Evolution of multiplicity fluctuations in heavy ion collisions

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Motivation

- Measured moments of the multiplicity distribution $\longrightarrow$ determination of the hadronisation parameters of hot QCD matter
- There are still some inelastic scatterings after hadronisation that may drive the multiplicity distribution out of equilibrium
- We demonstrate how the different moments depart away from their equilibrium values
- We would obtain different apparent temperatures from different moments
- For the description of the evolution of the multiplicity distribution we use a master equation

Master equation

- We consider a binary process $a_1 a_2 \longleftrightarrow b_1 b_2$ with $a \neq b$, this equation is suitable for studying rare particles
- Master equation for $P_n(t)$, the probability of finding n pairs $b_1 b_2$ has the following form

$$\frac{dP_n}{dt}(t) = \frac{G}{V} \langle N_{a_1} \rangle \langle N_{a_2} \rangle \left[P_{n-1}(t) - P_n(t) \right] - \frac{L}{V} \left[n^2 P_n(t) - (n+1)^2 P_{n+1}(t) \right], \tag{1}$$

where $n = 0, 1, 2, 3...$, V - proper volume of the reaction

- For thermal distribution of particle momentum $\rightarrow$ $G \equiv \langle \sigma_G v \rangle$ - gain term (creation), $L \equiv \langle \sigma_L v \rangle$ - loss term (anihilation) $\Rightarrow$ thermally averaged cross-sections

Relaxation of factorial moments

- When we want to study thermalisation $\longrightarrow$ then it is usefull overwrite the equation into dimensionless time variable $\tau = t\ L/V$, $V/L = \tau_0^c$ - relaxation time
- We use then constant $\epsilon = G \langle N_{a_1} \rangle \langle N_{a_2} \rangle / L$
- From the master equation we can derive equilibrium distribution of the factorial moments
- Equilibrium values of factorial moments

$$F_{2,eq} = -\frac{1}{2}\sqrt{\varepsilon}\frac{\mathrm{I}_1(2\sqrt{\varepsilon})}{\mathrm{I}_0(2\sqrt{\varepsilon})} + \frac{1}{2}\varepsilon\frac{\mathrm{I}_2(2\sqrt{\varepsilon}) + \mathrm{I}_0(2\sqrt{\varepsilon})}{\mathrm{I}_1(2\sqrt{\varepsilon})}$$

$$F_{3,eq} = \frac{3}{4}\sqrt{\varepsilon}\frac{\mathrm{I}_1(2\sqrt{\varepsilon})}{\mathrm{I}_0(2\sqrt{\varepsilon})} - \frac{3}{4}\varepsilon\left(1 + \frac{\mathrm{I}_2(2\sqrt{\varepsilon})}{\mathrm{I}_0(2\sqrt{\varepsilon})}\right) + \frac{1}{4}\varepsilon^{3/2}\frac{\mathrm{I}_3(2\sqrt{\varepsilon}) + 3\mathrm{I}_1(2\sqrt{\varepsilon})}{\mathrm{I}_0(2\sqrt{\varepsilon})}$$

$$F_{4,eq} = \ldots \tag{2}$$

Equilibrium values of factorial moments

- We let the distribution of the multiplicity relax with the help of master equation
- We will study the evolution of factorial moments
- Scaled moments

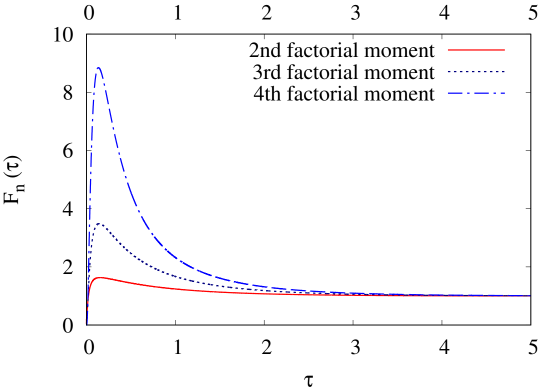
$$F_2(\tau) = \langle N(N-1) \rangle \,/\, \langle N \rangle^2 \,, \tag{3}$$

$$F_3(\tau) = \langle N(N-1)(N-2) \rangle \,/\, \langle N \rangle^3 \,, \tag{4}$$

$$F_4(\tau) = \langle N(N-1)(N-2)(N-3) \rangle \,/\, \langle N \rangle^4 \tag{5}$$

