



Prospects for Precision Lattice QCD

Andreas S. Kronfeld*
Fermilab & IAS TU München

Towards the Ultimate Precision in Flavor Physics
IPPP, Durham University | April 2, 2019



**DIVA = Durham International Visiting Academic*



Prospects for Precision Lattice $QCD \oplus QED$

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Introduction

- Where are we? Where do we need to be? What's cool and interesting?
 - Quark masses = $v_{\text{ev}} \times$ eigenvalues of the Yukawa matrices.
 - CKM matrix elements = mismatch of up-down Yukawa eigenvectors:
 - Decay constants for leptonic decays;
 - Form factors for semileptonic decays.
 - Flavor-changing neutral currents — \exists BSM contributions?

CKM and Lattice QCD

$$V = \begin{pmatrix}
 |V_{ud}| & |V_{us}| & |V_{ub}| \\
 \pi^+ \rightarrow l^+ \nu & K^+ \rightarrow l^+ \nu & B^+ \rightarrow \tau^+ \nu \\
 |V_{cd}| & |V_{cs}| & |V_{cb}| \\
 \pi^+ \rightarrow \pi^0 e^+ \nu & K \rightarrow \pi l^+ \nu & B \rightarrow \pi l^+ \nu \\
 D^+ \rightarrow l^+ \nu & D_s^+ \rightarrow l^+ \nu & B_c^+ \rightarrow \tau^+ \nu \\
 D \rightarrow \pi l^+ \nu & D \rightarrow K l^+ \nu & B \rightarrow \pi l^+ \nu \\
 |V_{td}| & |V_{ts}| & |V_{tb}| \\
 B^0 \rightarrow \pi^0 l^+ l^- & B^0 \rightarrow K^0 l^+ l^- & \\
 B^0 \leftrightarrow \bar{B}^0 & B_s^0 \leftrightarrow \bar{B}_s^0 &
 \end{pmatrix} \begin{array}{l}
 e^{-i\delta_{\text{KM}}} \\
 K^0 \leftrightarrow \bar{K}^0 \quad [\epsilon] \\
 K \rightarrow \pi\pi \quad [\epsilon']
 \end{array}$$

B2TiP Forecasts

T. Kaneko, ASK, S. Simula

N_f	forecast	f_B [MeV]	f_{B_s} [MeV]	f_{B_s}/f_B
	current (2017)	188(3)	227(4)	1.203(7)
	5 yr w/o EM	188(1.5)	227(2.0)	1.203(0.0035)
2+1+1	5 yr w/ EM	188(2.4)	227(3.0)	1.203(0.0125)
	10 yr w/o EM	188(0.60)	227(0.80)	1.203(0.0014)
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N_f	forecast	$f_B \sqrt{B_B^{(1)}}$	$f_B \sqrt{B_B^{(2)}}$	$f_B \sqrt{B_B^{(3)}}$	$f_B \sqrt{B_B^{(4)}}$	$f_B \sqrt{B_B^{(5)}}$
	current (2017)	169(8)	169(8)	200(19)	197(7)	190(9)
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2+1	5 yr w/ EM	169(4.3)	169(4.3)	200(9.7)	197(4.0)	190(4.9)
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Outline

- Introduction
- Quark masses
- Decay constants for leptonic decays
- Form factors for semileptonic decays
- Neutral-meson mixing
- Outlook

Other talks on lattice QCD

- Prospects for D decays [in a box] on the lattice: Max Hansen, Tu, 15:30
- News on kaon physics from the lattice: Chris Sachrajda, Wed, 16:45
- Theory developments in B mixing: Tobias Tsang, Th, 8:30

Quark masses



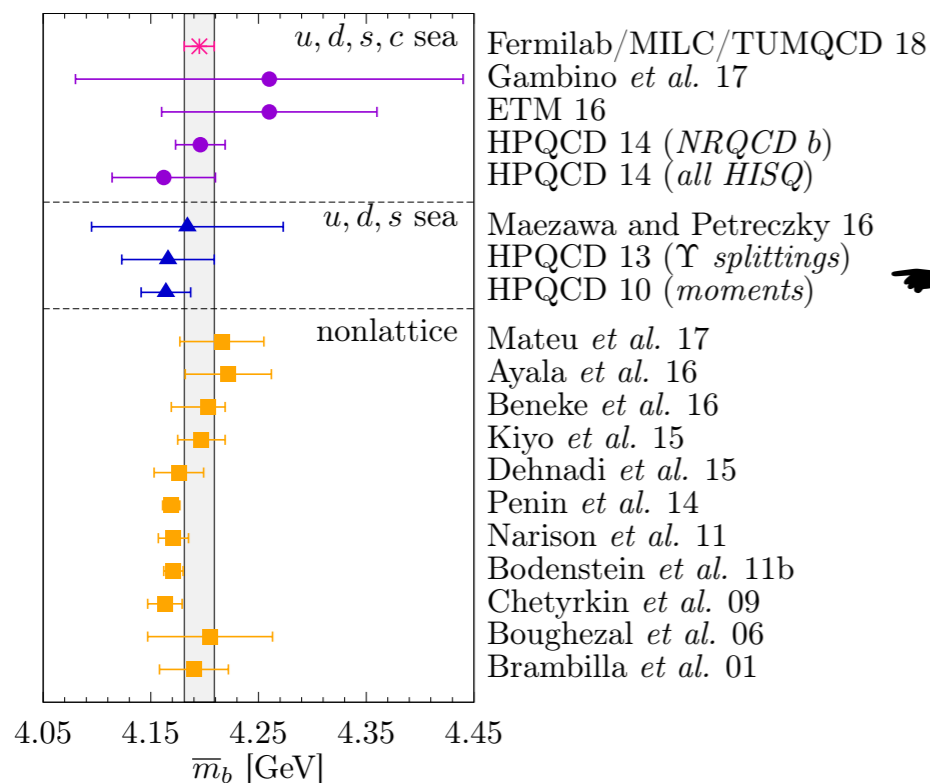
Charm and bottom masses

for averages, see
HPQCD or FLAG

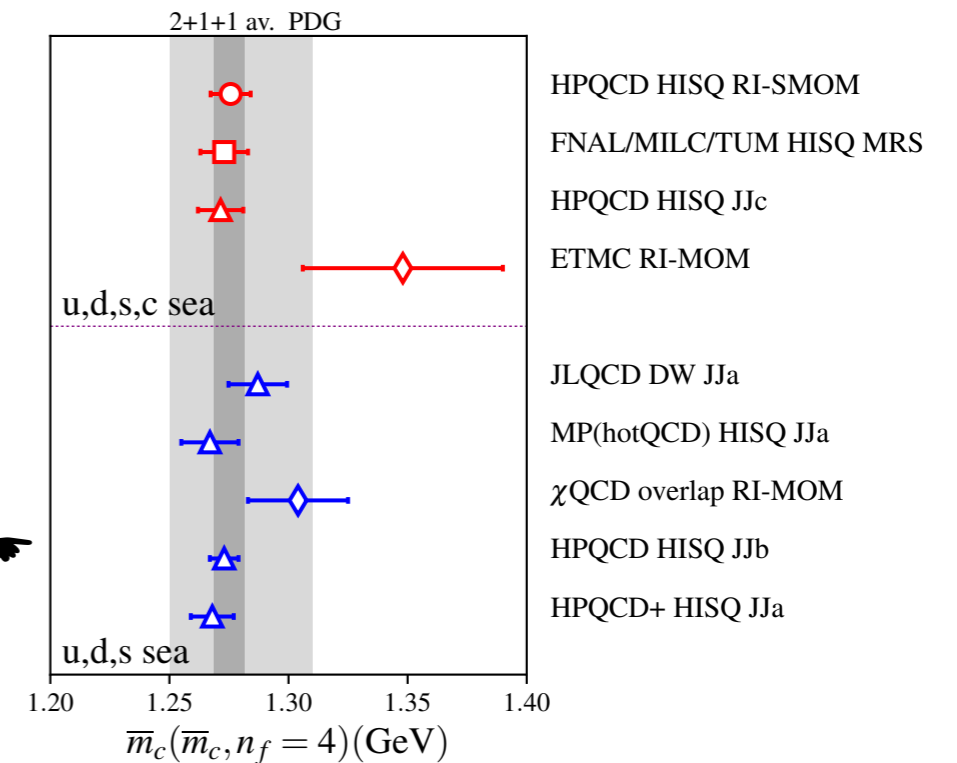
- For charm, precise results with three different methods:

$$m_{c,\overline{\text{MS}}}(3 \text{ GeV}) = \begin{cases} 989.6(3.2)_{\text{corr}}(5.2)_{\text{uncorr}} \text{ MeV} & m_{0c} & \text{RI-SMOM} & \text{HPQCD} \\ 983.7(4.3)_{\text{stat}}(3.6)_{\alpha_s+\text{syst}} \text{ MeV} & M_{D_q}, M_{B_q} & \text{MRS} & \text{F/M/T} \\ 985.1(2.2)_{\text{corr}}(5.9)_{\text{uncorr}} \text{ MeV} & \langle J_{\bar{c}c}^5(t) J_{\bar{c}c}^5(0) \rangle & \text{pQCD} & \text{HPQCD} \end{cases}$$

$$m_{b,\overline{\text{MS}}}(\bar{m}_b) = 4201(12)_{\text{stat}}(8)_{\alpha_s+\text{syst}} \text{ MeV} \quad M_{D_q}, M_{B_q} \quad \text{MRS} \quad \text{F/M/T}$$



0.34%
[arXiv:1802.04248](https://arxiv.org/abs/1802.04248)
[arXiv:1805.06225](https://arxiv.org/abs/1805.06225)
 {0.62, 0.57, 0.64}%



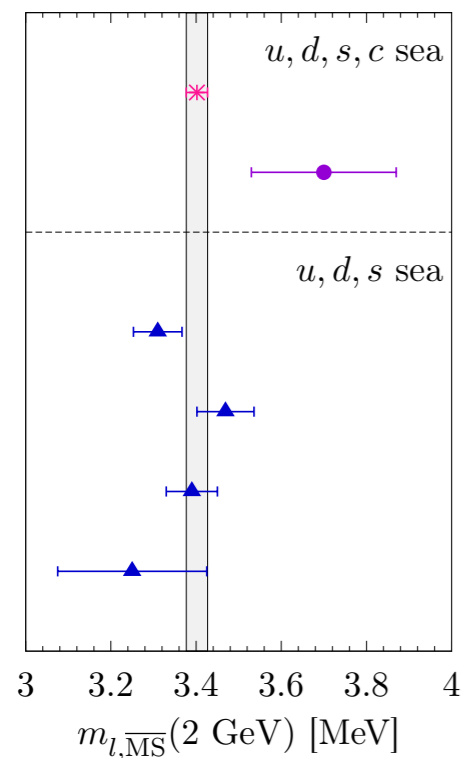
Up, down, strange quarks

for averages, see
HPQCD or FLAG

- For strange, precise results with three different methods:

$$m_{s,\overline{\text{MS}}}(2 \text{ GeV}) = \begin{cases} 94.49(79)_{\text{corr}}(63)_{\text{uncorr}} \text{ MeV} & m_{0s} & \text{RI-SMOM} & \text{HPQCD} \\ 92.47(39)_{\text{stat}}(56)_{\alpha_s+\text{syst}} \text{ MeV} & m_c m_{0s}/m_{0c} & \text{MRS} & \text{F/M/T} \\ 93.54(44)_{\text{corr}}(69)_{\text{uncorr}} \text{ MeV} & m_c m_{0s}/m_{0c} & \text{pQCD} & \text{HPQCD} \end{cases}$$

$$m_{u,\overline{\text{MS}}}(2 \text{ GeV}) = 2.130(41) \text{ MeV} \quad m_c m_{0u}/m_{0c} \quad \text{MRS} \quad \text{F/M/T}$$



Fermilab/MILC/TUMQCD 18

ETM 14

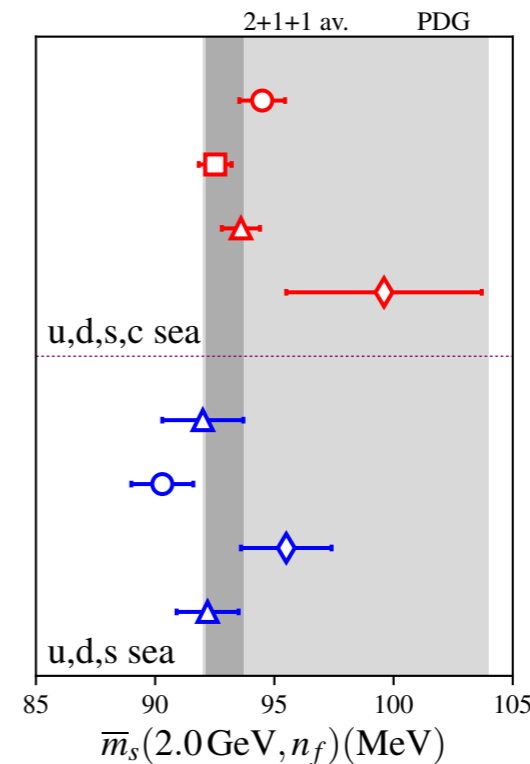
RBC/UKQCD 14

BMW 10

HPQCD 10

MILC 09

0.74% 1.9% ↑
arXiv:1802.04248
arXiv:1805.06225
{1.1, 0.74, 0.87}%



2+1+1 av. PDG

HPQCD HISQ RI-SMOM

FNAL/MILC/TUM HISQ MRS + m_h/m_s

HPQCD HISQ JJ + m_c/m_s

ETMC RI-MOM

MP(hotQCD) HISQ JJ + m_c/m_s

RBC/UKQCD DW RI-SMOM

BMW clover RI-MOM

HPQCD HISQ JJ + m_c/m_s

Remarks

- Up-quark mass settles the strong CP problem, modulo sharpening the controversy around instanton-induced additive mass renormalization.
- The precision for m_b, m_c is more than good enough for testing the Higgs boson's effect on quark masses at LHC and almost what ILC needs;
 - probably also for flavor physics.
- The treatment of QED is not yet rigorous and though not a dominant systematic uncertainty, it's a squishy one.
- Precision from analyses of MILC's 2+1+1 HISQ ensembles:
 - need comparable precision from different 2+1(+1) ensembles.

