Pions in Electromagnetic Field

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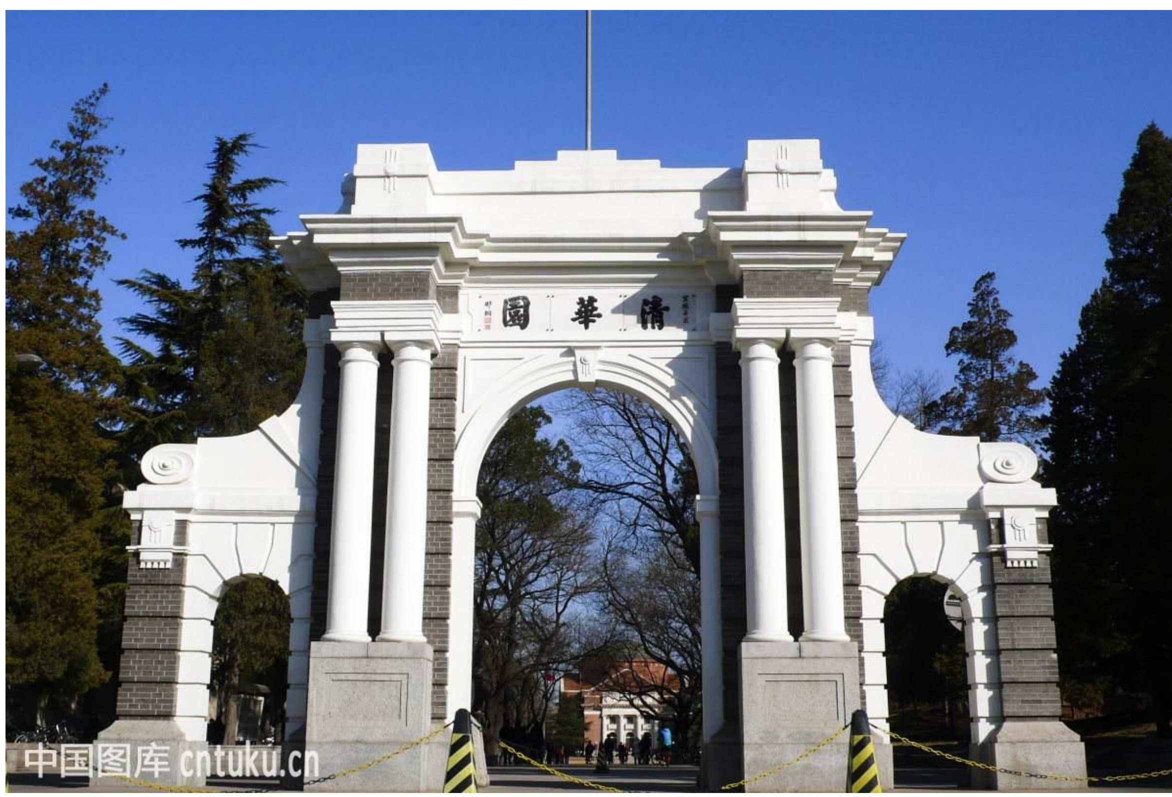
Chirality Workshop 2019

The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

The 1st (2015), 2nd(2016), and 3rd (2017) workshops at UCLA
The 4th workshop at Florence

