Prospects for D decays on the lattice

Maxwell T. Hansen

April 2nd, 2019

Flavor anomalies

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- Improving **QCD** predictions is crucial to **confirming the significance** and **interpreting** the anomalies

experiment = **SM** x *perturbative QCD* x **(non-perturbative QCD)**
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<u>*experiment*</u> = **SM** x *perturbative QCD* x **(non-perturbative QCD)**
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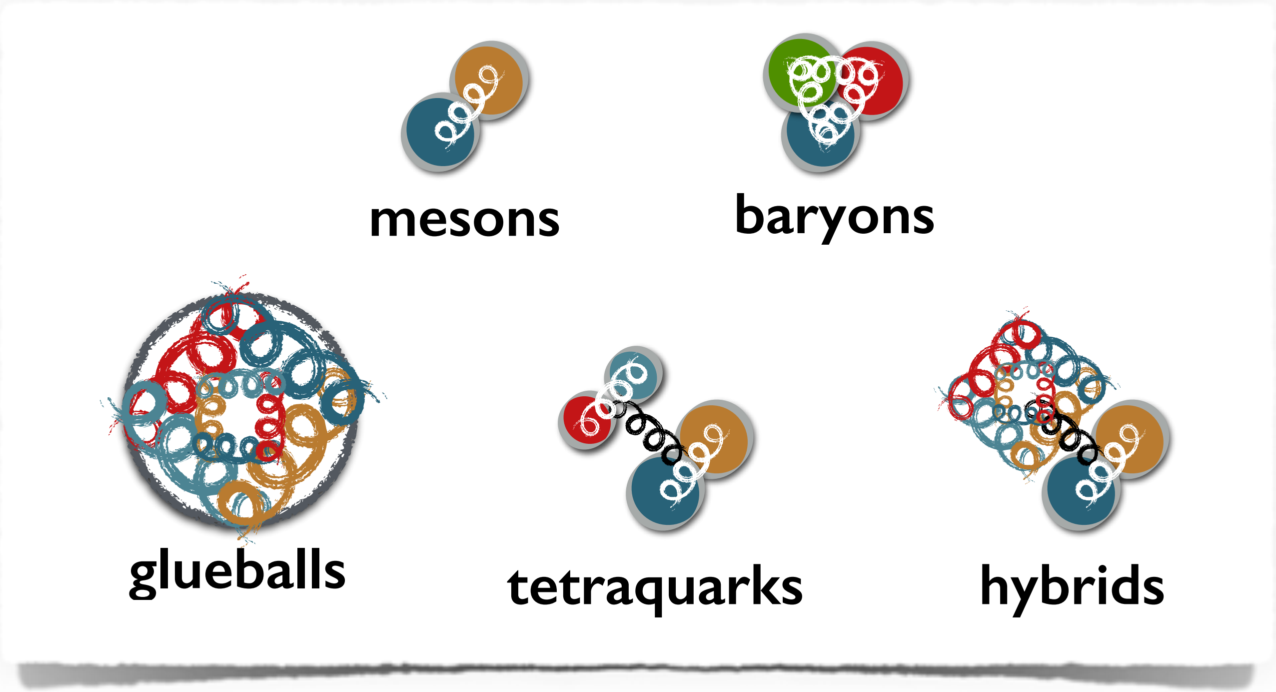
- **QCD** is complicated
- Difficult to extract rigorous and systematic predictions from the **fundamental theory**



Lattice QCD is a powerful tool for extracting QCD predictions

LQCD = evaluating a difficult integral numerically

$$\text{observable} = \int \mathcal{D}\phi \; e^{iS} \left[\begin{array}{c} \textbf{interpolator} \\ \textbf{for observable} \end{array} \right]$$

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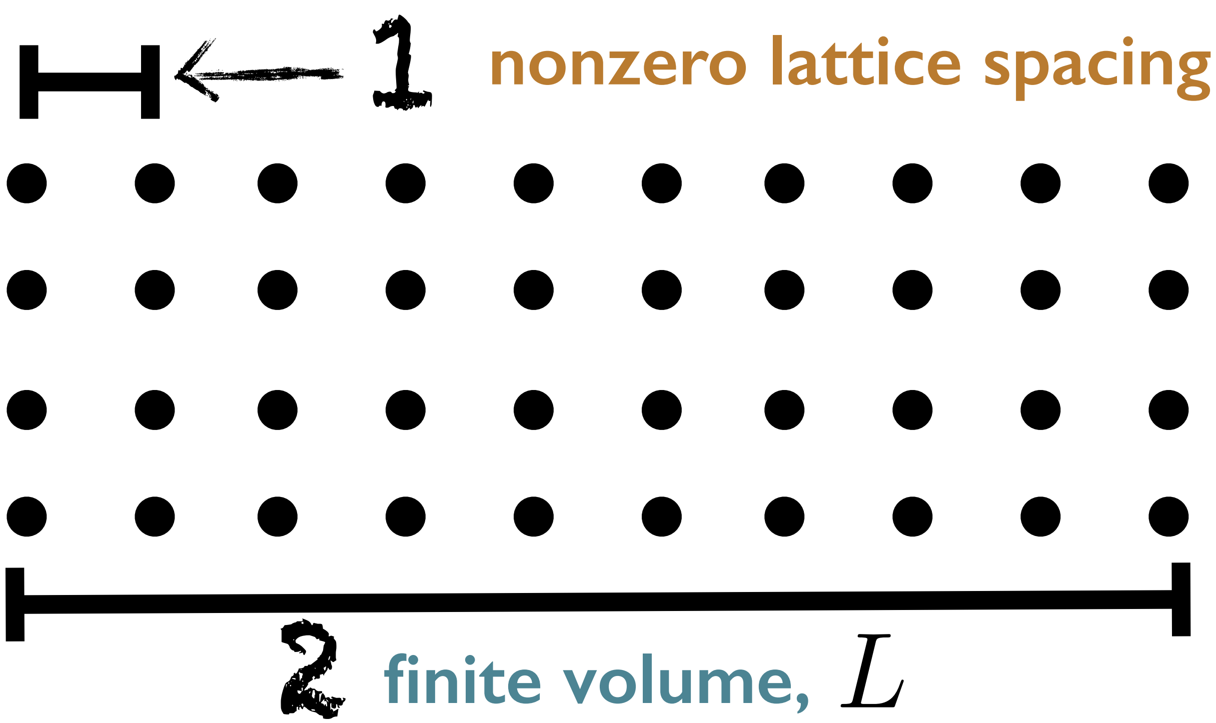
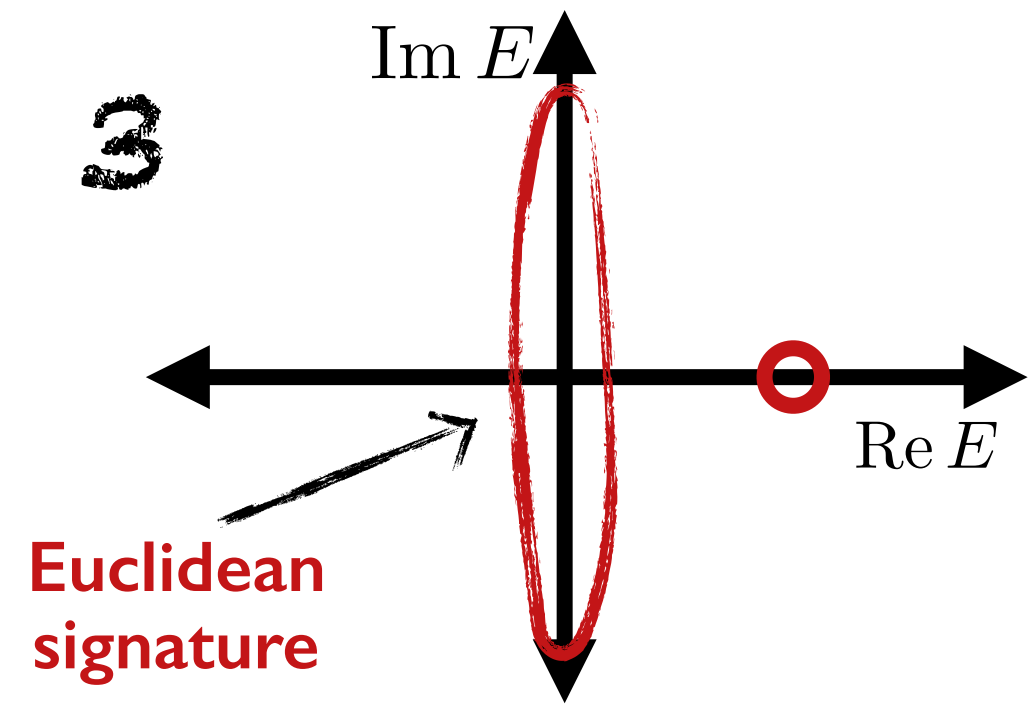
$$\left(\begin{array}{c}\textbf{observable?}\\ \text{discretized, finite volume,}\\ \text{Euclidean, heavy pions}\end{array}\right) = \int \prod_{i}^{N} d\phi_i \; e^{-S} \left[\begin{array}{c}\textbf{interpolator}\\ \textbf{for observable}\end{array}\right]$$

To do so we have to make three compromises



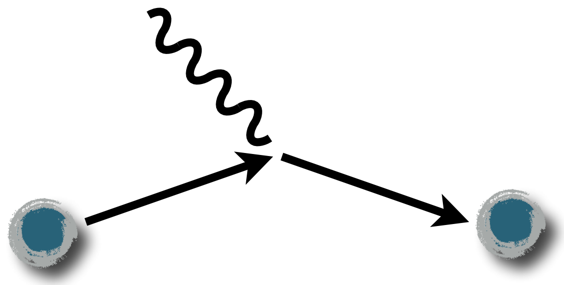


Also… Unphysical pion masses $M_{\pi,\mathrm{lattice}} > M_{\pi,\mathrm{our\ universe}}$

But calculations at the physical pion are becoming common

Multi-hadron transitions and LQCD

- Single-hadron initial and final states
  - Conceptually straightforward
  - Calculated directly in **LQCD**
    - Euclidean time poses no problem / lattice → 0 / volume → ∞
  - See FLAG averages



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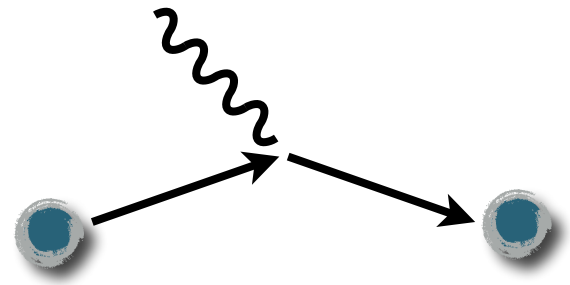
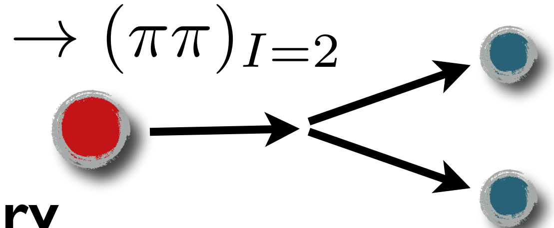


- A (non-resonant) multi-hadron final state
  - Significantly more challenging **LQCD** observables
  - Need to access multi-hadron states from a finite-volume theory
  - Alternative, (lattice supplemented) effective theory



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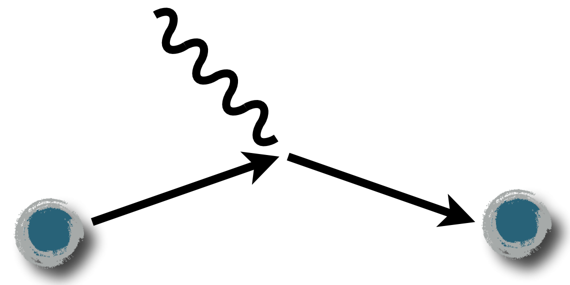

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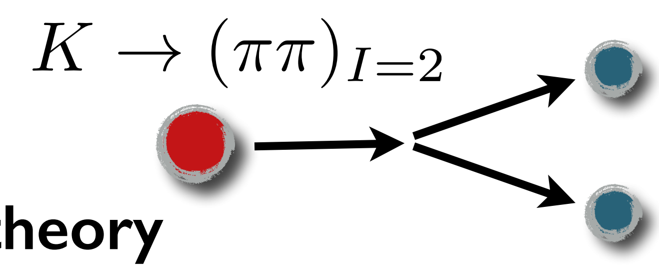

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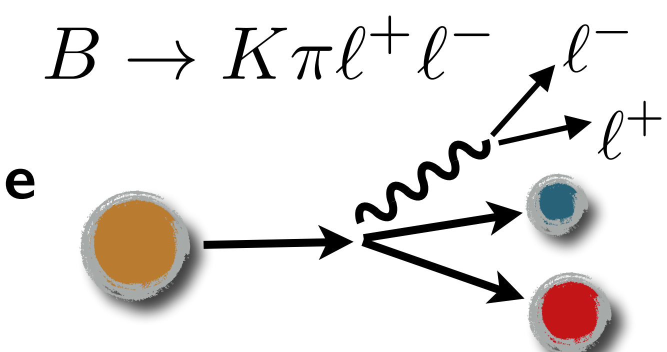
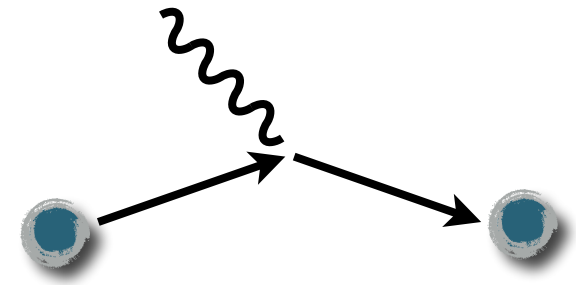


- A resonant final state
  - All the above issues still hold!
  - No rigorous way to treat the resonance like a stable particle

  … unless it is stabilized via heavy pions

Multi-hadron transitions and LQCD



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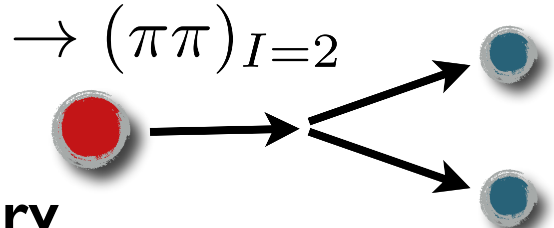

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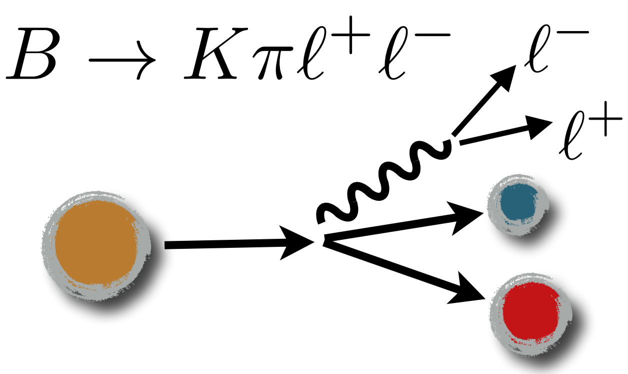

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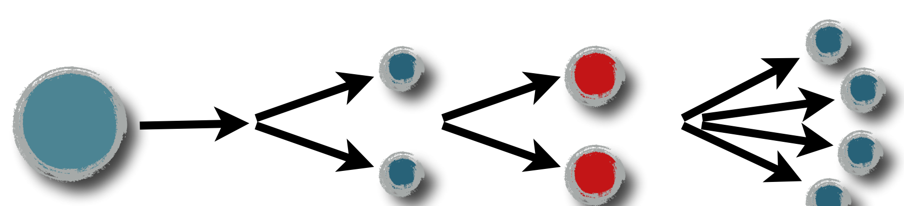
- Multi-hadron states for $\sqrt{s} > 4M_\pi$
  - Volume mixes the two-, four-, six-particle contributions
  - All or nothing (must constrain the entire S-matrix for a prediction)



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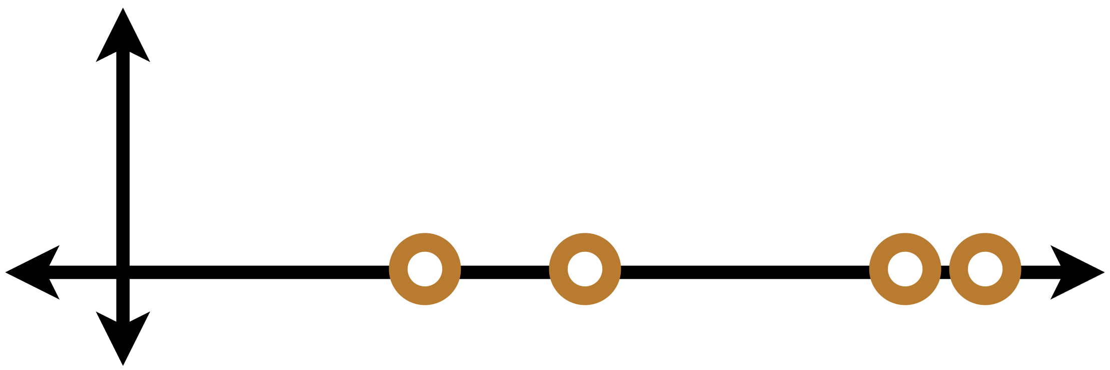
Lattice observables

'On the lattice' one can calculate finite-volume energies and matrix elements



$$\langle \mathcal{O}_j(\tau)\mathcal{O}_i^\dagger(0)\rangle = \sum_n \langle 0|\mathcal{O}_j(\tau)|E_n\rangle\langle E_n|\mathcal{O}_i^\dagger(0)|0\rangle = \sum_n e^{-E_n(L)\tau} Z_{n,j} Z_{n,i}^*$$

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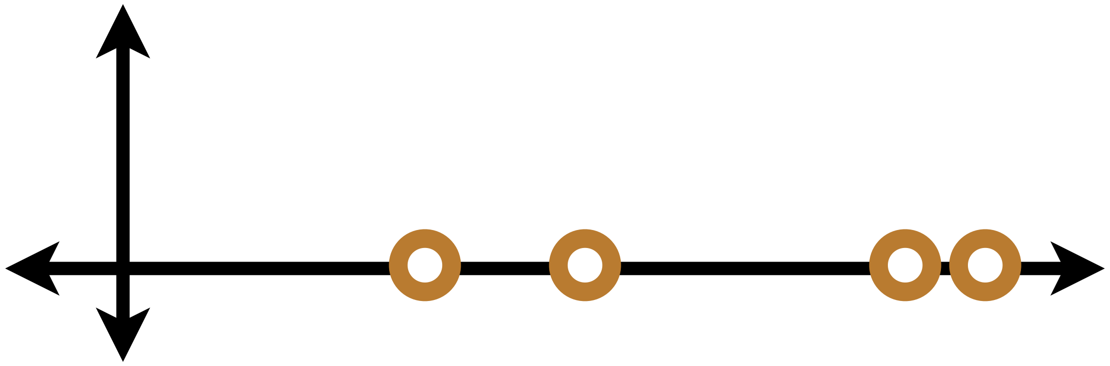

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Can determine optimized operators by ‘diagonalizing’ the correlator matrix (GEVP)

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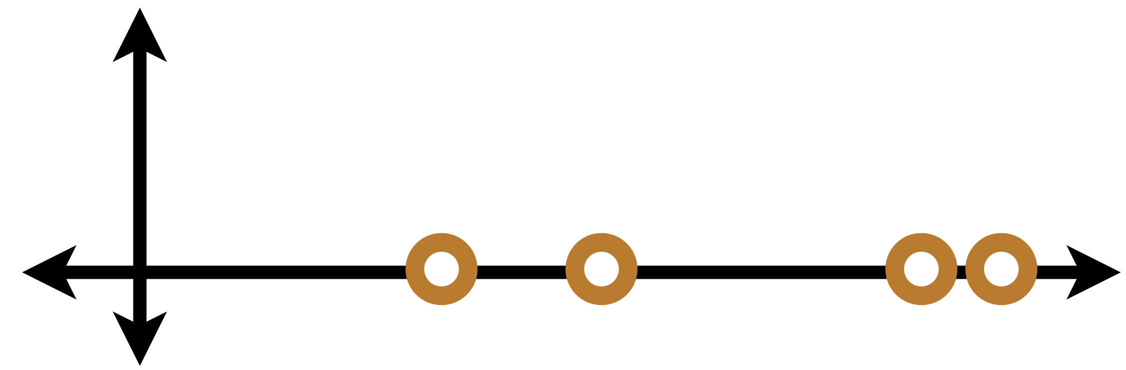

'On the lattice' one can calculate finite-volume **energies** and **matrix elements**

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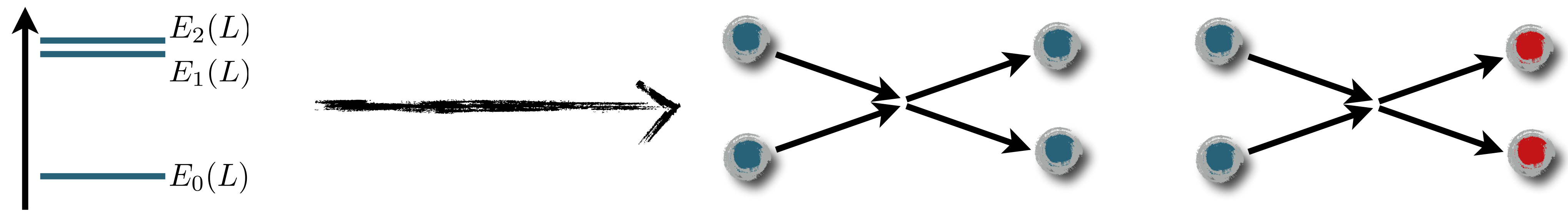
- **Our task is relate $E_n(L)$ and $\langle E_{m'}|\mathcal{J}(0)|E_m\rangle$ to experimental observables**
- **In this talk: less concerned with discretization effects**

  (Take lattice effects to be small/included in systematic uncertainty)