Leptonic decay constants



Heavy-light mesons

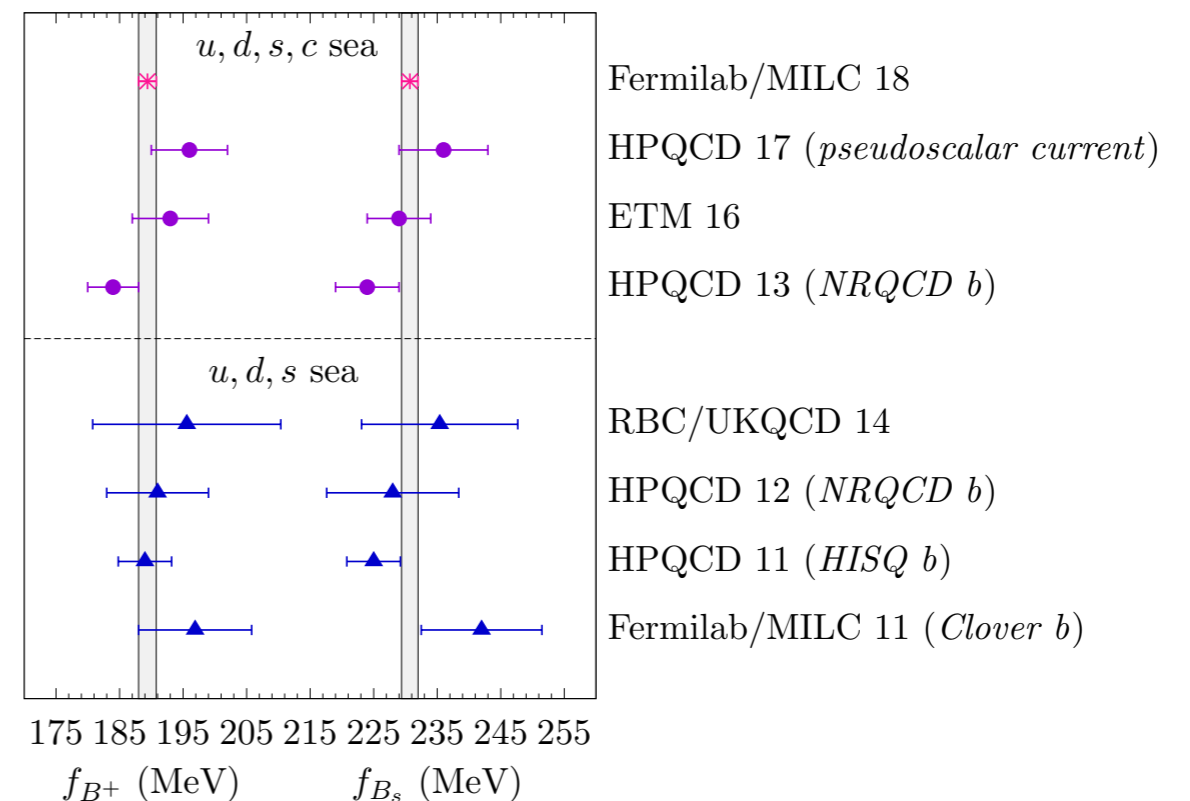
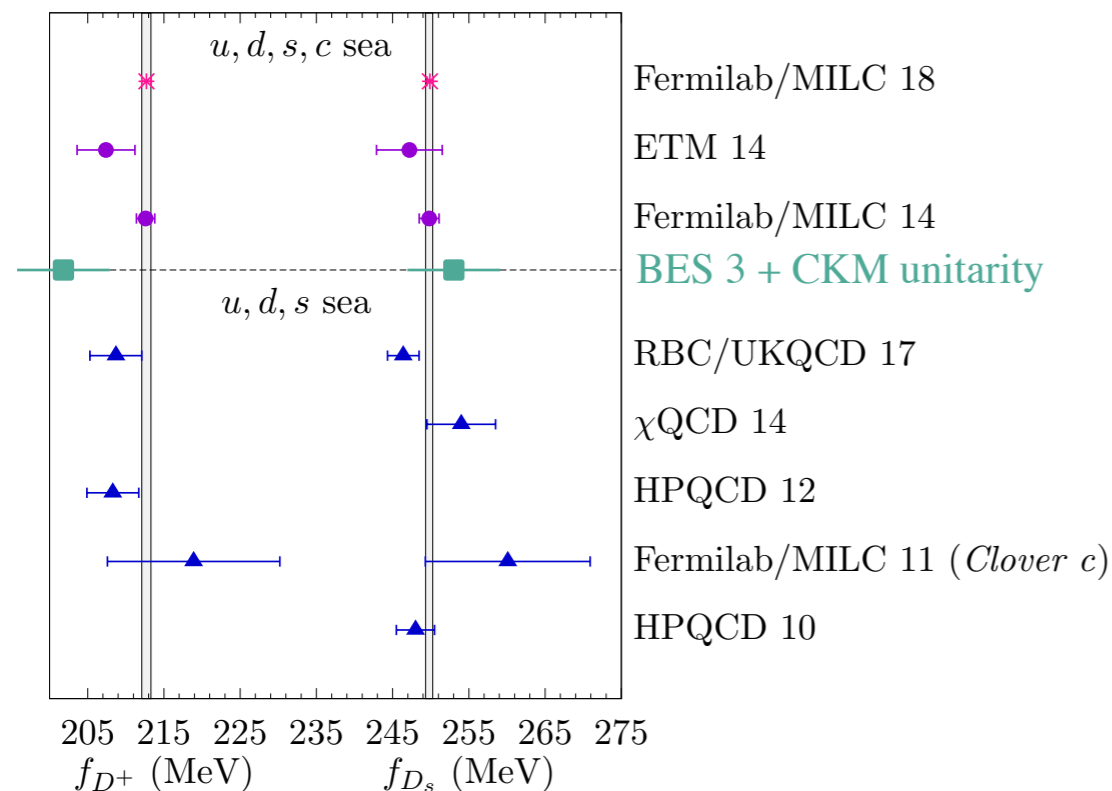
- Most precise results from [Fermilab/MILC](#) (.: dominate FLAG averages).

$$f_{D^+} = 212.7(0.3)_{\text{stat}}(0.4)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}} [0.2]_{\text{EM scheme}} \text{ MeV} \quad 0.3\%$$

$$f_{D_s} = 249.9(0.3)_{\text{stat}}(0.2)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}} [0.2]_{\text{EM scheme}} \text{ MeV} \quad 0.2\%$$

$$f_{B^+} = 189.4(0.8)_{\text{stat}}(1.1)_{\text{syst}}(0.3)_{f_{\pi,\text{PDG}}} [0.1]_{\text{EM scheme}} \text{ MeV} \quad 0.7\%$$

$$f_{B_s} = 230.7(0.8)_{\text{stat}}(1.0)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}} [0.2]_{\text{EM scheme}} \text{ MeV} \quad 0.6\%$$



Pion and kaon

- Numerous precise results; averages from FLAG:

$$f_{K^\pm}/f_{\pi^\pm} = 1.1932(19) \quad \text{Fermilab/MILC}$$

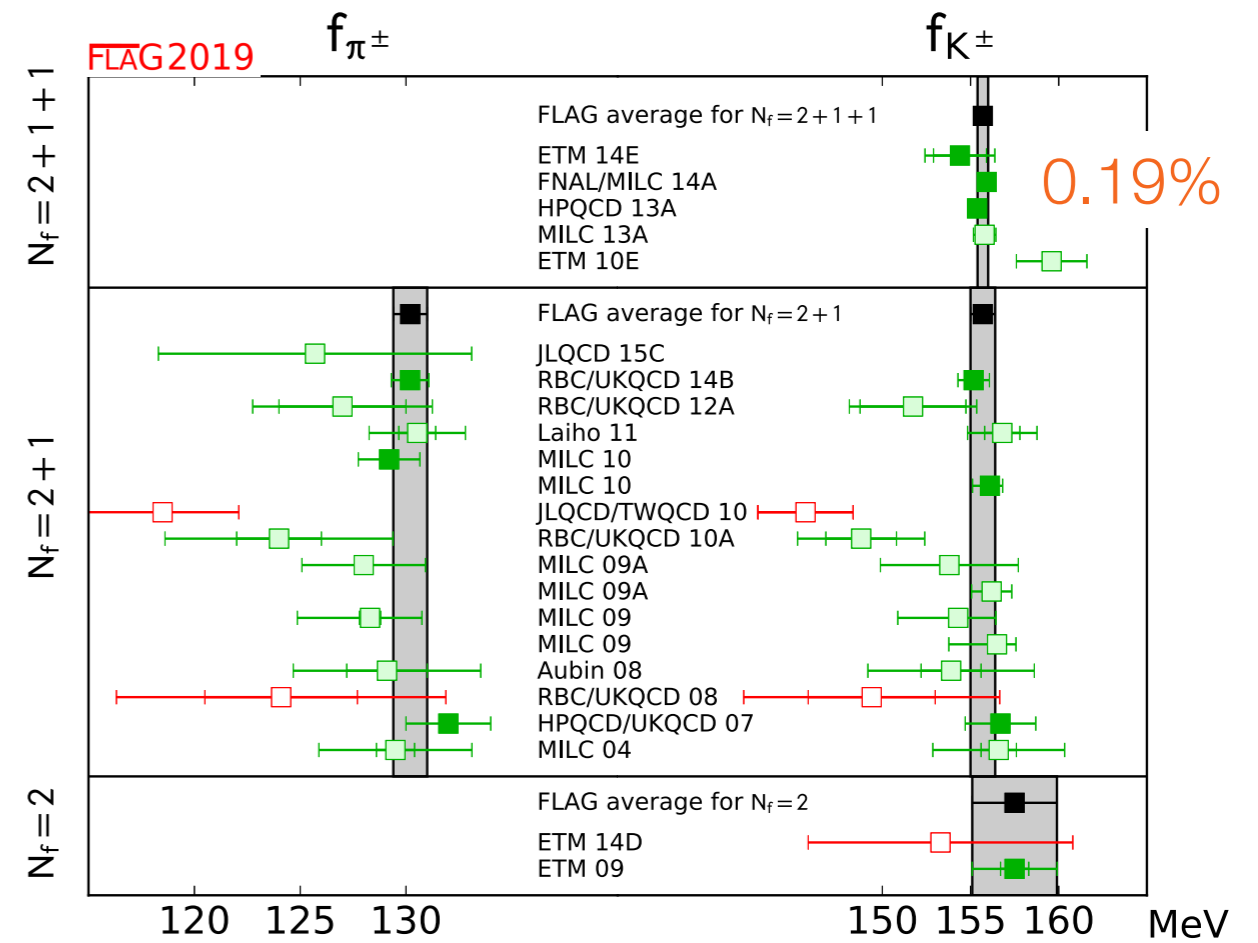
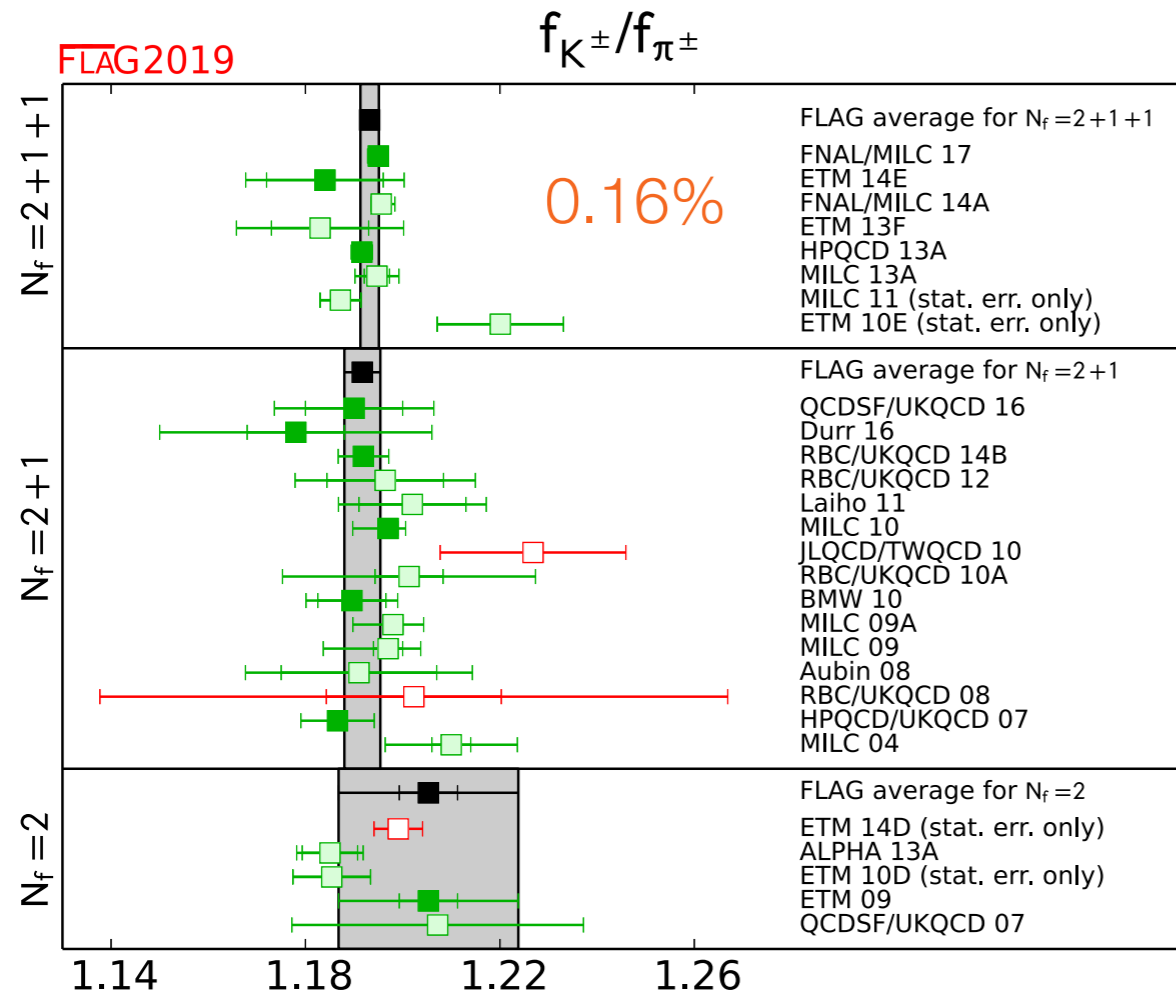
$$n_f = 2+1+1$$