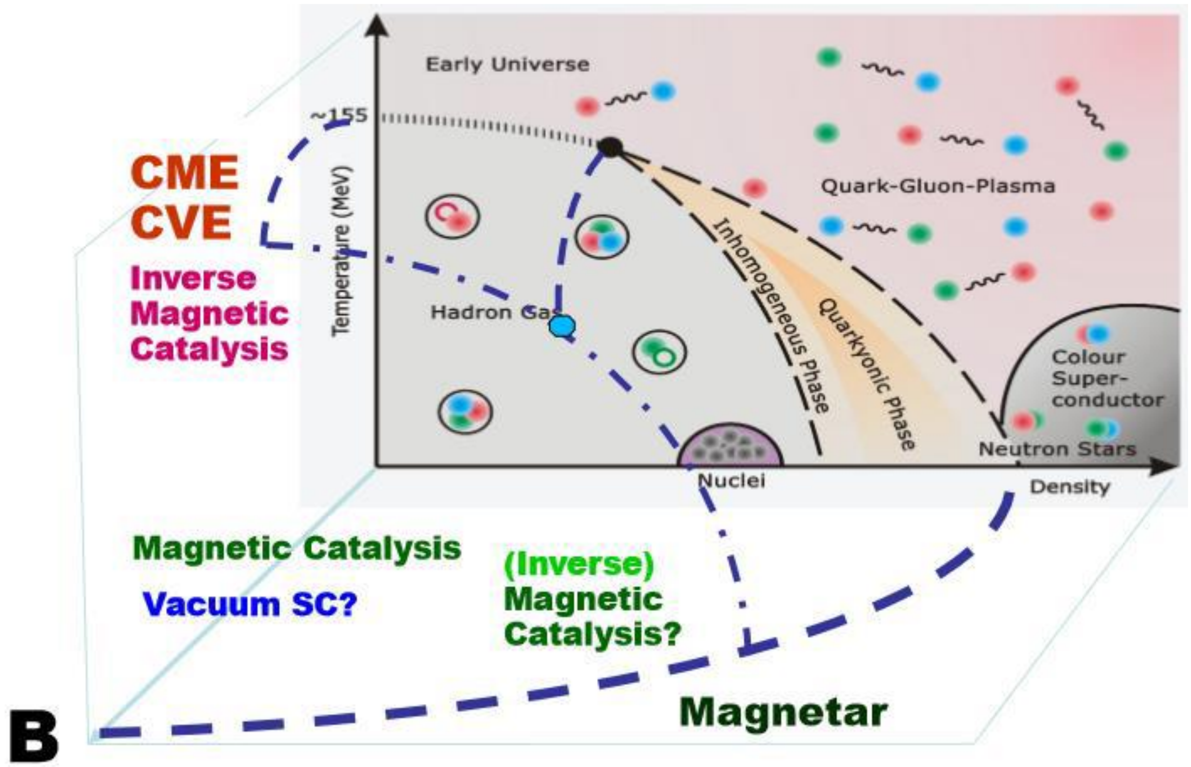
- *Place:* *Tsinghua University, Beijing*
- *Date:* *Beginning of April, 2019 (not yet precisely determined)*

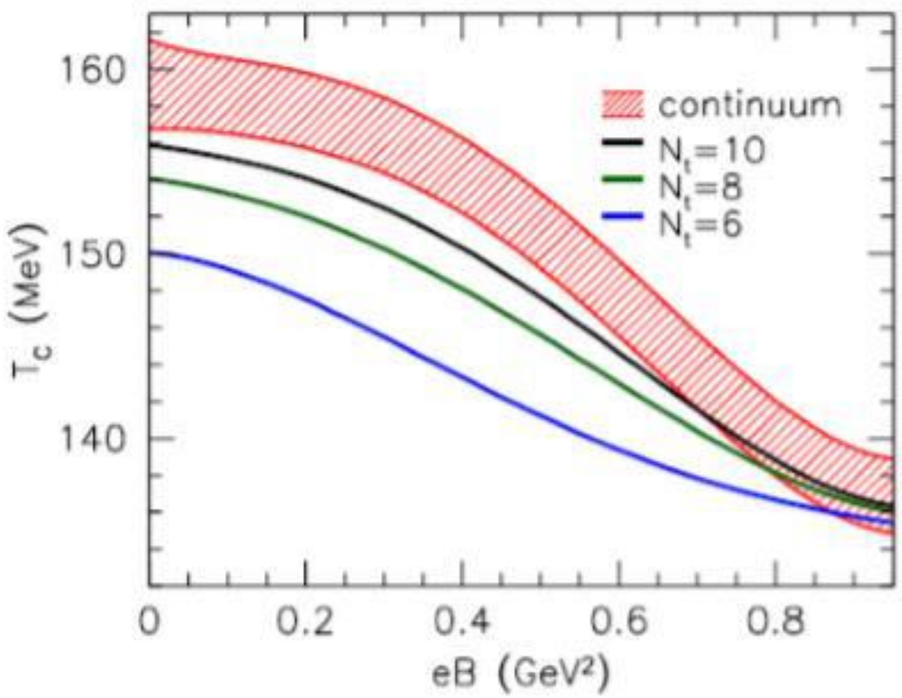


You are welcome to join the workshop!

