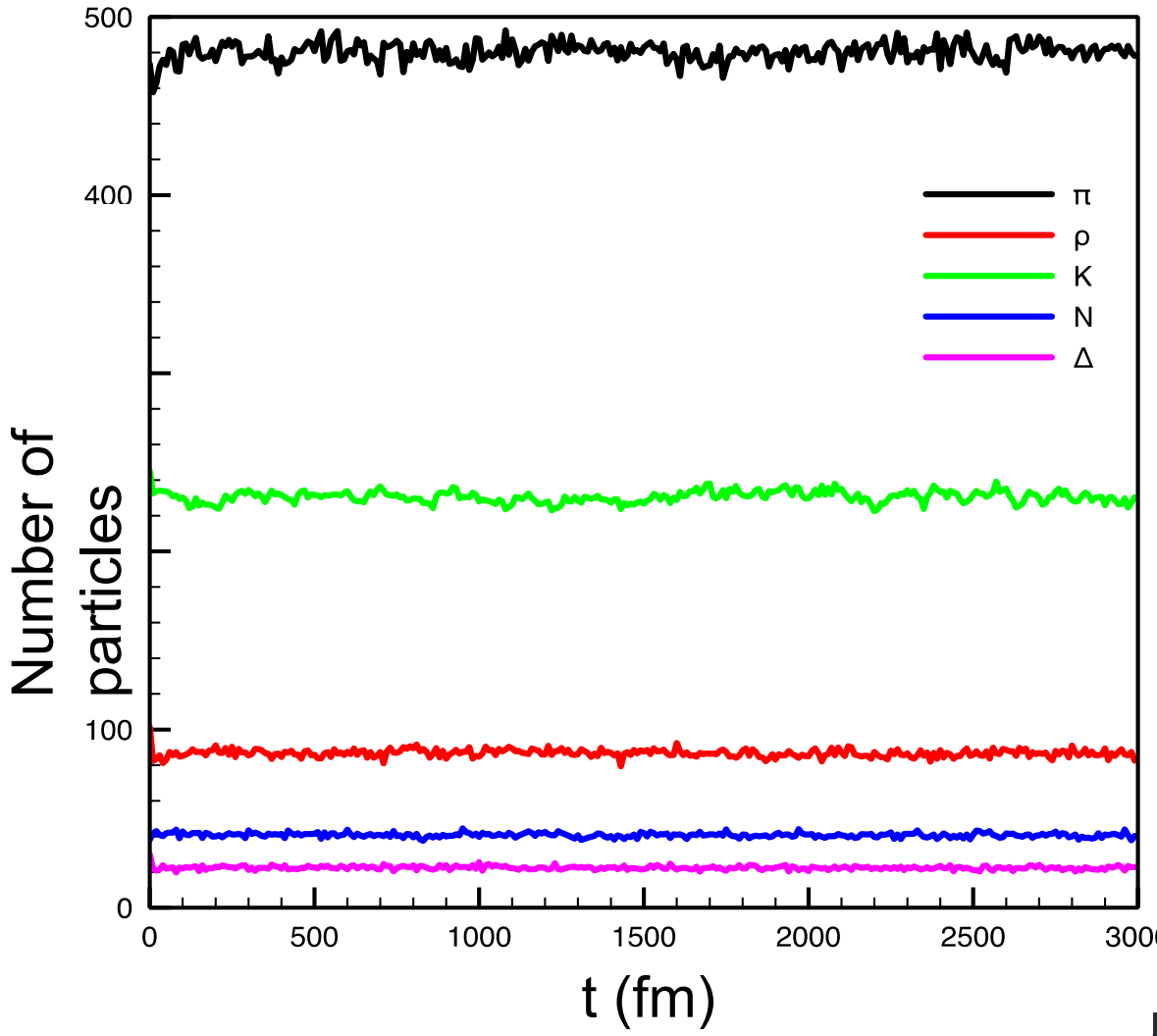
$$\eta = \frac{V C^{xy}(0)\tau}{T}$$



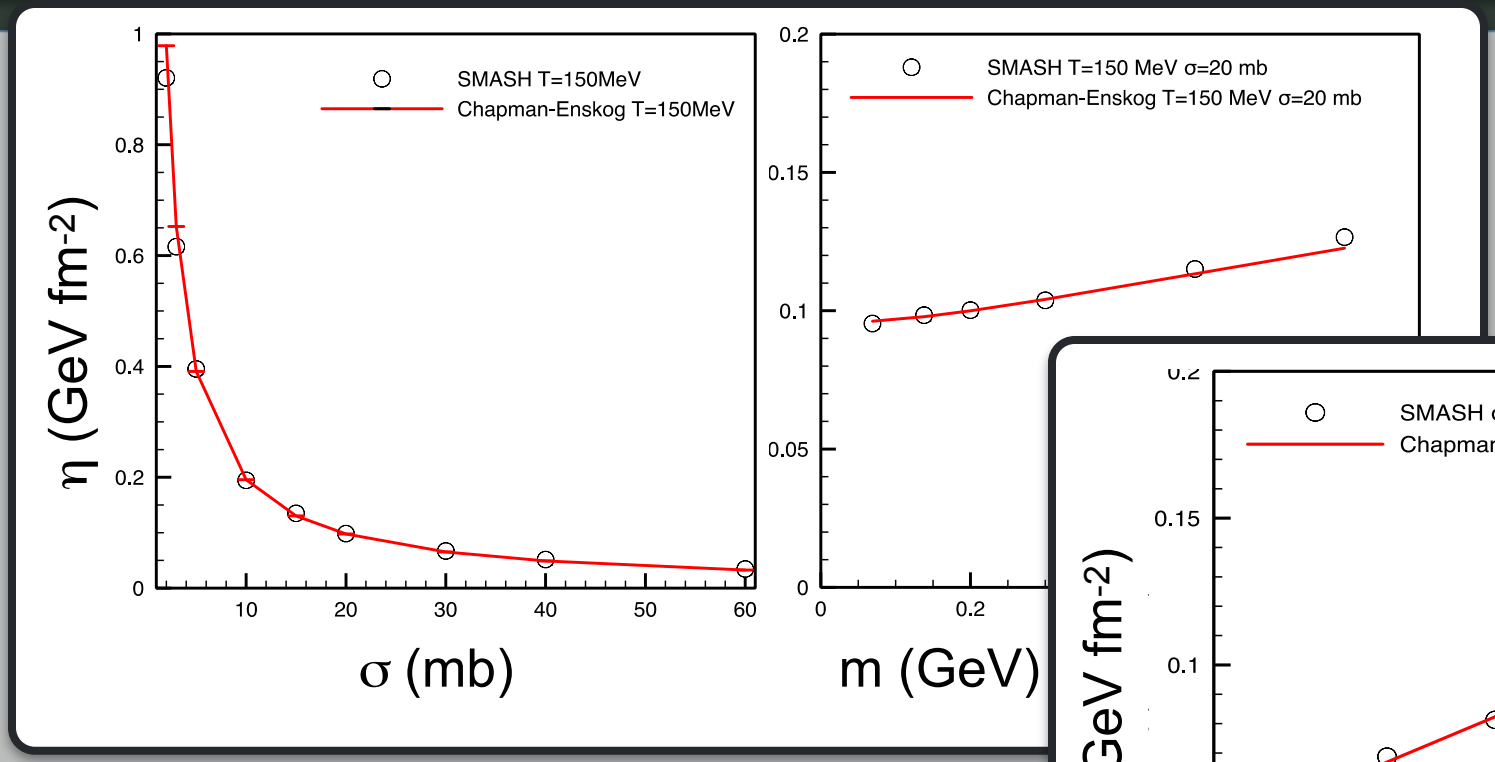
Equilibrium in SMASH

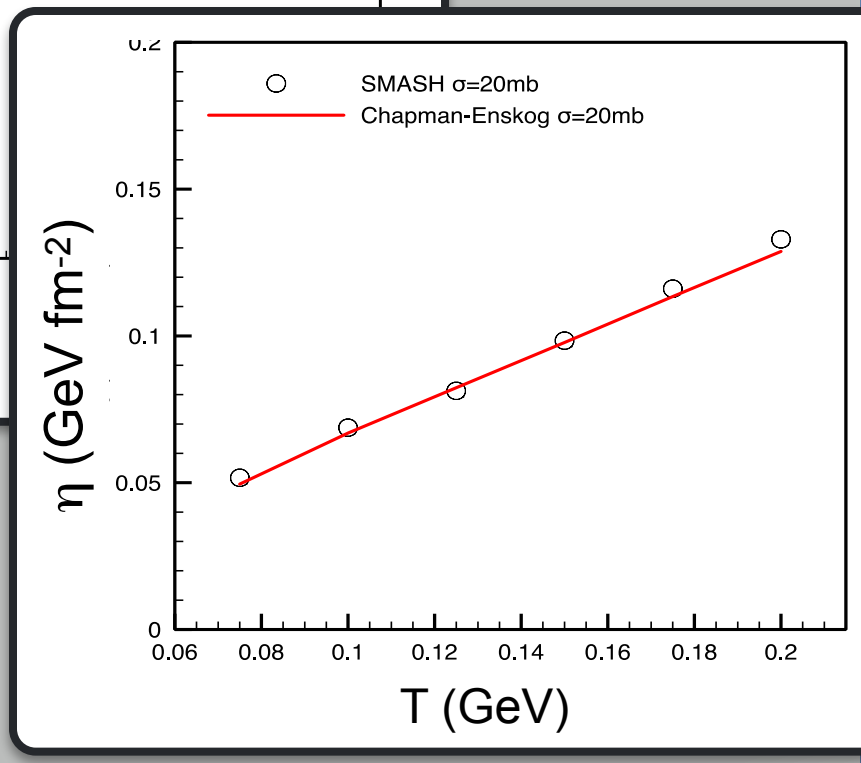
- Box calculations simulating infinite matter to apply the Green-Kubo procedure
- MUST have thermal & chemical equilibrium
- Baryon/antibaryon annihilation implemented to conserve detailed balance via an average decay to 5π





Test case #1: π with constant σ



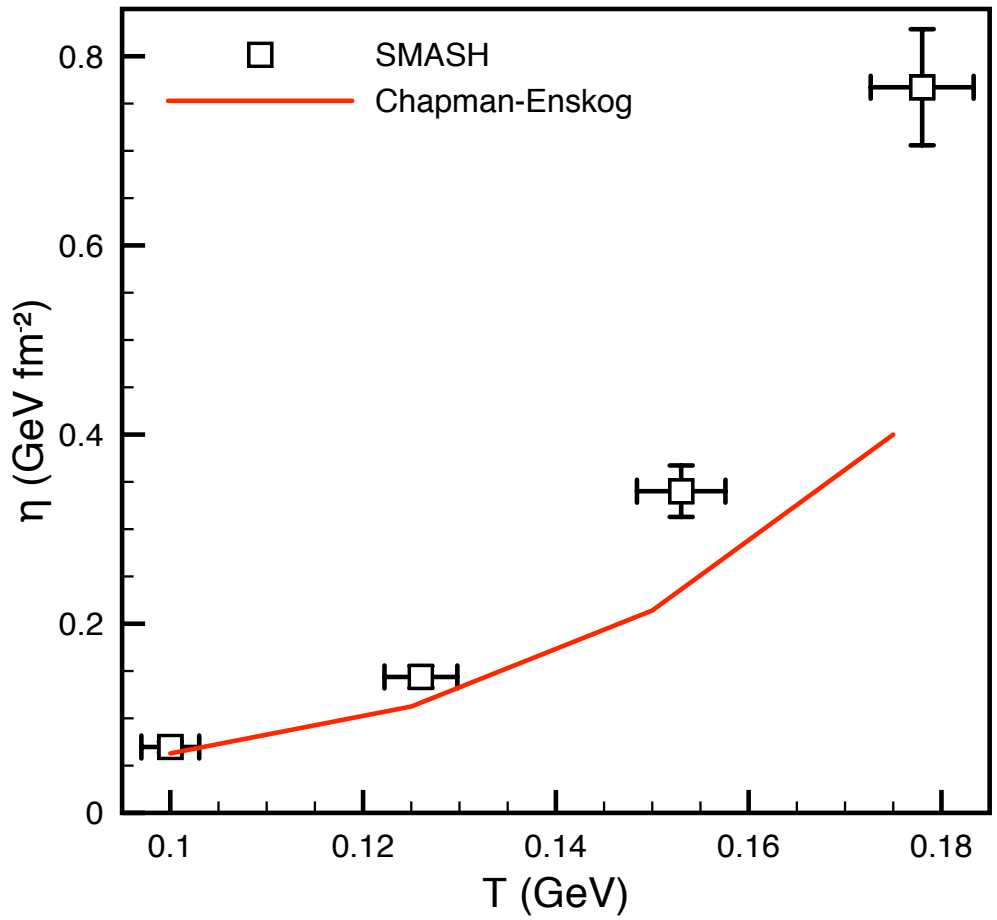


Main take-away:
The method is relatively insensitive to variations of parameters; maximum error is less than 10%