ETM
HPQCD

$$f_{K^\pm} = 155.7(0.3) \text{ MeV} \quad \text{Fermilab/MILC}$$

$$n_f = 2+1+1$$

ETM
HPQCD



Remarks

- The precision for $f_{D(s)}, f_{B(s)}$ is more than good enough for BES 3, Belle II.
- The treatment of QED is not yet rigorous and though not a dominant systematic uncertainty, it's a squishy one.
- The observed rates have soft photons:
 - for pions and kaons, see Sachrajda's talk;
 - for $B_{(s)}, D_{(s)}$ the soft scale $\Lambda^2/M_{B,D}$ (HFS) requires more thinking.
- Precision from analyses of MILC's 2+1+1 HISQ ensembles:
 - need comparable precision from different 2+1(+1) ensembles.

QED
in a
box

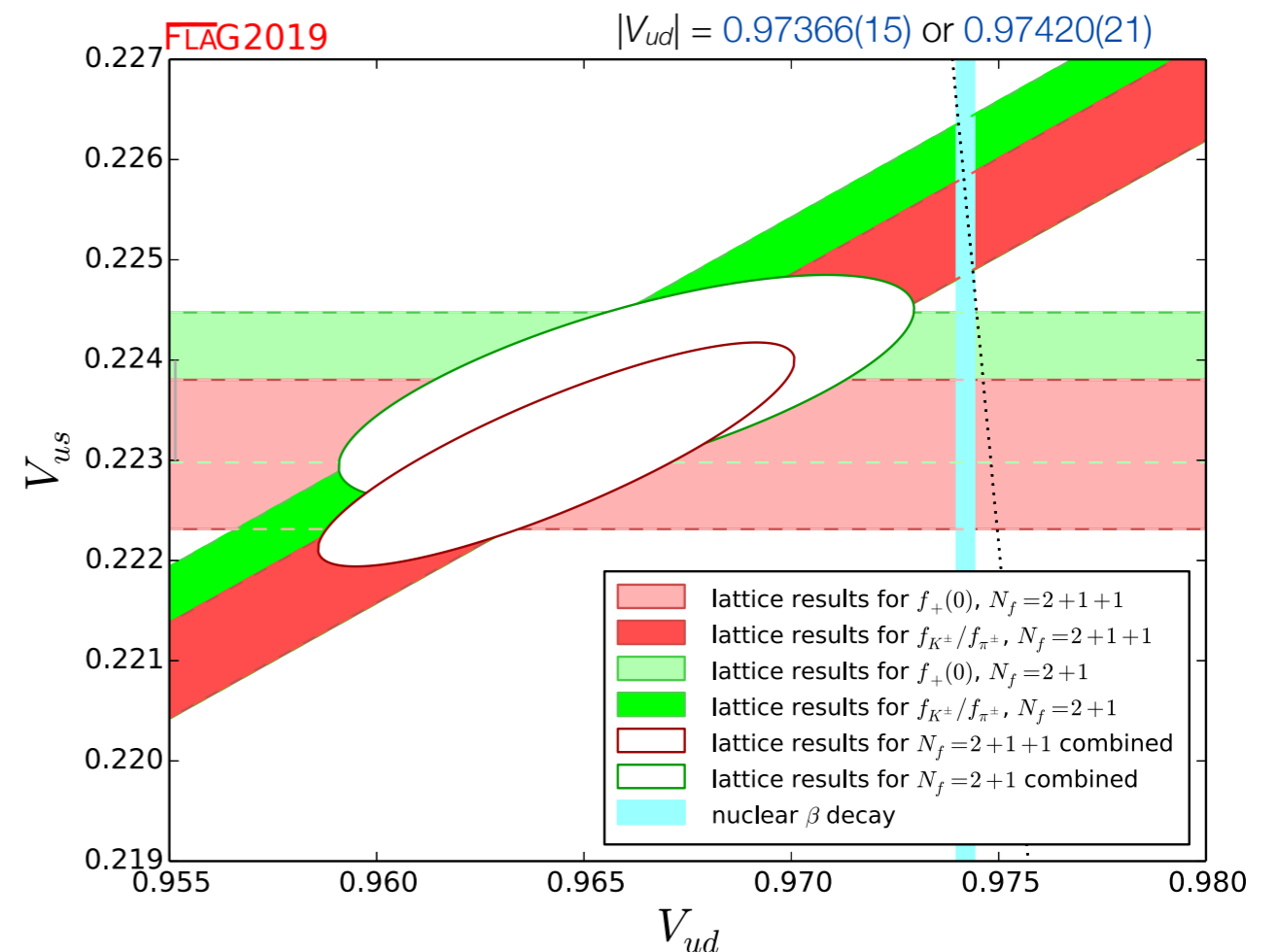
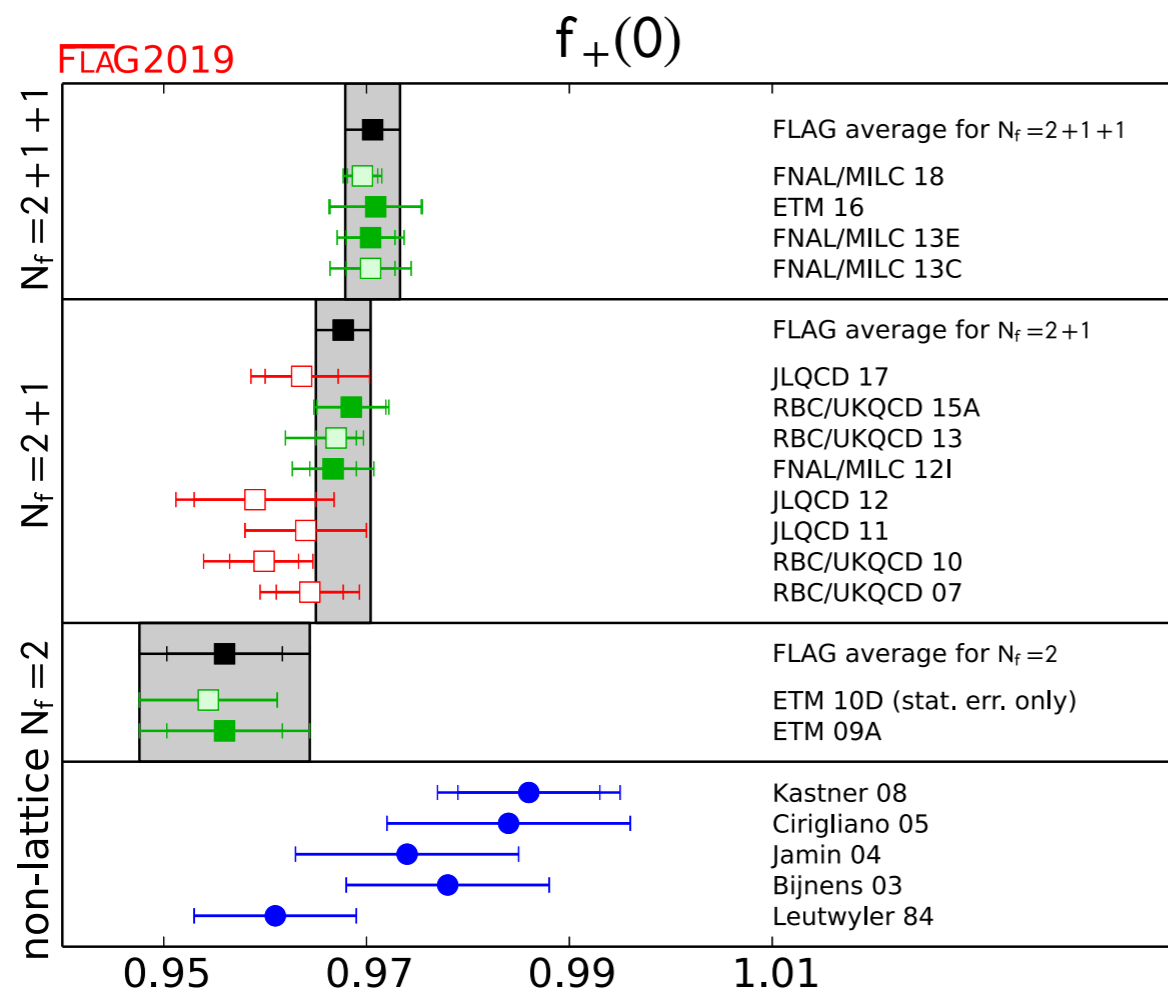
Semileptonic form factors



Kaon: unitarity crisis?

- QED and isospin-breaking contributions with chiral PT.