Chiral Symmetry in Magnetic Field

- *Chiral symmetry in external magnetic field*





inverse magnetic catalysis

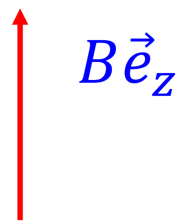
- *Pions (as Goldstone modes) in external magnetic field*

$SU(2) \times SU(2)$ *chiral symmetry* $\rightarrow$ $U(1) \times U(1)$

3 Goldstone modes $(\pi_0, \pi_\pm) \rightarrow$ *1 Goldstone mode* (π_0)

What is the change in pion properties in electromagnetic field?

Problem



There exists a special direction,

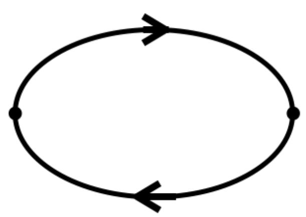
There is no more translation invariant !

- *Quark propagator in Schwinger formalism,*

$$S(x,y) = e^{i\Phi(x,y)}\tilde{S}(x-y),$$

the Schwinger phase $\Phi(x,y)$ *is not translation invariant !*

How to construct hadrons with Schwinger quark propagators?



A usually used way is to neglect the phase and take only the invariant part, see, for instance, PRD94, 113006(2016) and EPJC76, 307(2016).

- *Quark propagator in Ritus formalism*

Fourier transformed momentum $\tilde{p} = (p_0, 0, p_2, p_3)$,

Conserved Ritus momentum $\bar{p} = \left(p_0, 0, -sgn(Q_q B)\sqrt{2n|Q_q B|}, p_3\right)$.

for mesons in Ritus formalism, see arXiv:1808.10242 by Shijun Mao

- *In this talk, we include Schwinger phase in constructing mesons.*

Quarks at mean field level

- *NJL model at quark level*

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu - m_0\right)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\vec{\tau}\psi)^2]$$

$$D_\mu = \partial_\mu + iQA_\mu, \quad Q = diag(Q_u = 2e/3, Q_d = -e/3), \quad A_\mu = (0, 0, Bx, 0)$$

- *Introducing scalar and pseudo-scalar meson fields*

$$M = (\sigma,\ \vec{\pi}) = -2G(\bar{\psi}\psi,\ \bar{\psi}i\gamma^5\vec{\tau}\psi)$$

and integrating out the fermion fields,

$$\mathcal{L} = -\sum_M \frac{(g_M M)^2}{4G} - i\,Tr\,Ln\left(i\gamma^\mu D_\mu - m_0 - \sum_M \Gamma_M M\right)$$

$$\Gamma_M = (\Gamma_\sigma, \Gamma_{\vec{\pi}}) = (g_\sigma, ig_{\vec{\pi}}\gamma_5\vec{\tau})$$

- *Introducing chiral condensate $\sigma \rightarrow \langle\sigma\rangle + \sigma$,*

$$\mathcal{L}_{MF} = -\frac{(m - m_0)^2}{4G} - i\,Tr\,Ln\,S^{-1}, \qquad m = m_0 + g_\sigma\langle\sigma\rangle$$

$$S^{-1} = i\gamma^\mu D_\mu - m = diag(S_u, S_d)$$

- *Mean field quark propagator in Schwinger formalism*

$$S_q(x, y) = e^{i\Phi_q(x,y)}\tilde{S}_q(x - y)$$

Nucl. Phys. B462, 249(1996) by V.Gusynin, V.Miransky, and I.Shovkovy

Going beyond mean field

- *Including σ and $\vec{\pi}$ fluctuations,*

$$\mathcal{L} = \mathcal{L}_{MF} - \frac{2(m-m_0)\sigma + \sum_M (g_M M)^2}{4G} + \mathcal{L}_{FD}, \qquad \mathcal{L}_{FD} = -i\, Tr\, Ln\,(1 - S\sum_M \Gamma_M M)$$

- *Derivative expansion*

$$\mathcal{L}_{FD} = \sum_{n=1}^{\infty} \mathcal{L}^{(n)}, \qquad \mathcal{L}^{(n)} = \frac{i}{n} Tr(S\sum_M \Gamma_M M)^n$$