J. Torres-Rincon, PhD dissertation (2012), *Hadronic Transport Coefficients from Effective Field Theories*

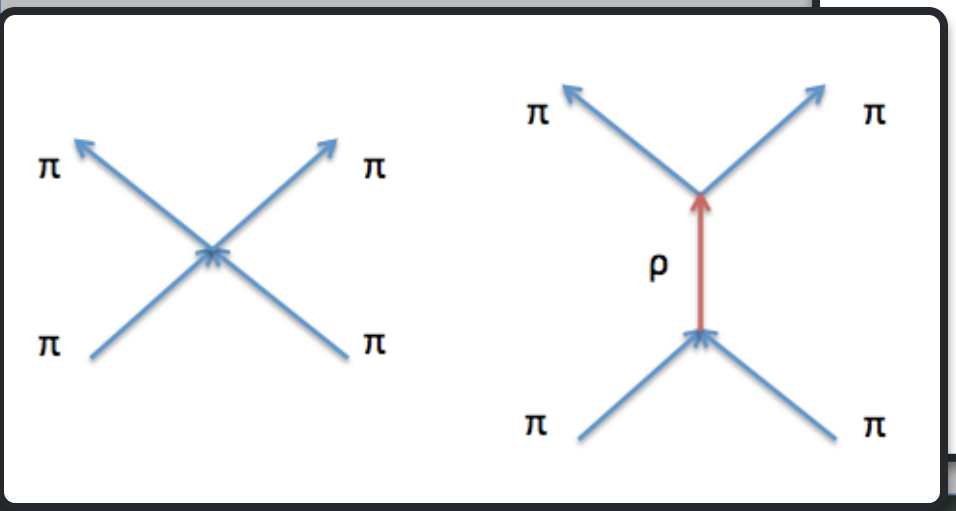
Test case #2: π-ρ gas

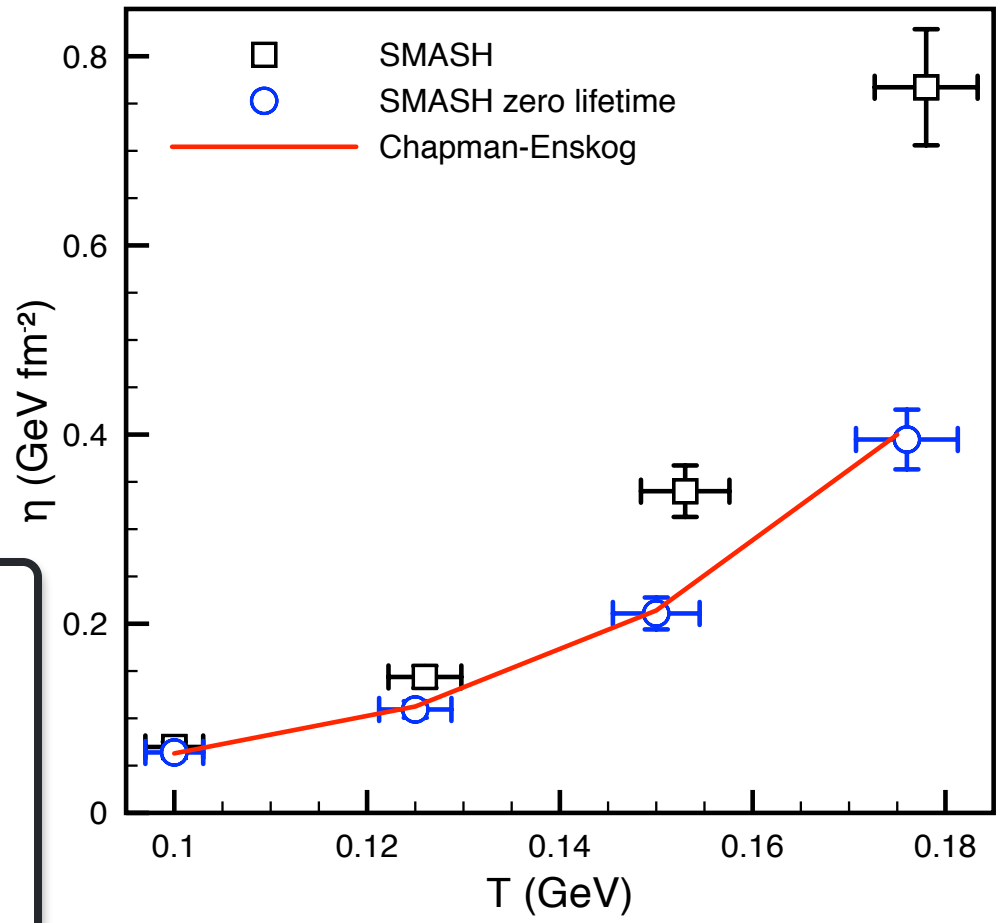
- Normal SMASH run does not coincide directly with Chapman-Enskog



Test case #2: π-ρ gas

- Normal SMASH run does not coincide directly with Chapman-Enskog
  - Resonance lifetimes





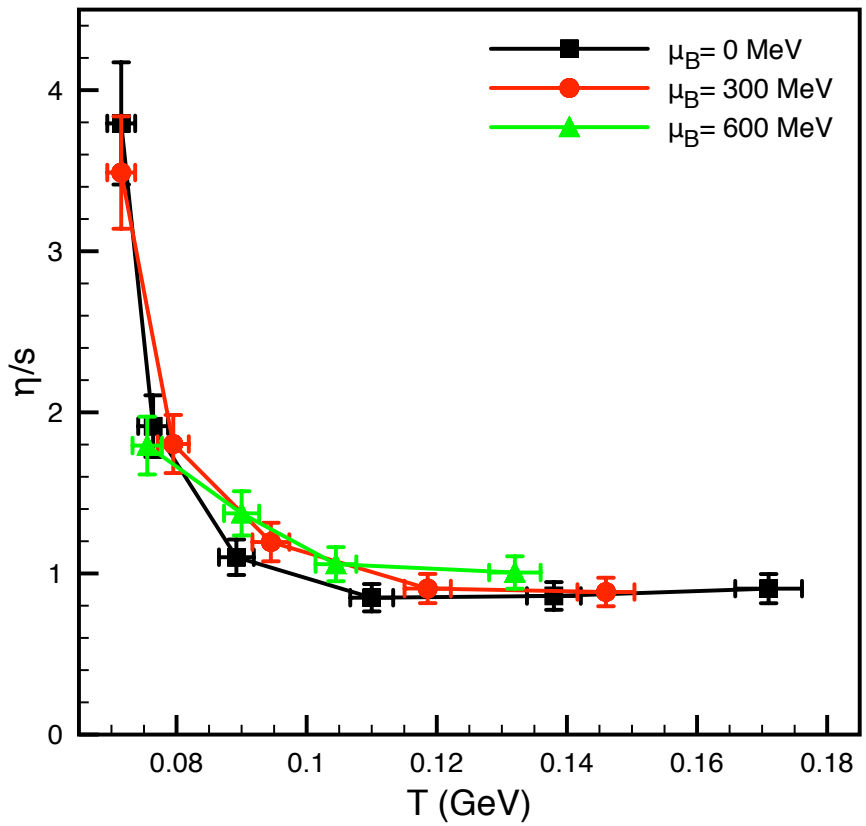
Hadron Gas: Degrees of freedom

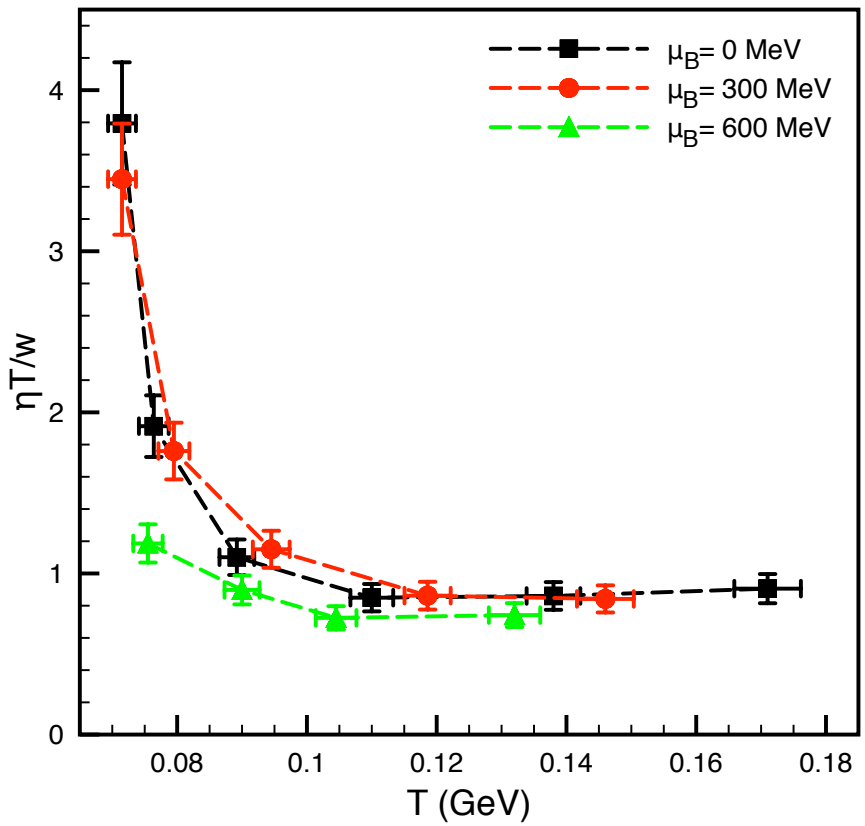
N	Δ	Λ	Σ	Ξ	Ω
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^-_{1672}
N_{1440}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	Ω^-_{2250}
N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}	
N_{1535}	Δ_{1905}	Λ_{1600}	Σ_{1670}	Ξ_{1820}	
N_{1650}	Δ_{1910}	Λ_{1670}	Σ_{1750}	Ξ_{1950}	
N_{1675}	Δ_{1920}	Λ_{1690}	Σ_{1775}	Ξ_{2030}	
N_{1680}	Δ_{1930}	Λ_{1800}	Σ_{1915}		
N_{1700}	Δ_{1950}	Λ_{1810}	Σ_{1940}		
N_{1710}		Λ_{1820}	Σ_{2030}		
N_{1720}		Λ_{1830}	Σ_{2250}		
N_{1875}		Λ_{1890}			
N_{1900}		Λ_{2100}			
N_{1990}		Λ_{2110}			
N_{2080}		Λ_{2350}			
N_{2190}					
N_{2220}					
N_{2250}					

- Isospin symmetry
- Perturbative treatment of non-hadronic particles (photons, dileptons)