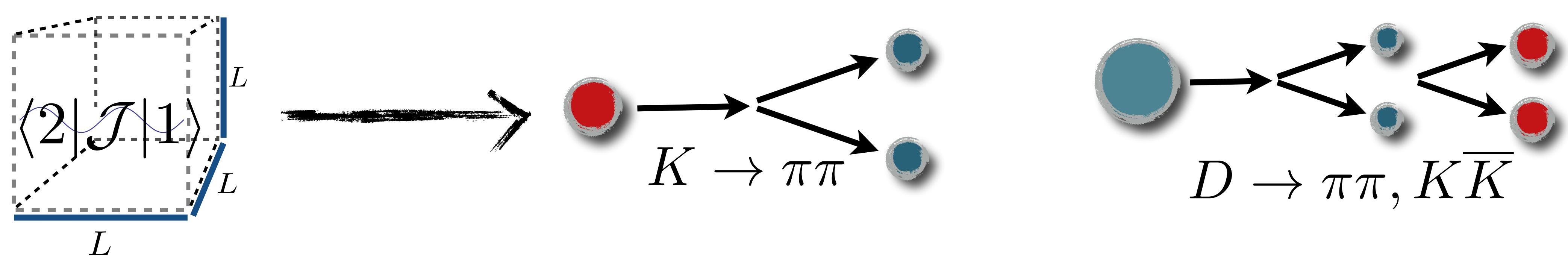
Multi-hadron processes from LQCD

KEY IDEA: We can use the finite volume as a **tool** to extract multi-hadron observables

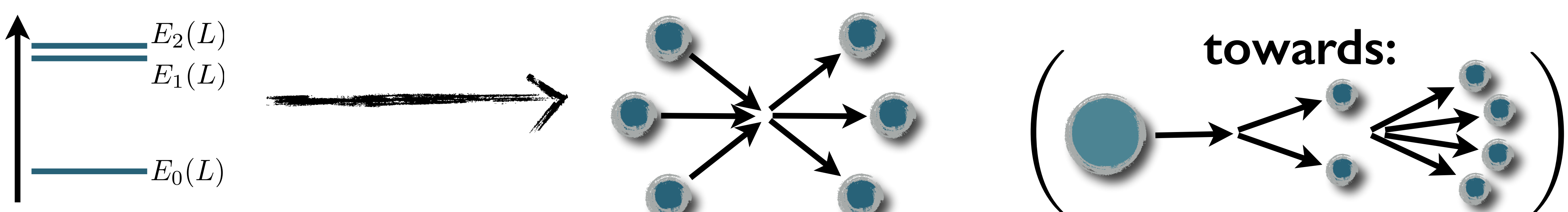
- **Two-to-two scattering**



- **One-to-two transitions**

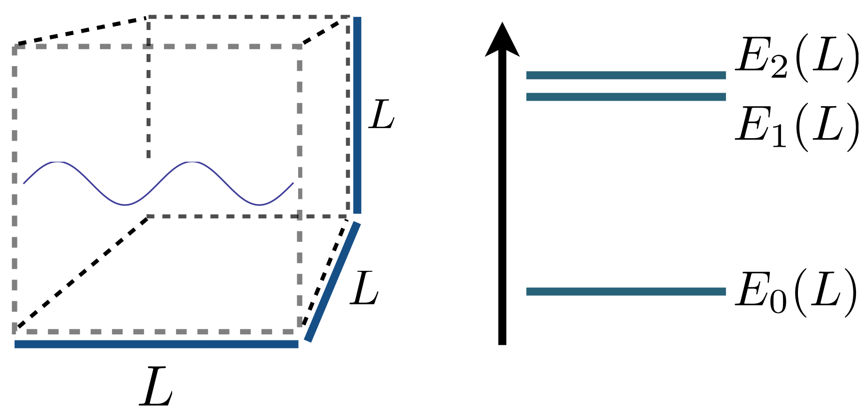


- **Two-to-three and three-to-three scattering**



The finite-volume as a tool

- **Finite-volume set-up**



- **cubic, spatial volume (extent *L*)**
- **periodic boundary conditions**

$$\vec{p} = \frac{2\pi}{L}\vec{n}\,, \quad \vec{n} \in \mathbb{Z}^3$$

- ***L* is large enough to neglect** $e^{-M_\pi L}$

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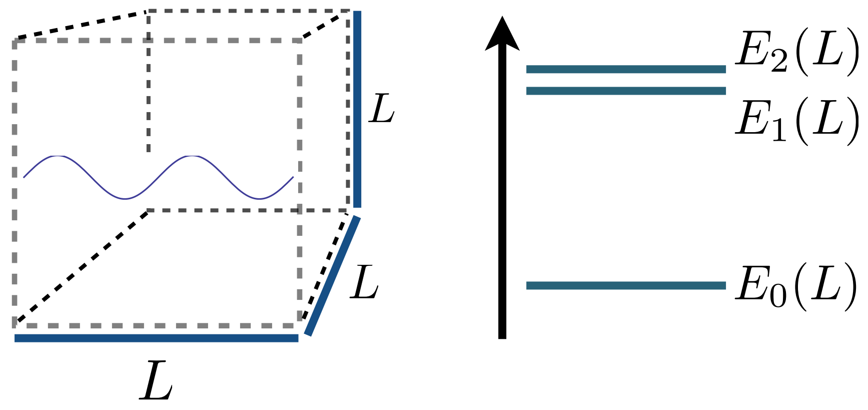

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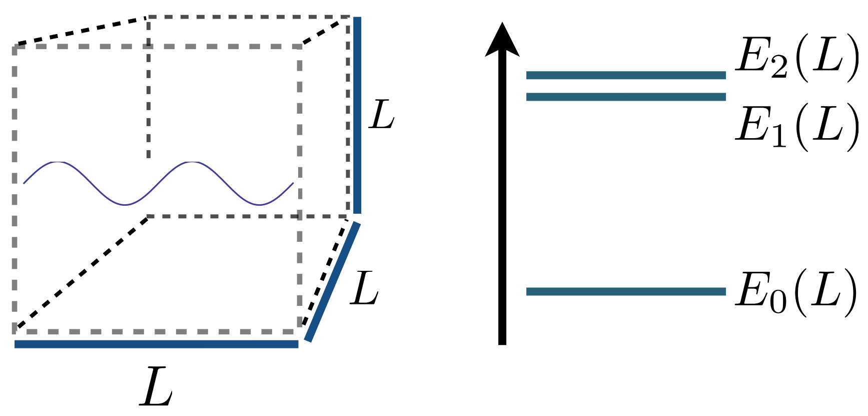

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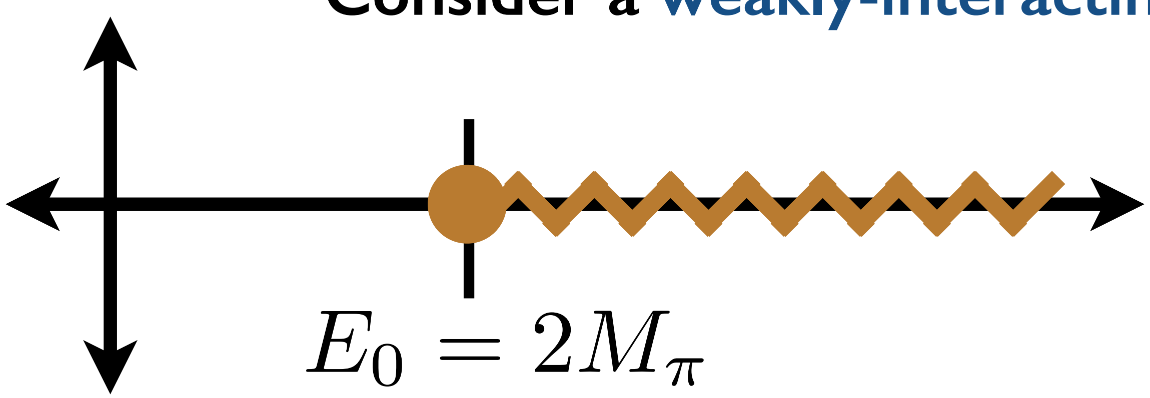
Scattering observables leave an *imprint* on finite-volume quantities

Consider a weakly-interacting, two-body system with no bound states



$$E_0 = 2M_\pi$$

Infinite-volume ground state

$$\mathcal{M}_{\ell=0}(2M_\pi) = -32\pi M_\pi a$$

Information is in the scattering amplitude

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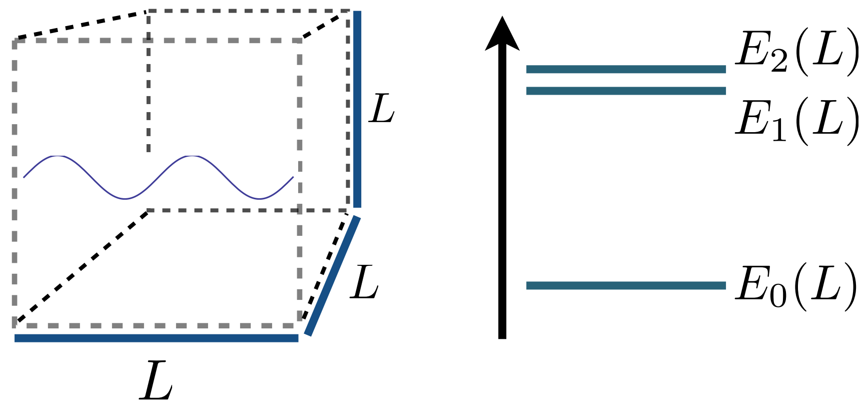

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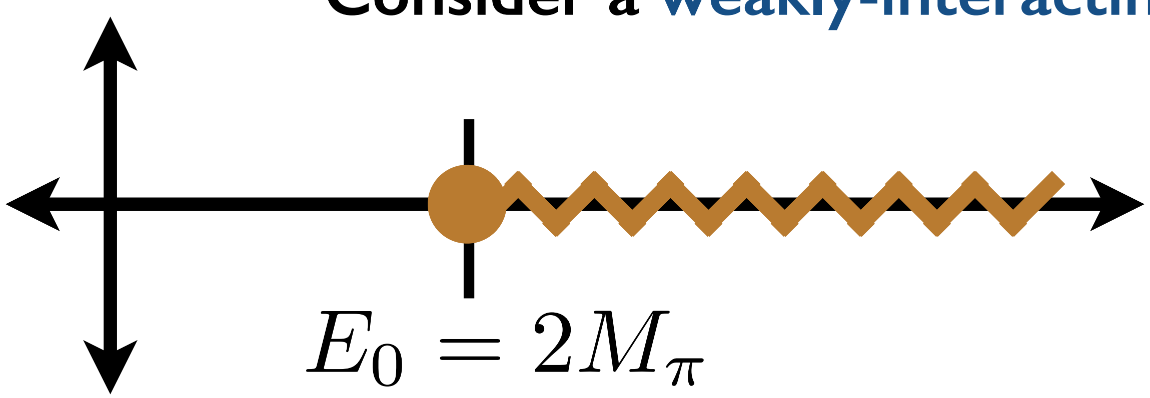

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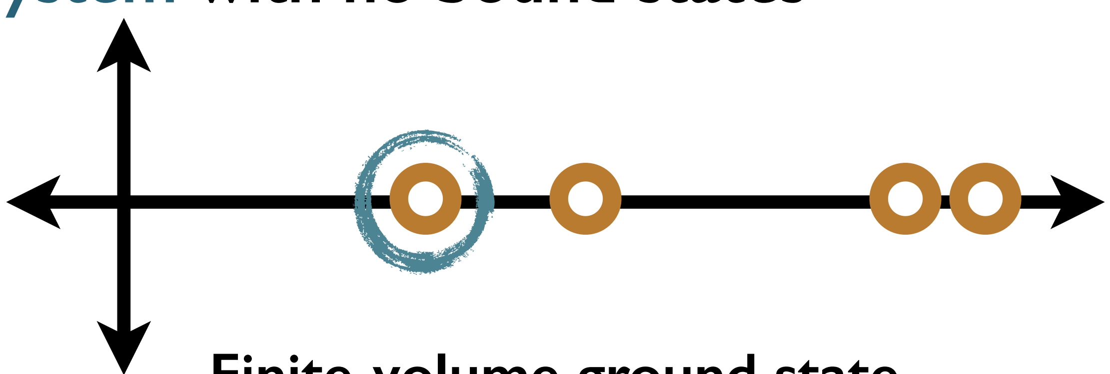

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Huang, Yang (1958)

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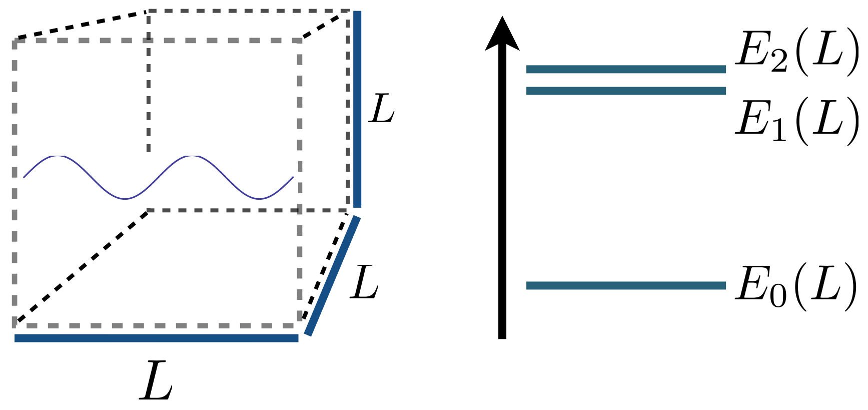

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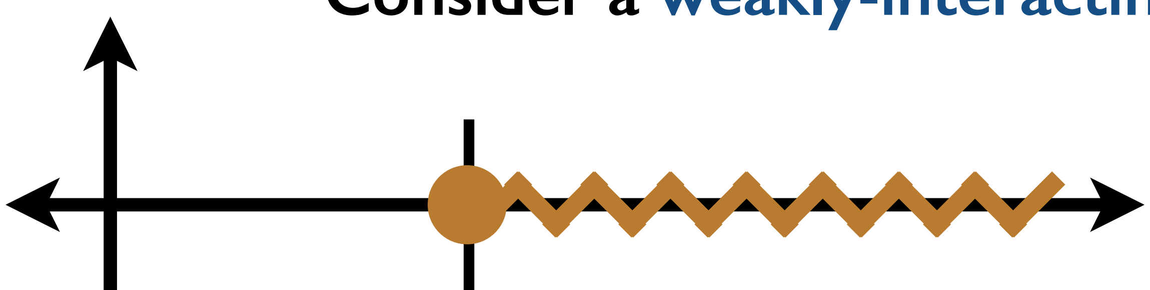

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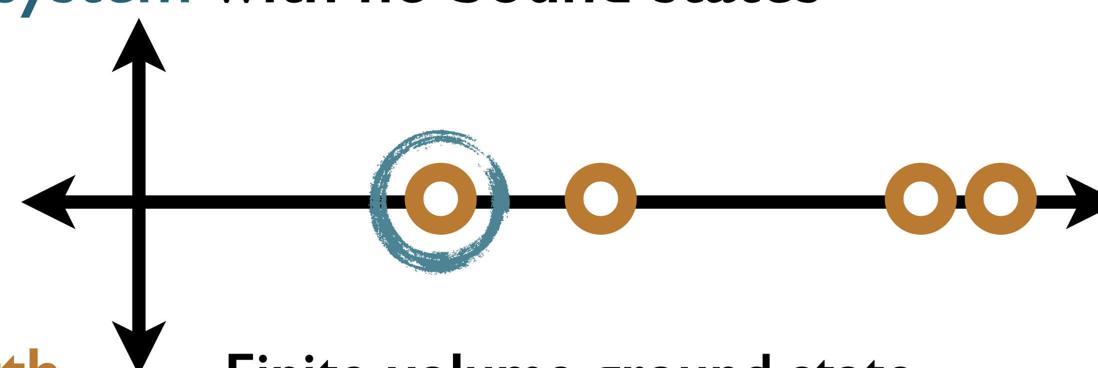
scattering length



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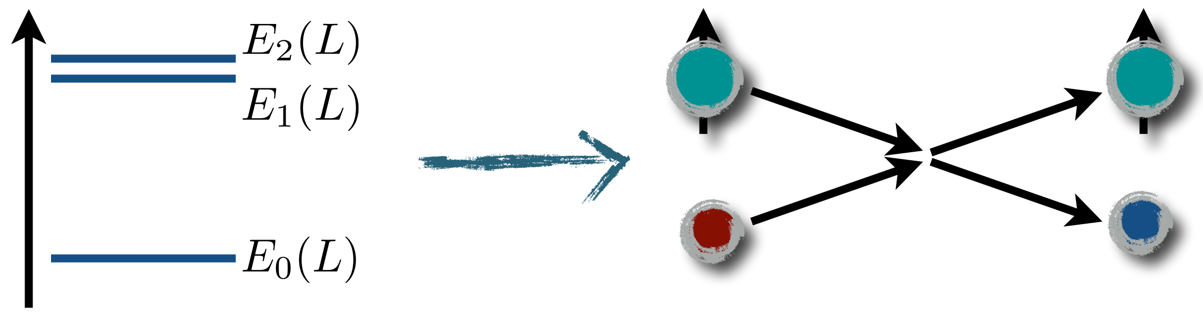
General two-to-two scattering

- **Lüscher's formalism + extensions give a general mapping**



- **All results are contained in a generalized quantization condition**

$$\det\left[\mathcal{M}_2^{-1}(E_n^*) + F(E_n, \vec{P}, L)\right] = 0$$

scattering amplitude **known geometric function**

Matrices in angular momentum, spin and channel space

Huang, Yang (1958) ○ Lüscher (1986, 1991) ○ Rummukainen, Gottlieb (1995)
Kim, Sachrajda, Sharpe (2005) ○ Christ, Kim, Yamazaki (2005) ○ He, Feng, Liu (2005)
Beane, Detmold, Savage (2007) ○ Tan (2008) ○ Leskovec, Prelovsek (2012) ○ Bernard *et. al.* (2012)
MTH, Sharpe (2012) ○ Briceño, Davoudi (2012) ○ Li, Liu (2013) ○ Briceño (2014)

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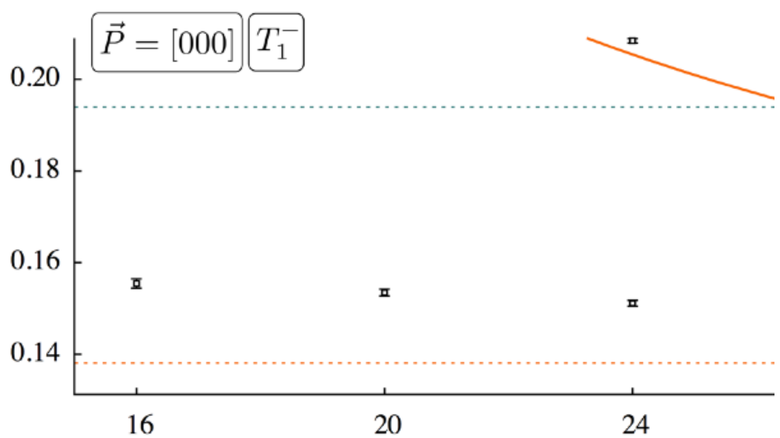

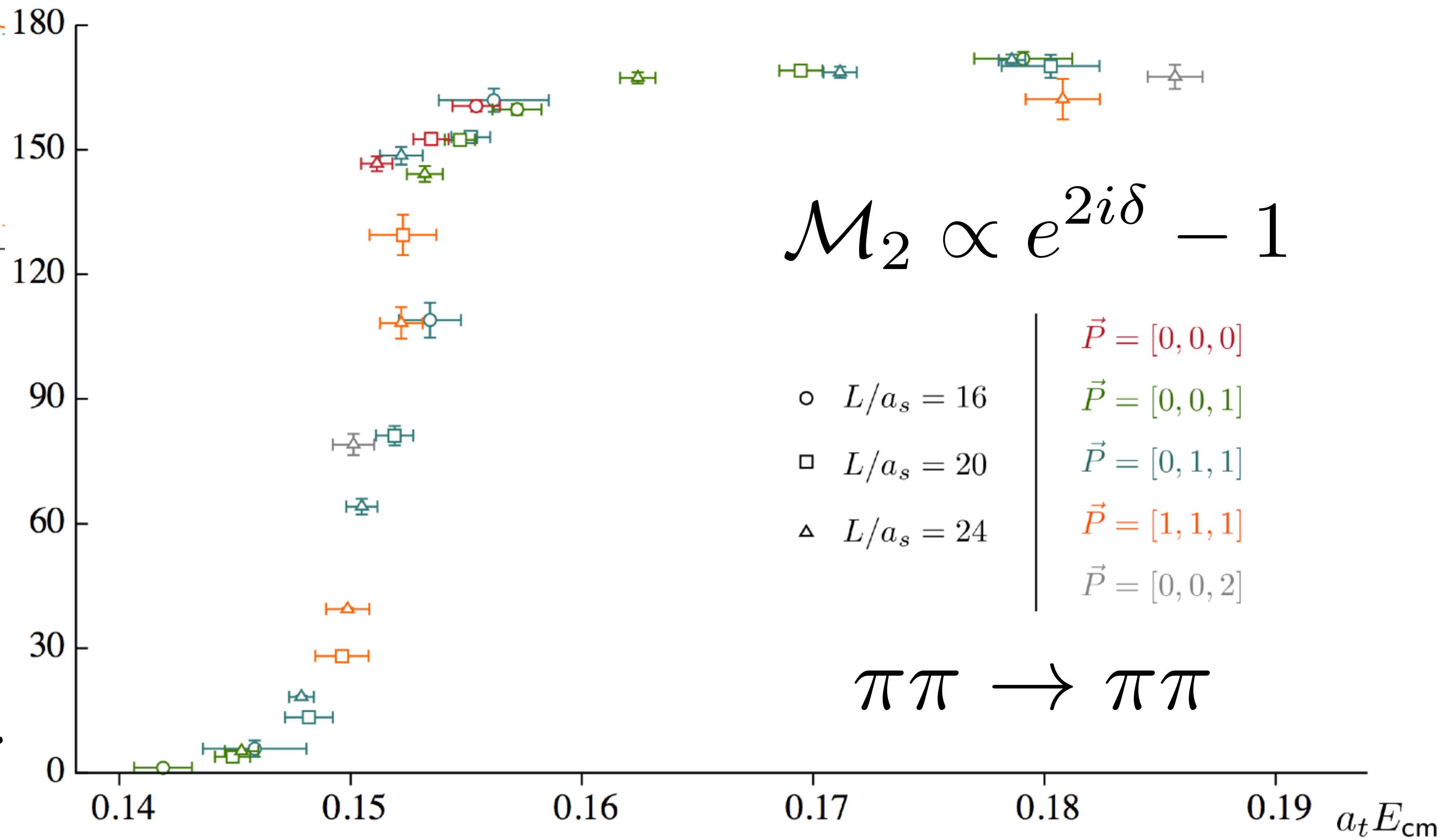

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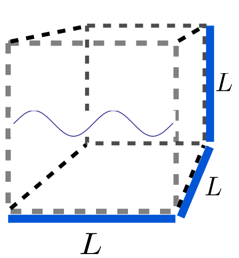
$$\pi\pi \to \pi\pi$$



from Dudek, Edwards, Thomas in *Phys.Rev.* D87 (2013) 034505

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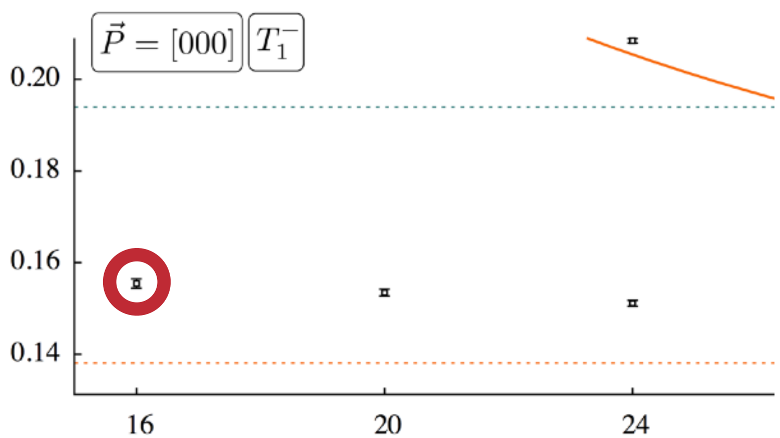