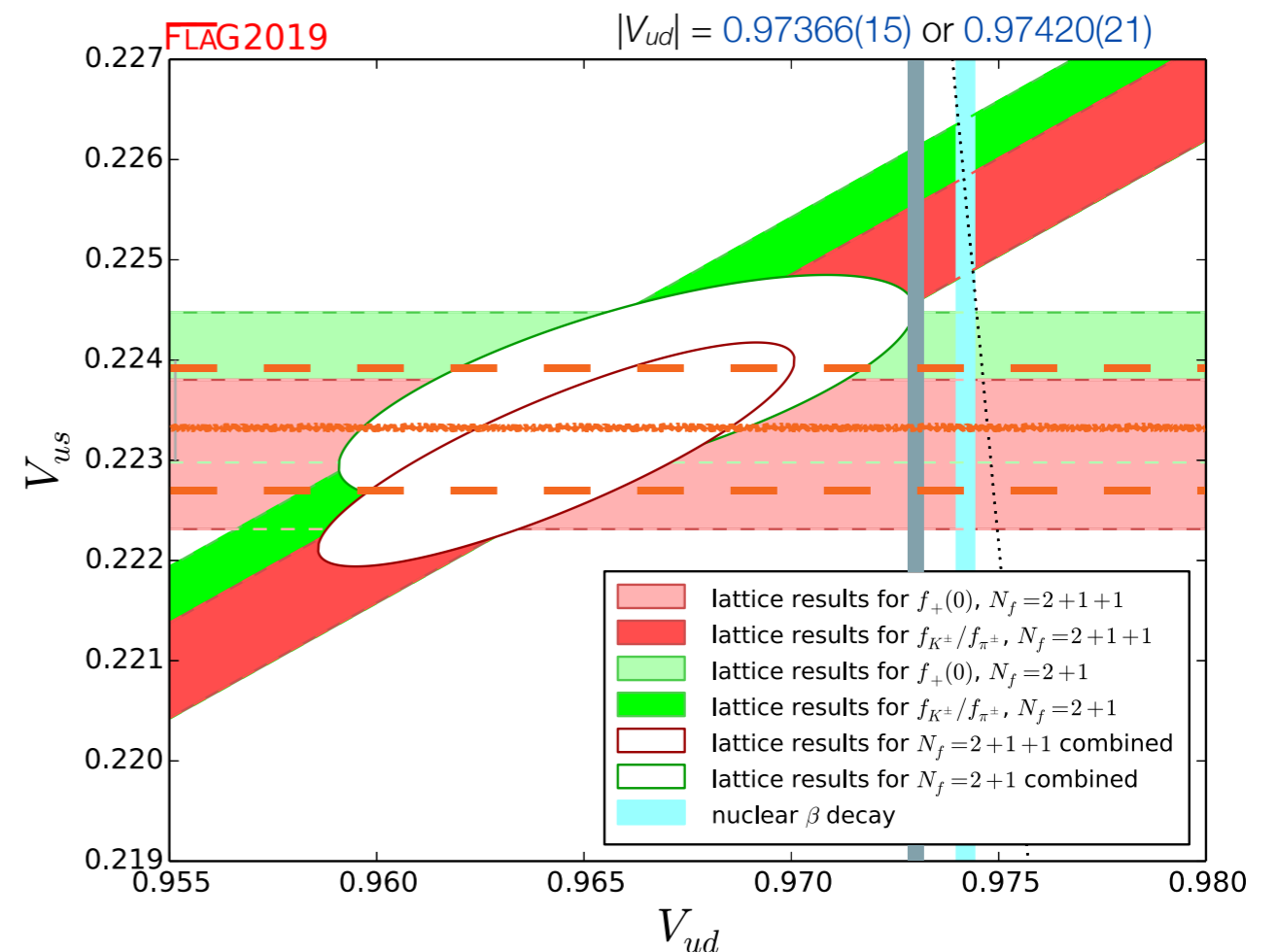
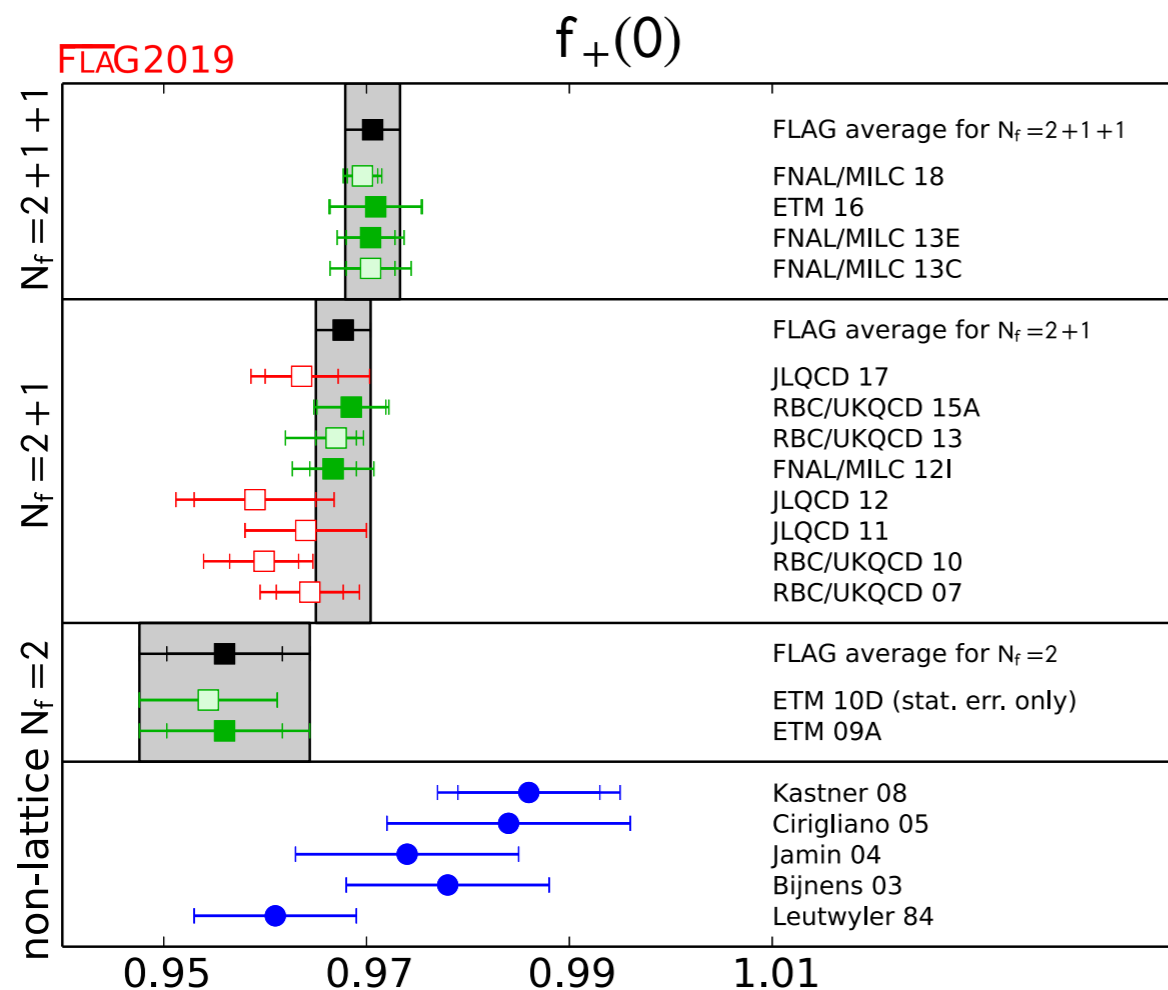
$$f_+^{K^0\pi^-}(0) = \begin{cases} 0.9706(27) & \text{2019 FLAG average} & \text{expt: 0.2\%} \\ 0.9696(19) & \text{Fermilab/MILC 2018 (preprint)} & 0.2\% \end{cases}$$



Kaon: unitarity crisis?

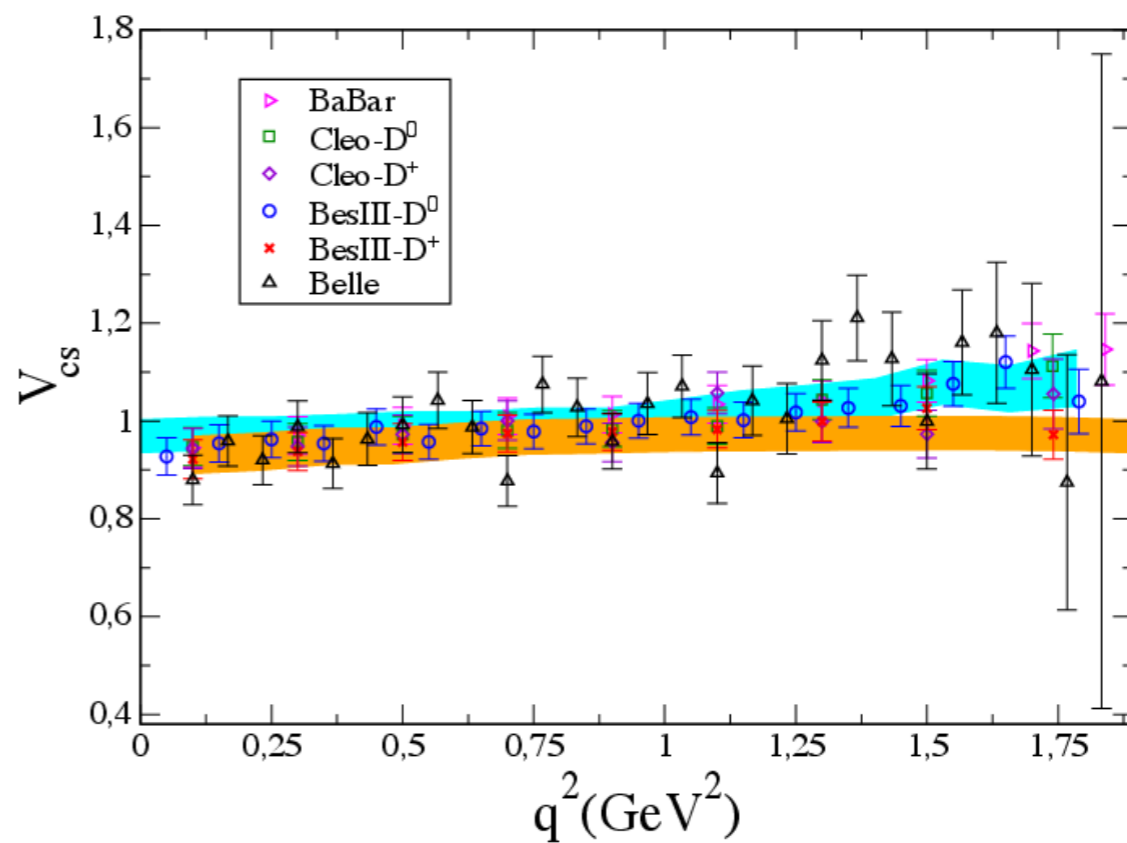
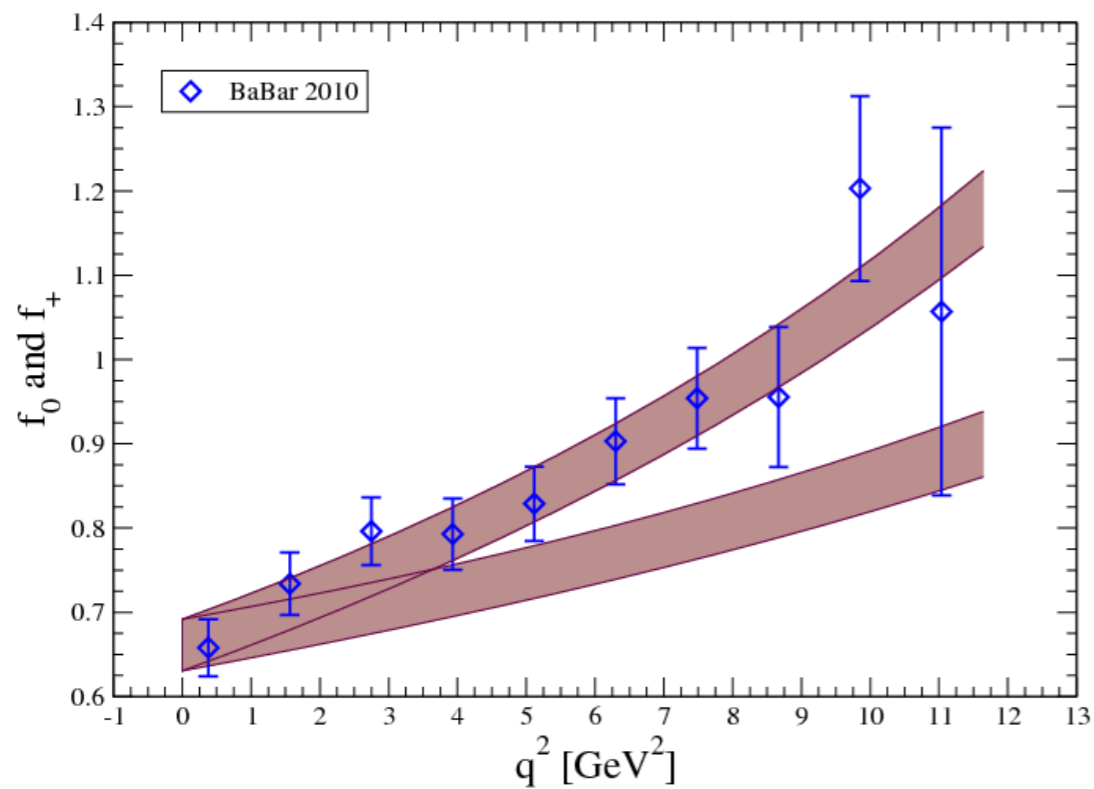
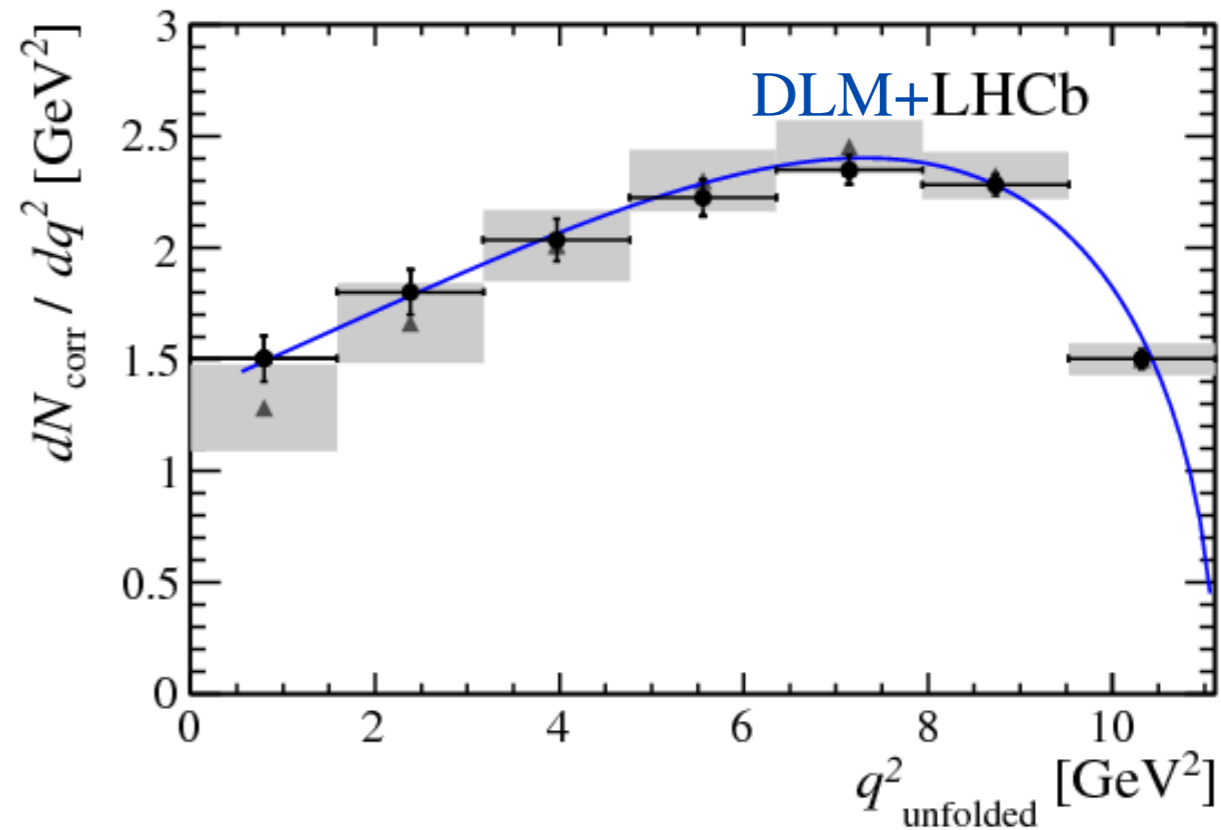
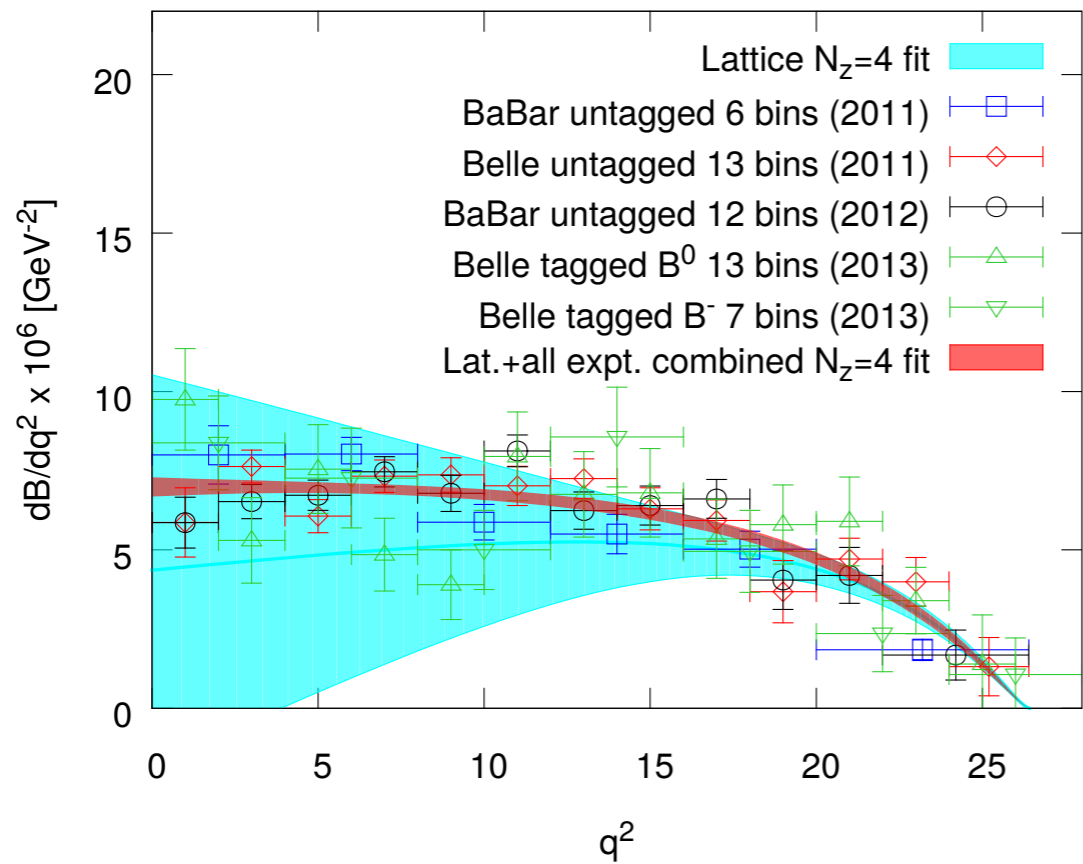
- QED and isospin-breaking contributions with chiral PT.