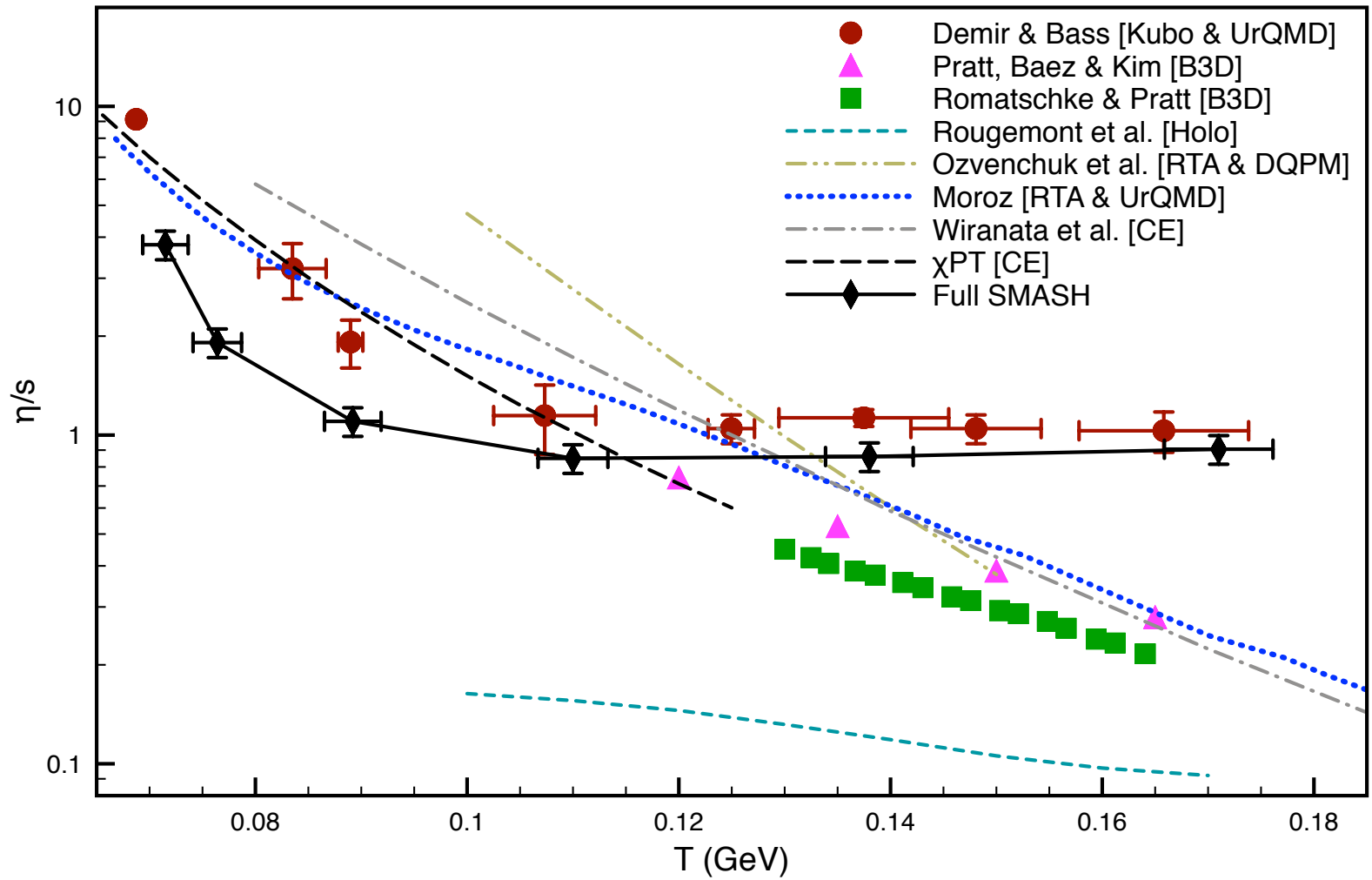
Unflavored				Strange
π_{138}	$f_{0\ 980}$	$f_{2\ 1275}$	$\pi_{2\ 1670}$	K_{494}
π_{1300}	$f_{0\ 1370}$	$f_{2\ 1525}'$		K^*_{892}
π_{1800}	$f_{0\ 1500}$	$f_{2\ 1950}$	$\rho_{3\ 1690}$	$K_{1\ 1270}$
	$f_{0\ 1710}$	$f_{2\ 2010}$		$K_{1\ 1400}$
η_{548}		$f_{2\ 2300}$	$\varphi_{3\ 1850}$	K^*_{1410}
η'_{958}	$a_{0\ 980}$	$f_{2\ 2340}$		$K_0^*{}_{1430}$
η_{1295}	$a_{0\ 1450}$		$a_{4\ 2040}$	$K_2^*{}_{1430}$
η_{1405}		$f_{1\ 1285}$		K^*_{1680}
η_{1475}	φ_{1019}	$f_{1\ 1420}$	$f_{4\ 2050}$	$K_{2\ 1770}$
	φ_{1680}			$K_3^*{}_{1780}$
σ_{800}		$a_{2\ 1320}$		$K_{2\ 1820}$
	$h_{1\ 1170}$			$K_4^*{}_{2045}$
ρ_{776}		$\pi_{1\ 1400}$		
ρ_{1450}	$b_{1\ 1235}$	$\pi_{1\ 1600}$		
ρ_{1700}				
	$a_{1\ 1260}$	$\eta_{2\ 1645}$		
ω_{783}				
ω_{1420}		$\omega_{3\ 1670}$		
ω_{1650}				

Hadron Gas: T and μ_B dependence

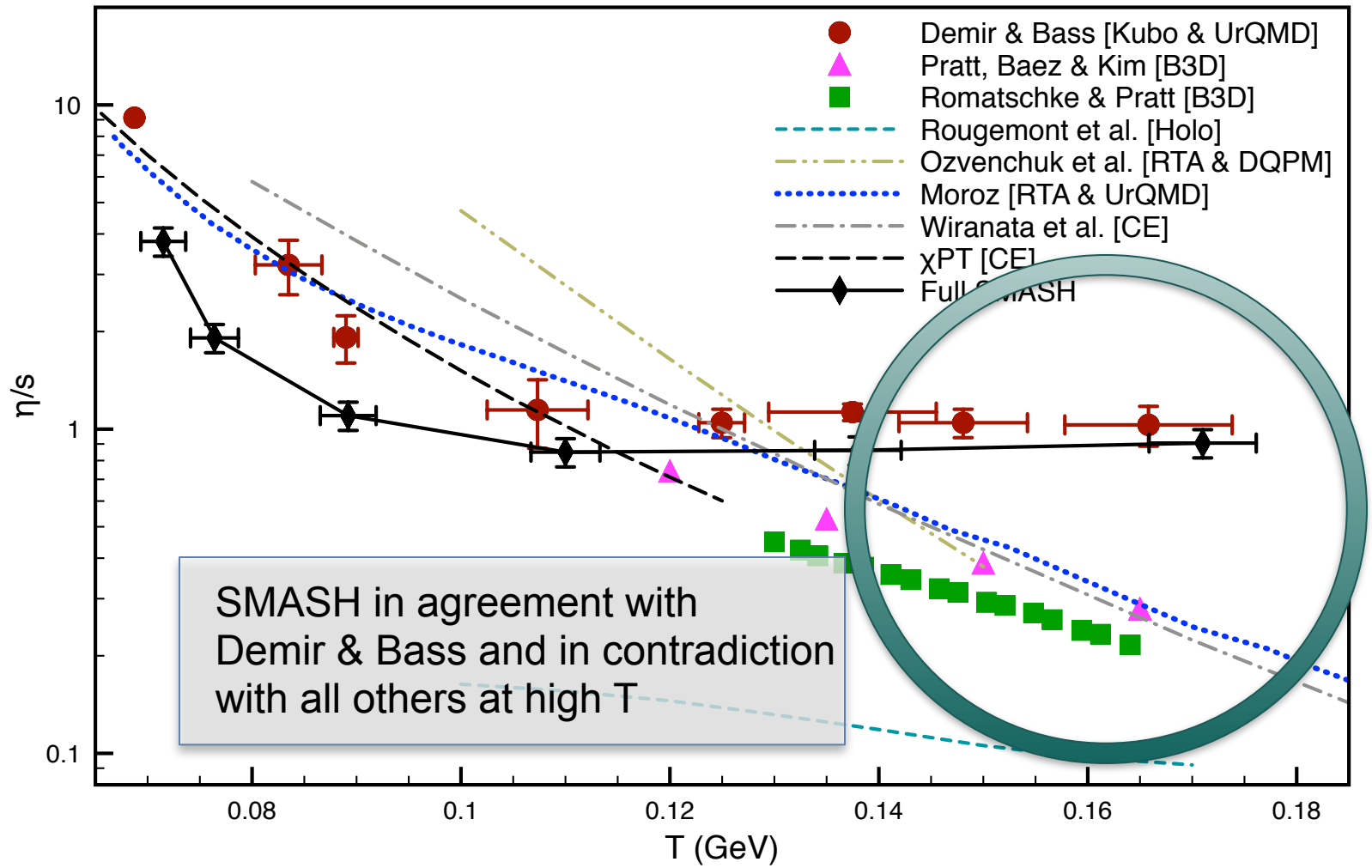




HG: Viscosity Comparison

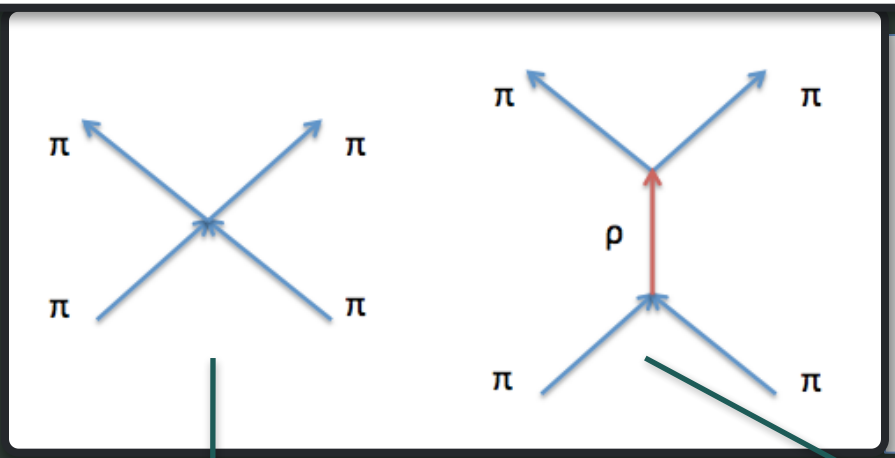


HG: Viscosity Comparison



SMASH in agreement with Demir & Bass and in contradiction with all others at high T

High temperature η/s : Resonance lifetimes

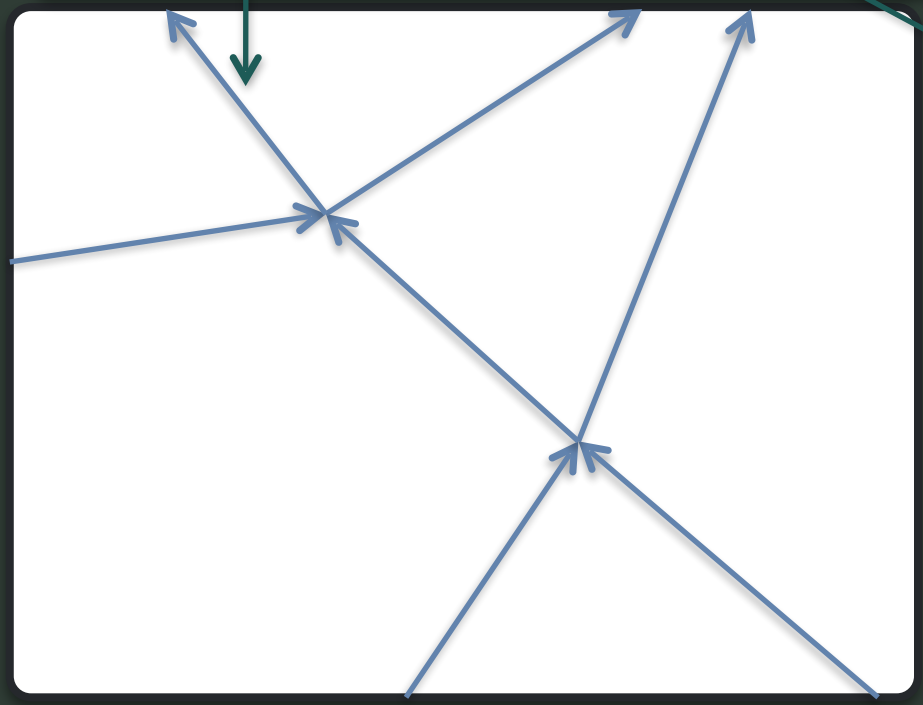


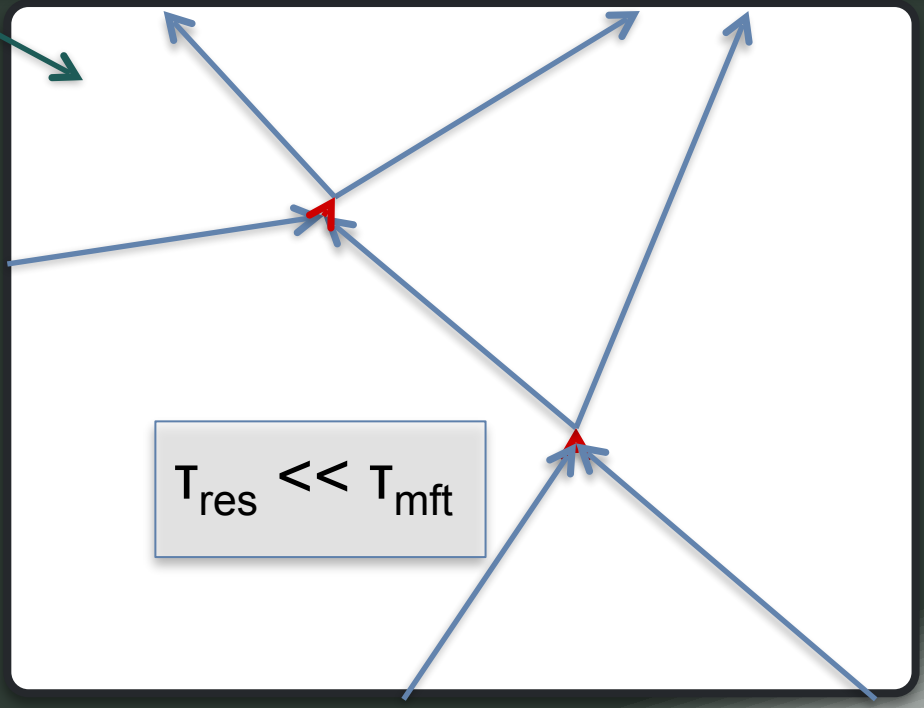
Must look at the microscopic picture from different descriptions