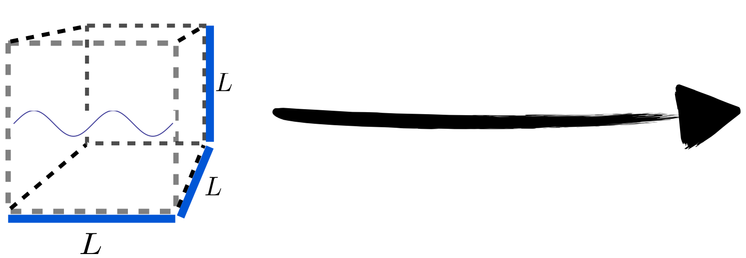

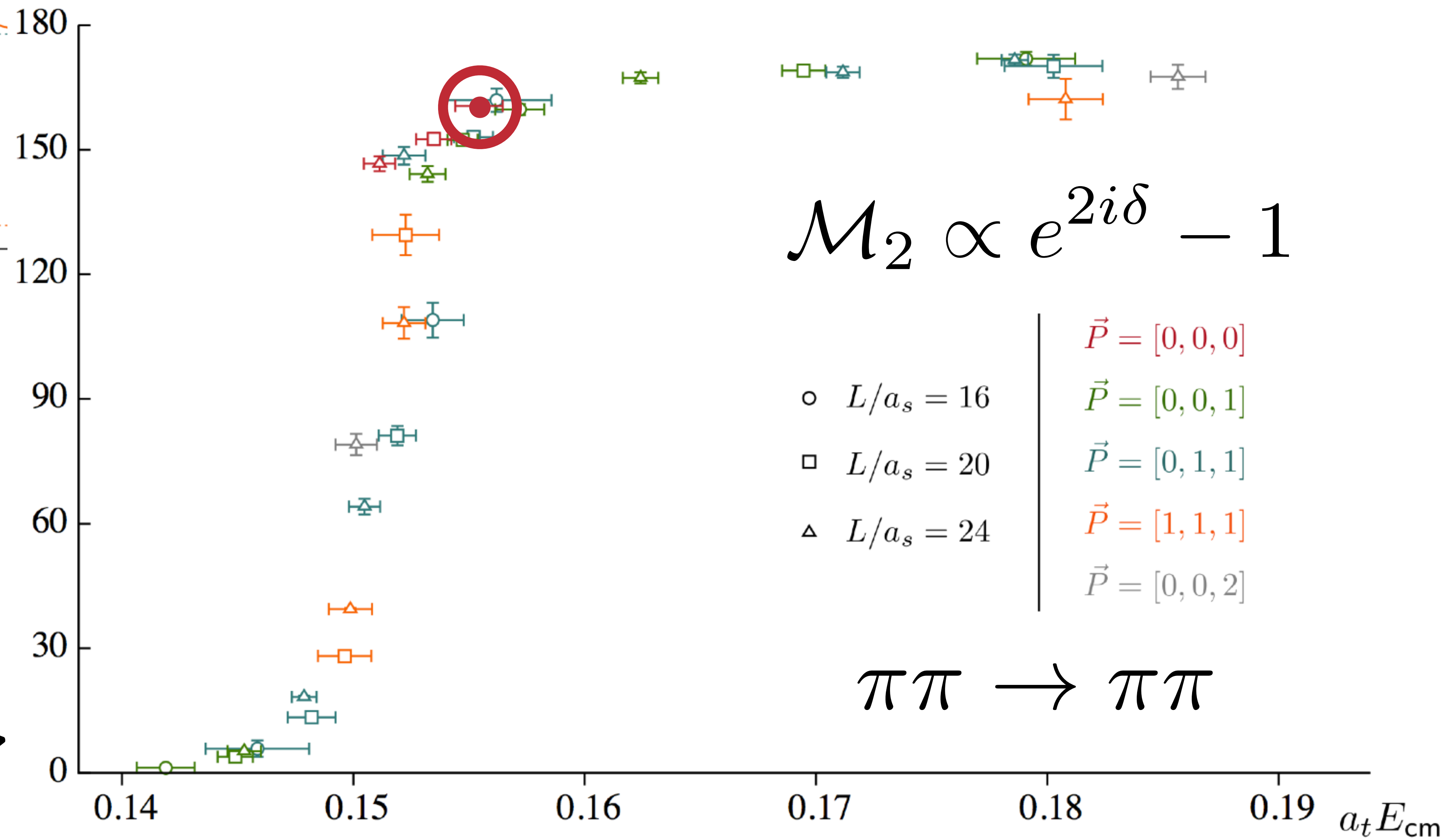

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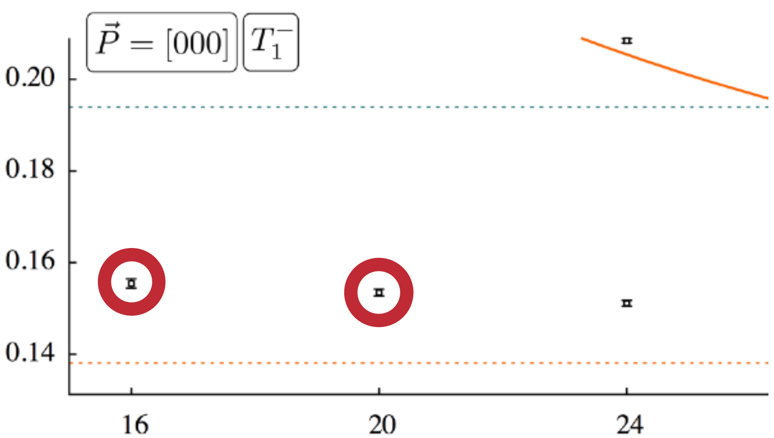

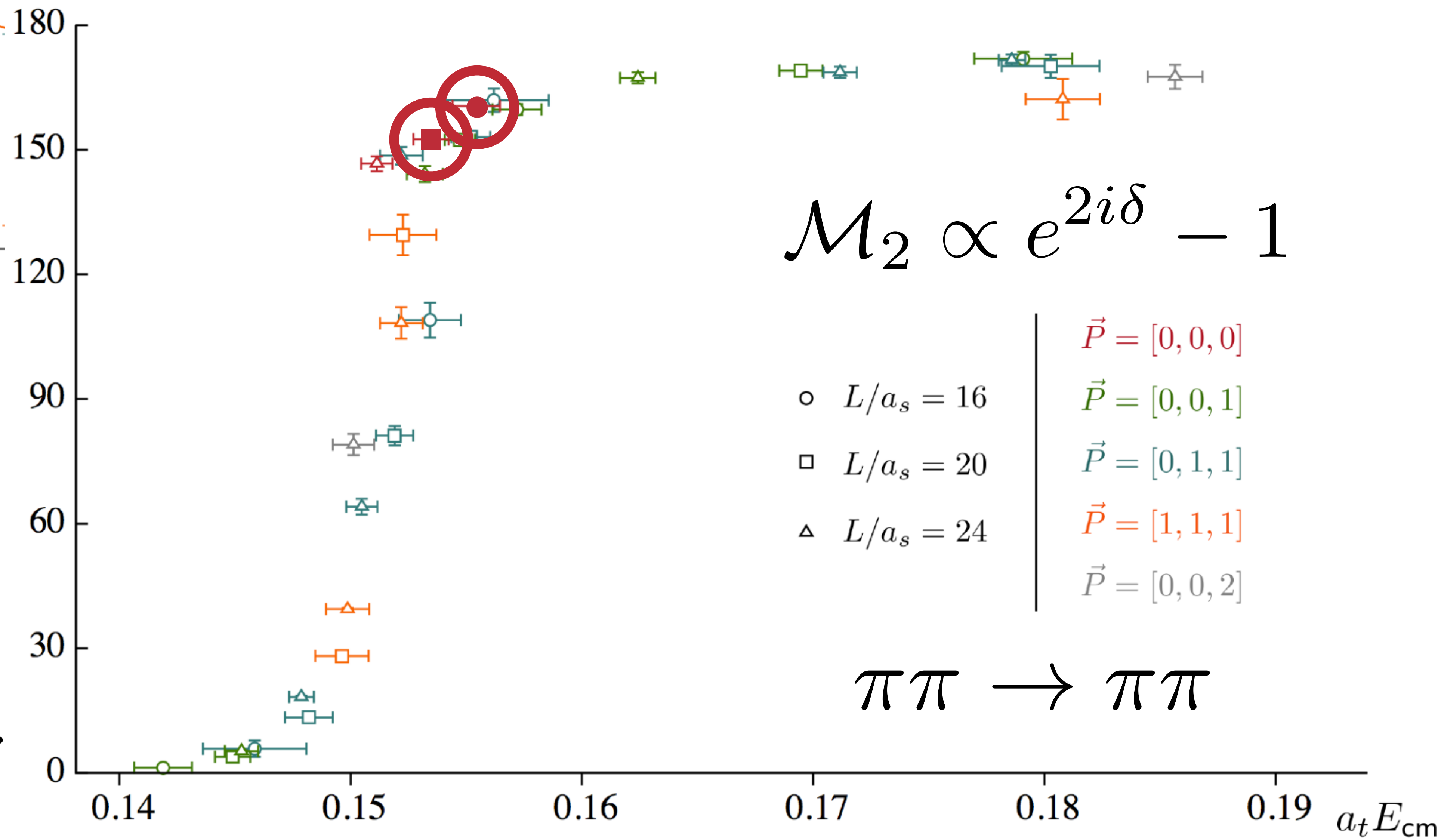

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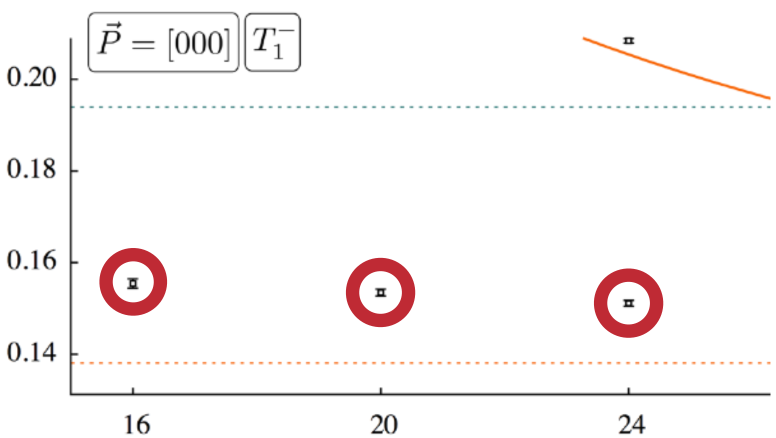

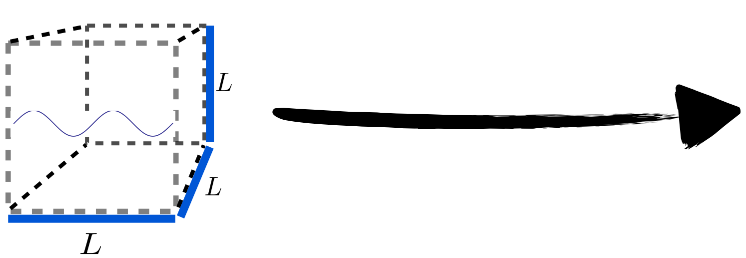

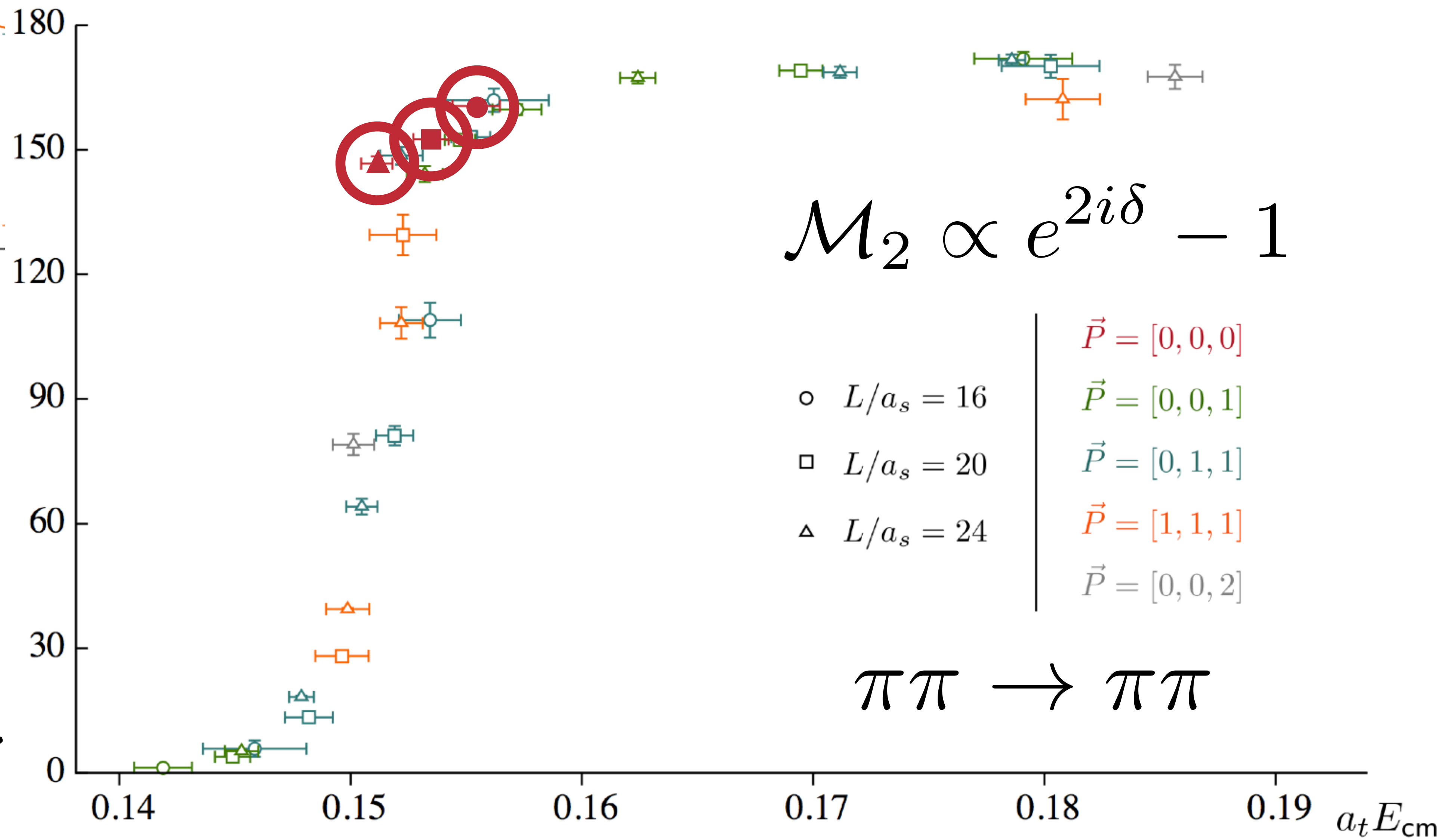

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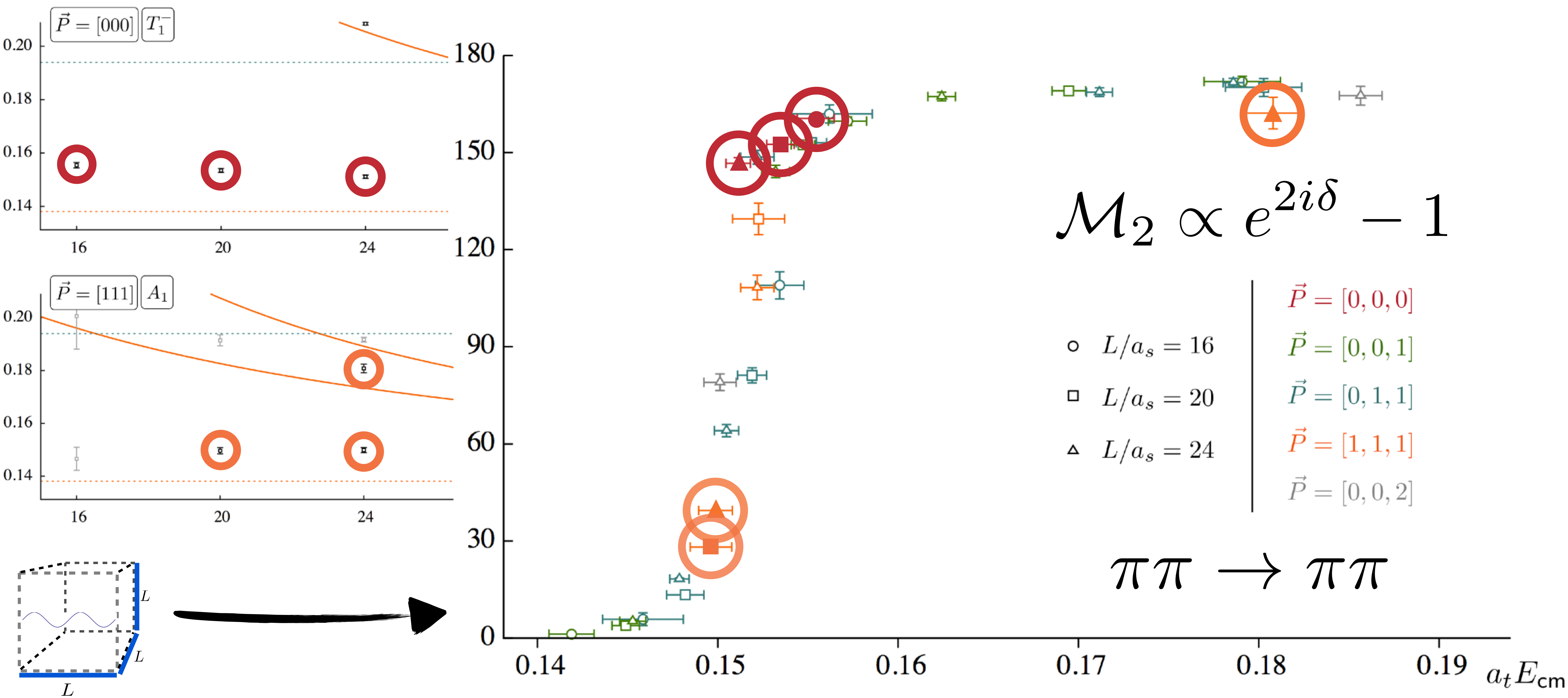