- For numerical calculations were used binomial initial conditions

Relaxation of 2nd, 3rd and 4th factorial moments



- All moments relax at the same dimensionless time τ
- Higher moments differ more from their equilibrium values than the lower moments
- For $\epsilon = 0.1$ and $N_0 = 0.005$

Real time dependent master equation

- Master equation defined in dimensionless time can be used only for constant temperature
- Now, fireball will cool down $\longrightarrow$ temperature will change $\longrightarrow$ relaxation time will change $\longrightarrow$ we use master equation which was defined on the 1st slide (1)
- We will calculate the evolution for inelastic process $\pi^+ + n \longleftrightarrow K^+ + \Lambda$
- At 165 MeV there is hadronisation $\longrightarrow$ fireball is still cooling down $\longrightarrow$ how does the distribution of multiplicity change?

Model for the evolution of the fireball

- Bjorken 1D expansion with initial temperature $T_0 = 165\ \mathrm{MeV}$
- Fireball cools down according to the relation

$$T^3 = T_0^3 \frac{t_0}{t} \tag{6}$$

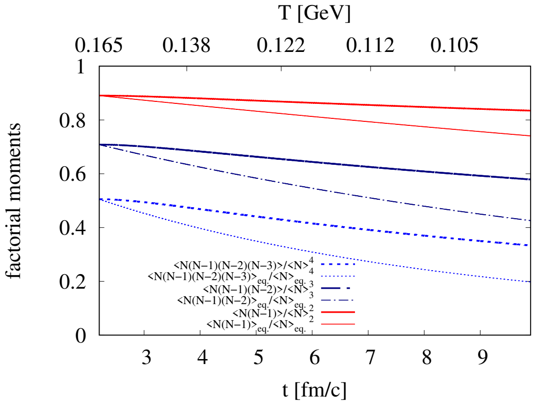
  down to temperature $T = 100\ \mathrm{MeV}$, $t_0 = 2.2\ \mathrm{fm/c}$

- System volume grows linearly

$$V = V_0 \frac{t}{t_0} \tag{7}$$

- The flux-averaged G and L-term are too small and lead to too low reaction rate $\rightarrow$ in order to proceed with qualitative studies we scale up the cross-section by hand
- The reaction rate might increase considerably if the threshold is lowered $\longrightarrow$ we investigate density dependent masses in the 2nd part

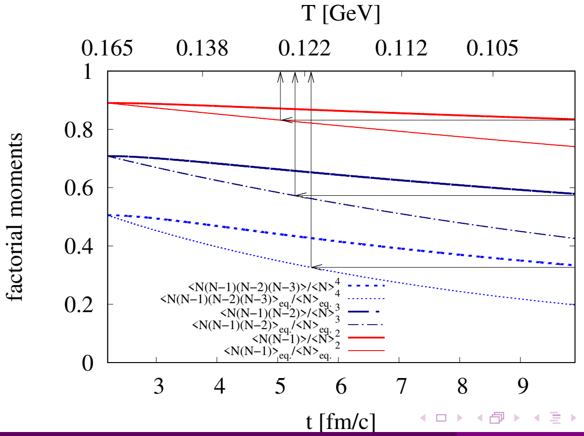
Scaled factorial moments for gradual change of temperature



- Gradual change of temperature from 165 MeV to 100 MeV
- $\tau = 6$ fm/c for $T = 165$ MeV
- For 15 pions and 10 neutrons
- 50times enlarged cross-section
- $V_0 = 125\ \mathrm{fm}^3$