$$f_+^{K^0\pi^-}(0) = \begin{cases} 0.9706(27) & \text{2019 FLAG average} & \text{expt: } 0.2\% \\ 0.9696(19) & \text{Fermilab/MILC 2018 (preprint)} & 0.2\% \end{cases}$$

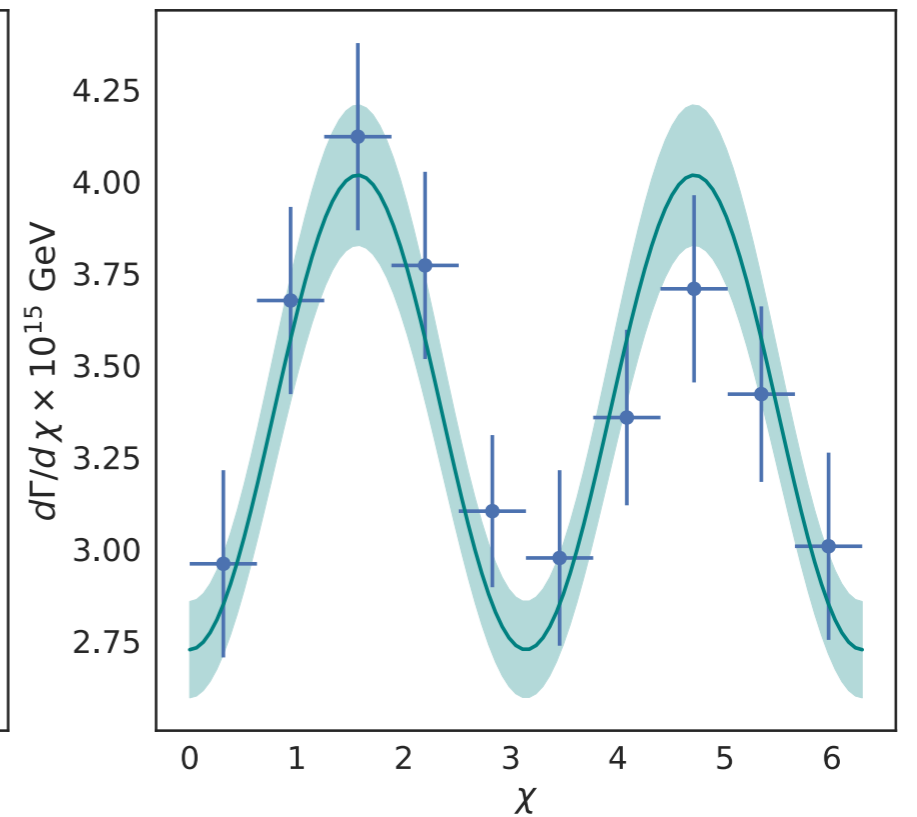
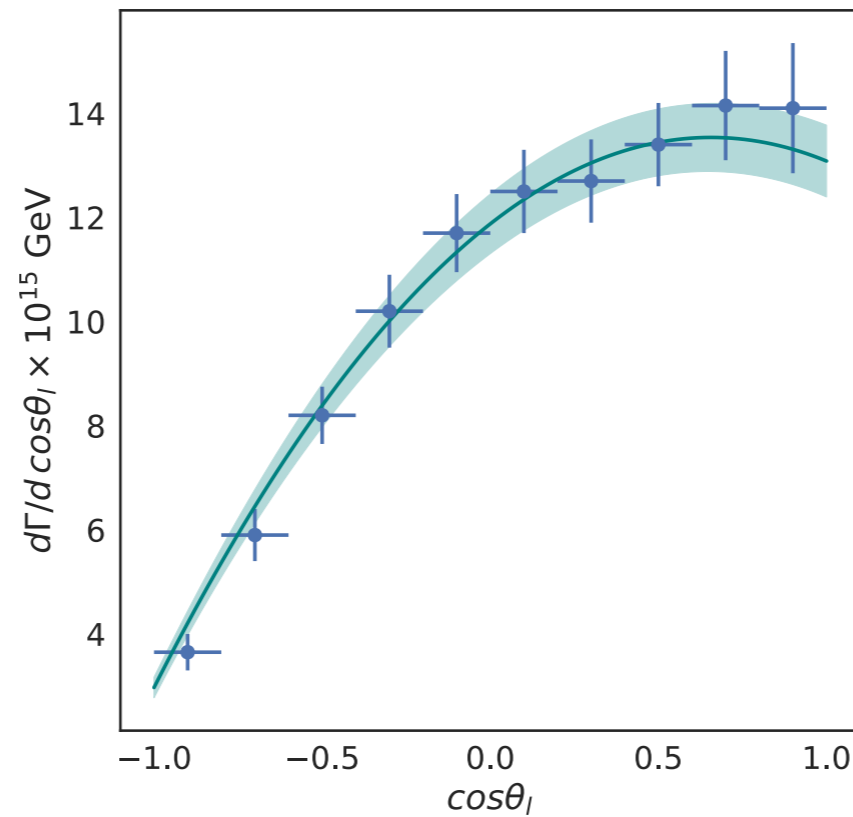
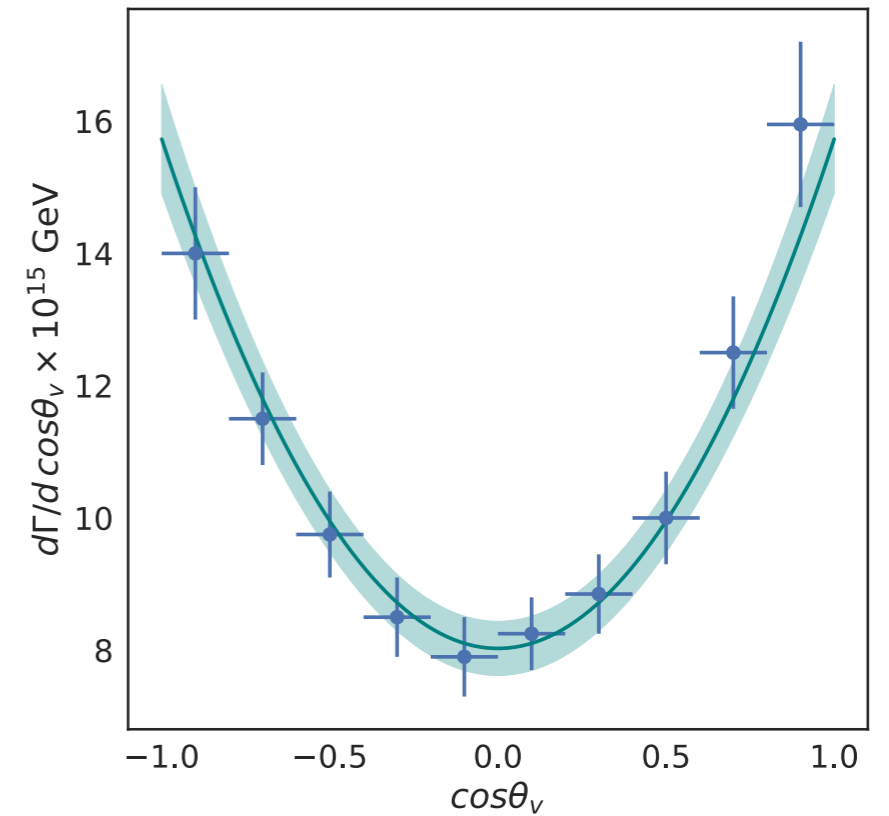
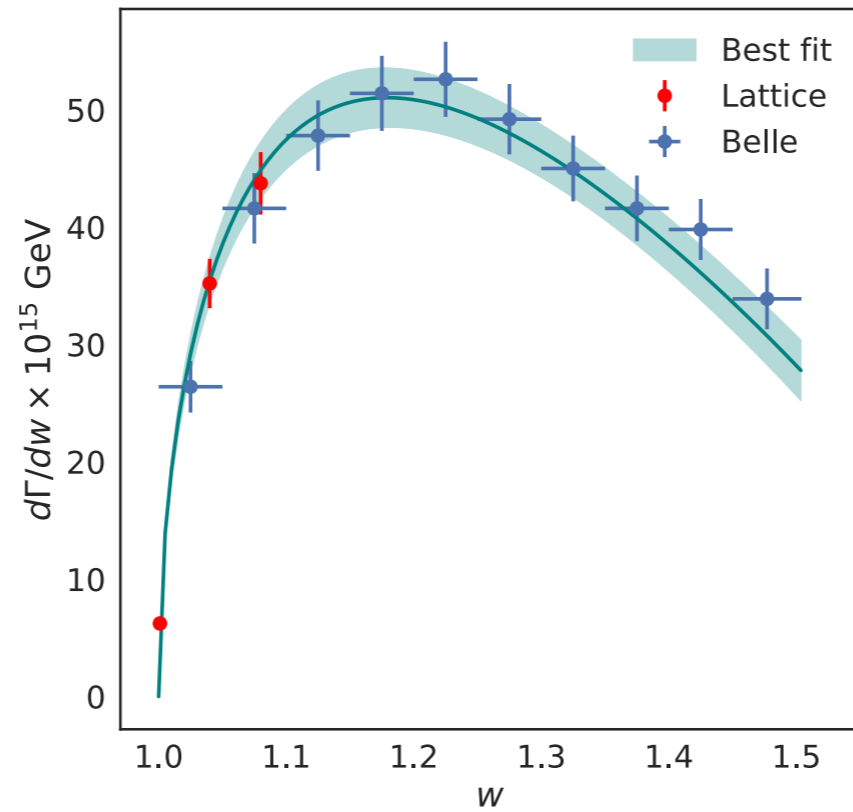


$B_{(s)}$ Mesons

- In general BSM, 3 (many) [many] form factors for $B \rightarrow P (V) [\Lambda_b \rightarrow \Lambda, p]$.
- Functions of q^2 .
- Use z expansion to extend lattice QCD to full kinematic range
- Report coefficients of z expansion plus correlations between them, between various form factors.
- Joint fit to lattice QCD and experimental data yields $|V_{xb}|$ [$|V_{ub}|/|V_{cb}|$].
- Similar approach for D semileptonic decays.