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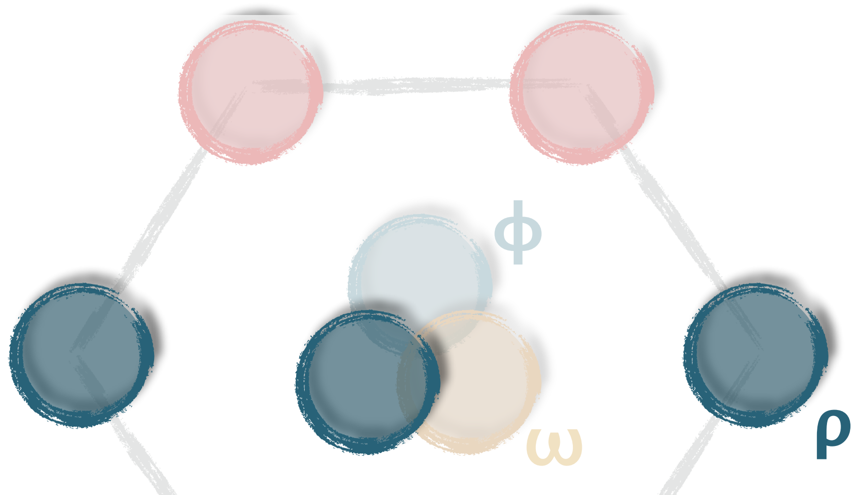
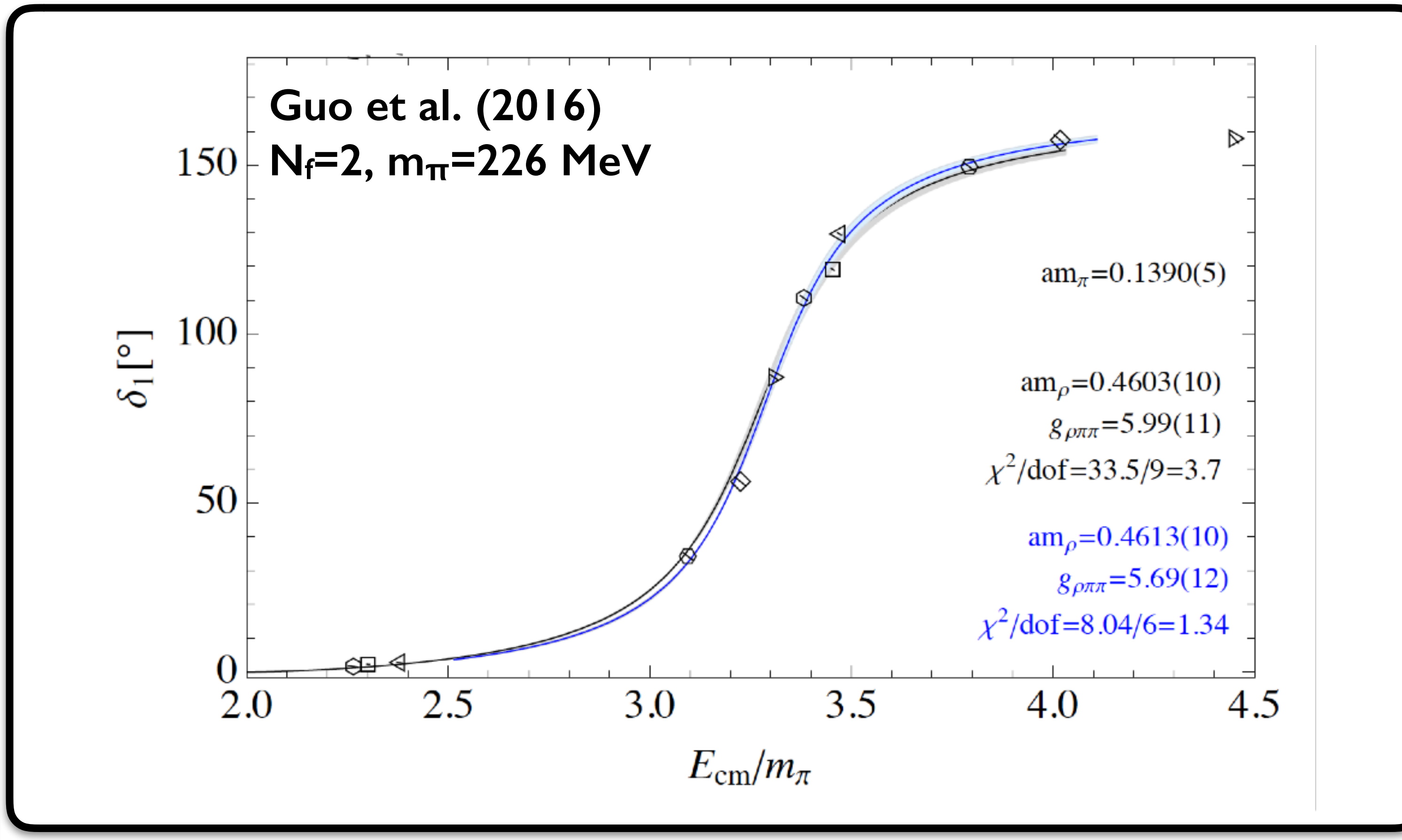
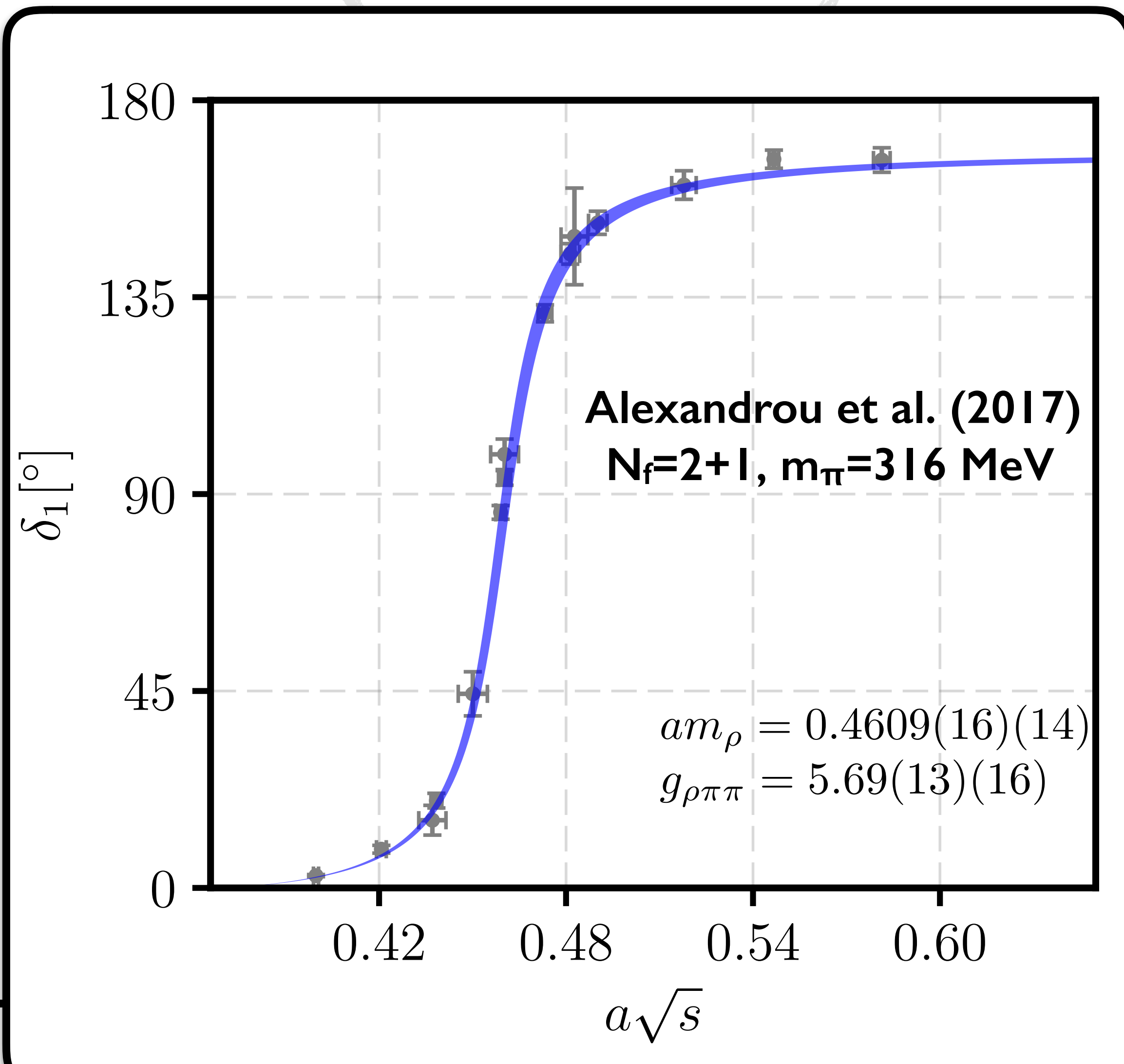
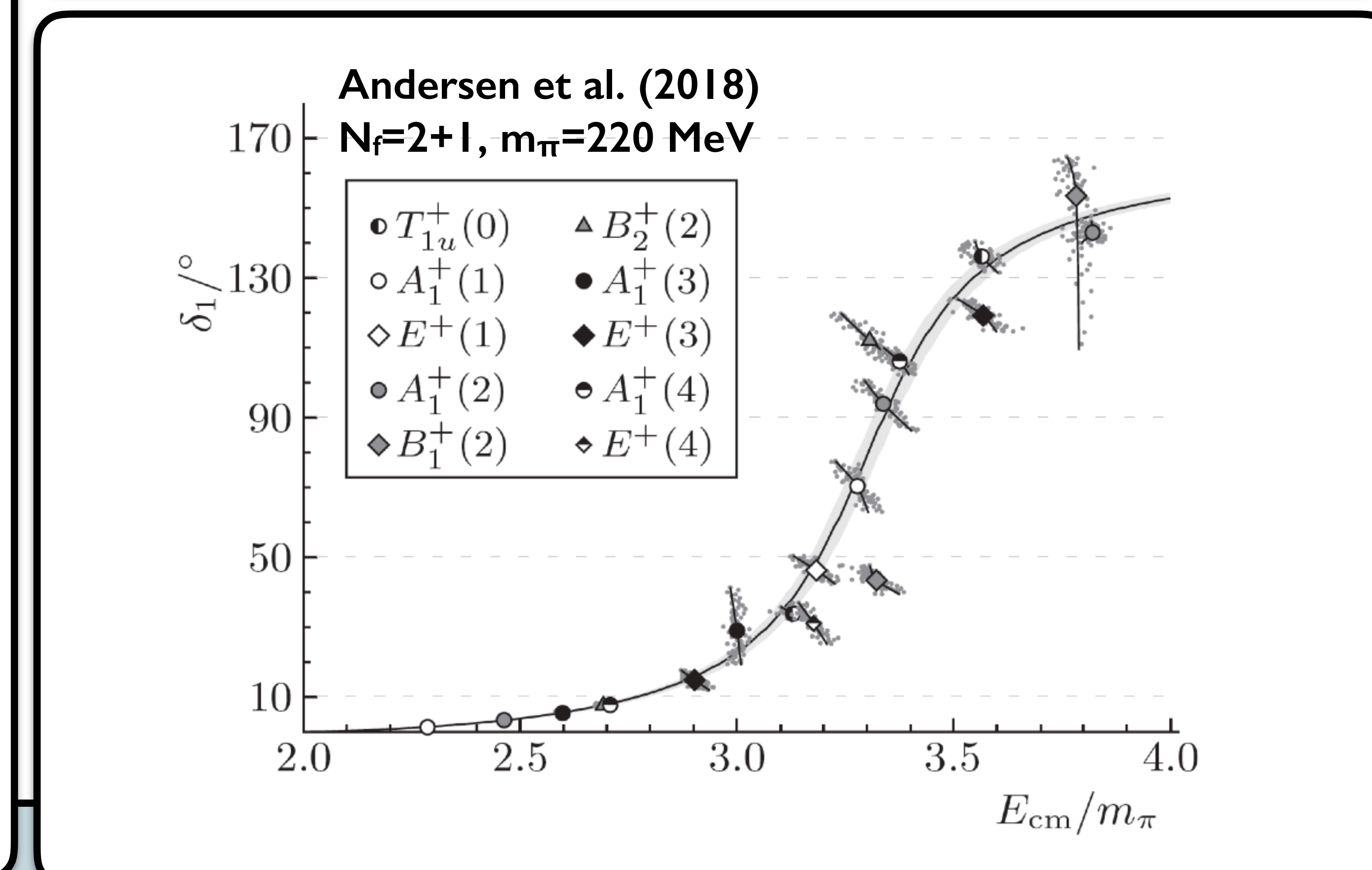
$$\rho \to \pi\pi$$

$$I^G(J^{PC}) = 1^+(1^{--})$$







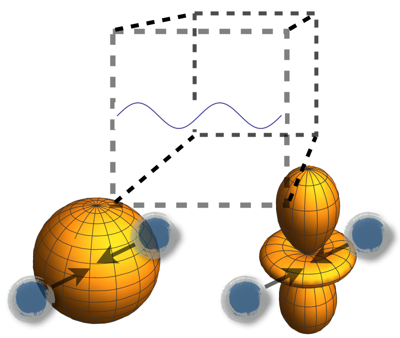


Coupled channels

- **The cubic volume mixes different partial waves…**

e.g. $K\pi \to K\pi$
$\vec{P} \neq 0$

$$\longrightarrow \det\left[\begin{pmatrix} \mathcal{M}_s^{-1} & 0 \\ 0 & \mathcal{M}_p^{-1} \end{pmatrix} + \begin{pmatrix} F_{ss} & F_{sp} \\ F_{ps} & F_{pp} \end{pmatrix}\right] = 0$$



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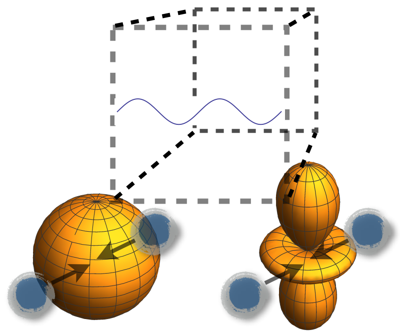

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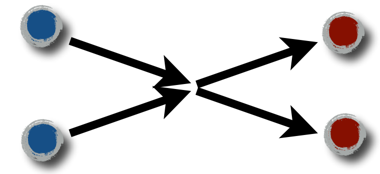
e.g. $a = \pi\pi$, $b = K\overline{K}$ $\longrightarrow$ $\det\left[\begin{pmatrix} \mathcal{M}_{a\to a} & \mathcal{M}_{a\to b} \\ \mathcal{M}_{b\to a} & \mathcal{M}_{b\to b} \end{pmatrix}^{-1} + \begin{pmatrix} F_a & 0 \\ 0 & F_b \end{pmatrix}\right] = 0$



MTH, Sharpe (2012) Briceño, Davoudi (2012)

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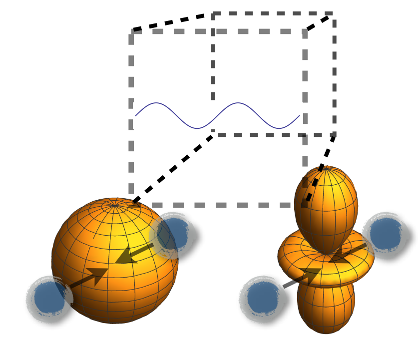

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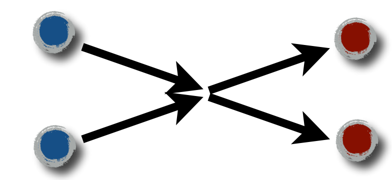


- **The road to physics…**

MTH, Sharpe (2012) ○ Briceño, Davoudi (2012)

Calculate a matrix of correlators with a large & varied operator basis

$\langle \mathcal{O}_a(\tau)\mathcal{O}_b^\dagger(0)\rangle$

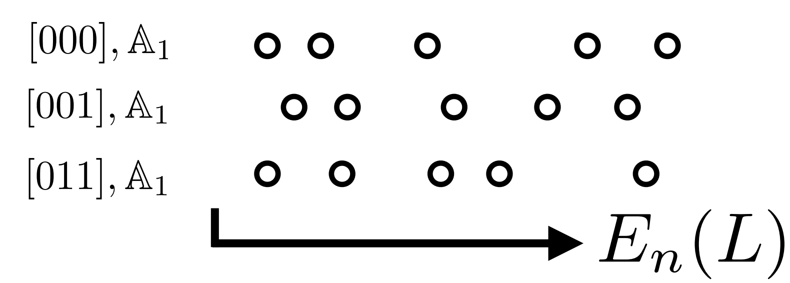
→

Diagonalize (GEVP) to reliably extract finite-volume energies

$\langle \Omega_m(\tau)\Omega_m^\dagger(0)\rangle \sim e^{-E_m(L)\tau}$

→

Vary *L* and *P* to recover a dense set of energies



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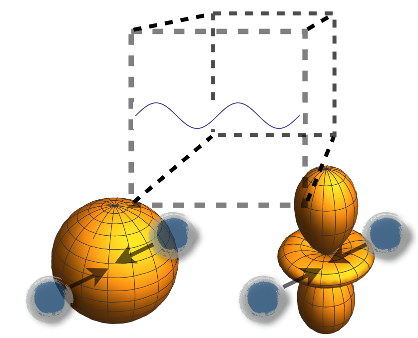

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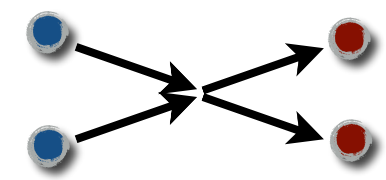

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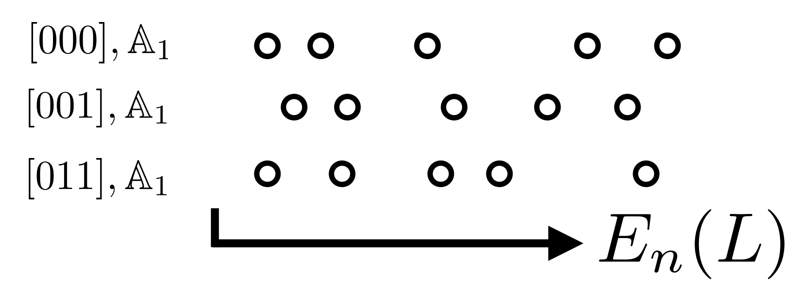

Identify a broad list of K-matrix parametrizations

polynomials and poles

EFT based

dispersion theory based

Coupled channels

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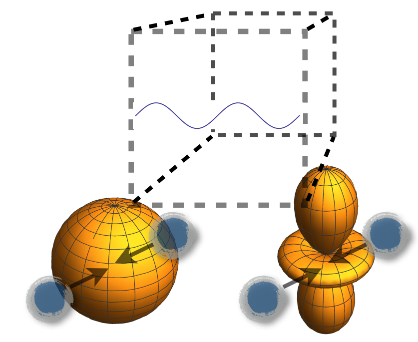

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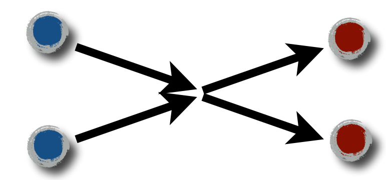

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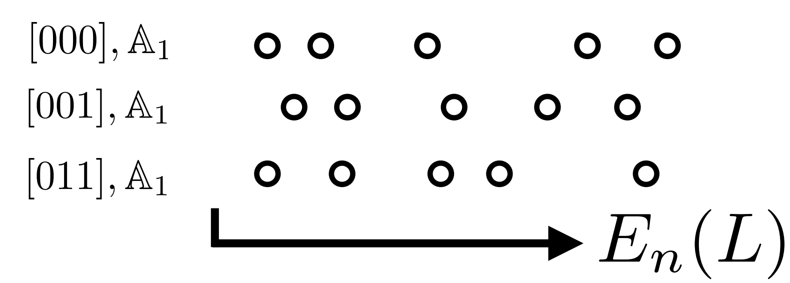


↓

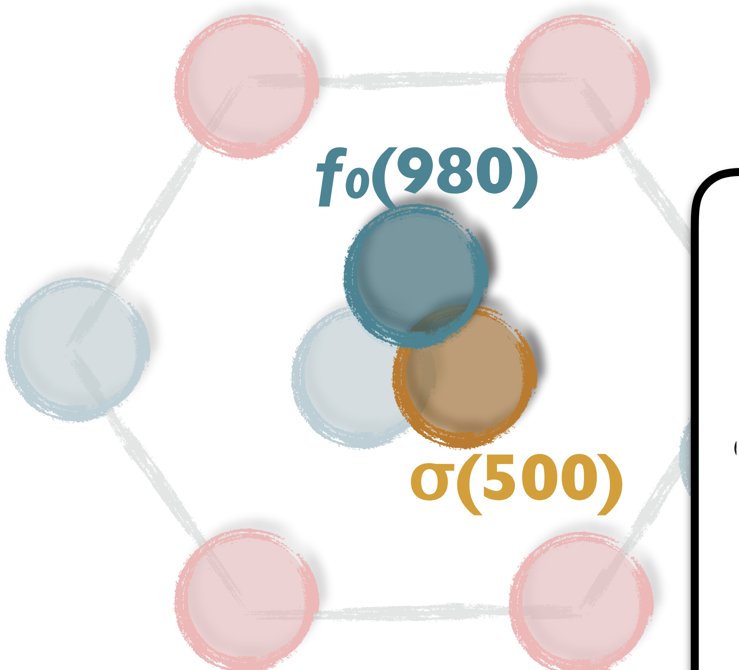
Identify a broad list of K-matrix parametrizations

polynomials and poles | EFT based | dispersion theory based

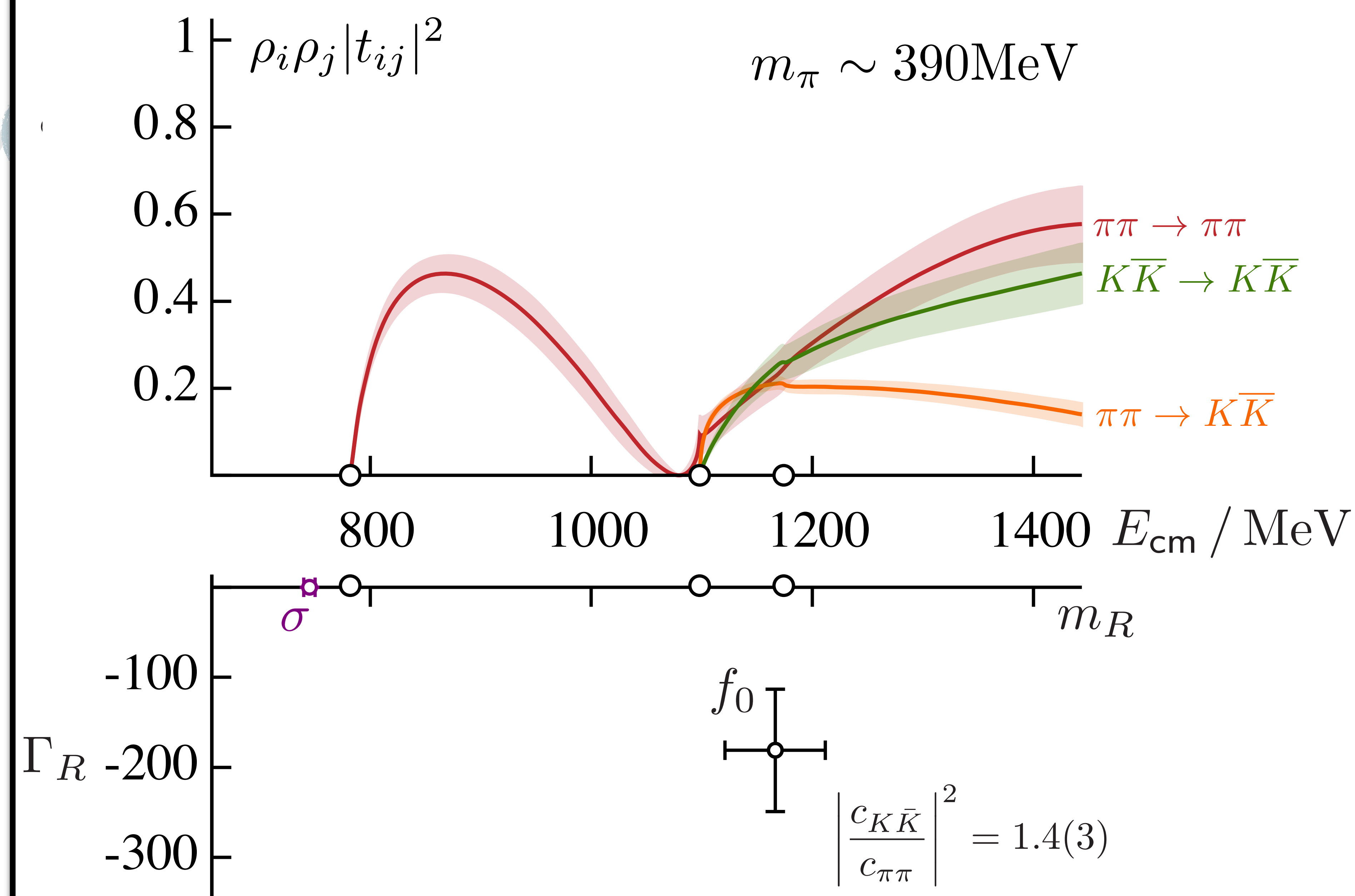
→

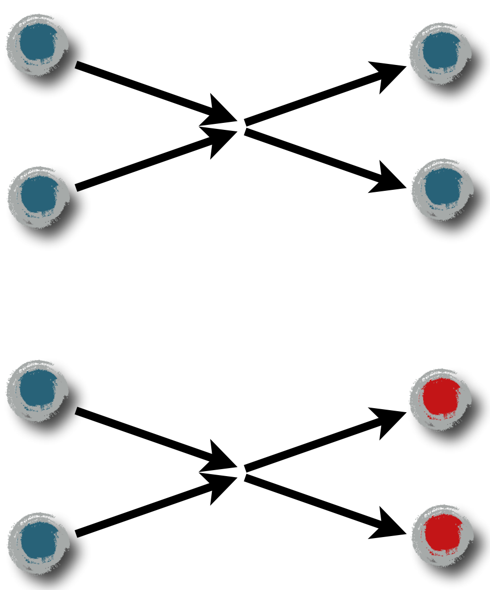
Perform global fits to the finite-volume spectrum

$$I^G(J^{PC}) = 0^+(0^{++})$$



Coupled-channel scattering



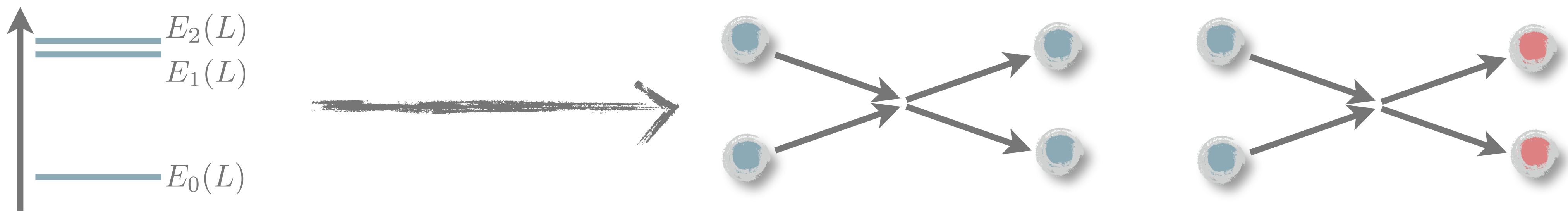


Briceño, Dudek, Edwards & Wilson (2017)