Apparent freeze-out temperature

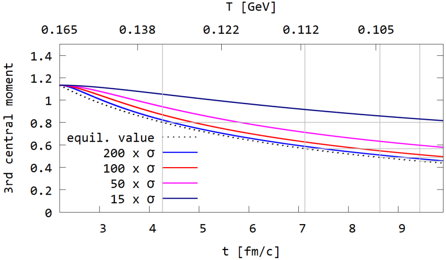
- Suppose we observe the final values of factorial moments (at 100 MeV) $\longrightarrow$ How we can extract the temperature from such a measurement? $\longrightarrow$ we assume thermalisation, although the system evolved along the thick lines, one assumes as if did so along the thin lines

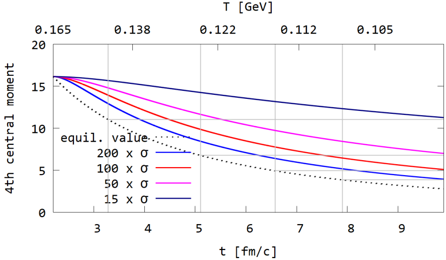


Central moments

- For data processing $\rightarrow$ central moments, event. their combination
- 2nd central moment $\mu_2 = \left\langle N^2 \right\rangle - \left\langle N \right\rangle^2$
- 3rd central moment $\mu_3 = \left\langle (N - \langle N \rangle)^3 \right\rangle$
- 4th central moment $\mu_4 = \left\langle (N - \langle N \rangle)^4 \right\rangle$
- Coefficient of skewness $S = \frac{\mu_3}{\sigma^3} = \frac{\left\langle (N - \langle N \rangle)^3 \right\rangle}{\left\langle (N - \langle N \rangle)^2 \right\rangle^{3/2}}$.
- Coefficient of kurtosis $\kappa = \frac{\mu_4}{\sigma^4} - 3 = \frac{\left\langle (N - \langle N \rangle)^4 \right\rangle}{\left\langle (N - \langle N \rangle)^2 \right\rangle^2} - 3$

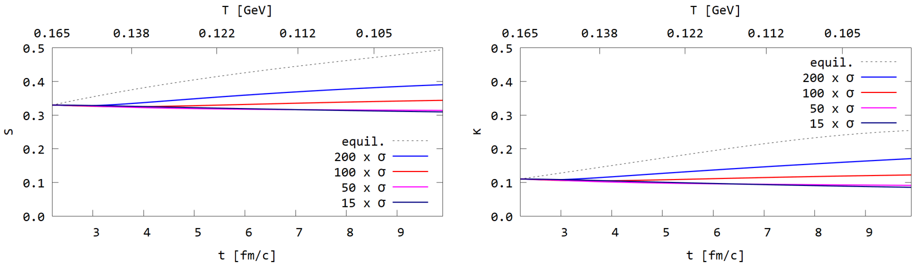
Time evolution of central moments





- Gradual change of temperature from 165 MeV to 100 MeV for different cross-sections, for 15 pions and 10 neutrons
- What happens with the moments when the fireball cools down? $\longrightarrow$ we have nonequilibrium values $\longrightarrow$ it is difficult to extract the exact freeze-out temperature

Time evolution of skewness and kurtosis



- Gradual change of temperature from 165 MeV to 100 MeV for different cross-sections, for 15 pions and 10 neutrons

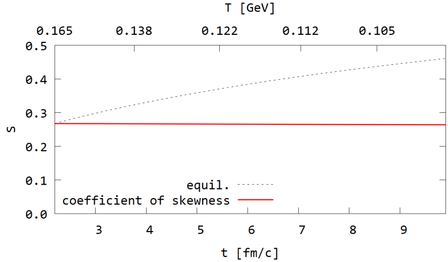
Decreasing mass of Λ hyperon

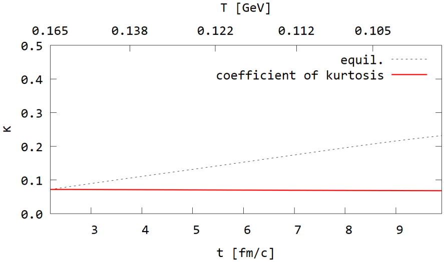
- By lowering the mass of the hyperon $\longrightarrow$ threshold for the reaction is lowered $\longrightarrow$ its rate may grow due to the increase of the available phase space
- Rather extreme and simplifed dependence of the hyperon mass on baryon density