τ_{res} = resonance lifetime

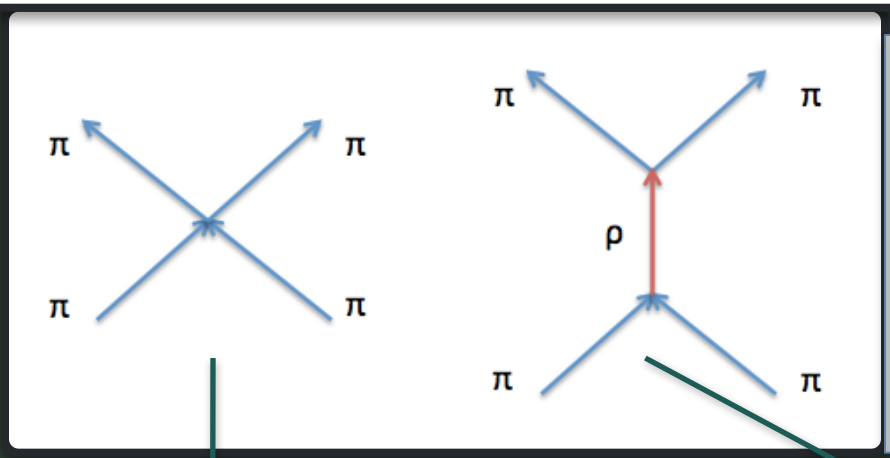
τ_{mft} = mean free time

At low T and density:





High temperature η/s : Resonance lifetimes

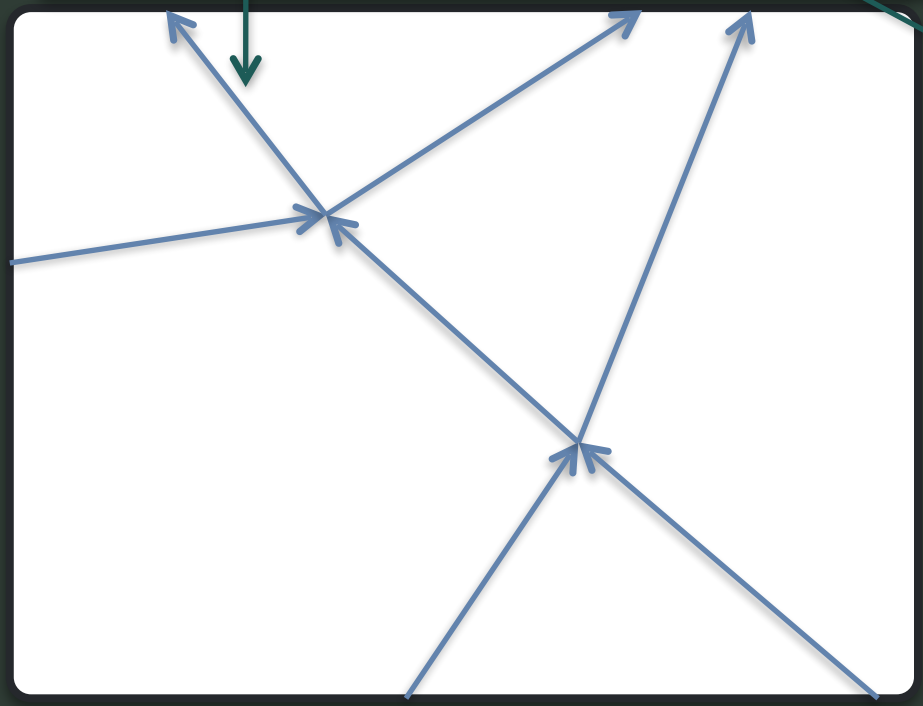


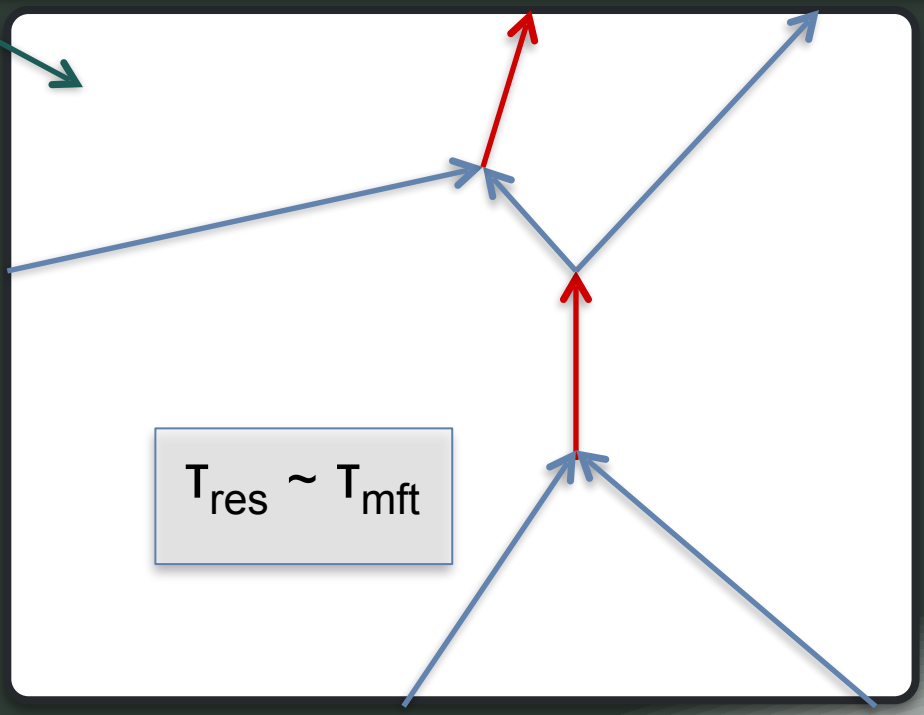
Must look at the microscopic picture from different descriptions

τ_{res} = resonance lifetime

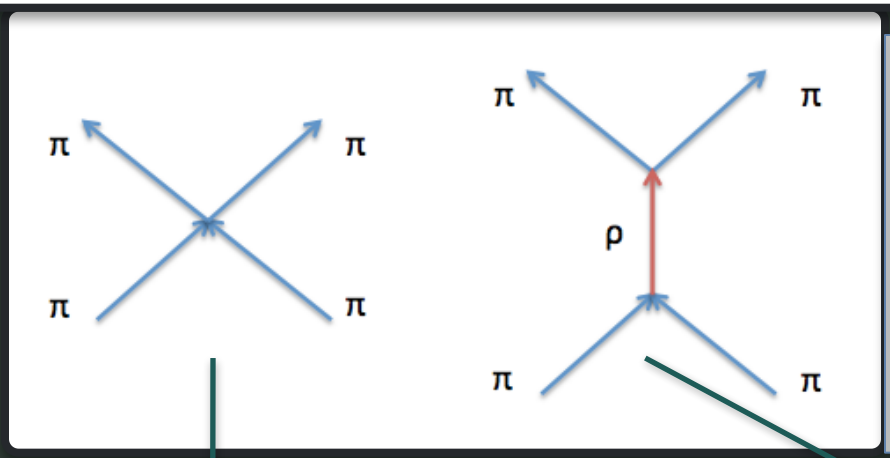
τ_{mft} = mean free time

At high T and density:





High temperature η/s : Resonance lifetimes

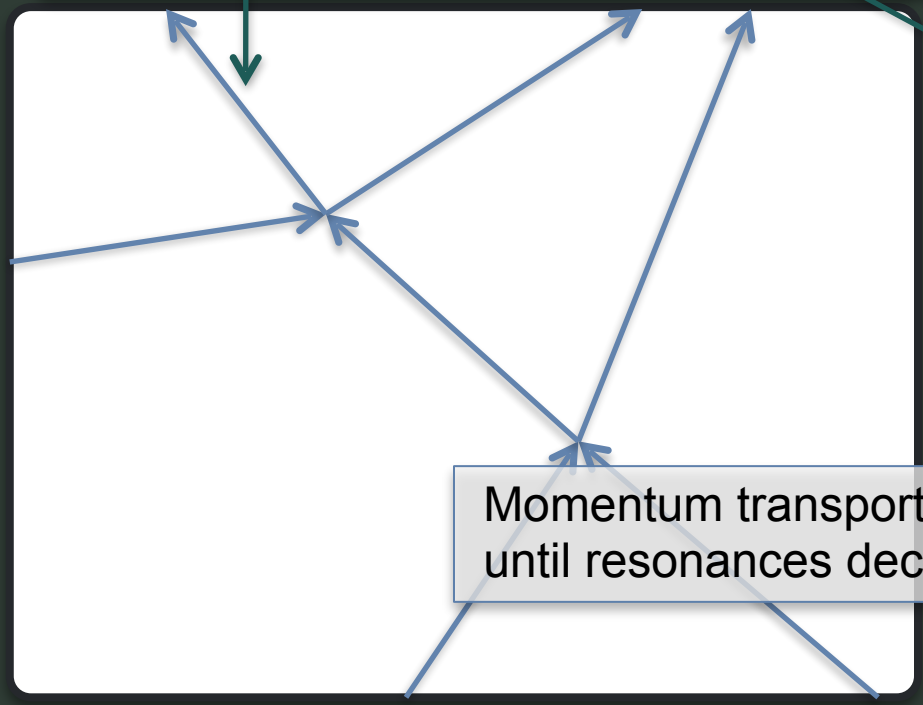


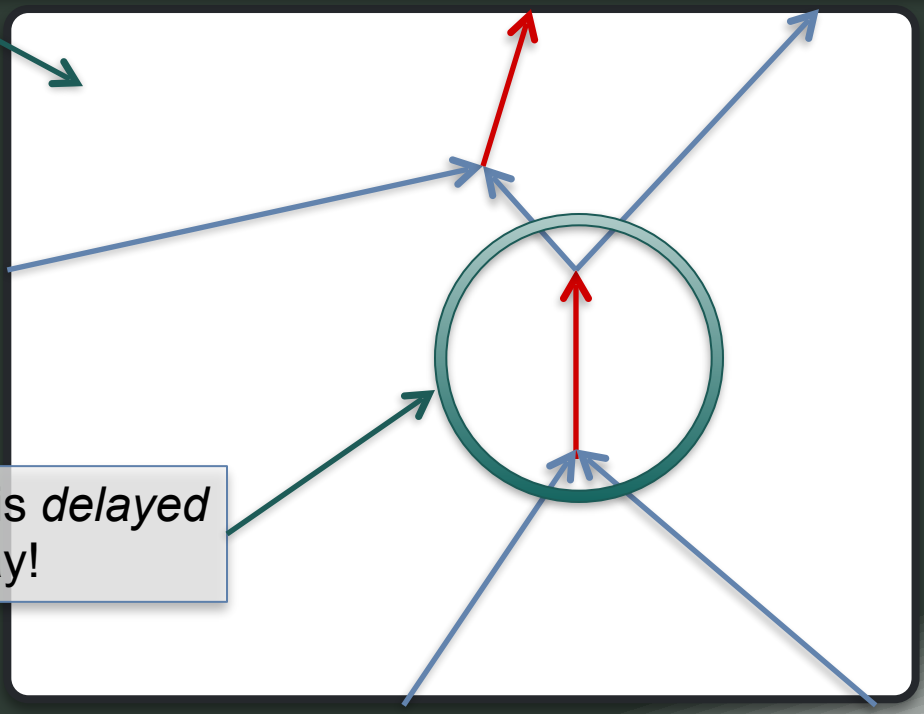
Must look at the microscopic picture from different descriptions

τ_{res} = resonance lifetime

τ_{mft} = mean free time

At high T and density:

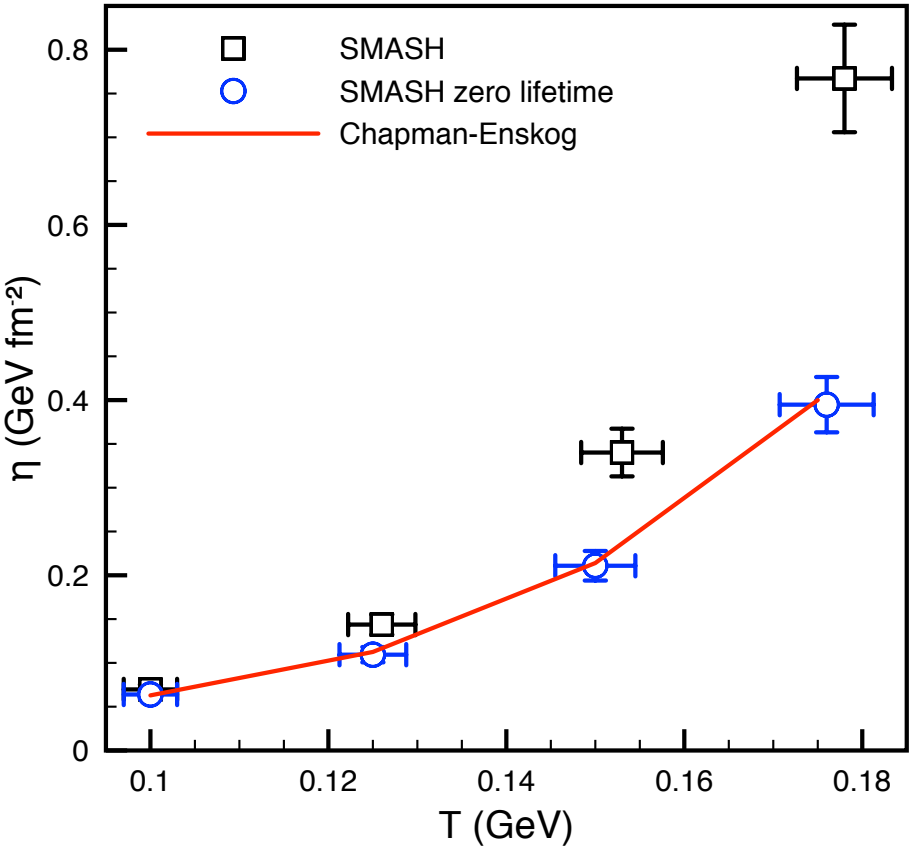


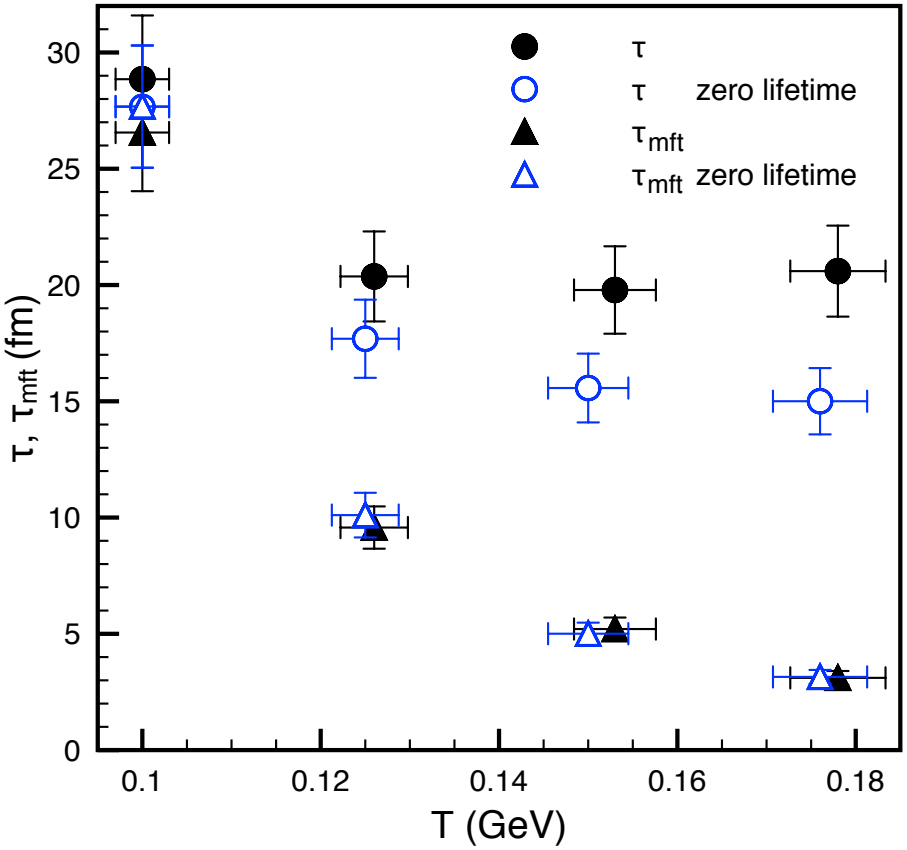


Momentum transport is *delayed* until resonances decay!

π-ρ: Zero lifetimes vs relaxation time

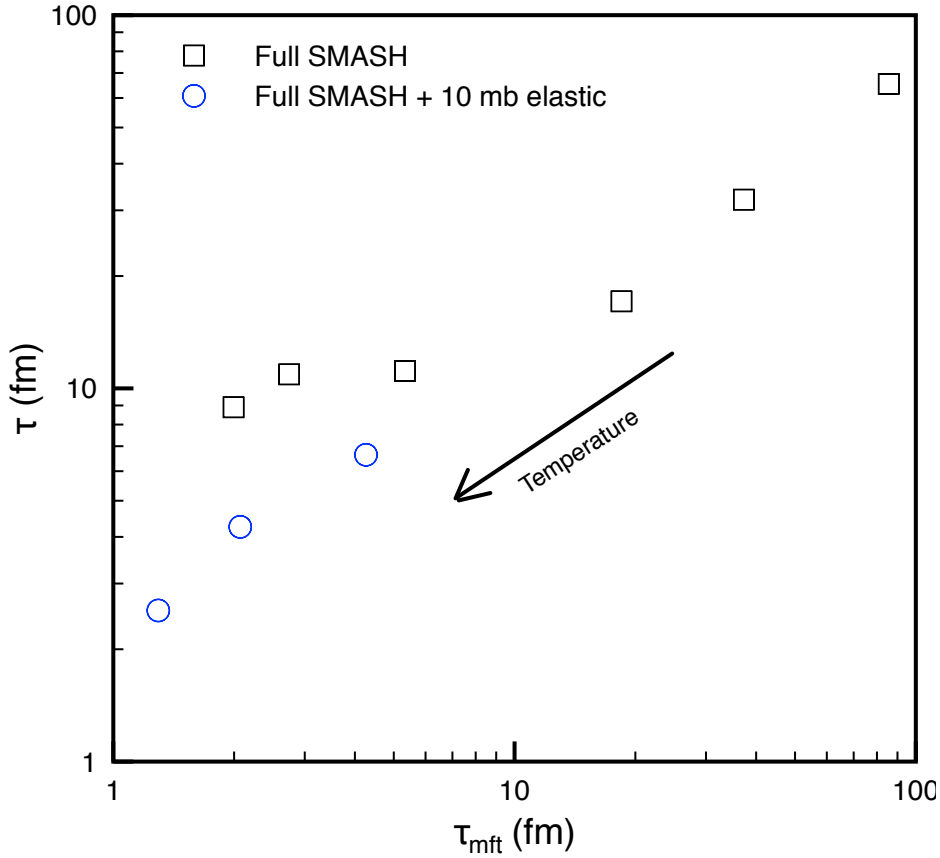
Large part of the difference explained from eliminating lifetimes

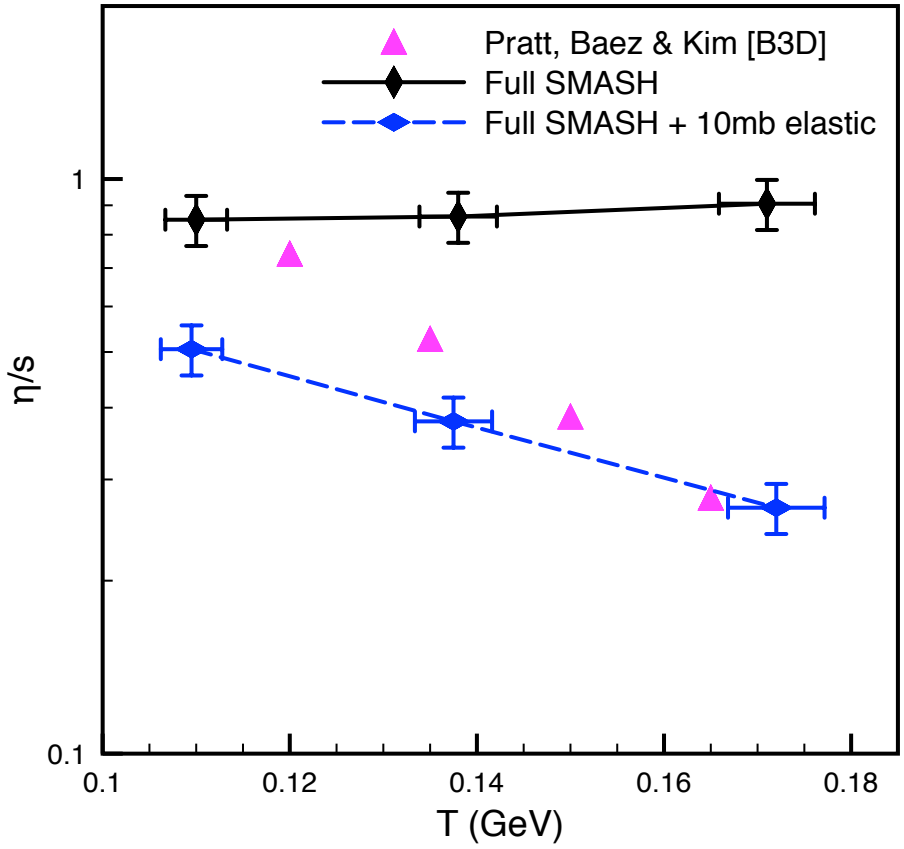




Effect of many non-resonant interactions

Introduce a constant elastic cross-section between all particles to add many non-resonant interactions

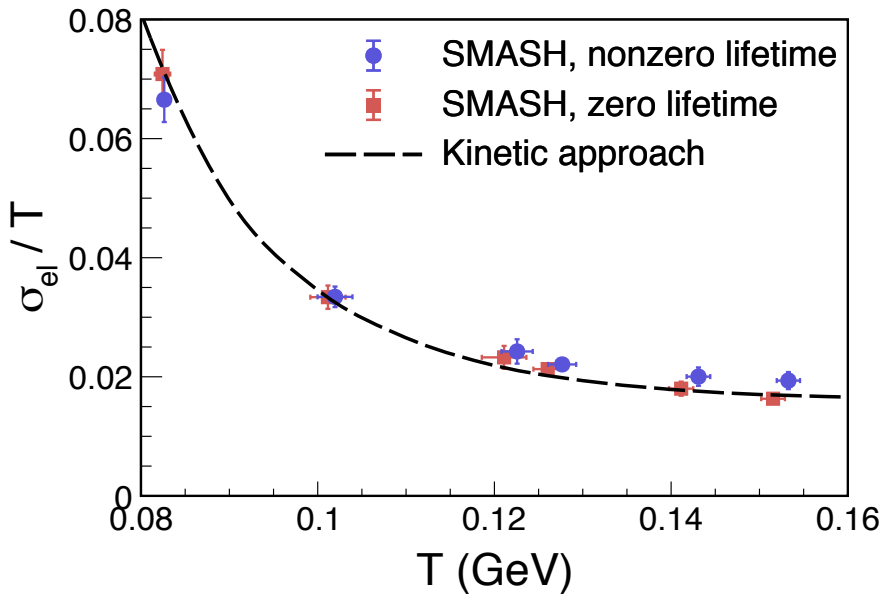


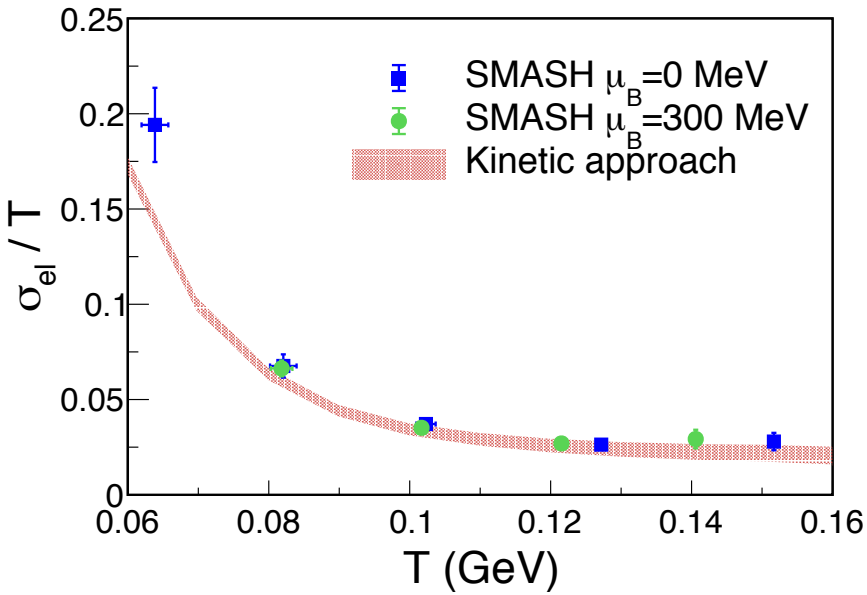


Summary & Outlook

- **Investigated temperature, cross-section and mass dependence of the shear viscosity in an elastic pion box**
  - Very good agreement with Chapman-Enskog approximation (within 10%)
  - Resonance lifetimes need to be considered
- **Full hadron gas *η/s* calculated at zero and non-zero μ_B**
  - High T discrepancy explained by looking at microscopic details of resonance modelling; finite lifetime <u>increases</u> viscosity
  - Could be used to constrain the treatment of resonances
- **Outlook:**
  - Investigation of angular dependent interactions on viscosity
  - At temperatures close to the phase transition, inclusion of multi-particle interaction will play a role, and needs to be investigated
  - Other transport coefficients (electrical conductivity, bulk viscosity, etc.)