Progress on $B \rightarrow D^*$
 SM form factors
 (A. Vaquero,
 Fermilab/MILC):



Remarks I

- Better precision needed for **BES 3**, LHCb, and Belle II:
 - await D form factors on 2+1+1 HISQ ensembles (*cf.* decay constants);
 - kaon $f_+(0)$ precision now commensurate with (legacy) experiments.
- Averages of functions are tricky:
 - in collider phenomenology, people choose a default set of PDFs and compare with different choices;
 - same could apply to form factors in flavor phenomenology;
 - phenomenological ff shape could come from joint fit (for $f_+(q^2)$).

Remarks II

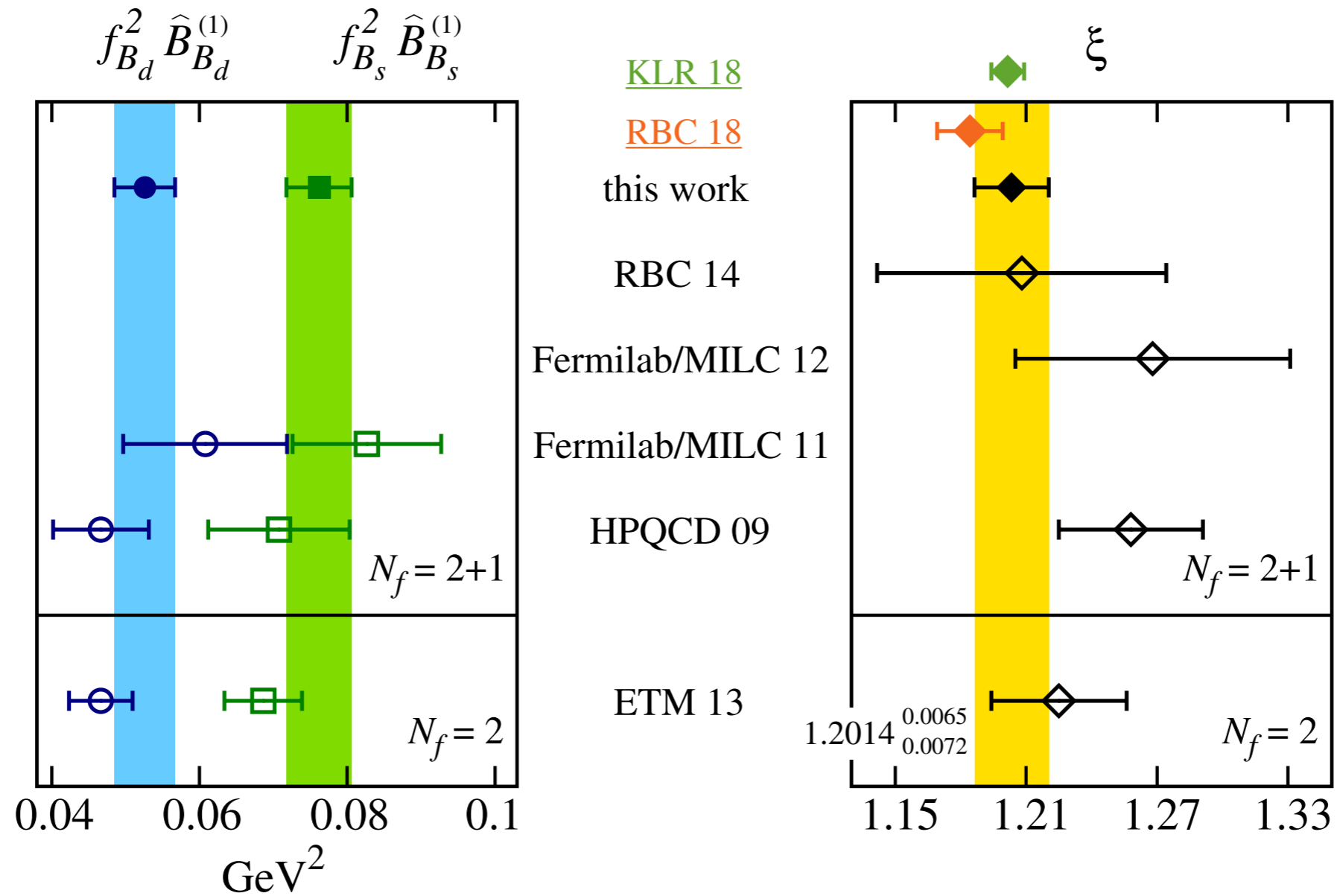
- Rigorous calculations with unstable hadrons ($B \rightarrow K^*$) are starting.
- QED uncertainties are relevant & the observed rates have soft photons:
 - solve the soft-scale " $\Lambda^2/M_{B,D}$ " problem for leptonic decays first;
 - Coulomb photons in final state of neutral-hadron semileptonic decays should be understood better.
- Form factors for FCNC semileptonic decays are the same as those for CKM, up to isospin corrections:
 - non-factorizable QCD contributions will be more uncertain.

Neutral-meson mixing & Semileptonic FCNC

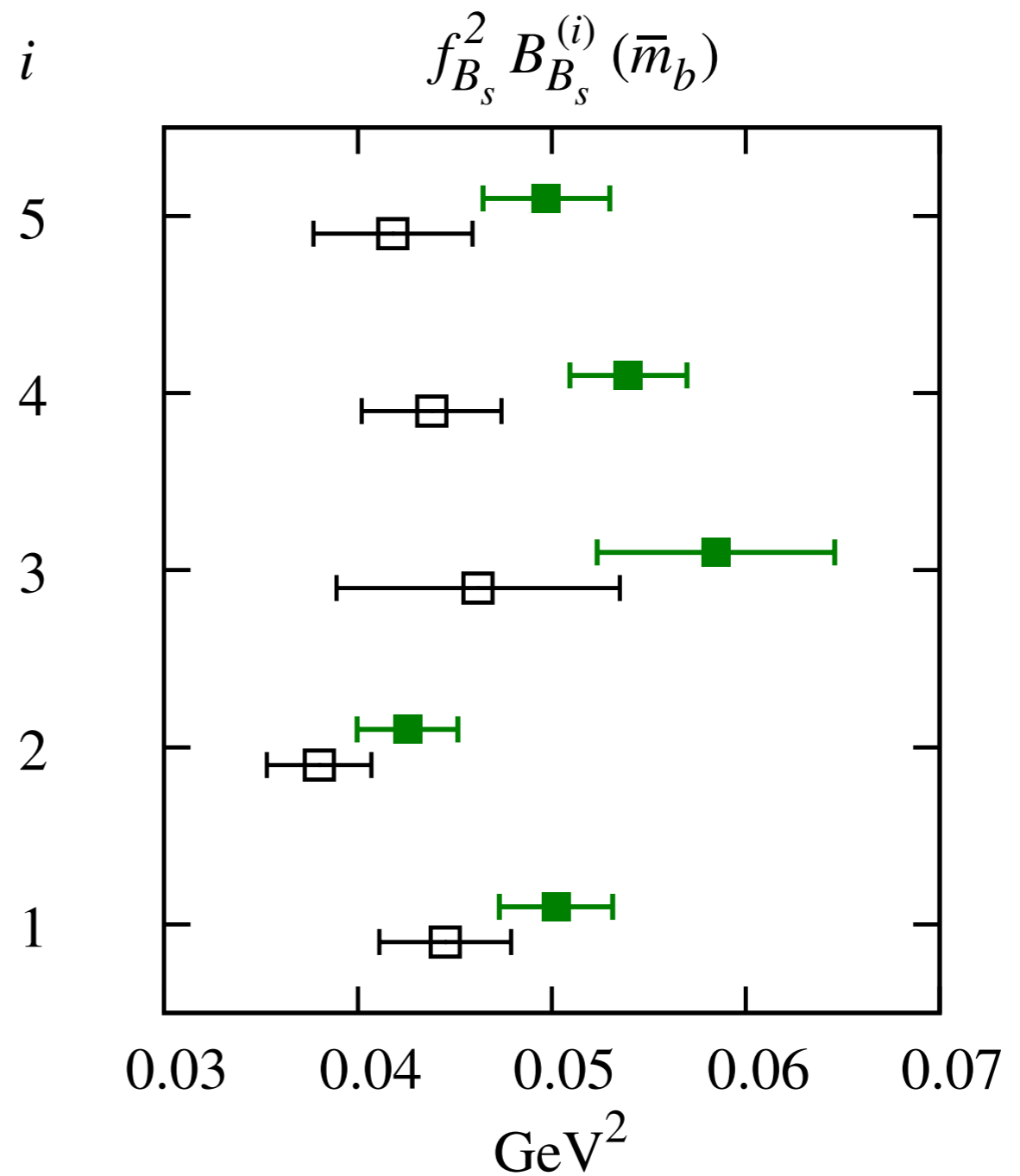
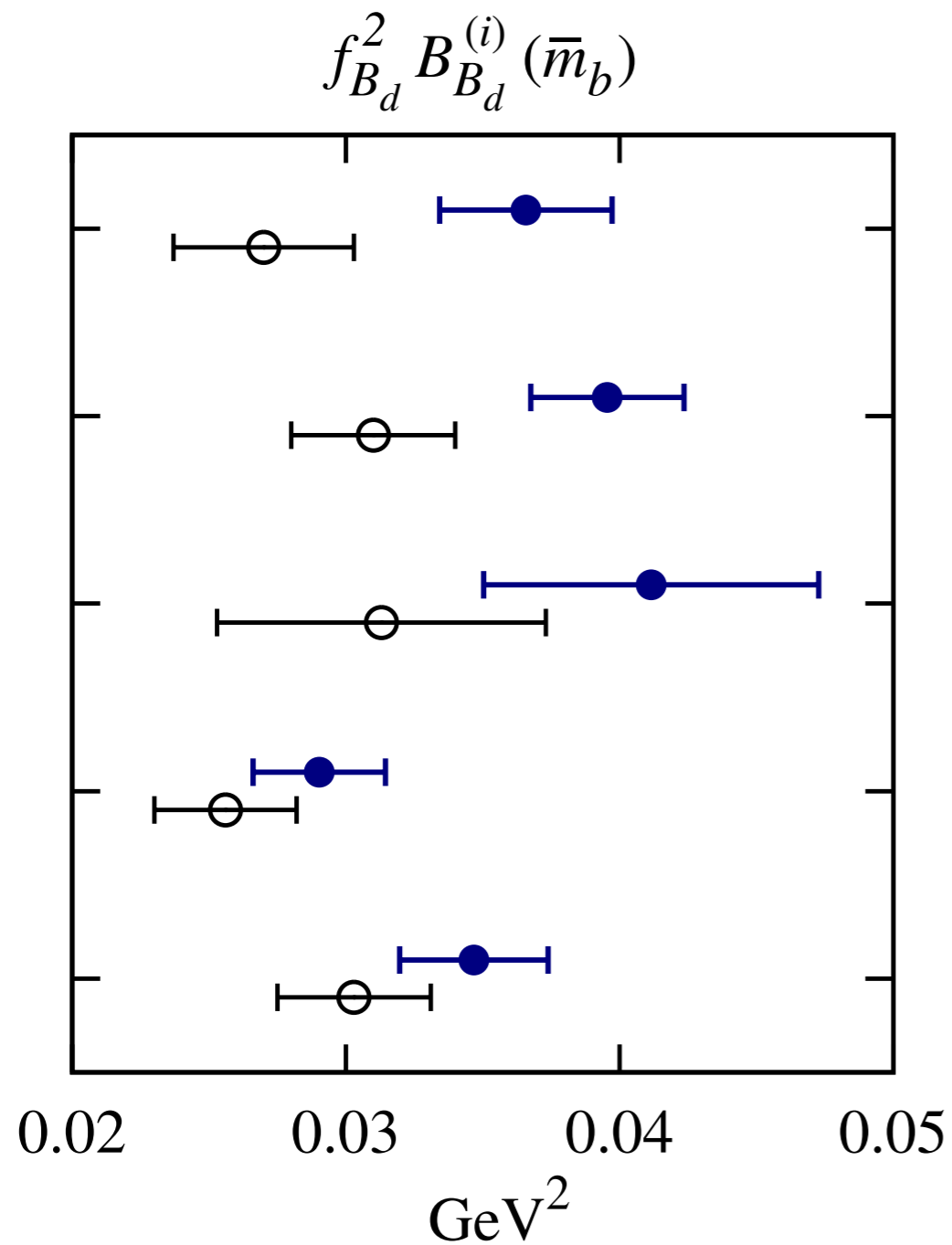