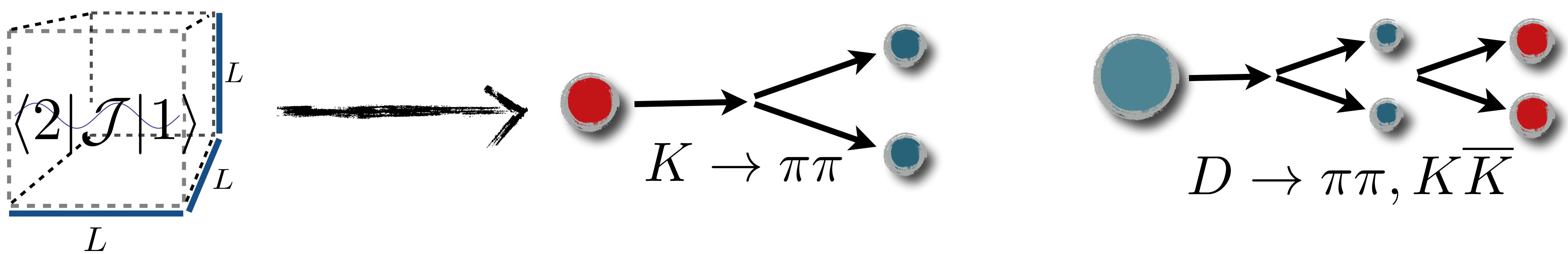
Multi-hadron processes from LQCD

KEY IDEA: We can use the finite volume as a tool to extract multi-hadron observables

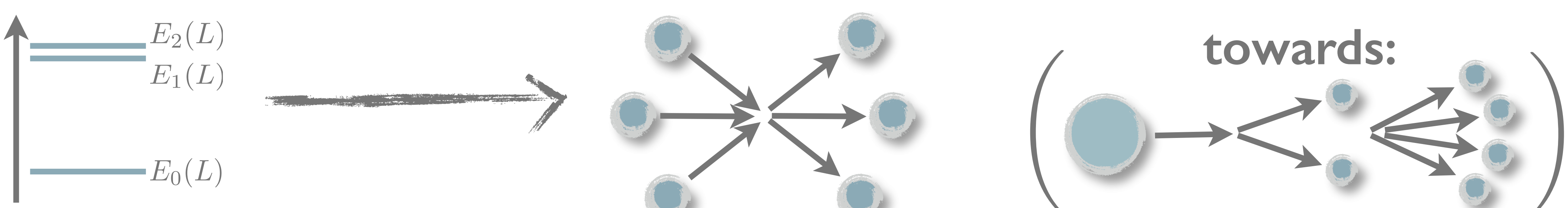
- [x] Two-to-two scattering



- [ ] **One-to-two transitions**



- [ ] Two-to-three and three-to-three scattering



Weak decays…

- So far: QCD scattering from finite-volume energies
- Now: Weak decays from finite-volume matrix elements

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- ☐ **Now: Weak decays from finite-volume matrix elements**

get this from the lattice **experimental observable**

$$|\langle n, L|\mathcal{H}_W|K\rangle|^2 = \mathcal{R}(E_n, L)|\langle \pi\pi, \mathrm{out}|\mathcal{H}_W|K\rangle|^2$$

depends on scattering phase shift

Lellouch, Lüscher (2001)

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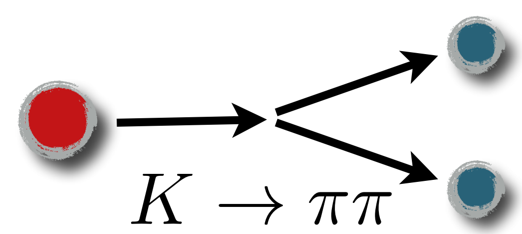
Lellouch, Lüscher (2001)

- ☐ **Three steps to lattice weak decay**
  - ☐ Calculate finite-volume energies → ππ scattering phase → *$R(E_n, L)$*
  - ☐ Calculate renormalized finite-volume matrix elements
  - ☐ Combine *$R(E_n, L)$* with f.v. matrix elements → **decay amplitudes**



- ☑ **Complete numerical calculation by RBC/UKQCD**

RBC/UKQCD, e.g. PRL 2015, ***(1505.07863)***

D decays…

get this from the lattice

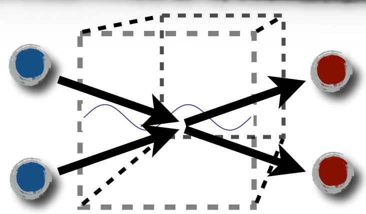
experimental observables

$$\langle n, L|\mathcal{H}_W|D\rangle = \begin{pmatrix}\mathcal{C}_{\pi\pi} & \mathcal{C}_{K\overline{K}} & \mathcal{C}_{\eta\eta}\end{pmatrix}\begin{pmatrix}\langle \pi\pi, \mathrm{out}|\mathcal{H}_W|D\rangle \\ \langle K\overline{K}, \mathrm{out}|\mathcal{H}_W|D\rangle \\ \langle \eta\eta, \mathrm{out}|\mathcal{H}_W|D\rangle\end{pmatrix}$$

depends on scattering matrix

MTH, Sharpe (2012)

- [ ] **Coupled channels mix in the finite volume**



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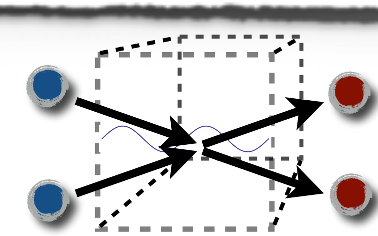
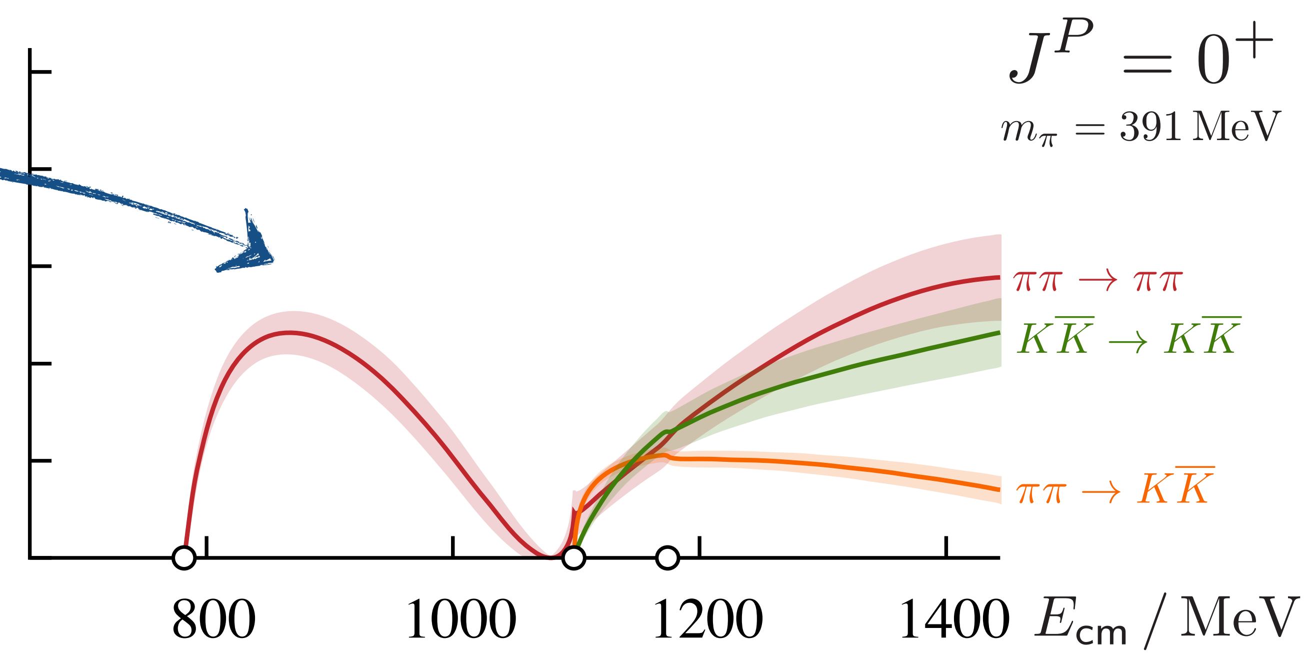


- ❑ **Three steps to D decays**
  - ❑ **Calculate *many* finite-volume energies → coupled scattering → C_{xy}**



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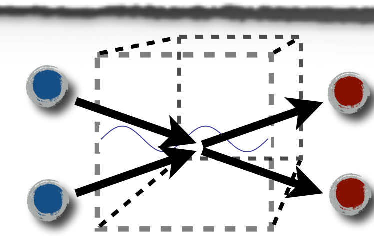
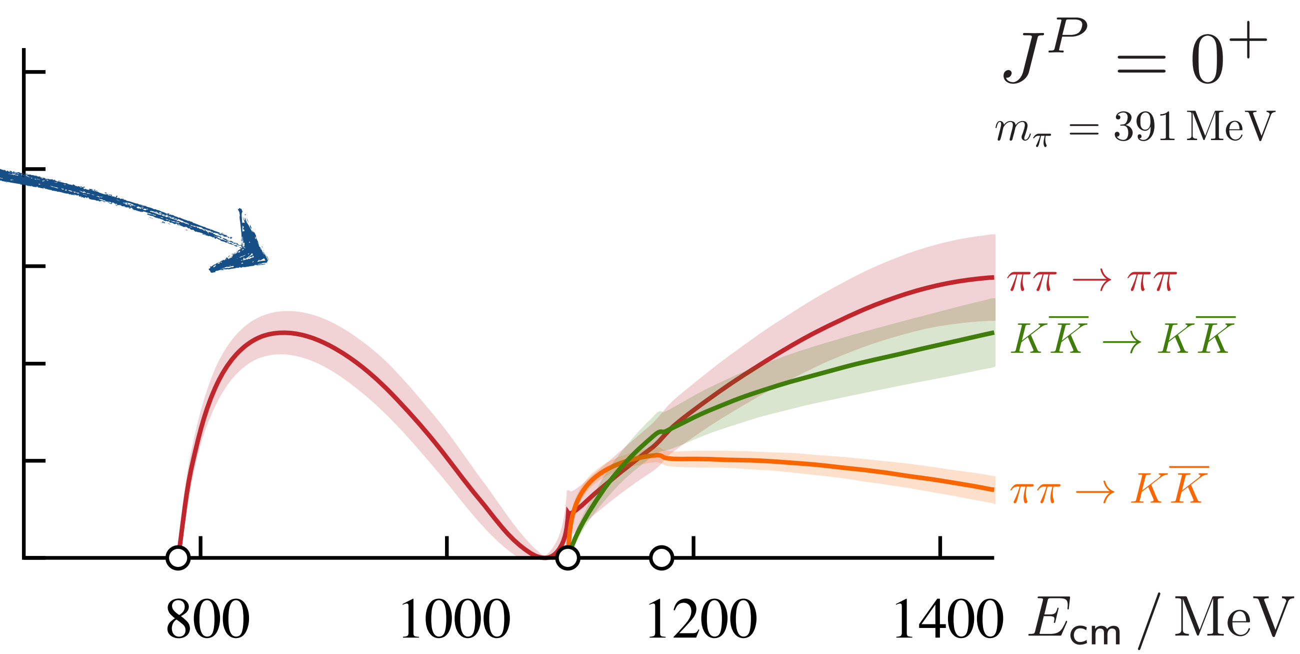
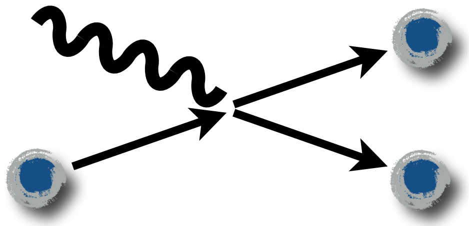


- Three steps to D decays
  - Calculate ***many*** finite-volume energies → coupled scattering → C_{xy}
  - Calculate ***many*** renormalized finite-volume matrix elements
  - Extract **amplitudes** in a global fit



- Important caveat: The relation ignores ππππ states

Pion photo-production

$\langle \pi\pi, \mathrm{out}|\mathcal{J}_\mu|\pi\rangle \equiv$ 

- **Method also applies when current injects energy and momentum**

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Briceño, MTH, Walker-Loud (2015)

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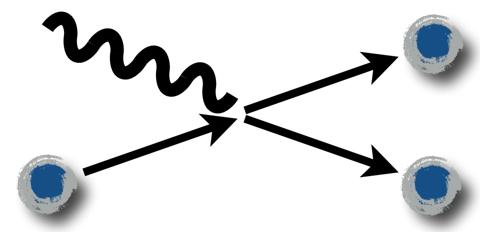

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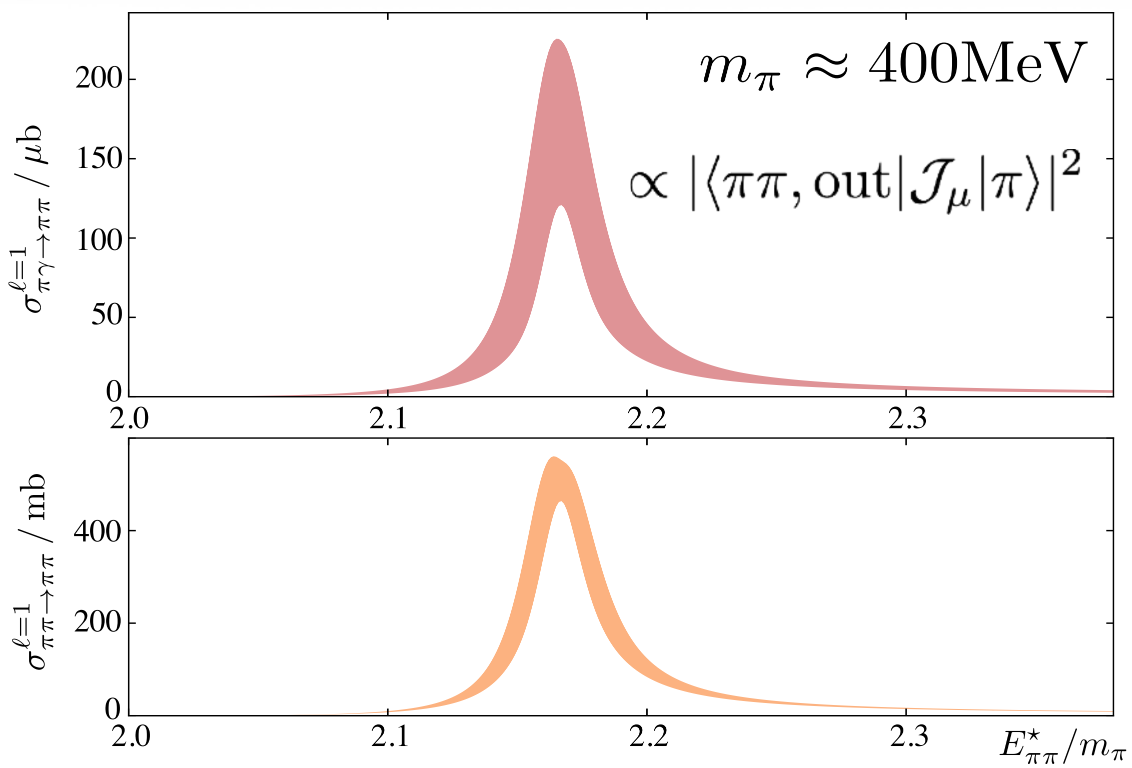
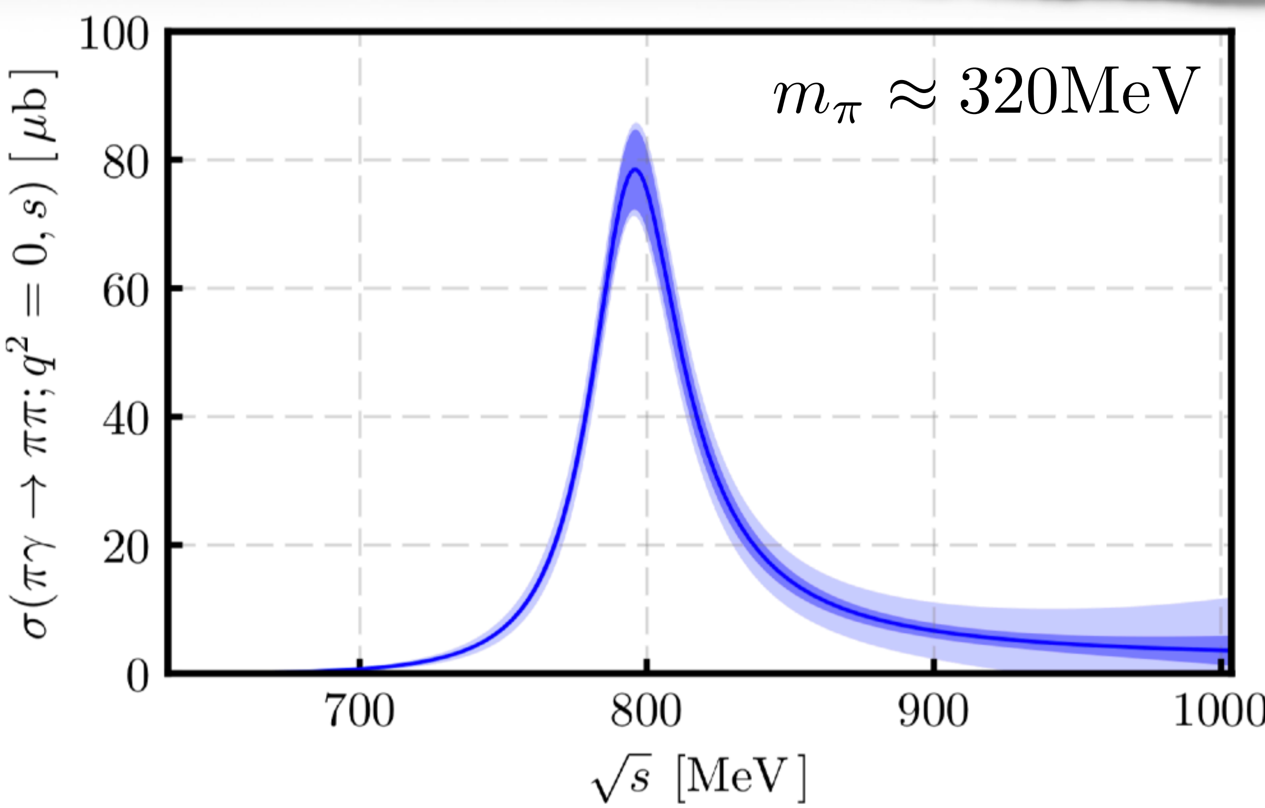


Briceño et. al., Phys. Rev. D93, 114508 (2016)



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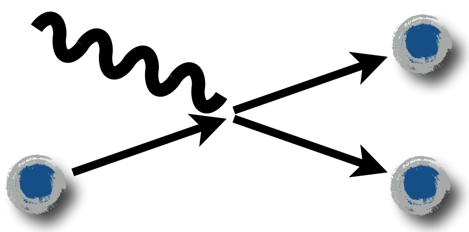
$\langle \pi\pi, \mathrm{out} | \mathcal{J}_\mu | \pi \rangle \equiv$ 

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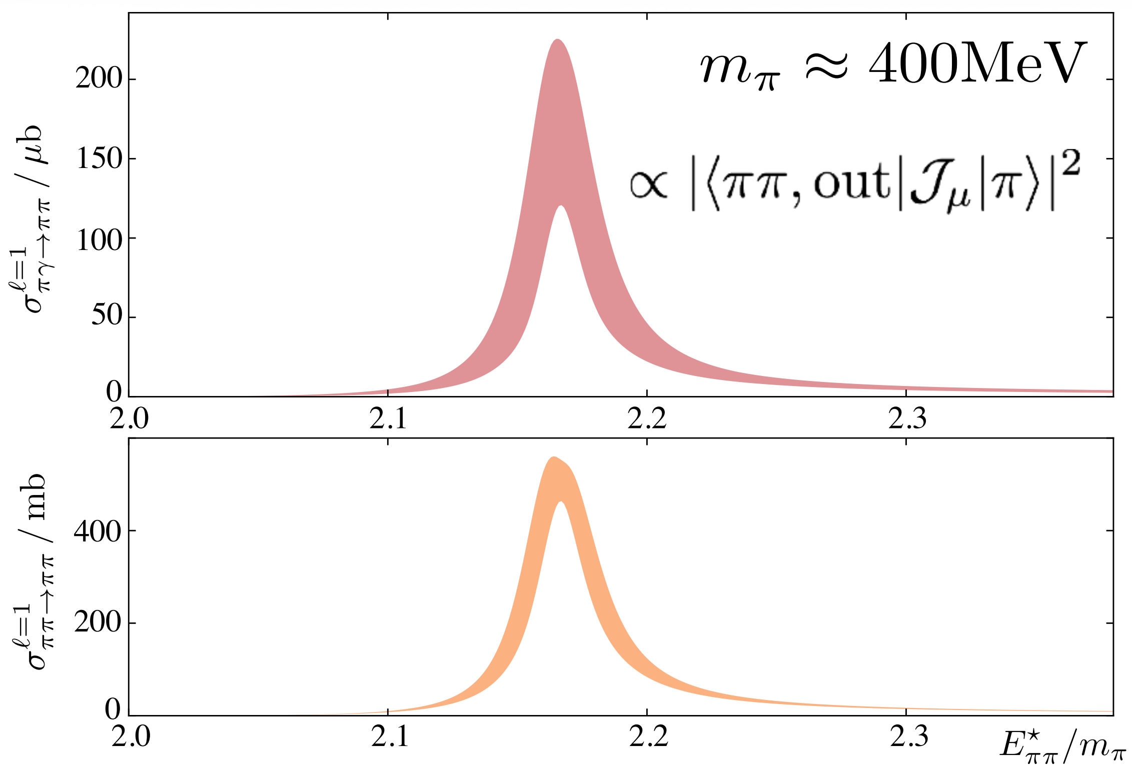