Outlook: Electric conductivity





Resonant case shows lower sensitivity to resonance lifetimes

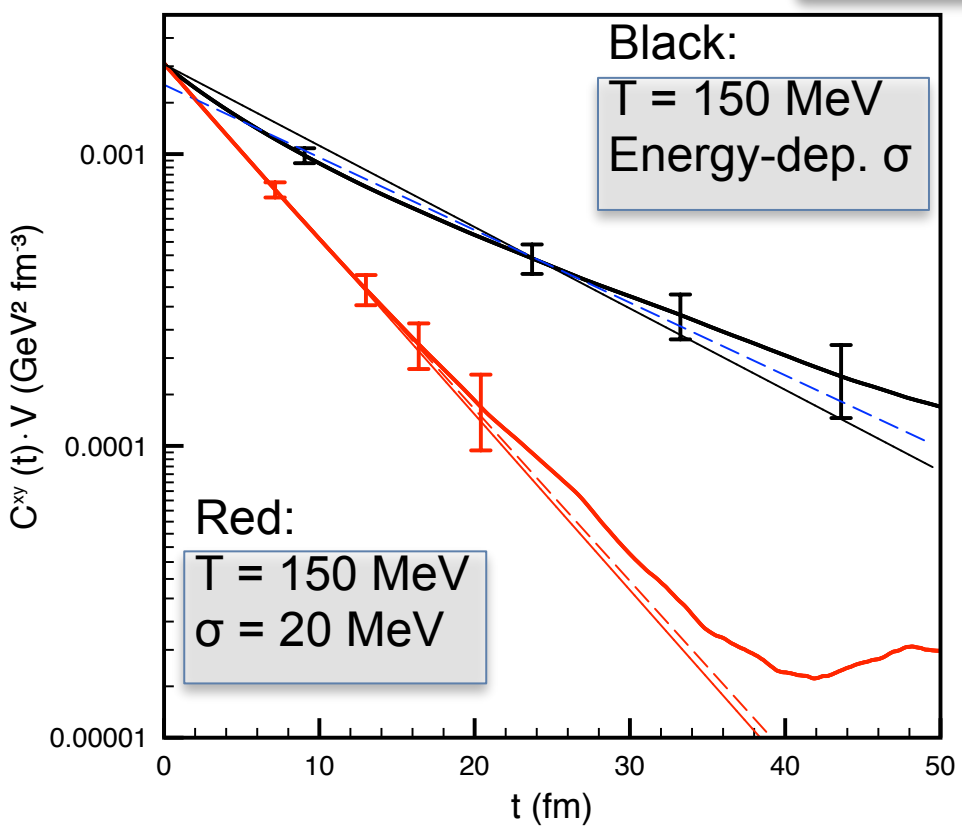
Multi-species case qualitatively comparable to previous calculations

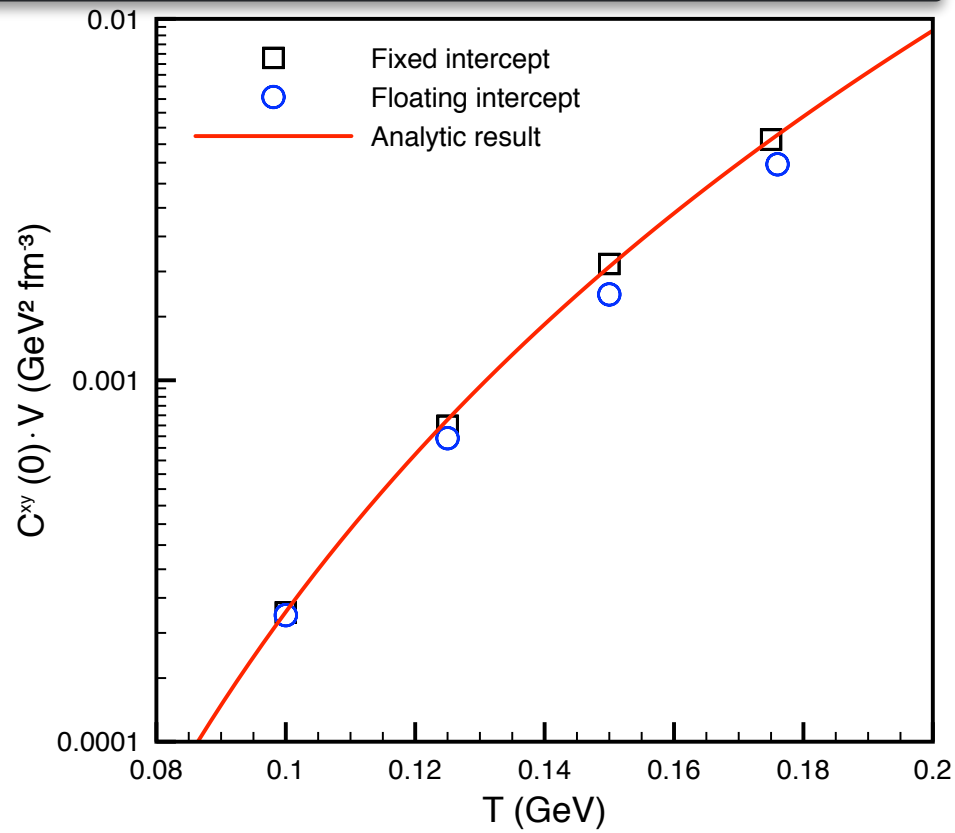
M. Greif, C. Greiner, G. S. Denicol, Phys. Rev D93 (2016) 096012, arXiv:1602.05085

Backup slides

How to fit?

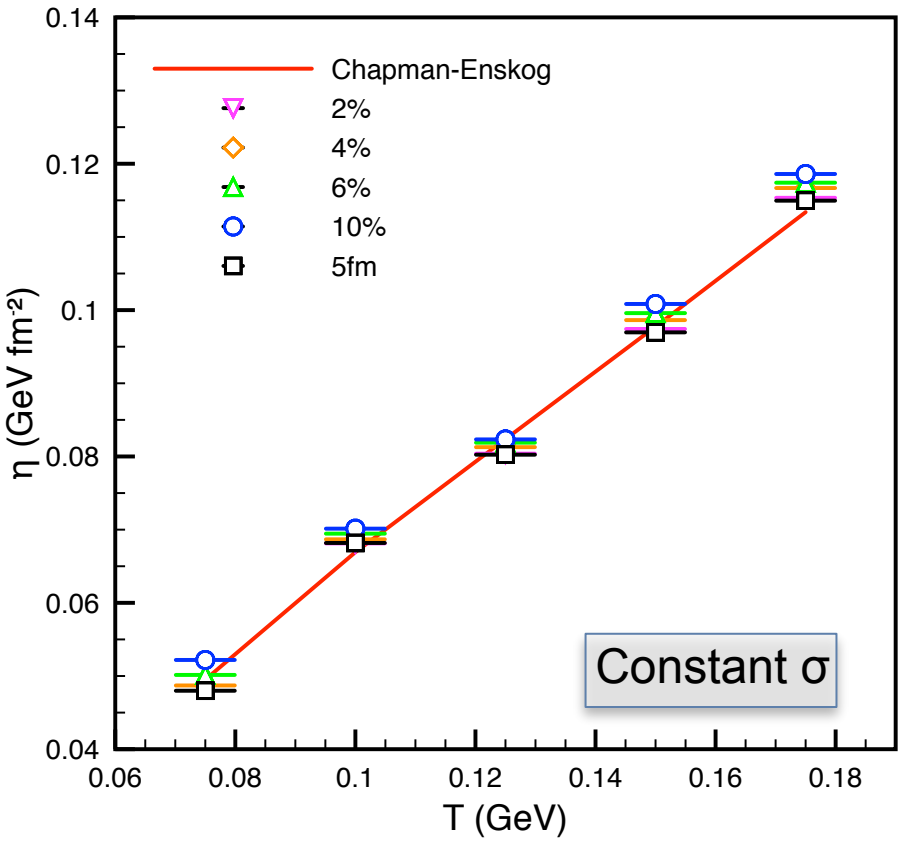
$$C^{xy}(0) = \frac{g \exp\left(\frac{\mu}{T}\right)}{30\pi^2 V} \int_0^\infty dp \, \frac{p^6}{m^2 + p^2} \exp\left(-\frac{\sqrt{m^2 + p^2}}{T}\right)$$

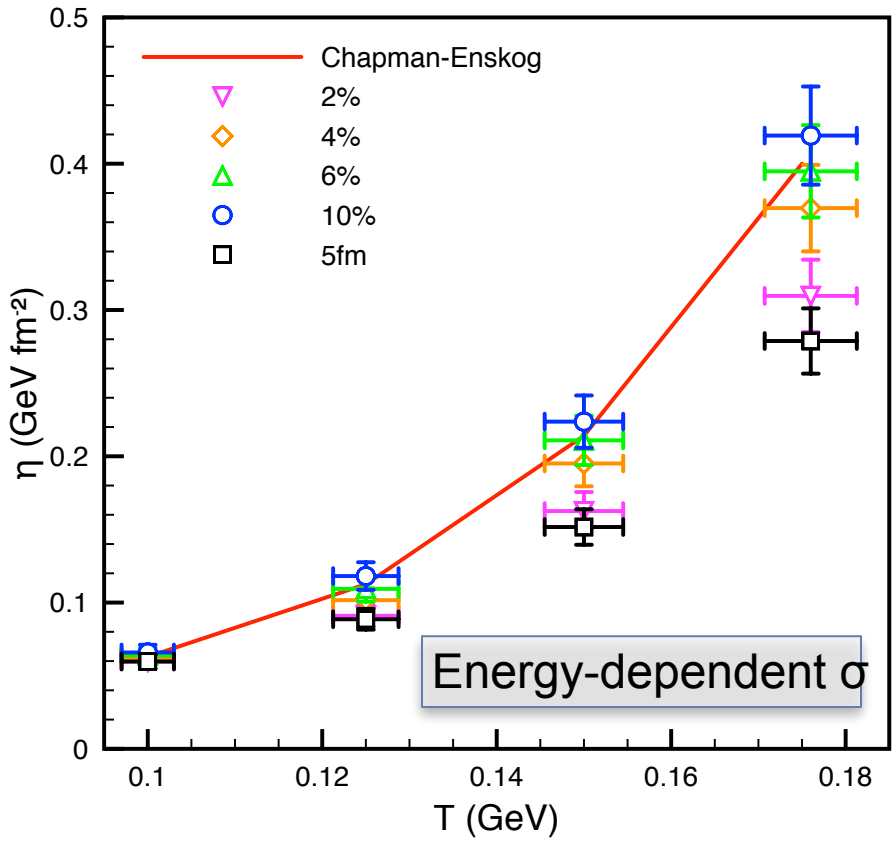




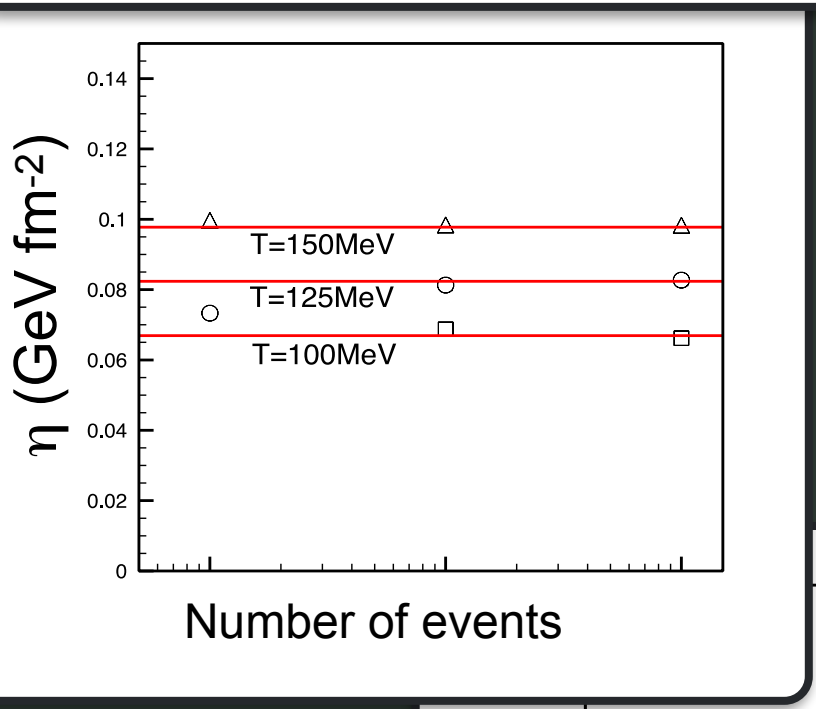
Where to stop the fitting?