$$m_\Lambda(\rho_B) = \frac{\rho_0 - \rho_B}{\rho_0} m_{\Lambda 0} + \frac{\rho_B}{\rho_0} m_{\rho 0} \tag{8}$$

  so that the hyperon mass becomes identical to that of the proton at the highest baryon density ρ_0 at which our calculation starts, and returns to the vacuum value $m_{\Lambda 0}$ if baryon density vanishes
- Density also behave like in 1D longitudinally boost-invariant expansion $\rho = \rho_0 \frac{t_0}{t}$, $\rho_0 = 0.08\ \mathrm{fm}^{-3}$

Time evolution for the density dependent mass





- The relaxation time in this case is about 100 $\mathrm{fm/c}$ for 165 MeV
- Thanks to the increase of the cross-section the system is driven away from the initial state at 165 MeV
- Other channels can change the numbers of kaons (lambdas) $\longrightarrow$ moments may change more then indicated by these calculations

Conclusion

- If equilibrium is broken, higher factorial moments of multiplicity distribution differ more from their equilibrium values than the lower moments
- Evolution of chemical reaction off equilibrium may show different temperatures for different orders of the (factorial or central) moments (we demonstrated this on the reaction $\pi^+ + n \longleftrightarrow K^+ + \Lambda$)
- The behavior of the combination of the central moments depends on the combination of moments we choose
- We should be very careful when we want to extract the freeze-out temperature from higher moments
- This work was published: arXiv:1809.09653

Backup

Generating function

- The master equation can be converted into the partial differential equation for the generating function

$$g(x,\tau) = \sum_{n=0}^{\infty} x^n P_n(\tau) \tag{9}$$

- From the derivative of the generating function we can easily determine the moments.
- Multiplying eq. (1) by x^n and summing over n, we find

$$\frac{\partial g(x,\tau)}{\partial \tau} = \frac{L}{V}(1-x)(xg'' + g' - \epsilon g), \tag{10}$$

where $g' \equiv \partial g / \partial x$

- The equilibrium solution, $g_{eq.}(x)$, thus obeys the following equation:

$$xg_{eq}^{''} + g_{eq}^{'} - \epsilon g_{eq} = 0. \tag{11}$$

- The solution that is regular at $x = 0$ is then given by

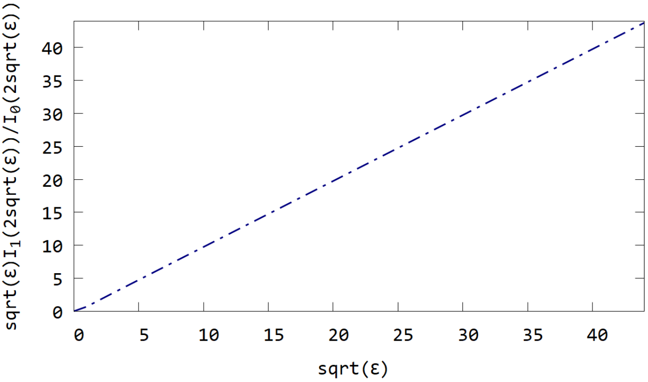
$$g_{eq}(x) = \frac{I_0(2\sqrt{\epsilon x})}{I_0(2\sqrt{\epsilon})} \tag{12}$$

- The average number of $b_1 b_2$ pairs per event in equilibrium is given by

$$\langle N \rangle_{eq} = g_{eq}'(1) = \sqrt{\epsilon}\frac{I_1(2\sqrt{\epsilon})}{I_0(2\sqrt{\epsilon})} \tag{13}$$

- 2nd derivative gives us than equilibrium value of the 2nd factorial moment, ...