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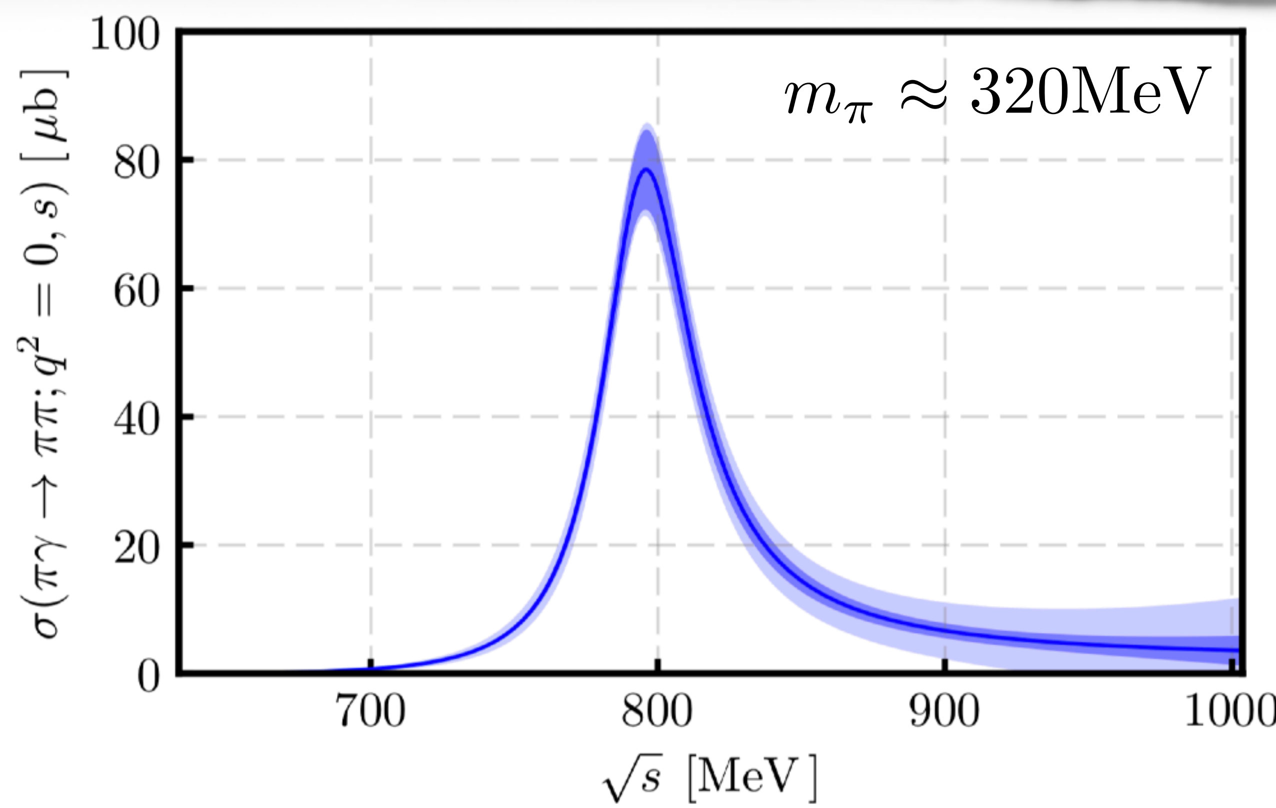


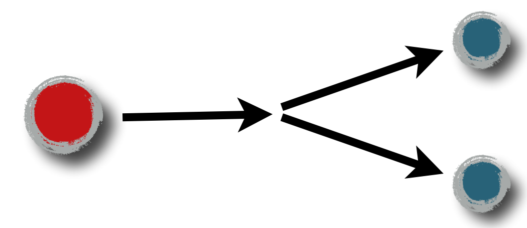
Alexandrou et. al., Phys. Rev. D98, 074502 (2018)

Could be used to extract weak decays at unphysical kinematics (explore resonance enhancements, fine tuning, etc.)

Same basic idea in many different contexts…

Weak decay

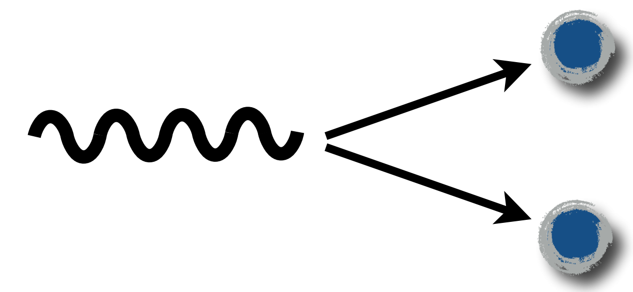
$$\langle \pi\pi, \mathrm{out} | \mathcal{H} | K \rangle \equiv$$



Lellouch, Lüscher (2001) ○ Kim, Sachrajda, Sharpe (2005) ○ Christ, Kim, Yamazaki (2005) ○ MTH, Sharpe (2012) ○

Time-like form factors

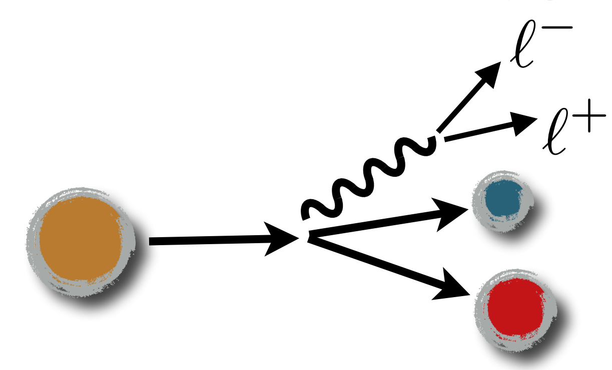
$$\langle \pi\pi,\ \mathrm{out} | \mathcal{J}_\mu | 0 \rangle \equiv$$



○ Meyer (2011) ○

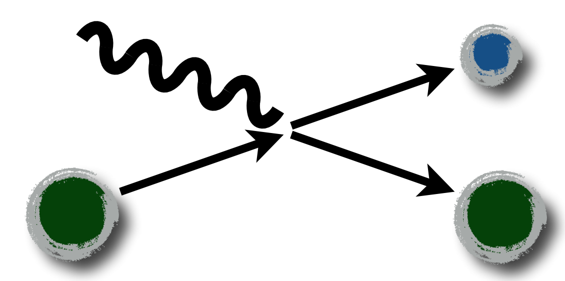
Resonance transition amplitudes

$$\langle K\pi, \mathrm{out} | \mathcal{J}_{\alpha\beta} | B \rangle \equiv$$



Particles with spin

$$\langle N\pi, \mathrm{out} | \mathcal{J}_\mu | N \rangle \equiv$$



Agadjanov *et al.* (2014) ○ Briceño, MTH, Walker-Loud (2015) ○ Briceño, MTH (2016)

Multi-hadron processes from LQCD

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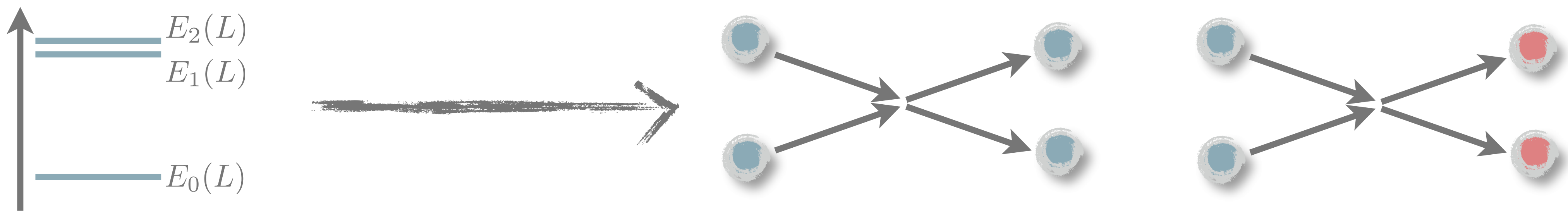
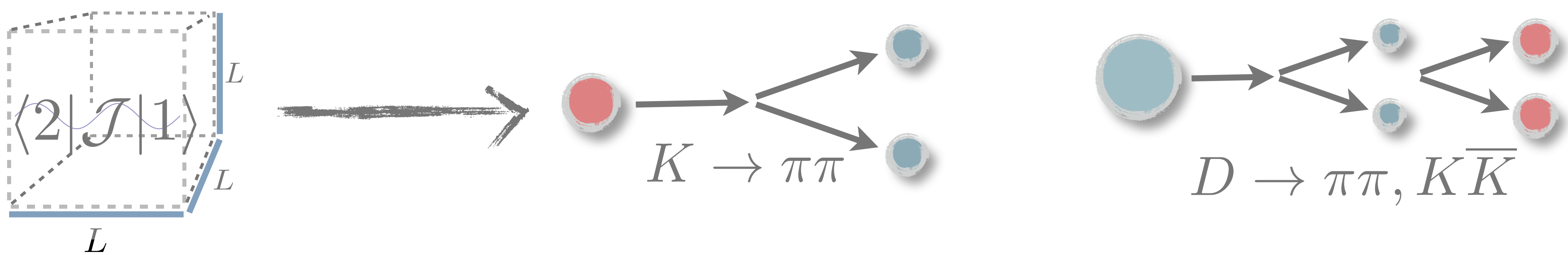
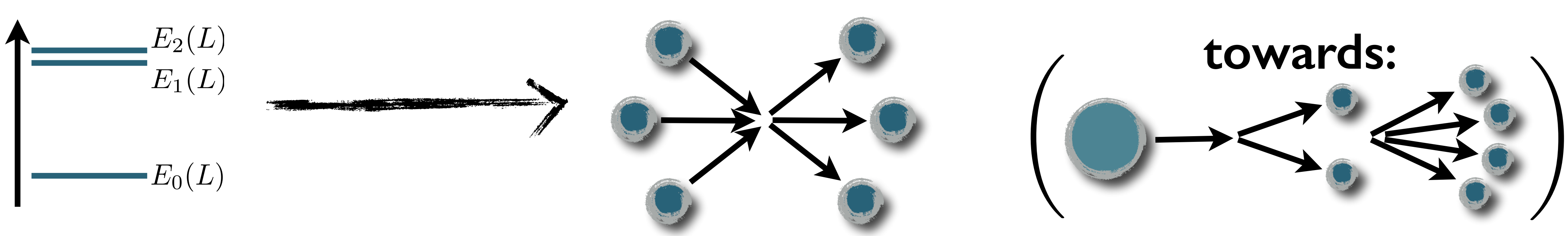


One-to-two transitions



Two-to-three and three-to-three scattering



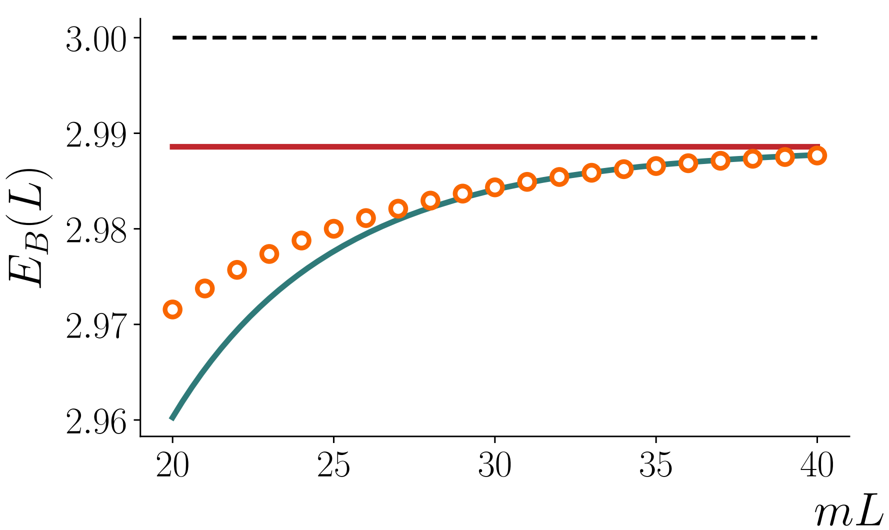
Three-hadron scattering

- **Formalism is complete for two and three (identical) scalars**

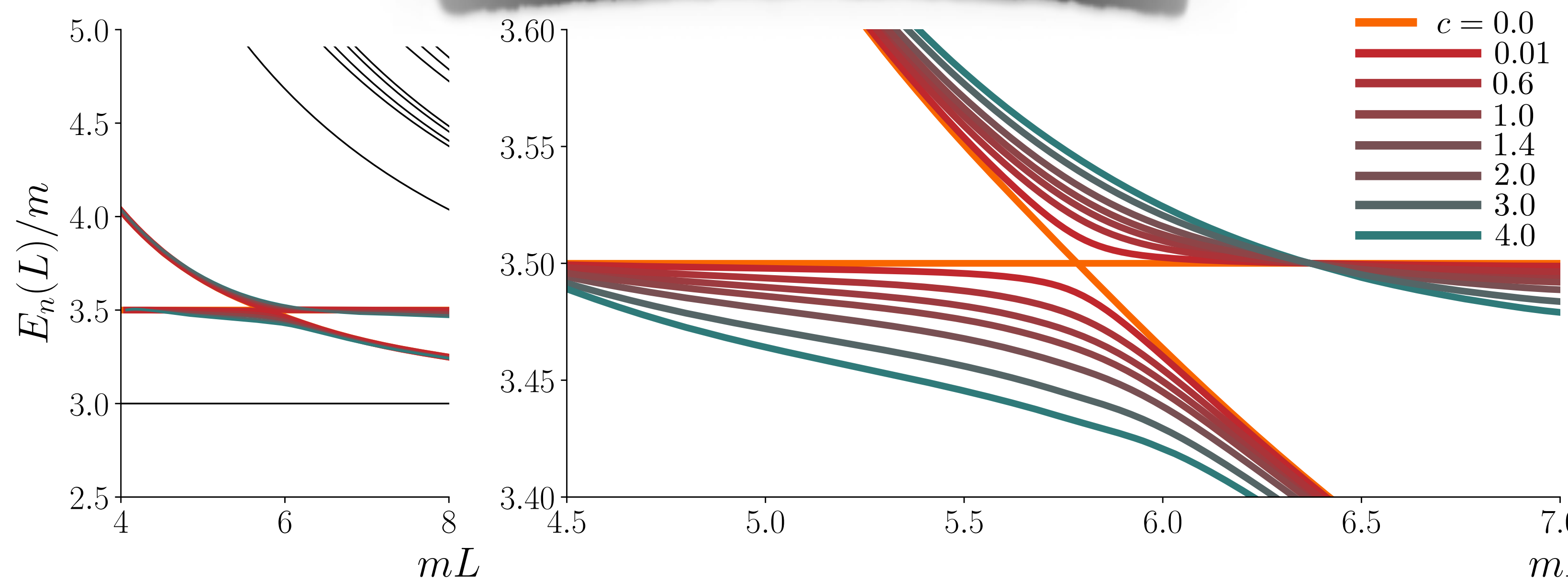
MTH, Sharpe (2014-2016) ○ Briceño, MTH, Sharpe (2017, 2018)

- **Currently exploring utility through numerical toy examples**

Volume effects on an Efimov state



Model of a 3-particle resonance



Briceño, MTH, Sharpe (2017)

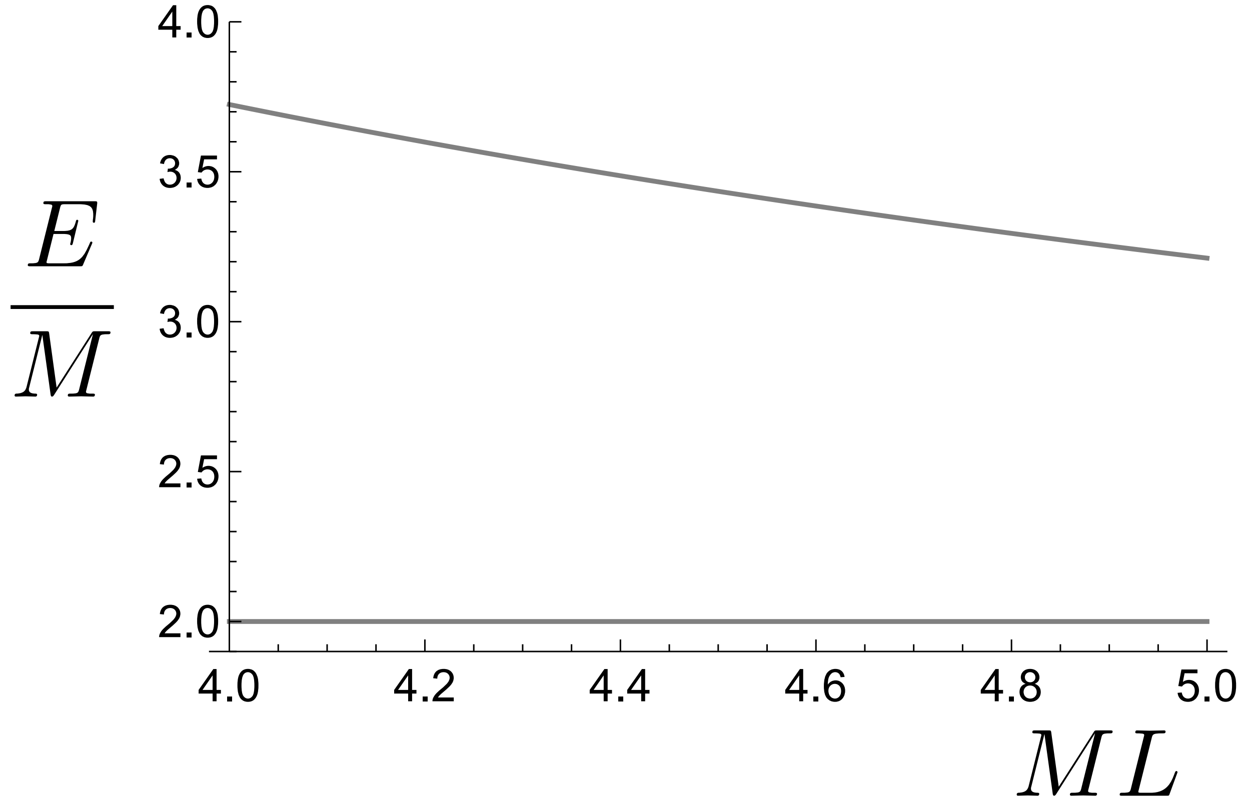
- **Recent review for *Annual Review of Nuclear and Particle Physics***

MTH, Sharpe (2019) *[1901.00483]*

Towards a new paradigm…

- **Assume we work out general finite-volume relations:**
  Application will be challenging due to growing density of states

2-particle states



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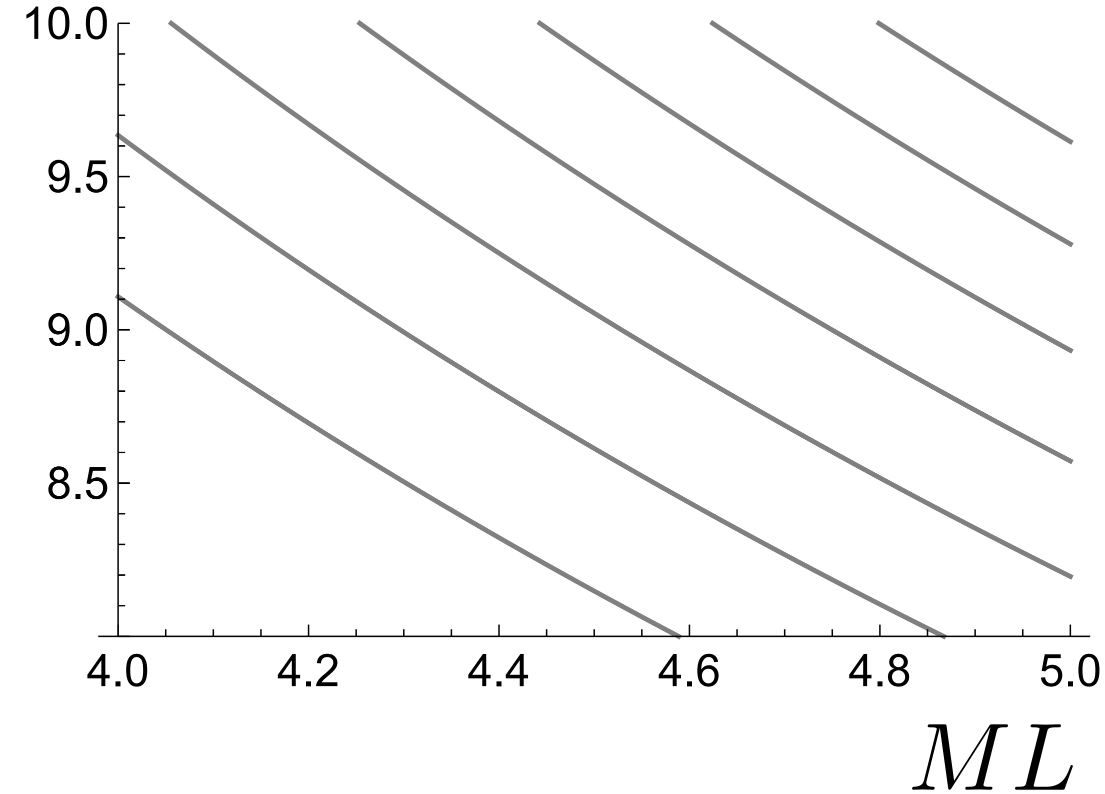