- *Linear term in meson fields*

$$\mathcal{L}^{(1)} = iN_c \sum_{q,p} Tr\tilde{S}_q(p) g_\sigma \sigma$$

the disappearance of the whole linear term from $\mathcal{L}$ leads to the gap equation for m:

$$m = m_0 + 2iGN_c \sum_{q,p} Tr\,\tilde{S}_q(p)$$

- *Quadratic term in meson fields*

$$\mathcal{L}^{(2)} = \frac{i}{2} Tr\left(S\sum_M \Gamma_M M\right)^2 = \mathcal{L}_\sigma^{(2)} + \mathcal{L}_{\pi_0}^{(2)} + \mathcal{L}_{\pi_\pm}^{(2)}$$

$$\mathcal{L}_\sigma^{(2)} = \frac{i}{2} g_\sigma^2\, Tr(S_u\sigma S_u\sigma + S_d\sigma S_d\sigma),$$

$$\mathcal{L}_{\pi_0}^{(2)} = \frac{i}{2} g_\pi^2\, Tr(S_u i\gamma_5\pi_0 S_u i\gamma_5\pi_0 + S_d i\gamma_5\pi_0 S_d i\gamma_5\pi_0)$$

$$\mathcal{L}_{\pi_\pm}^{(2)} = ig_\pi^2\, Tr\,(S_u i\gamma_5\pi_- S_d i\gamma_5\pi_+ + S_d i\gamma_5\pi_+ S_u i\gamma_5\pi_-)$$

Wave function renormalization

- $$\mathcal{L}_\sigma^{(2)} = \frac{i}{2} g_\sigma^2\, Tr(S_u\sigma S_u\sigma + S_d\sigma S_d\sigma)$$
$$= \frac{i g_\sigma^2 N_c}{2} \sum_{q,y} e^{i\left(\Phi_q(x,y)+\Phi_q(y,x)\right)} Tr\, \tilde{S}_q(x-y)\tilde{S}_q(y-x)\sigma(y)\sigma(x)$$

the two Schwinger phases cancel to each other,

$$\Phi_q(x,y) + \Phi_q(y,x) = 0,$$

only the translation invariant part $\tilde{S}_q$ contributes to neutral mesons.

- *Local expansion*

$$\sigma(y) = \sigma(x) + (y-x)^\mu \partial_\mu \sigma(x) + \frac{1}{2}(y-x)^\mu (y-x)^\nu \partial_\mu \partial_\nu \sigma(x) + \cdots$$

- *mass term in $\mathcal{L}_\sigma^{(2)}$:* $\sim\sigma^2(x)$

  surface term: $\sim\sigma(x)\partial_\mu\sigma(x)$, *disappears after integration over* y

  kinetic term: $\sim\sigma(x)\partial_\mu\partial_\nu\sigma(x)$

- $$\mathcal{L}_\sigma^{(2)} = -\frac{1}{2}\bar{m}_\sigma^2 \sigma^2(x) + \frac{1}{2} F_\sigma^{\mu\nu} \partial_\mu\sigma(x)\partial_\nu\sigma(x)$$

Mass term in terms of $\tilde{S}_q$: $\bar{m}_\sigma^2 = -i g_\sigma^2 N_c \sum_{q,p} Tr\ \tilde{S}_q(p)\tilde{S}_q(p)$

Wave function renormalizations in terms of $\tilde{S}_q$: $F_\sigma^{\mu\nu} = \frac{i g_\sigma^2 N_c}{2} \sum_{q,p} Tr \left[\tilde{S}_q(p) \frac{\partial^2}{\partial p_\mu \partial p_\nu} \tilde{S}_q(p)\right]$

Similar treatment for π_0

Schwinger phases for charged mesons

- $\mathcal{L}_{\pi_-}^{(2)} = ig_\pi^2\, Tr\,(S_u i\gamma_5 \pi_- S_d i\gamma_5 \pi_+)$
  $= ig_\pi^2 N_c \sum_y e^{i\Phi_{\pi_-}(y,x)} Tr\left[\tilde{S}_u(x-y) i\gamma_5 \tilde{S}_d(y-x) i\gamma_5 \pi_-(y)\pi_+(x)\right]$

- *Schwinger phase*

$$\begin{aligned}\Phi_{\pi_-}(y,x) &= \Phi_u(x,y) + \Phi_d(y,x)\\ &= Q_u \int_y^x A^\mu(x')dx'_\mu + Q_d \int_x^y A^\mu(x')dx'_\mu\\ &= -e\int_x^y A^\mu(x')dx'_\mu\end{aligned}$$