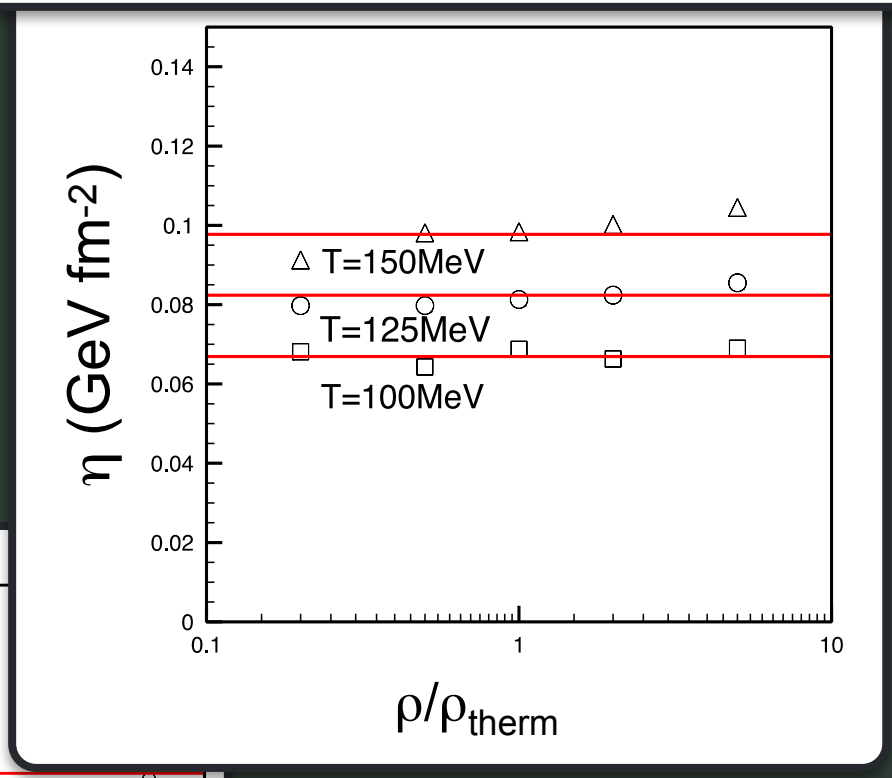
J. Torres-Rincon, PhD dissertation (2012), *Hadronic Transport Coefficients from Effective Field Theories*

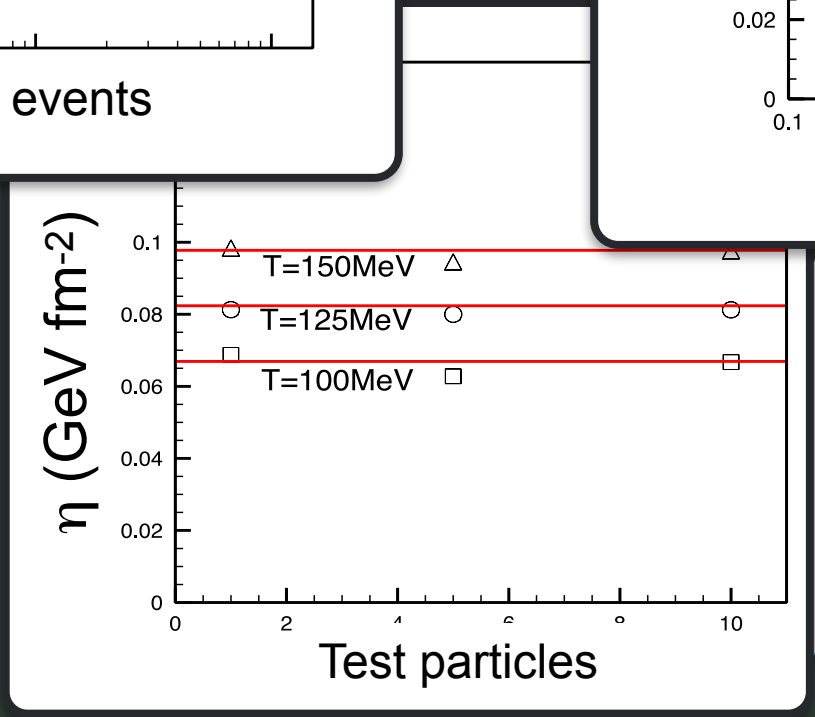




Constant σ: Systematics





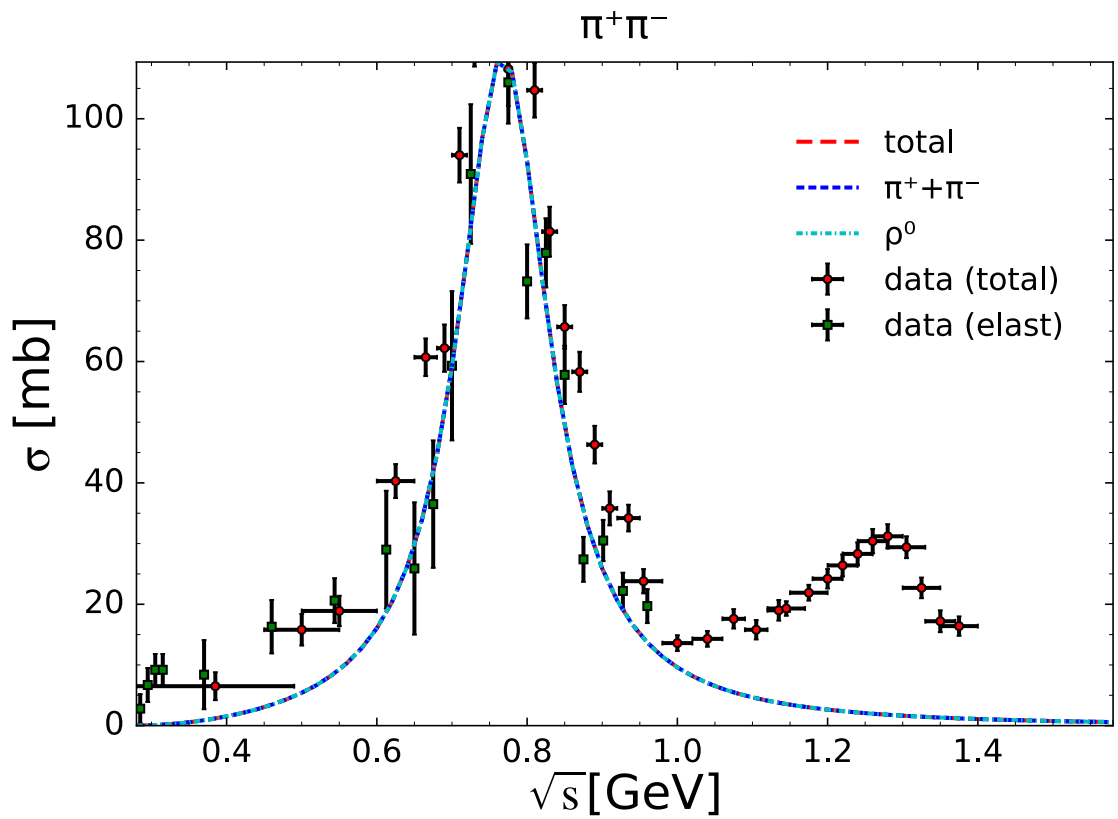


Main take-away:

The method is relatively inelastic to variations of most parameters; maximum error is less than 10%

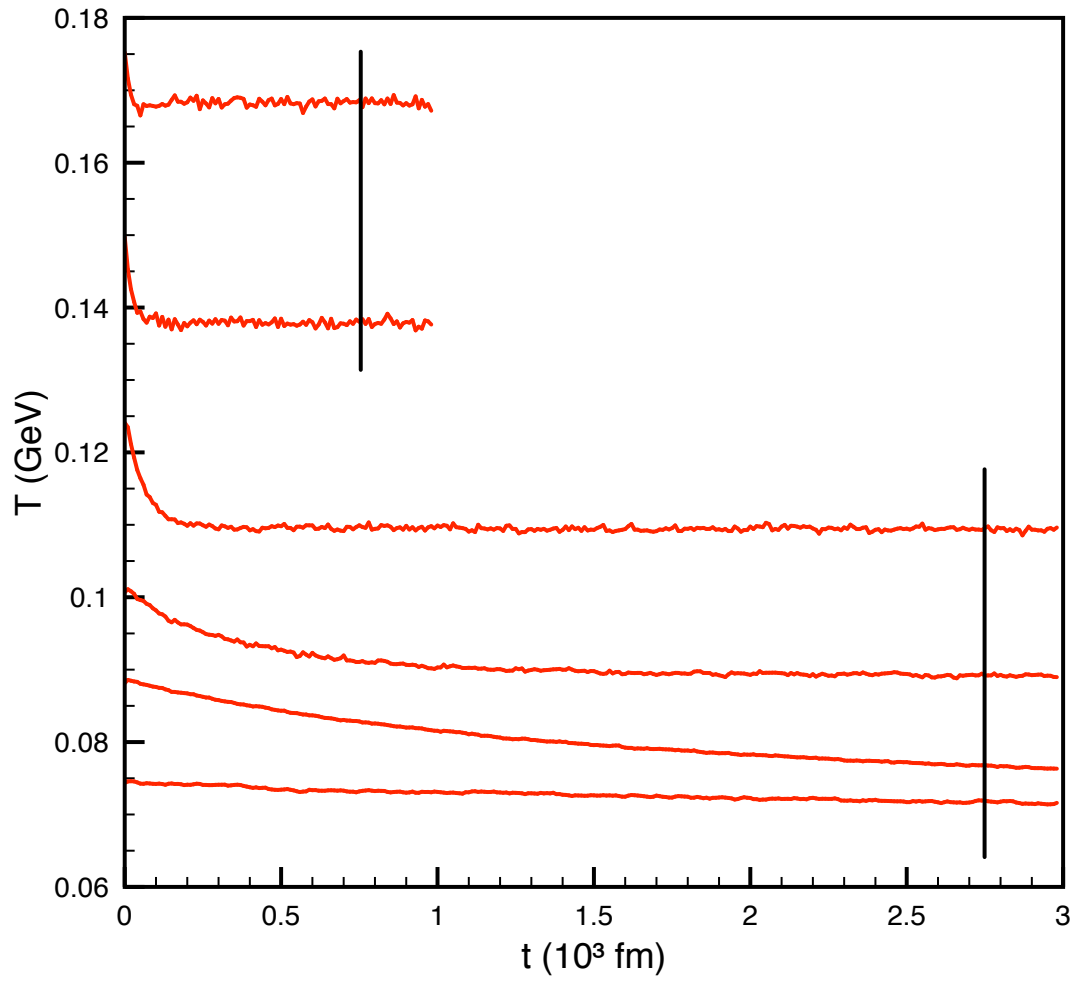
Test case #2: Energy-dependent σ

- Pions in a $(20\ \mathrm{fm})^3$ box simulating infinite matter
- Cross-section uses ρ resonance
- Runs for t_{max}=200 fm
- Initialized with initial densities consistent with Boltzmann ideal gas



Hadron Gas

- All particles and resonances initialized to thermal multiplicities (at the pole mass)
- Must wait for equilibration and compute *T, μ* once in equilibrium from most abundant particles
  - T fitted from weighted momentum spectra of *π, K & N*
  - μ_B obtained from *N* / anti-*N* ratio



What about entropy?