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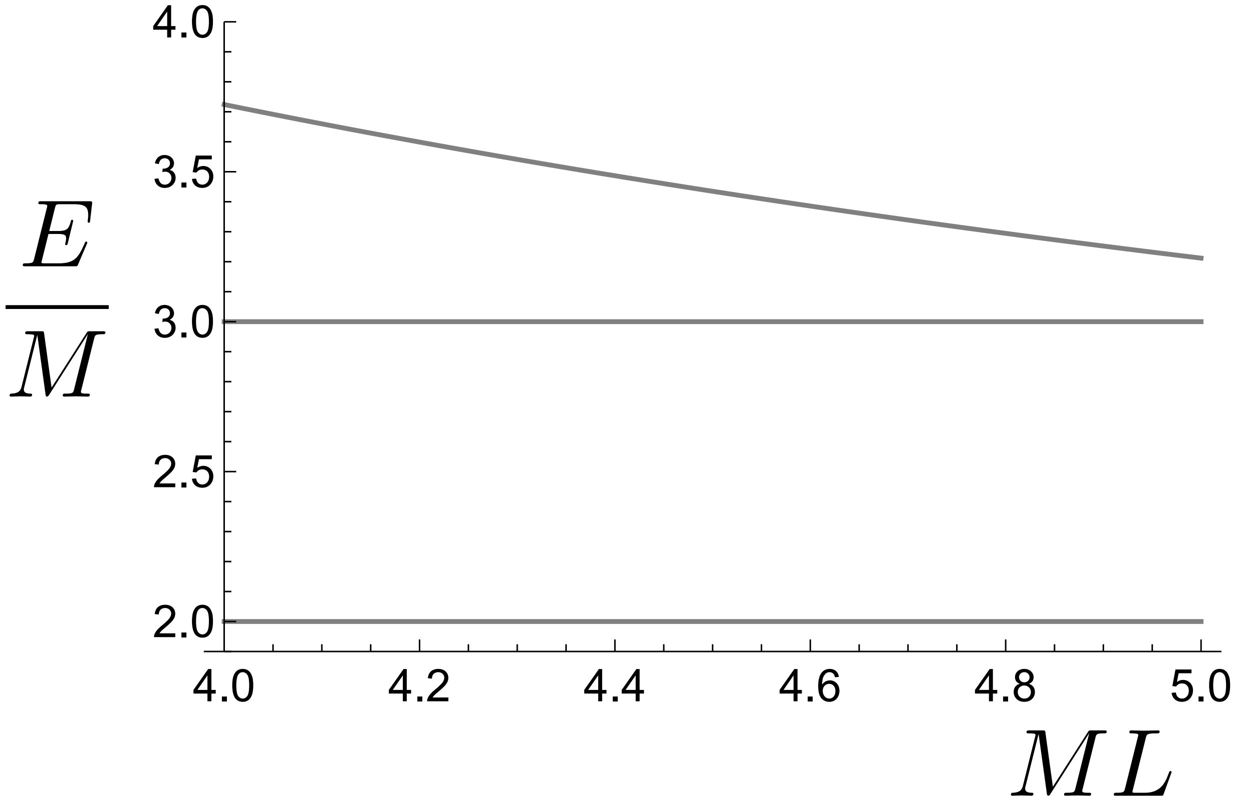

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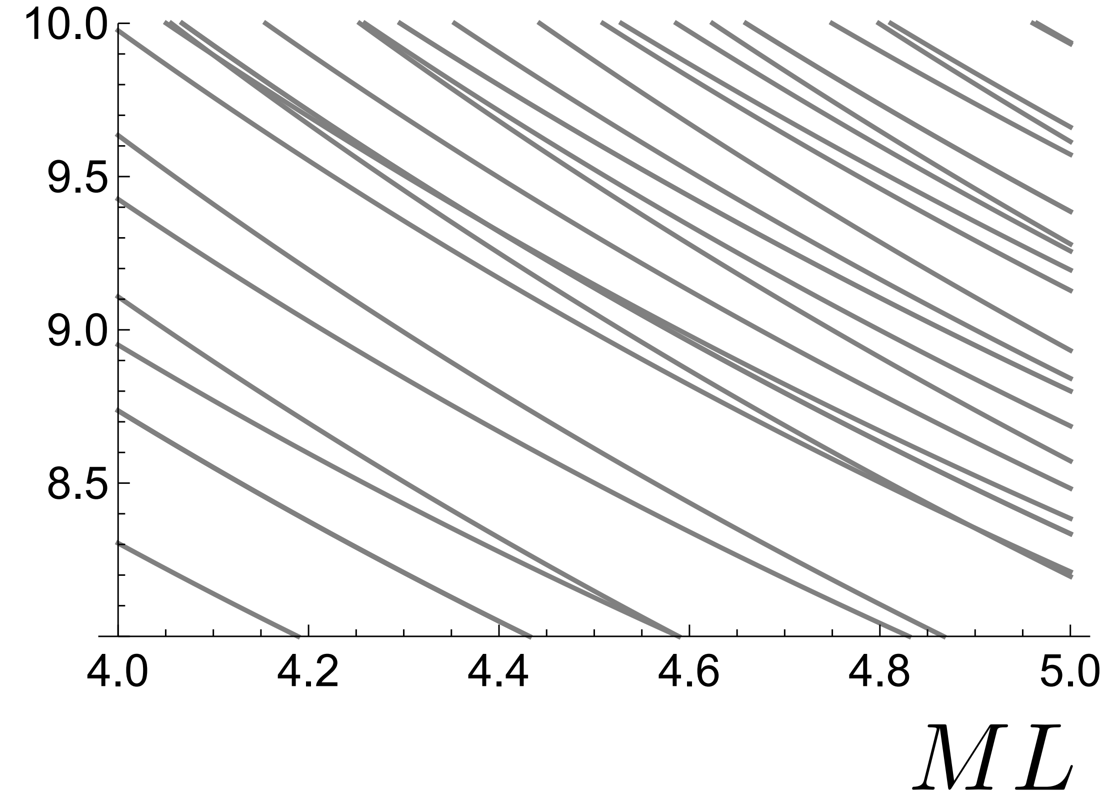

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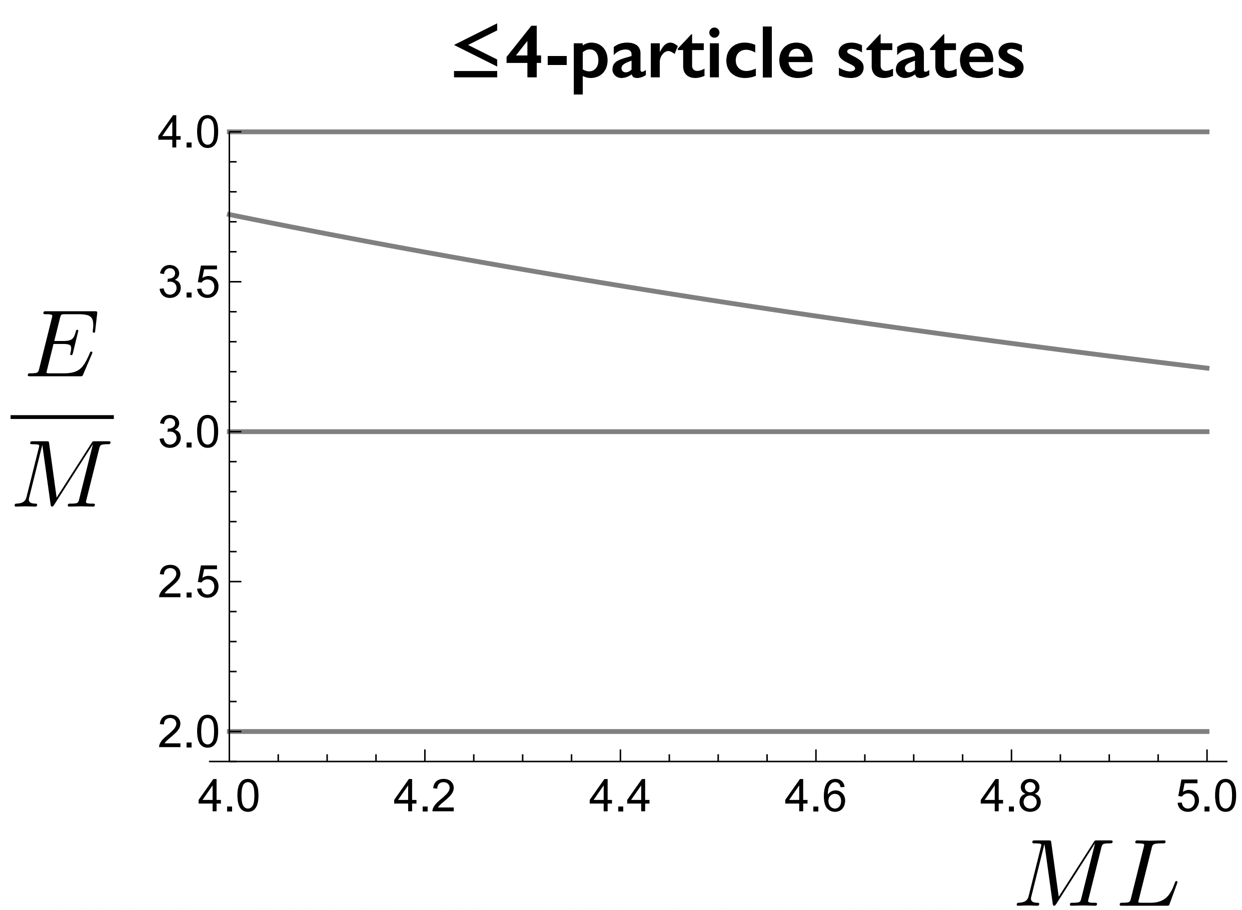
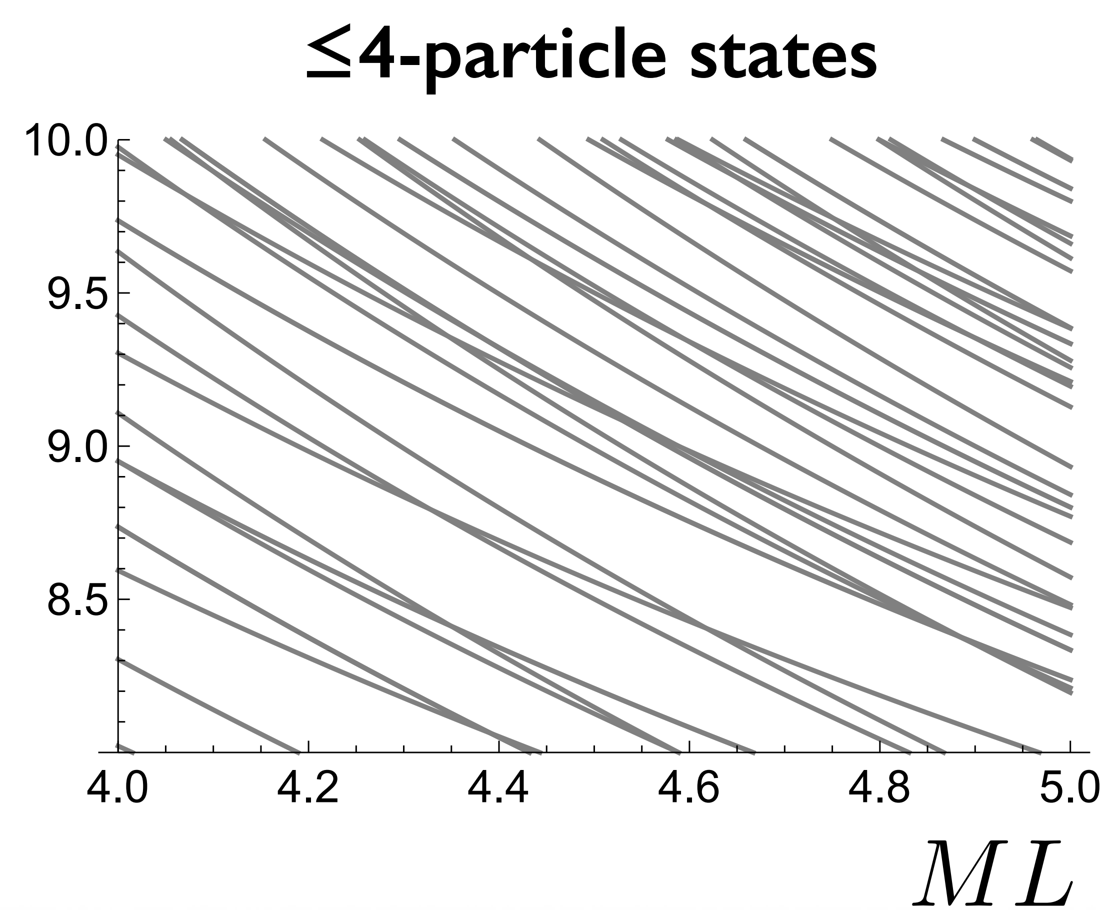
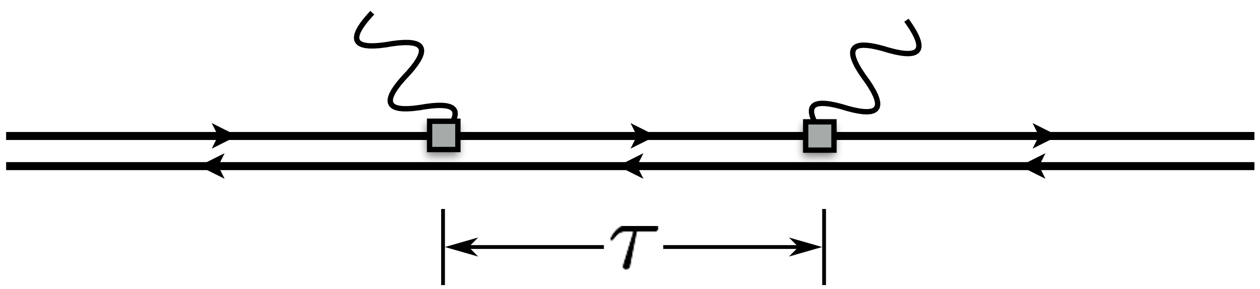




$E_n(L)$ ← **Contains information about all open channels with given QCD quantum numbers** $\pi\pi, \pi\pi\pi\pi, K\overline{K}, \cdots$

Scattering via spectral functions

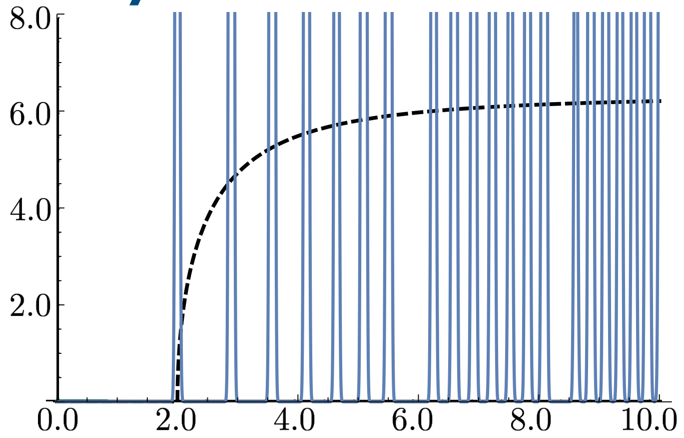
- **Recently proposed new methods of extracting scattering via 'smeared' finite-volume spectral functions**



$$G_L(\tau) = \int d\omega\, e^{-\omega\tau}\, \rho_L(\omega)$$

spectral function appears in an ill-posed inverse problem

spectral function is dominated by volume effects



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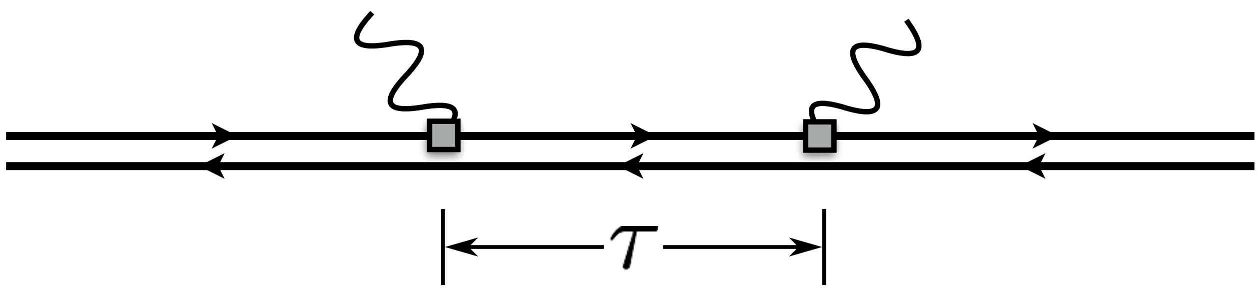

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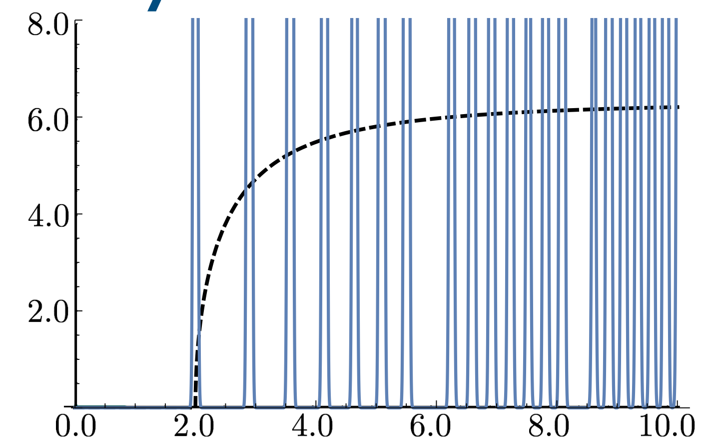

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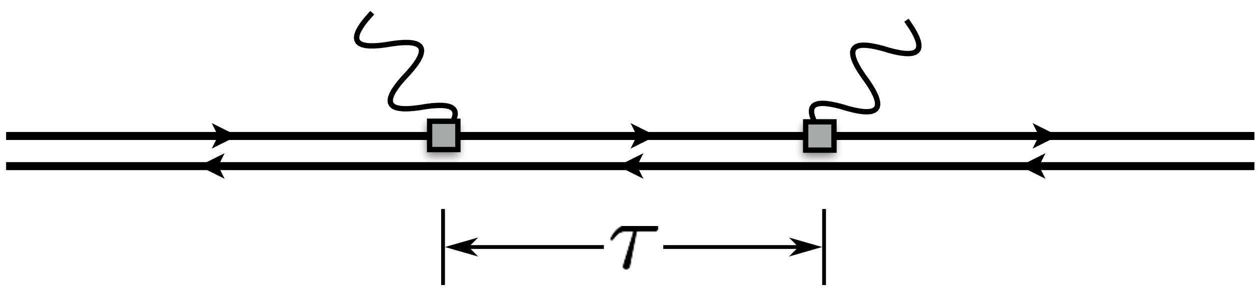

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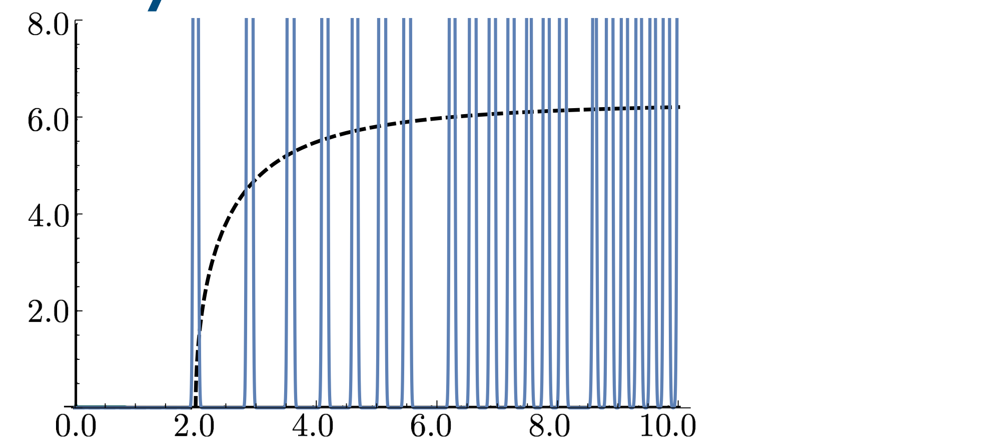

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- **Observable is recovered by:**

Saturating $L \to \infty$ at fixed Δ, then extrapolating $\Delta \to 0$

Narrow resolution function:
Total production rates, particle lifetimes
MTH, Robaina, Meyer (2017)

Complex pole resolution function:
Scattering + transition amplitudes
Bulava, MTH, (2019)

Perturbative exploration

- **Complex-pole kernel implements the standard *iε* prescription (LSZ)**

$$\widehat{\rho}_{L,\epsilon}(E_{\rm cm}) \equiv \int d\omega\ \widehat{\delta}_{\epsilon}(E_{\rm cm},\omega)\ \rho_L(\omega)$$

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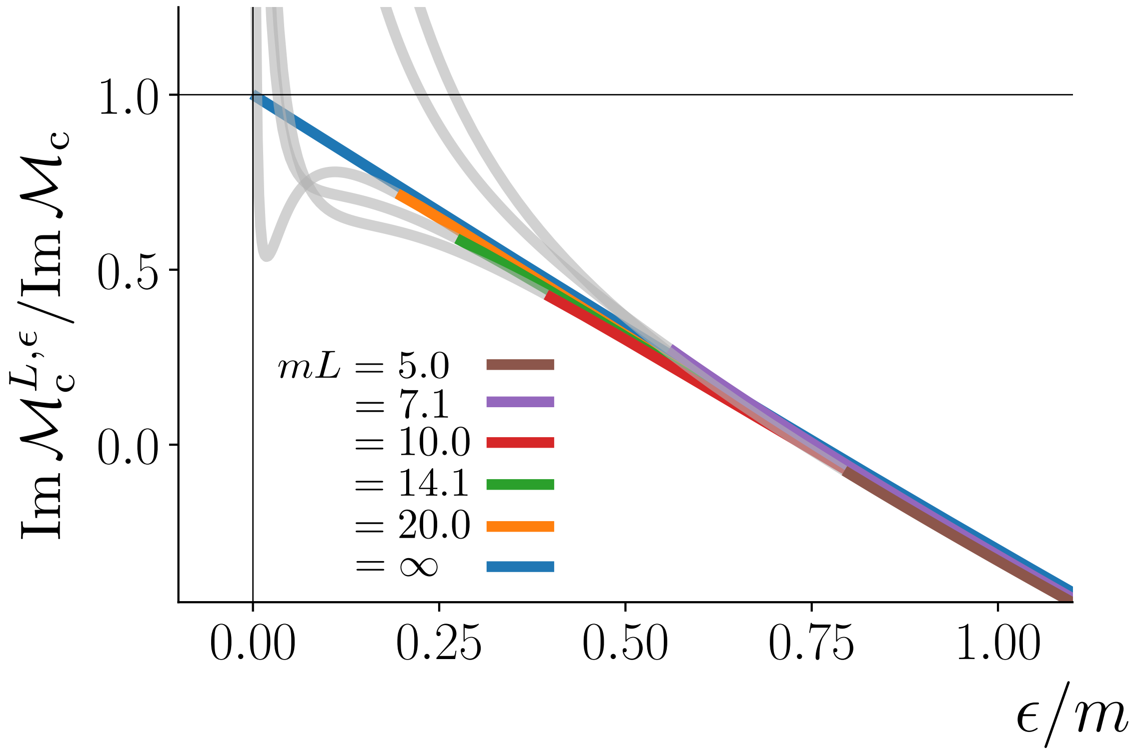