- *Local expansion*

$$\pi_-(y) = \pi_-(x) + (y-x)^\mu \partial_\mu \pi_-(x) + \frac{1}{2}(y-x)^\mu (y-x)^\nu \partial_\mu \partial_\nu \pi_-(x) + \cdots$$

$$e^{i\Phi_{\pi_-}(y,x)} = 1 - ieA^\mu(x)(y-x)_\mu - \frac{ie}{2}\partial^\nu A^\mu(x)(y-x)_\mu (y-x)_\nu + \frac{(-ie)^2}{2}\left(A^\mu(x)(y-x)_\mu\right)^2 + \cdots$$

- *Covariant derivatives acting on charged mesons*

$$\mathcal{L}_{\pi_+}^{(2)} + \mathcal{L}_{\pi_-}^{(2)} = \frac{1}{2}\left(F_{\pi_+}^{\mu\mu}\left|D_\mu^+ \pi_+(x)\right|^2 + F_{\pi_-}^{\mu\mu}\left|D_\mu^- \pi_-(x)\right|^2\right) - \frac{1}{2}\left(\bar{m}_{\pi_+}^2 |\pi_+|^2 + \bar{m}_{\pi_-}^2 |\pi_-|^2\right)$$

Covariant derivatives $D_\mu^\pm = \partial_\mu \pm ieA_\mu$ coming from the Schwinger phases

Mass terms in terms of $\tilde{S}_q$: $\bar{m}_{\pi_+} = \bar{m}_{\pi_-}$

Wave function renormalizations in terms of $\tilde{S}_q$: $F_{\pi_+}^{\mu\mu} = F_{\pi_-}^{\mu\mu}$

Curvature mass

- *Effective Lagrangian of the quark-meson plasma*

$$\mathcal{L} = \mathcal{L}_{MF} + \sum_{M}\left[\frac{1}{2}F_M^{\mu\mu}\left|D_\mu^M M(x)\right|^2 - \frac{1}{2}\left(\frac{g_M^2}{2G} + \bar{m}_M^2\right)|M(x)|^2\right]$$

$$D_\mu^\sigma = D_\mu^{\pi_0} = \partial_\mu\,, \qquad D_\mu^\pm = \partial_\mu \pm ieA_\mu$$

- *Curvature mass*

$$m_M^2 = \frac{g_M^2}{2G} + \bar{m}_M^2$$

- *Comparing with the gap equation*

$$m_{\pi_0} = 0$$

in chiral breaking phase, π_0 is the Goldstone mode of chiral symmetry breaking in magnetic field.

Pole mass

$$\mathcal{L} = \mathcal{L}_{MF} + \sum_M \left[\frac{1}{2} F_M^{\mu\mu} \left|D_\mu^M M(x)\right|^2 - \frac{1}{2}\left(\frac{g_M^2}{2G} + \bar{m}_M^2\right) |M(x)|^2\right]$$

- *On-shell condition for non-interacting hadrons*

$$p_0^2 - \vec{p}^2 = m_M^2 \ ,$$

in electromagnetic field,

$$F_M^{00} p_0^2 - F_M^{11} p_1^2 \text{-} F_M^{22} p_2^2 - F_M^{33} p_3^2 = m_M^2 \qquad \text{for } M = \sigma, \pi_0$$

$$F_M^{00} p_0^2 - F_M^{11} (2n+1)|eB| - F_M^{33} p_3^2 = m_M^2 \qquad \text{for } M = \pi_\pm$$

- *Pole mass*

$$p_0 = m_M^{(0)} \ \text{ at } \ p_1 = p_2 = p_3 = 0 \quad \text{for } \ M = \sigma, \pi_0$$

$$n = p_3 = 0 \qquad \text{for } \ M = \pi_\pm$$

$$m_M^{(0)} = \begin{cases} \dfrac{m_M}{\sqrt{F_M^{00}}} & for \ \ M = \sigma, \pi_0 \\ \sqrt{(m_M^2 + F_M^{11}|eB|)/F_M^{00}} & for \ \ M = \pi_\pm \end{cases}$$

Screening mass and screening radius

$$\mathcal{L} = \mathcal{L}_{MF} + \sum_M \left[\frac{1}{2} F_M^{\mu\mu} \left| D_\mu^M M(x) \right|^2 - \frac{1}{2}\left(\frac{g_M^2}{2G} + \bar{m}_M^2\right) |M(x)|^2\right]$$

- *Without magnetic field,*

$$on - shell\ condition\ at\ |\vec{p}| = i m_M^{scr}\ and\ p_0 = 0$$

with electromagnetic field,

$p_j = i m_M^{(j)}$ in the direction $\vec{e}_j$ at $p_0 = p_k = 0$ for $k \neq j$

