Shear viscosity and resonance lifetimes in the hadron gas

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Viscosity in heavy ion collisions

- Investigating deconfinement requires a good knowledge of transport coefficients
Viscosity in heavy ion collisions

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- Hydrodynamics relatively successful at explaining this with small $\eta/s$ above the transition.

Luzum & Romatschke 10.1103/Phys. Rev. C 78.034915
Viscosity in heavy ion collisions

- Investigating deconfinement requires a good knowledge of transport coefficients.
- Hydrodynamics relatively successful at explaining this with small $\eta/s$ above the transition.
- Still not clear what the behavior of $\eta/s$ is at low energies (FAIR, late stage RHIC/LHC).

Previous HG viscosity calculations

- Demir & Bass [Kubo & UrQMD]
- Pratt, Baez & Kim [B3D]
- Romatschke & Pratt [B3D]
- Rougemont et al. [Holo]
- Ozvenchuk et al. [RTA & DQPM]
- Moroz [RTA & UrQMD]
- Wiranata et al. [CE]
- χPT [CE]
Previous HG viscosity calculations

Multiple descriptions inconsistent with each other!
SMASH is a new semi-classical transport approach for the hadron gas.

Geometric collision criterion:

\[ d_{\text{trans}} < d_{\text{int}} = \sqrt{\frac{\sigma_{\text{tot}}}{\pi}} \]

Spectral functions of resonances are described by relativistic Breit-Wigner functions, with resonance lifetime:

\[ \tau_{\text{res}} = \frac{1}{\Gamma(m)} \]

Elastic scatterings parameterized for NN; all other elastic scatterings assumed to go through resonances.

Inelastic scatterings, currently include:
- NN\leftrightarrow NR, NN\leftrightarrow \Delta R
- KN\leftrightarrow KN, KN\leftrightarrow \pi H
- +antiparticles

Strings (turned off for detailed balance)
The shear viscosity is calculated from

$$\eta = \frac{V}{T} \int_0^\infty C^{xy}(t) dt$$

where

$$C^{xy}(t) = \frac{1}{N} \sum_{s} T^{xy}(s) T^{xy}(s+t)$$

and

$$T^{\mu\nu} = \frac{1}{V} \sum_{i}^{N_{\text{part}}} \frac{p_i^\mu}{p_i^0} \frac{p_i^\nu}{p_i^0}$$

$N$ is the number of time steps, and $N_{\text{part}}$ the number of particles.
Green-Kubo Formalism

It has been shown that the correlation function in

\[ \eta = \frac{V}{T} \int_0^{\infty} C^{xy}(t) dt \]

Follows

\[ C^{xy}(t) = C^{xy}(0) \exp\left(-\frac{t}{\tau}\right) \]

So that

\[ \eta = \frac{V C^{xy}(0) \tau}{T} \]
Equilibrium in SMASH

- Box calculations simulating infinite matter to apply the Green-Kubo procedure
- MUST have thermal & chemical equilibrium
- Baryon/antibaryon annihilation implemented to conserve detailed balance via an average decay to $5\pi$
Test case #1: $\pi$ with constant $\sigma$

Main take-away:
The method is relatively insensitive to variations of parameters; maximum error is less than 10%.

J. Torres-Rincon, PhD dissertation (2012), *Hadronic Transport Coefficients from Effective Field Theories*
Test case #2: $\pi$-$\rho$ gas

- Normal SMASH run does not coincide directly with Chapman-Enskog
Test case #2: $\pi$-$\rho$ gas

- Normal SMASH run does not coincide directly with Chapman-Enskog
  - Resonance lifetimes

![Diagram of particle interactions](image)
### Hadron Gas: Degrees of freedom

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- **Isospin symmetry**
- **Perturbative treatment of non-hadronic particles (photons, dileptons)**

Jean-Bernard Rose

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Hadron Gas: $T$ and $\mu_B$ dependence
HG: Viscosity Comparison

\[ \eta/s \text{ vs. } T \text{ (GeV)} \]

- Demir & Bass [Kubo & UrQMD]
- Pratt, Baez & Kim [B3D]
- Romatschke & Pratt [B3D]
- Rougemont et al. [Holo]
- Ozvenchuk et al. [RTA & DQPM]
- Moroz [RTA & UrQMD]
- Wiranata et al. [CE]
- \( \chi \text{PT} \) [CE]
- Full SMASH

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HG: Viscosity Comparison

SMASH in agreement with Demir & Bass and in contradiction with all others at high T
High temperature $\eta/s$ : Resonance lifetimes

Must look at the microscopic picture from different descriptions

$\tau_{\text{res}} = \text{resonance lifetime}$

$\tau_{\text{mft}} = \text{mean free time}$

At low $T$ and density:

$\tau_{\text{res}} \ll \tau_{\text{mft}}$
High temperature $\eta/s$ : Resonance lifetimes

Must look at the microscopic picture from different descriptions
$\tau_{\text{res}} = $ resonance lifetime
$\tau_{\text{mft}} = $ mean free time

At high T and density:

$T_{\text{res}} \sim T_{\text{mft}}$
High temperature $\eta/s$ : Resonance lifetimes

Momentum transport is delayed until resonances decay!