Neutral Meson Mixing

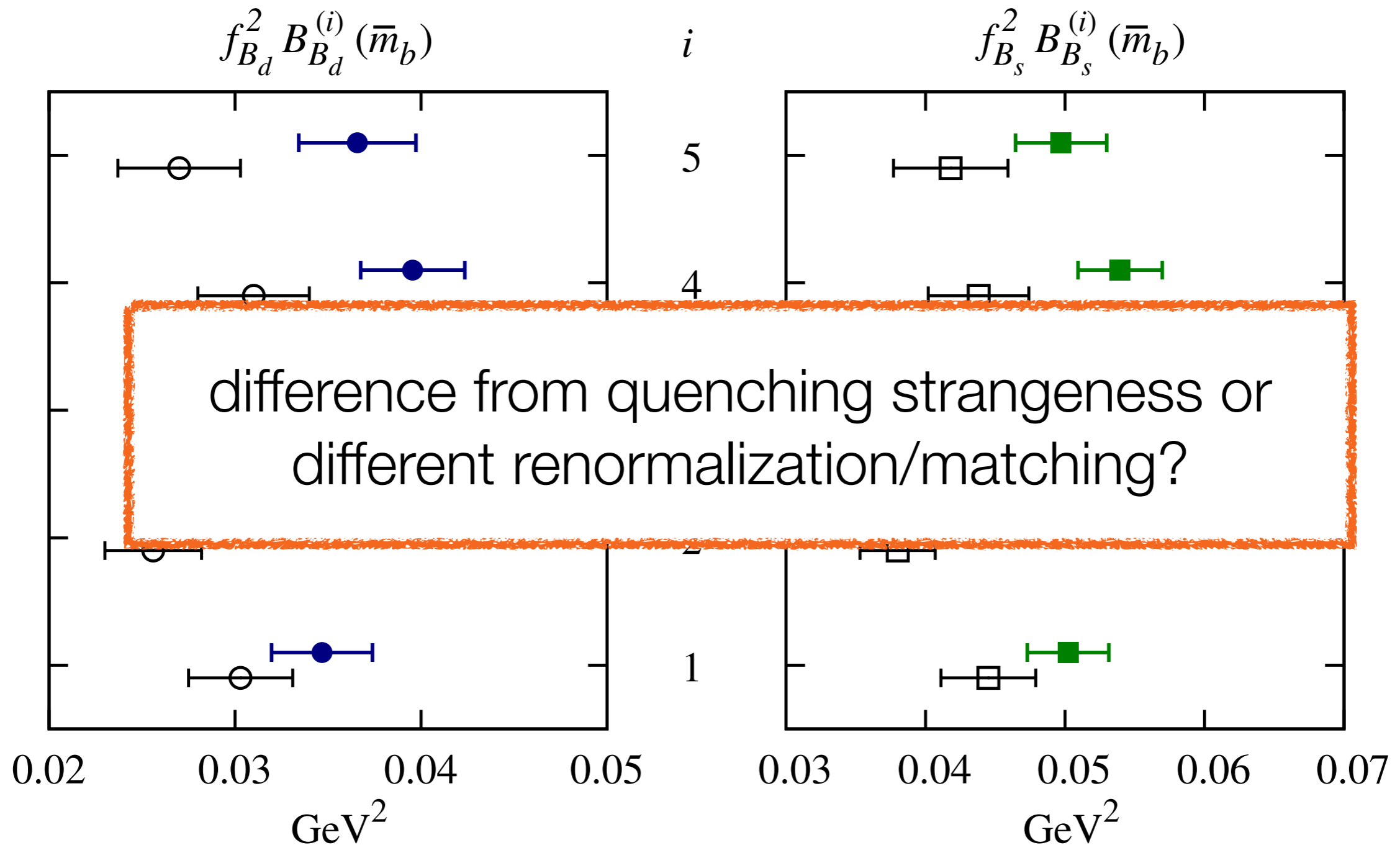
- Fermilab/MILC [[arXiv:1602.03560](https://arxiv.org/abs/1602.03560)] not in 2016 **FLAG** but in on-line update.



Comparison with ETM Calculation ($n_f = 2$)

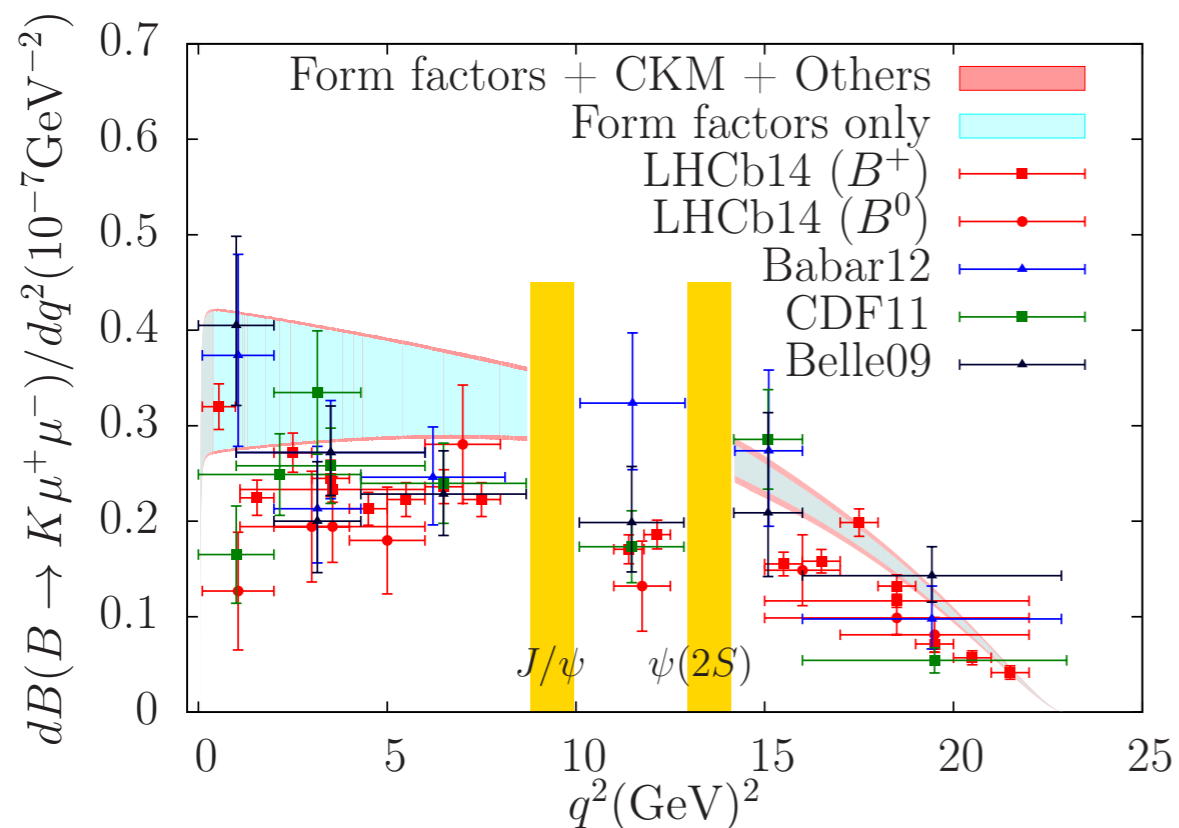


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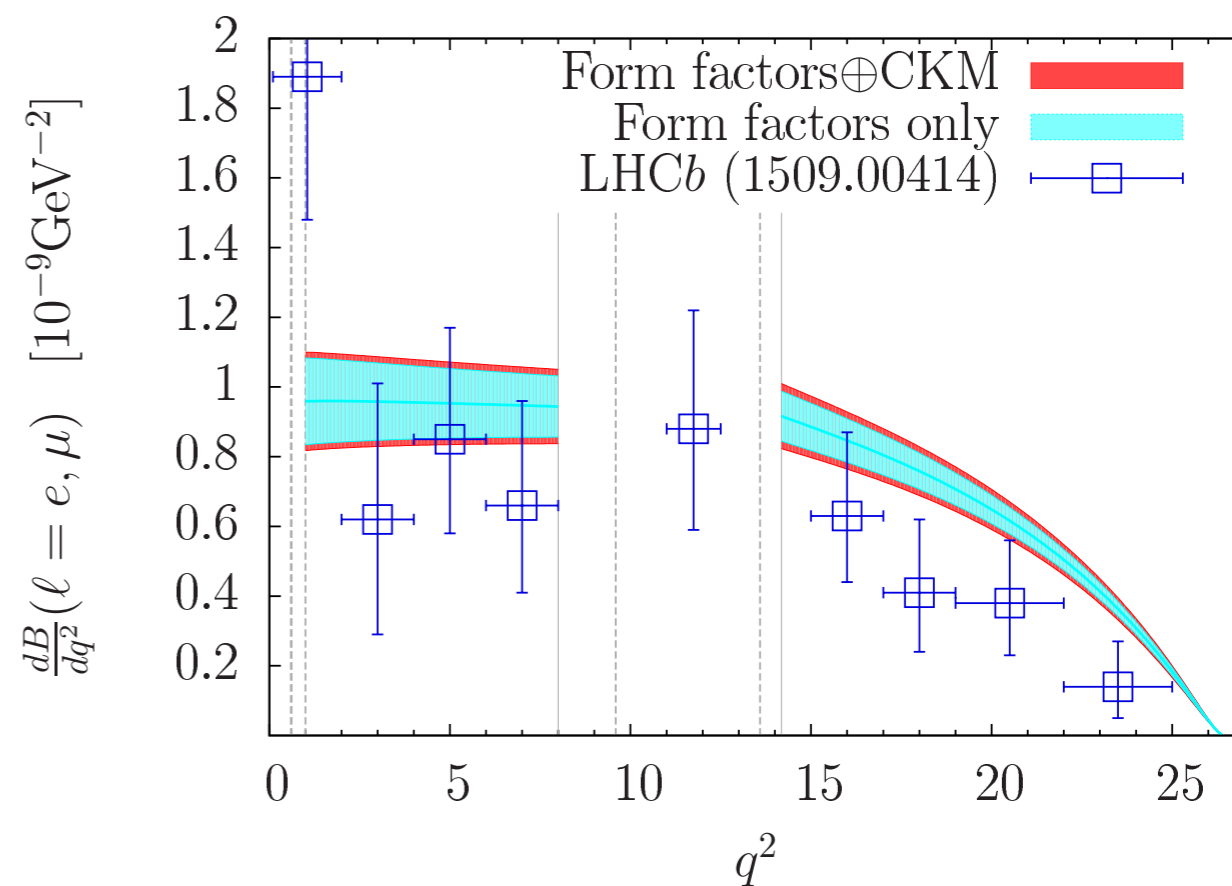


Kinematic Distributions for $B \rightarrow Kll$ and $B \rightarrow \pi ll$

- Experimental data from LHCb [[arXiv:1403.8044](https://arxiv.org/abs/1403.8044), [arXiv:1509.00414](https://arxiv.org/abs/1509.00414)] and earlier experiments; right plot's theory **preceded** measurement:



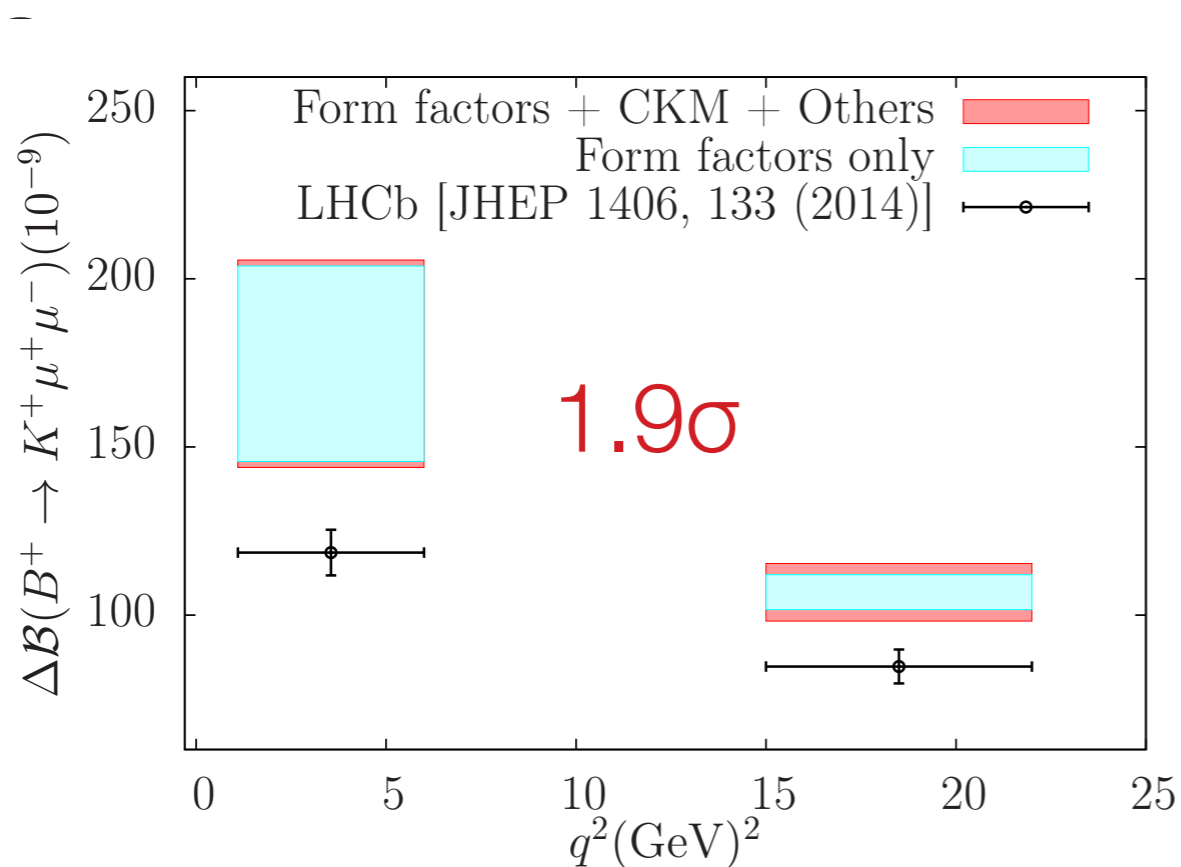
[arXiv:1510.02349](https://arxiv.org/abs/1510.02349)



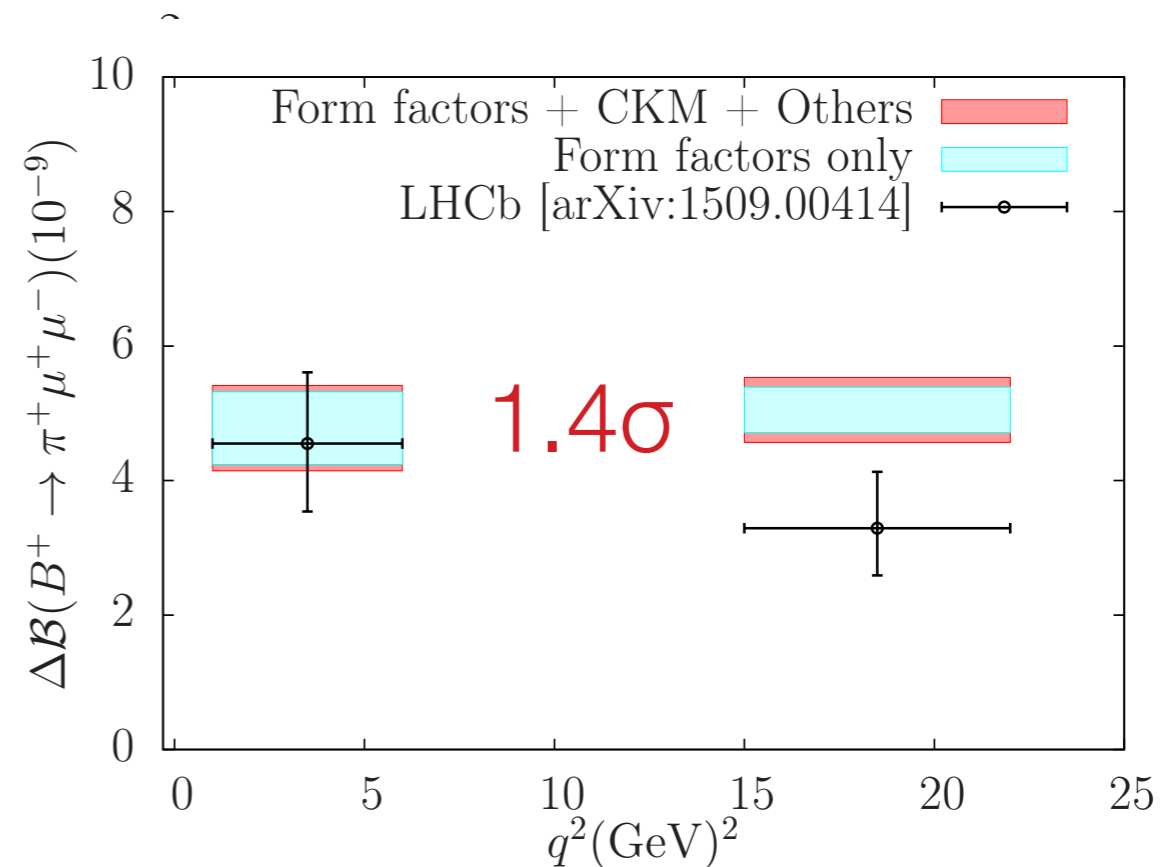
[arXiv:1507.01618](https://arxiv.org/abs/1507.01618)

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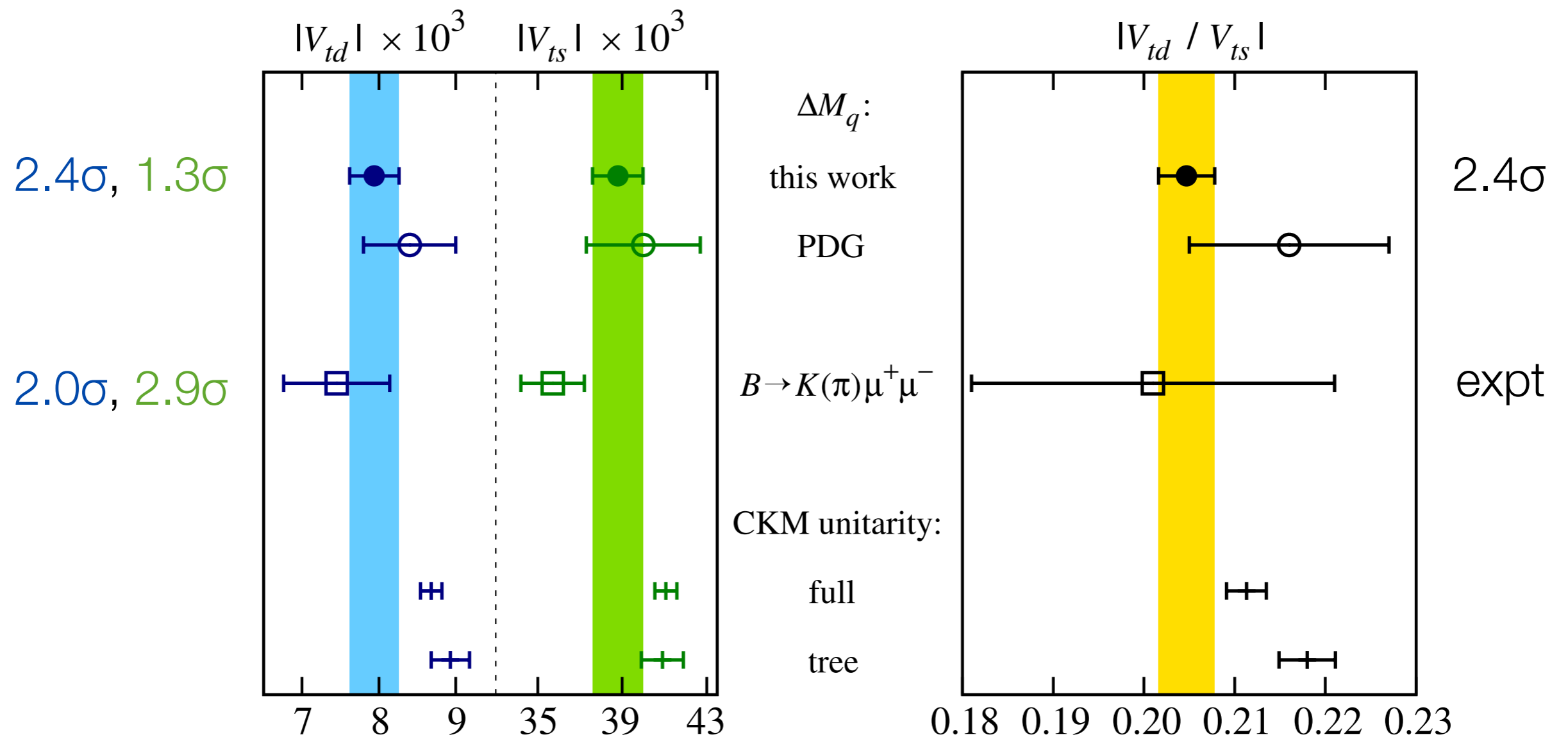
[arXiv:1510.02349](https://arxiv.org/abs/1510.02349)



[arXiv:1507.01618](https://arxiv.org/abs/1507.01618)

CKM Comparison

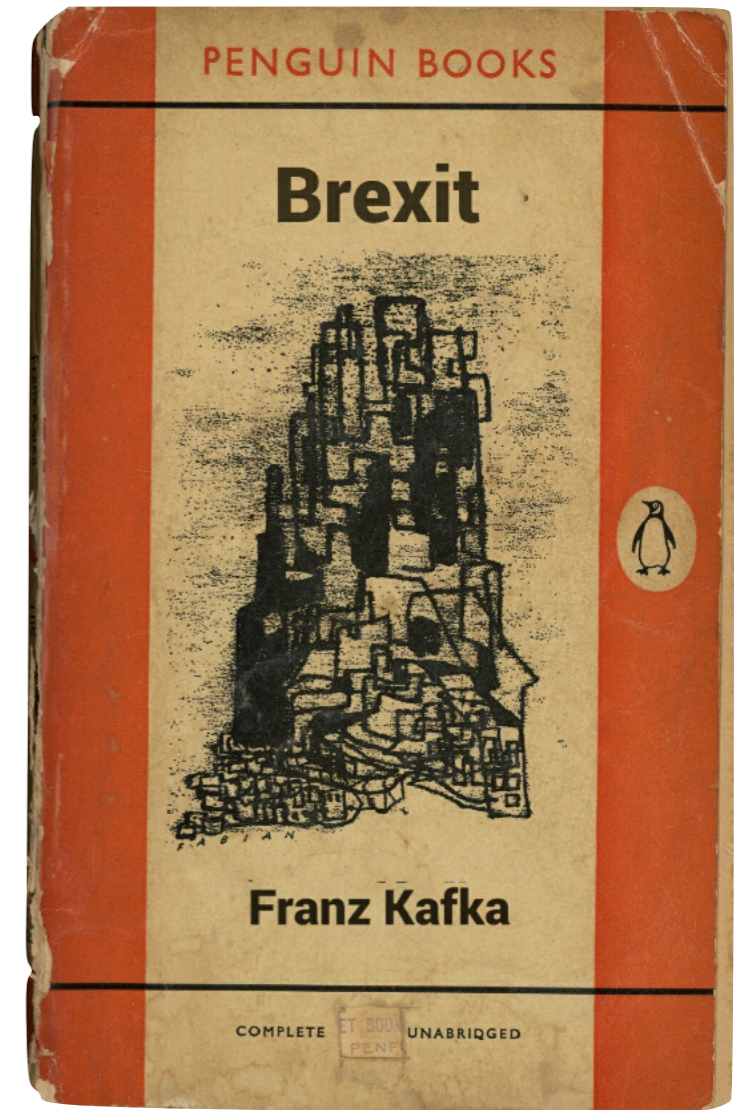
- CKM from FCNC are lower than determinations from trees and unitarity.



Remarks

- Once measured, frequencies are very precise. Thus,
 - mixing matrix elements are not precise enough for legacy experiments, let alone LHC: 3–4% vs 0.06–0.20%.
- A "heavier-than-charm" analysis of these four-quark operators will be
 - very difficult (in chiral perturbation theory) with all-staggered;
 - far off with all-domain-wall, because of the computing needed to generate a suitable range of ensembles and hadron correlators.
- Mixed-action approach: domain-wall valence on 2+1+1 HISQ sea?

Outlook



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Remarks

- Precision of various quantities (compared with experiment) is either "done for now (modulo confirmation)", "proceeding apace", or "way behind":
 - leptonic decays are simple but the rate is helicity suppressed;
 - semileptonic decays are easier experimentally, but harder with lattice QCD: *e.g.*, new discretization effects for $p > \mathbf{0}$;
 - frequencies are super precise but four-quark operators are hard.
- If you want lattice flavor physics to continue, don't just cite FLAG.
- Theoretical physics (QFT in a box) needed: QED and multi-body!

Thank you!

Towards the Ultimate Precision in Flavour Physics

2.- 4.4.2019 @Durham University



- Precision measurements of tree-level observables
- B decays to rare leptonic and semileptonic final states
- CP violation in the charm sector

- Organising Committee
- Simone Bifani (Birmingham)
 - Tim Gershon (Warwick)
 - Alexander Lenz (Durham)
 - Sneha Malde (Oxford)
 - Mark Williams (Manchester)