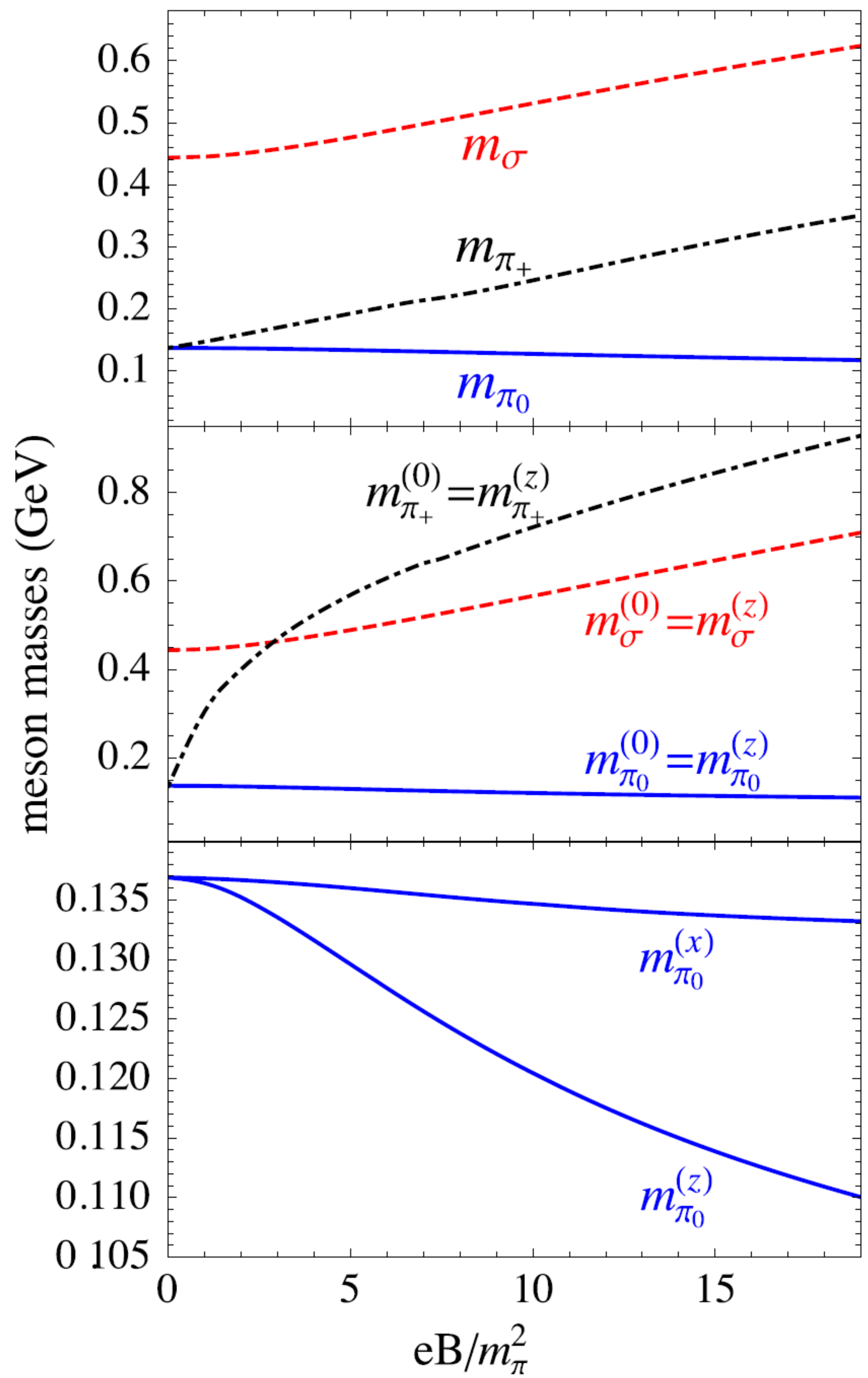
- *Screening radius*

$$r_M^{(j)} = \frac{1}{m_M^{(j)}}$$

- *The Goldstone mode π_0 propagates the long range interaction*

$$m_{\pi_0}^{(j)}, \qquad r_{\pi_0}^{(j)}$$

Numerical results (I)