- Dependence of $\sqrt{\epsilon}\dfrac{I_1(2\sqrt{\epsilon})}{I_0(2\sqrt{\epsilon})}$ on $\sqrt{\epsilon}$



Binomial initial conditions

- We can assume that initially there is at most one particle in given event
- Then the initial conditions are

$$P_0(\tau = 0) = 1 - N_0 \tag{14}$$

$$P_1(\tau = 0) = N_0 \tag{15}$$

$$P_n(\tau = 0) = 0, n > 1 \tag{16}$$

where N_0 is initial averaged number of particles

- In this case, the factorial moments then start at 0

Temperature dependent master equation

- Because of averaging over relative velocities, we will assume that the momenta are distributed according to Boltzmann distribution

$$n_i(p) \propto exp\left(-\frac{\sqrt{m_i^2 + p^2}}{T}\right) \tag{17}$$

- The averaged cross section is then obtained as

$$\left\langle v_{ij}\sigma_{ij}^X\right\rangle = \frac{\int_{\sqrt{s_0}}^{\infty} dx \sigma_{ij}^X(x) K_1(\frac{x}{T}) \left[x^2 - (m_i + m_j)^2\right] \left[x^2 - (m_i - m_j)^2\right]}{4 m_i^2 m_j^2 T K_2(m_i/T) K_2(m_j/T)} \tag{18}$$

where K_i's are the modified Bessel functions and $\sqrt{s_0} = max(m_i + m_j, \Sigma_{final} m_a)$ is the reaction threshold

- If we know cross section for the reactions $a_1 a_2 \to b_1 b_2$, the cross section for the inverse reactions follows from phase-space considerations as

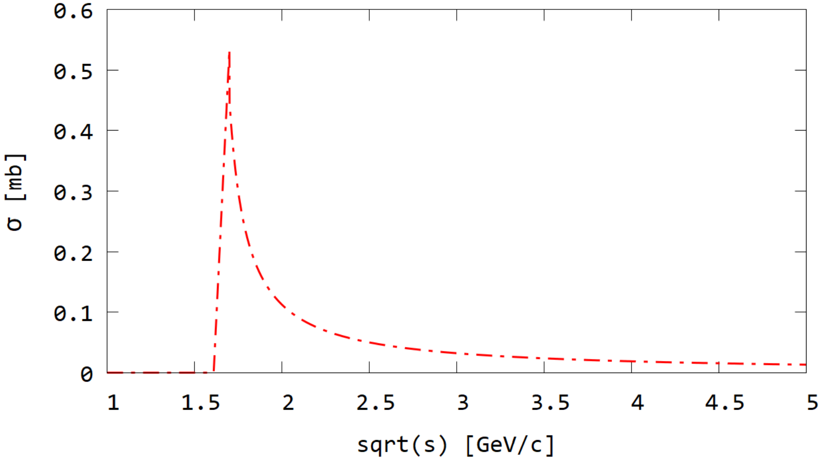
$$\sigma_{34 \longrightarrow 12}(\sqrt{s}) = \frac{(2J_3+1)(2J_4+1)}{(2J_1+1)(2J_2+1)} \frac{p_{cm}^2(s, m_1, m_2)}{p_{cm}^2(s, m_3, m_4)} \times \sigma_{12 \longrightarrow 34}(\sqrt{s}) \tag{19}$$

where J_i and m_i are spins and masses of the participating species, and p_{cm} is the center-of-mass momentum defined as

$$p_{cm}^2(s, m_1, m_2) = \frac{\left[s - (m_1^2 + m_2^2)\right]^2 - 4m_1^2 m_2^2}{4s} \tag{20}$$

Reaction $\pi^+ + n \longrightarrow K^+ + \Lambda^0$.

- Volume of the reaction is $V = 125\ \mathrm{fm}^3$
- Cross-section for this reaction is



Decreasing mass of Λ hyperon

- Dependance of the $m(\rho)$