The entropy density can be calculated from the Gibbs formula:

$$s = \frac{e + p - \mu n}{T} = \frac{w - \mu n}{T}$$

where the energy density and pressure can be taken from the average shear-stress tensor according to:

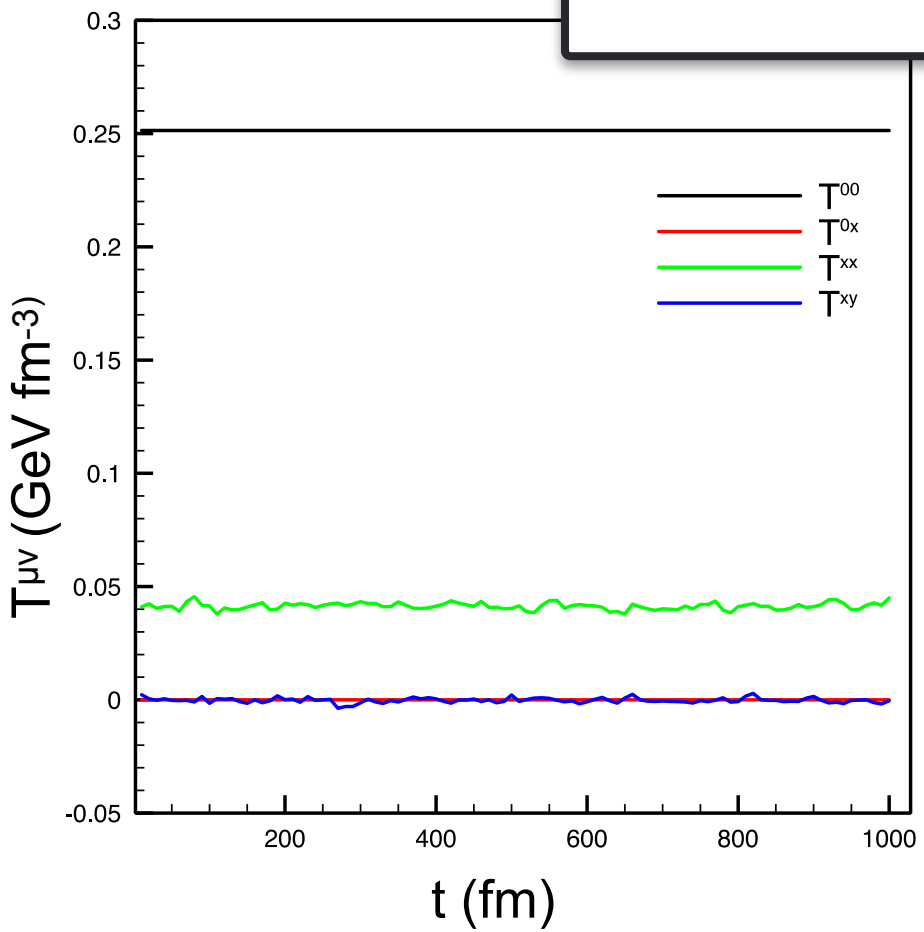
$$T^{\mu\nu} = diag(e, p, p, p)$$

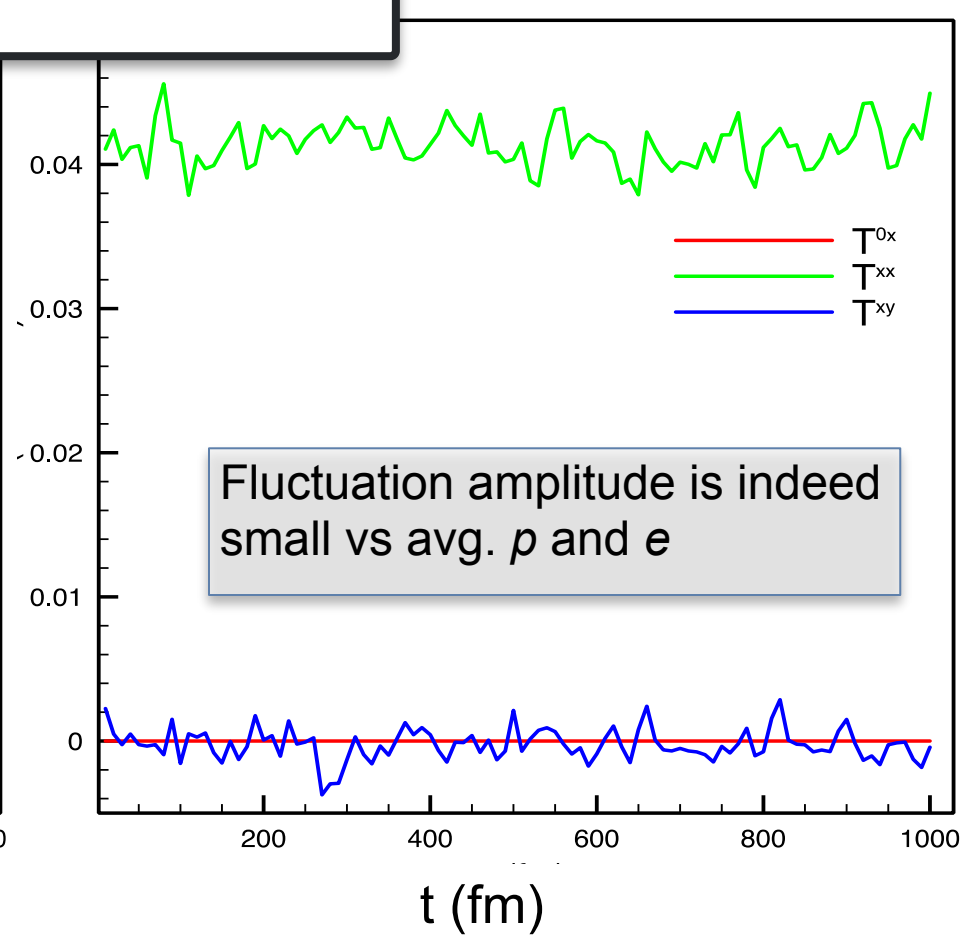
Assuming a nearly ideal gas, one can fit the temperature and chemical potential with momentum distributions:

$$\frac{dN}{dp} = \frac{g}{2\pi^2} V p^2 \exp\left(-\frac{\sqrt{p^2 + m^2} - \mu}{T}\right)$$

Energy density and pressure

$$T^{\mu\nu} = diag(e, p, p, p)$$

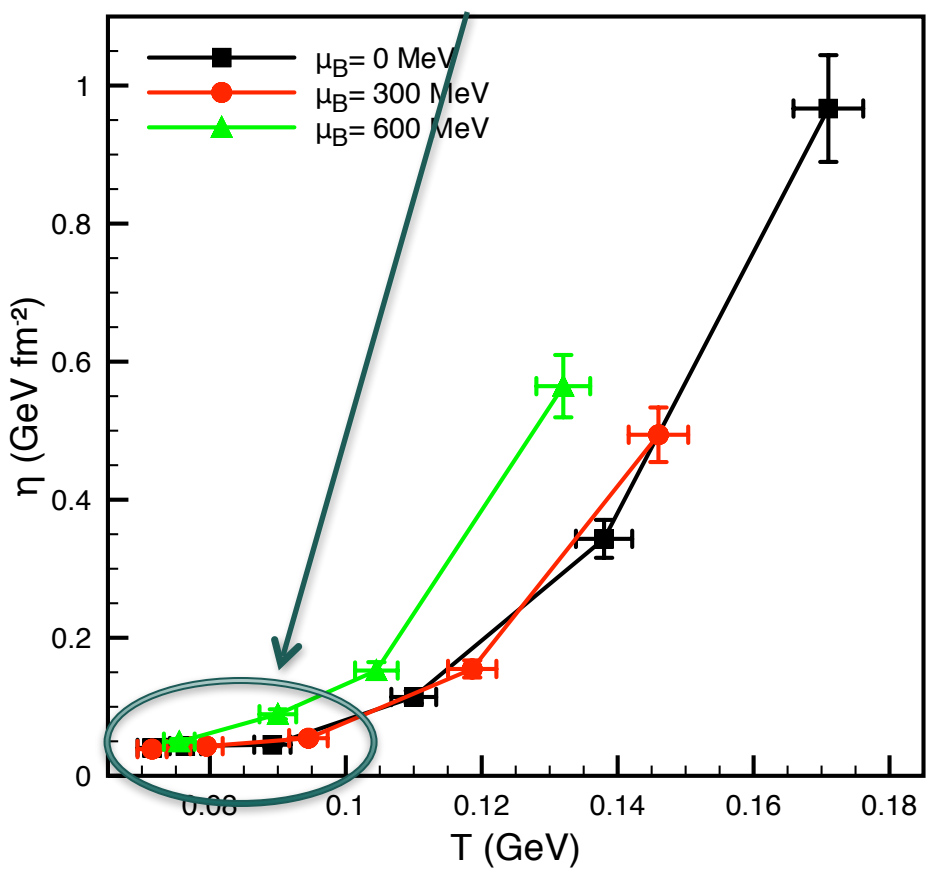


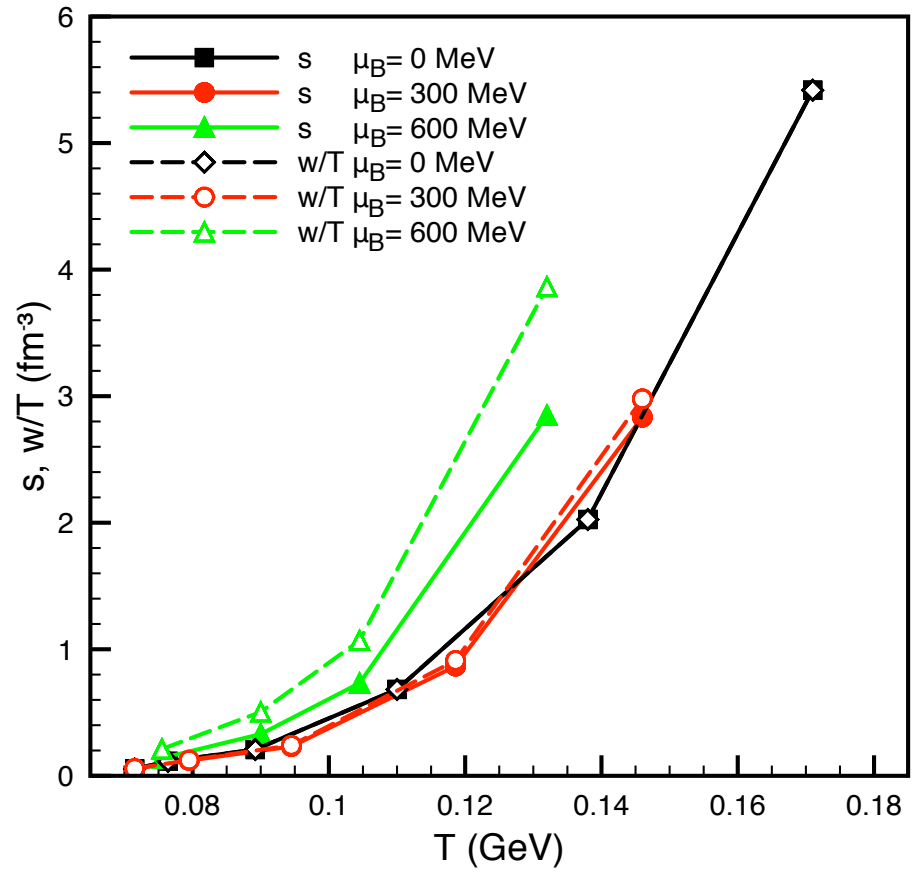


Fluctuation amplitude is indeed small vs avg. *p* and *e*

Hadron Gas: η, s and w/T

Viscosity decreases slower at small temperatures; explains rise of *η/s*





Hadron Gas: Low temperature η/s

- Low temperature hadron gas is composed almost exclusively of pions
- π-π cross-section is then most relevant
  - At very low energy, SMASH much higher than UrQMD/χPT
  - χPT includes angular dependence, UrQMD&SMASH don't; increases viscosity by factor up to 5/3 for ρ resonance