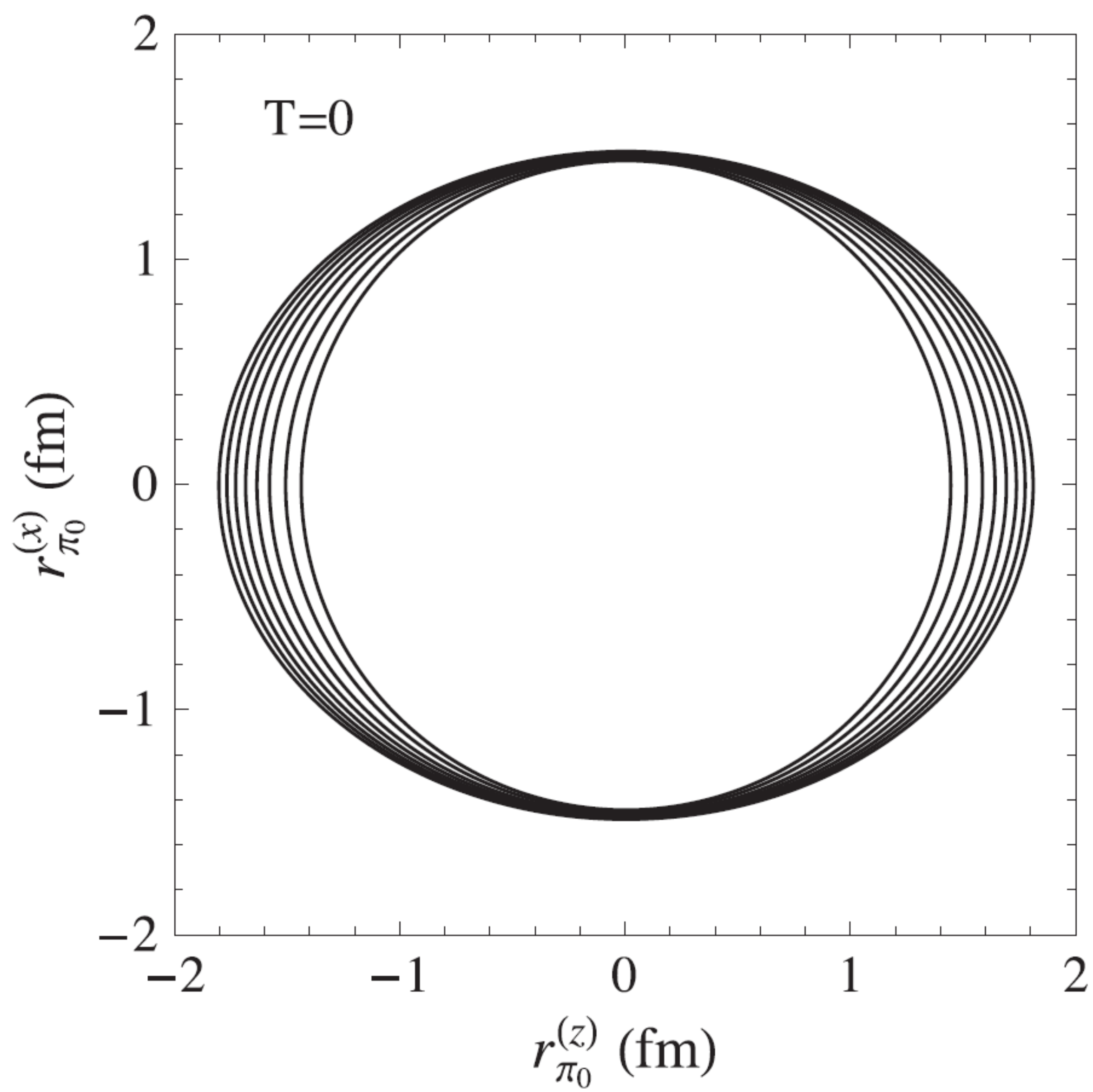
T=0

1) $\vec{B}$ *leads to the pion mass splitting*

2) *The difference between curvature and pole masses is due to the wave function renormalization.*