Must look at the microscopic picture from different descriptions:

- $\tau_{\text{res}} = \text{resonance lifetime}$
- $\tau_{\text{mft}} = \text{mean free time}$

At high T and density:

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Large part of the difference explained from eliminating lifetimes

\[ \pi - \rho: \text{Zero lifetimes vs relaxation time} \]
Effect of many non-resonant interactions

Introduce a constant elastic cross-section between all particles to add many non-resonant interactions

- Full SMASH
- Full SMASH + 10 mb elastic

![Graph showing temperature vs. τ_mft and η/s vs. T](image-url)
Summary & Outlook

• Investigated temperature, cross-section and mass dependence of the shear viscosity in an elastic pion box
  – Very good agreement with Chapman-Enskog approximation (within 10%)
  – Resonance lifetimes need to be considered

• Full hadron gas $\eta/s$ calculated at zero and non-zero $\mu_B$
  – High T discrepancy explained by looking at microscopic details of resonance modelling; finite lifetime increases viscosity
  – Could be used to constrain the treatment of resonances

• Outlook:
  – Investigation of angular dependent interactions on viscosity
  – At temperatures close to the phase transition, inclusion of multi-particle interaction will play a role, and needs to be investigated
  – Other transport coefficients (electrical conductivity, bulk viscosity, etc.)
Outlook: Electric conductivity

Resonant case shows lower sensitivity to resonance lifetimes

Multi-species case qualitatively comparable to previous calculations

Backup slides
How to fit?

\[ C_{xy}(0) = \frac{g \exp\left(\frac{\mu}{T}\right)}{30\pi^2V} \int_0^\infty dp \frac{p^6}{m^2 + p^2} \exp\left(-\frac{\sqrt{m^2 + p^2}}{T}\right) \]

**Graphs:**
- **Red:** 
  - \( T = 150 \) MeV
  - \( \sigma = 20 \) MeV
- **Black:** 
  - \( T = 150 \) MeV
  - Energy-dep. \( \sigma \)

**Legend:**
- Fixed intercept
- Floating intercept
- Analytic result
Where to stop the fitting?

J. Torres-Rincon, PhD dissertation (2012), *Hadronic Transport Coefficients from Effective Field Theories*
Constant $\sigma$: Systematics

Main take-away: The method is relatively inelastic to variations of most parameters; maximum error is less than 10%.
Test case #2: Energy-dependent $\sigma$

- Pions in a $(20 \text{ fm})^3$ box simulating infinite matter
- Cross-section uses $\rho$ resonance
- Runs for $t_{\text{max}}=200 \text{ fm}$
- Initialized with initial densities consistent with Boltzmann ideal gas
Hadron Gas

- All particles and resonances initialized to thermal multiplicities (at the pole mass)
- Must wait for equilibration and compute $T$, $\mu$ once in equilibrium from most abundant particles
  - $T$ fitted from weighted momentum spectra of $\pi$, $K$ & $N$
  - $\mu_B$ obtained from $N$/anti-$N$ ratio
What about entropy?

The entropy density can be calculated from the Gibbs formula:

\[ S = \frac{e + p - \mu n}{T} = \frac{w - \mu n}{T} \]

where the energy density and pressure can be taken from the average shear-stress tensor according to:

\[ T^{\mu\nu} = \text{diag}(e, p, p, p) \]

Assuming a nearly ideal gas, one can fit the temperature and chemical potential with momentum distributions:

\[ \frac{dN}{dp} = \frac{g}{2\pi^2} V p^2 \exp\left(-\frac{\sqrt{p^2 + m^2} - \mu}{T}\right) \]
Energy density and pressure

\[ T^{\mu\nu} = \text{diag}(e, p, p, p) \]

Fluctuation amplitude is indeed small vs avg. \( p \) and \( e \)
Hadron Gas: $\eta$, $s$ and $w/T$

Viscosity decreases slower at small temperatures; explains rise of $\eta/s$
Hadron Gas: Low temperature $\eta/s$

- Low temperature hadron gas is composed almost exclusively of pions

- $\pi-\pi$ cross-section is then most relevant
  
  - At very low energy, SMASH much higher than UrQMD/$\chi$PT
  
  - $\chi$PT includes angular dependence, UrQMD&SMASH don’t; increases viscosity by factor up to 5/3 for $\rho$ resonance