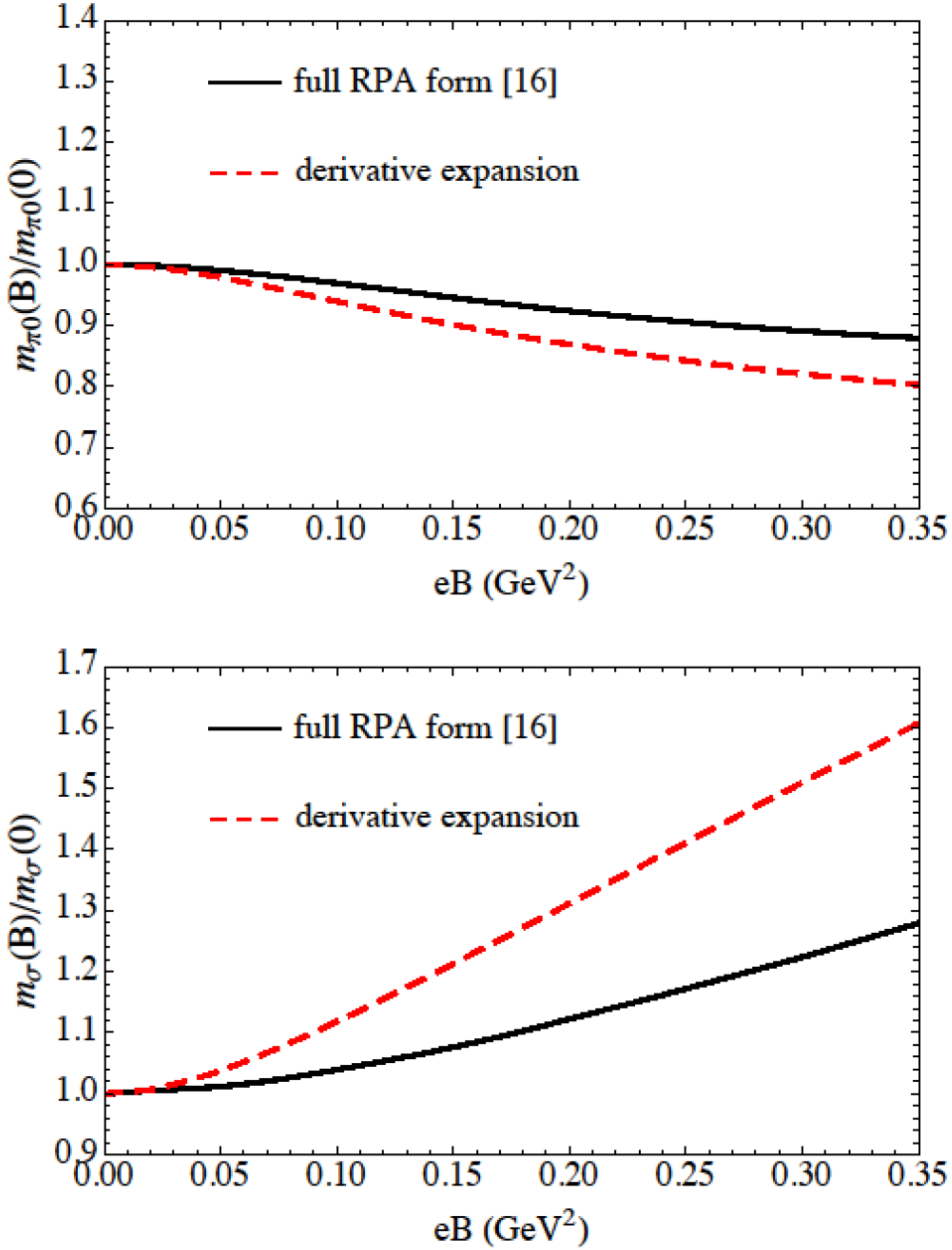
3) *Different magnetic field effect in the directions parallel and perpendicular to the magnetic field.*

Numerical results (II)



1) *Different screening in the directions parallel and perpendicular to the magnetic field.*
2) *The asymmetry increases with magnetic field strength.*
3) *Thermal motion at finite temperature will reduce the asymmetry.*

Comparison with RPA approach



T=0

1) *Derivative expansion is with only one quark loop, but RPA includes an finite number of quark loops.*

2) *Derivative expansion is good for light mesons, and the two approach to each other in weak electromagnetic field.*

3) *It is difficult to treat Schwinger phase in the bubble summation in RPA.*

Summary

- *We developed a systematical way to construct mesons in a quark model in electromagnetic field, including Schwinger phases leads to the minimum coupling between charged mesons and the gauge field.*

- *Wave function renormalization becomes anisotropic in electromagnetic field, which leads to anisotropic screening in quark matter.*

- *Higher order terms in derivative expansion determine interactions among the mesons $\rightarrow$ dynamical processes in electromagnetic field.*

Thank you for your attention !