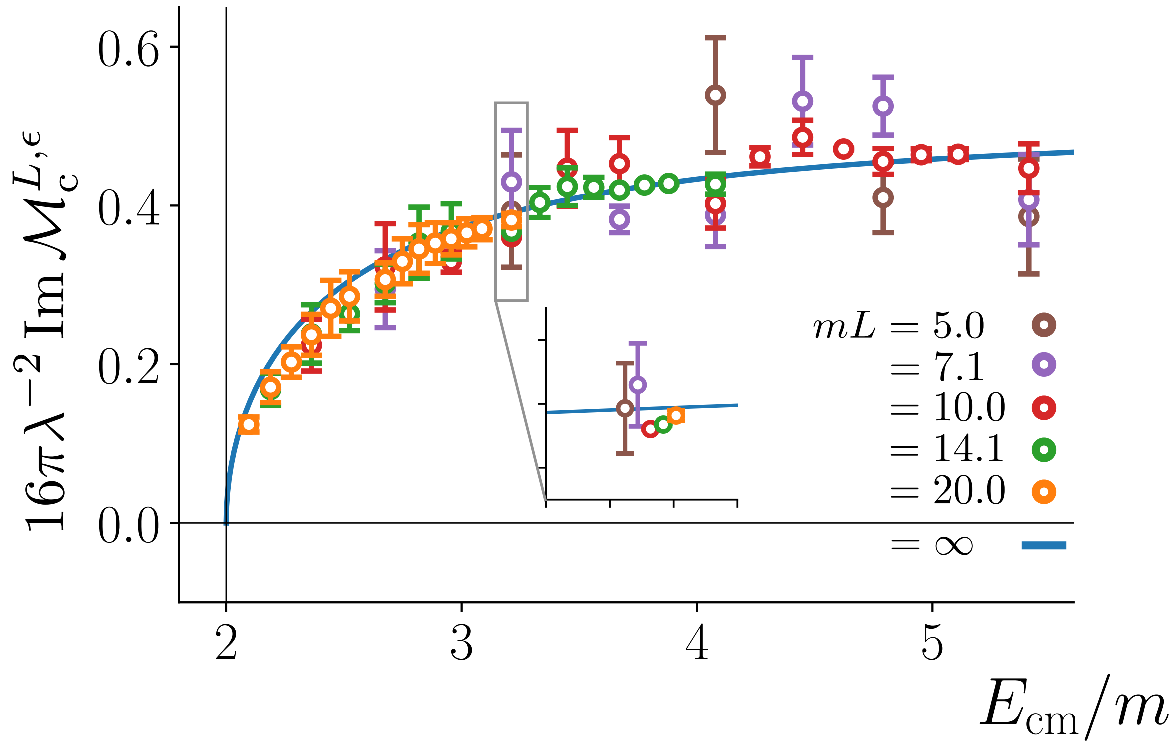

Bulava, MTH, (2019) *[1903.11735]*

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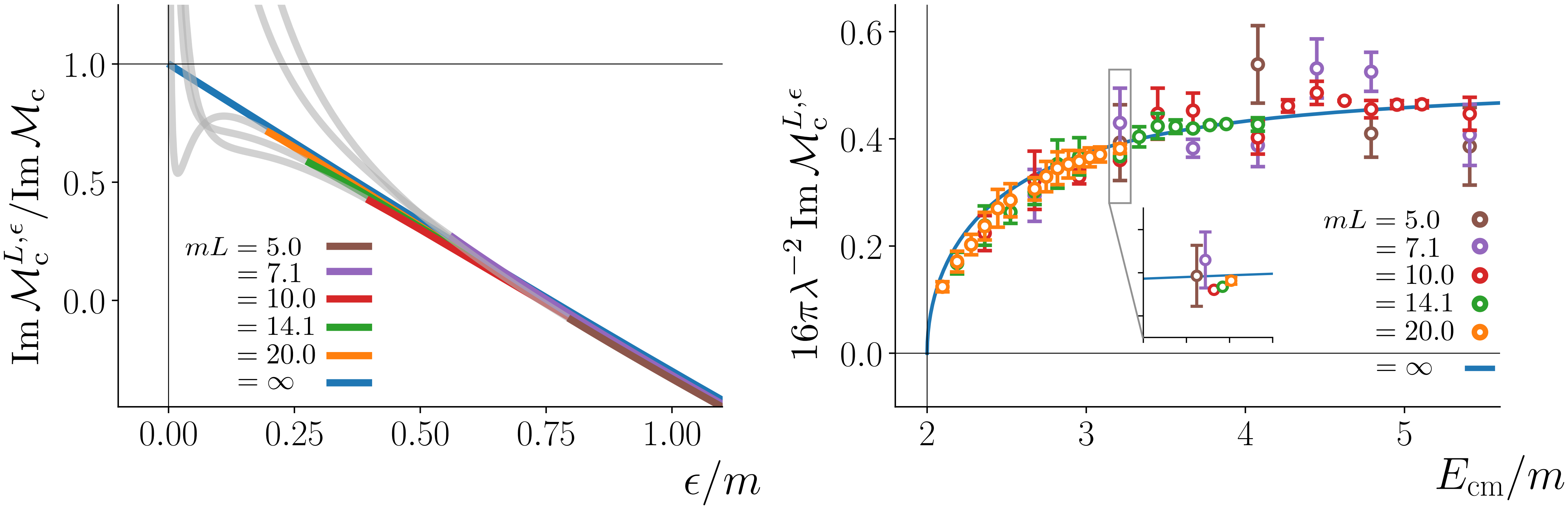


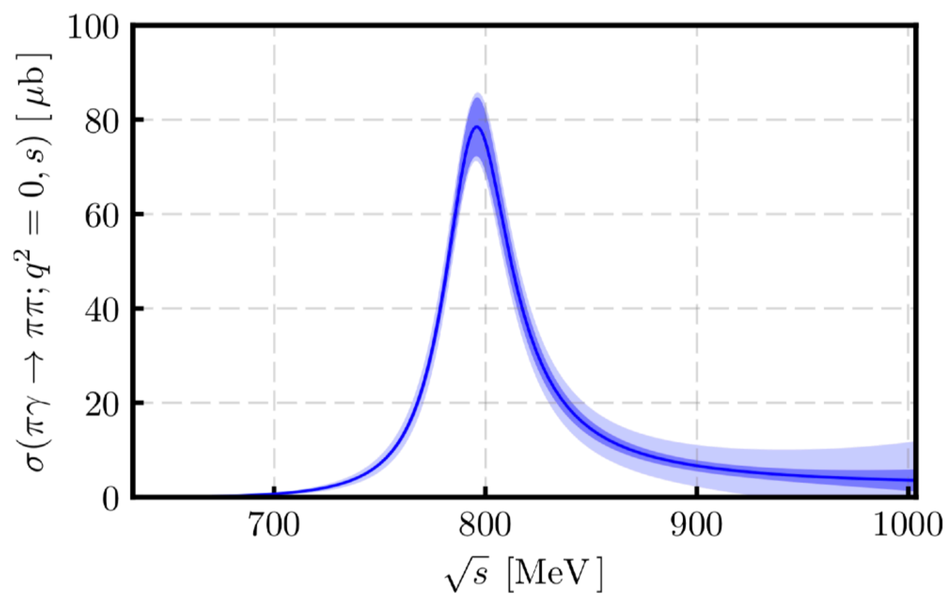
Bulava, MTH, (2019) *[1903.11735]*

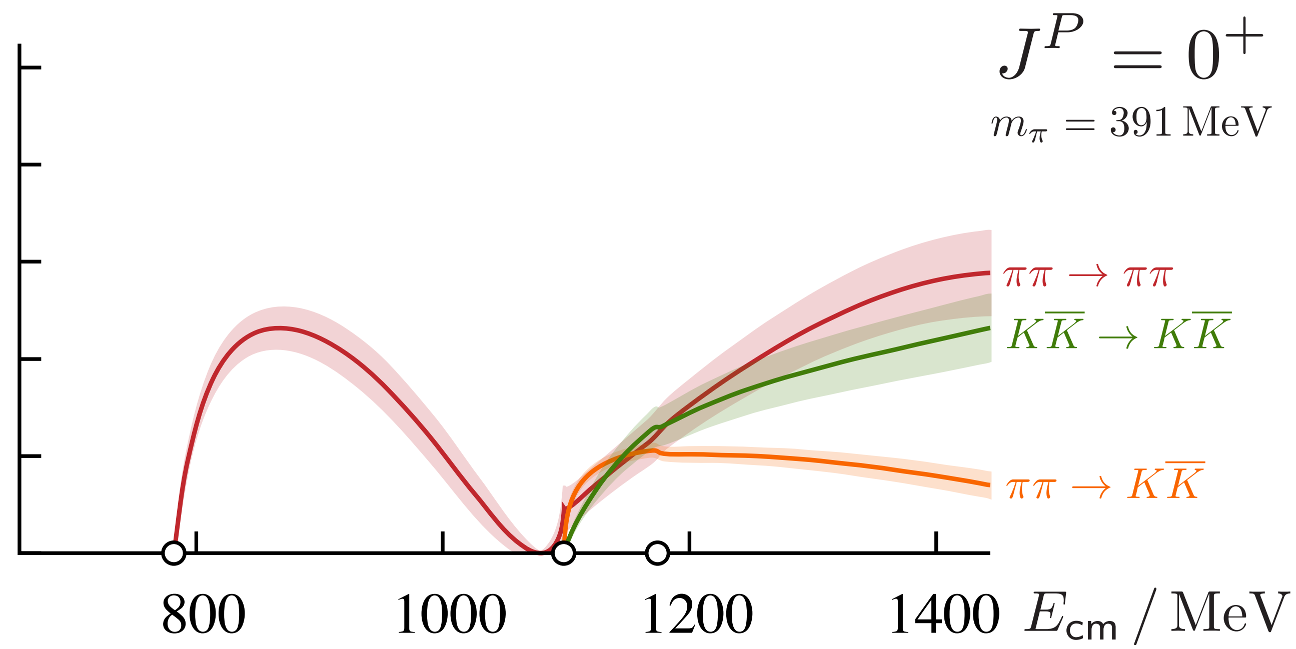
- **Directly applicable to D → ππ, KK**
- **But significant challenges remain:**

n-point functions, extracting ρ, saturating limits

Take home messages…

- ❑ **The finite volume is an asset (interactions leave imprint on *E(L)*)**





- ❑ **At high energies, many multi-particle channels open up**
  - ❑ Challenging finite-volume formalism, but seems achievable
  - ❑ Finite-volume energies feel the entire S-matrix… difficult to disentangle
  - ❑ **Many channels = central issue for lattice D decays**

- ❑ **Spectral-function-based methods are complimentary**
  - ❑ High precision Euclidean 4-pt functions $\Longrightarrow$ D → ππ, KK

Backup Slides

Inverse problem

- **The task is thus to identify an optimal linear combination…**

$$q_1(E)e^{-E' a_\tau} + q_2(E)e^{-E' 2a_\tau} + q_3(E)e^{-E' 3a_\tau} + \cdots = \widehat{\delta}_\Delta(E', E)$$

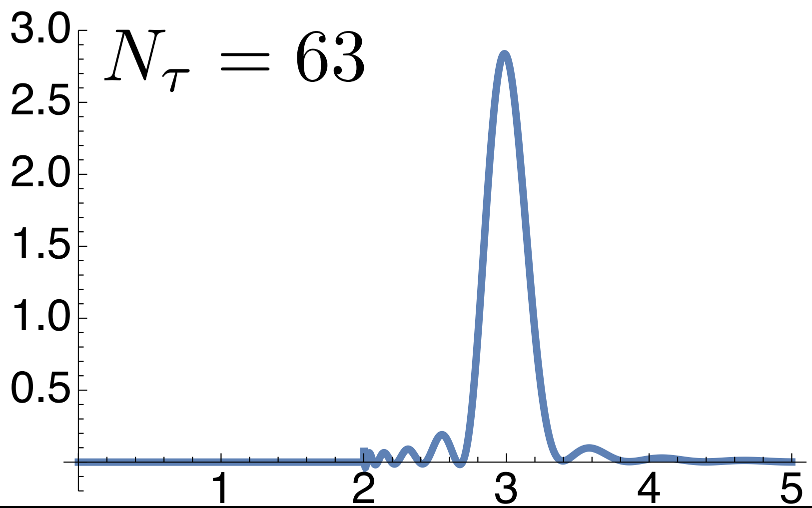
- **leading to a smeared out version of the spectral function**

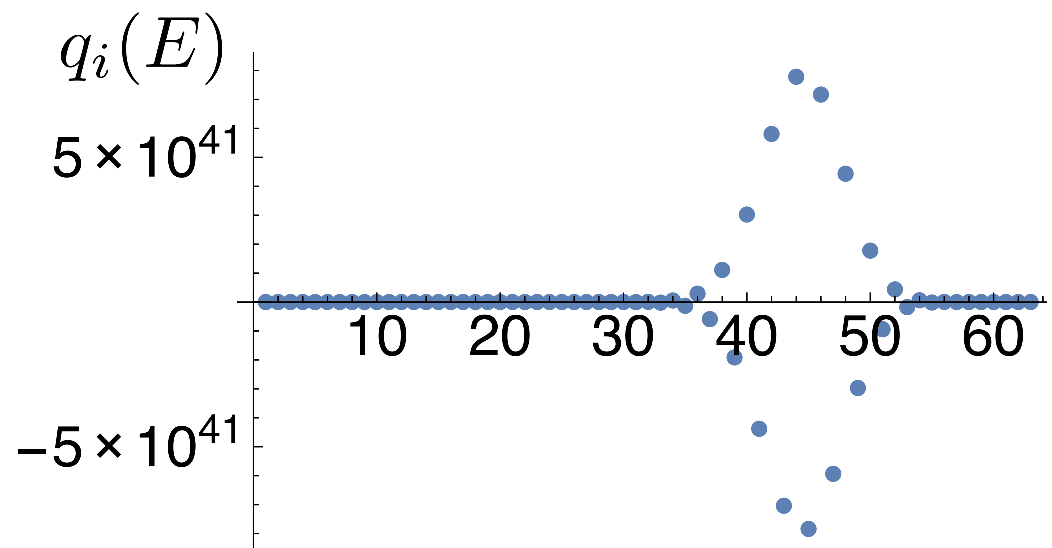
$$2\pi \sum_n q_n(E)\, G(na_\tau, L) = \int dE'\, \widehat{\delta}_\Delta(E', E)\, \rho(E', L) \equiv \widehat{\rho}_\Delta(E, L)$$

Optimal choice depends on target precision and competition of scales

$$1/L \ll \Delta \ll M_{\mathrm{QCD}}$$

- **An example of what *not* to do…**





Backus-Gilbert method

- Developed by geophysicists in 1967
- Linear, model-independent approach
- Spectral function smeared with a known resolution function
- The covariance matrix is used to stabilize the inverse

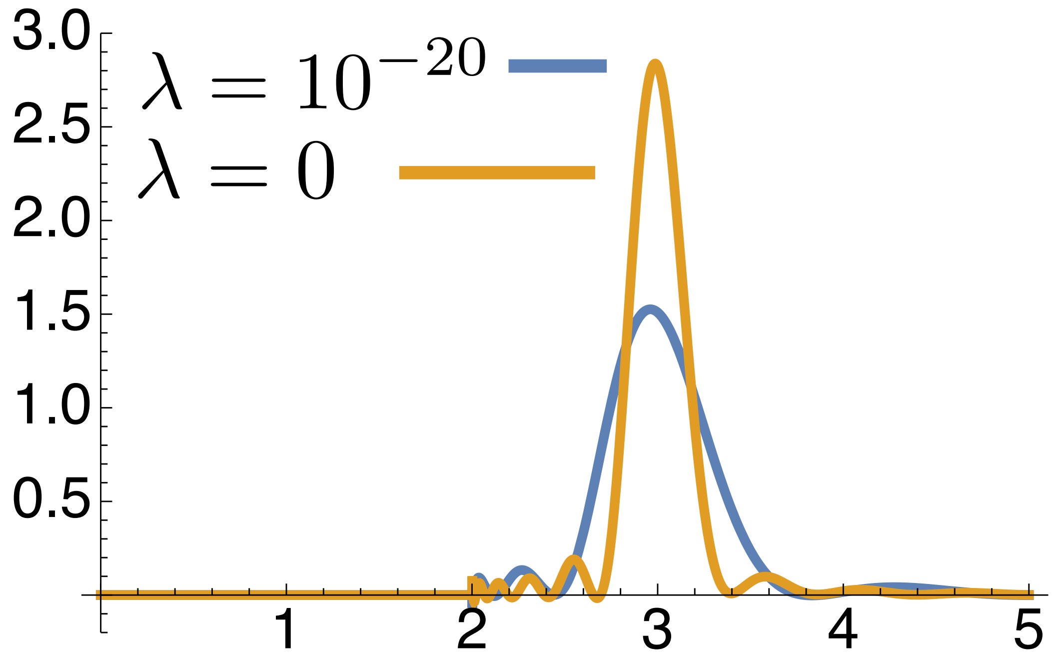
optimal coefficients

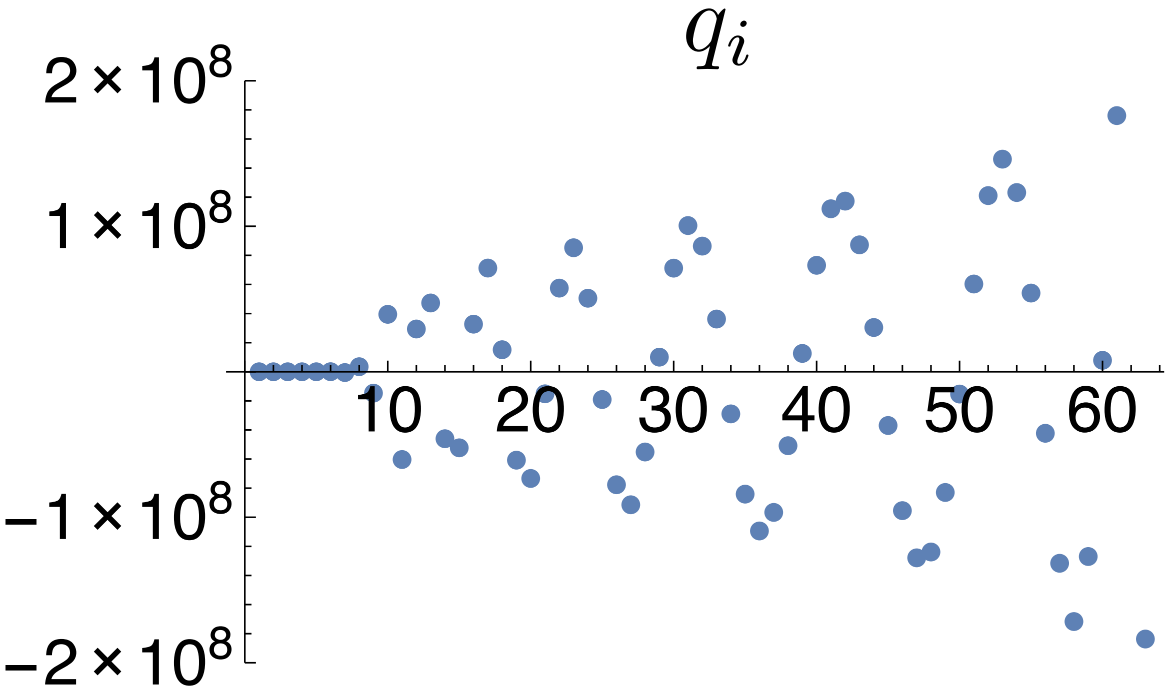
$$q_i(E) = \frac{[W(E) + \lambda S]^{-1} \cdot R}{R \cdot [W(E) + \lambda S]^{-1} \cdot R}$$

covariance

resolution function

$$\sum_i e^{-E\tau_i} q_i(E') = \widehat{\delta}_\Delta(E - E')$$





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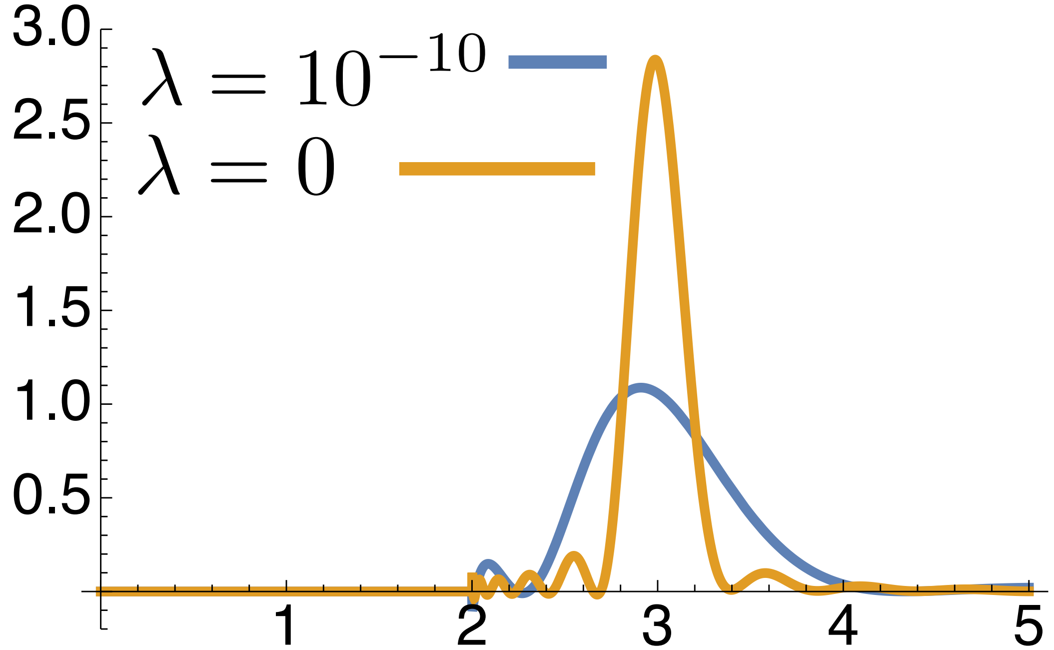

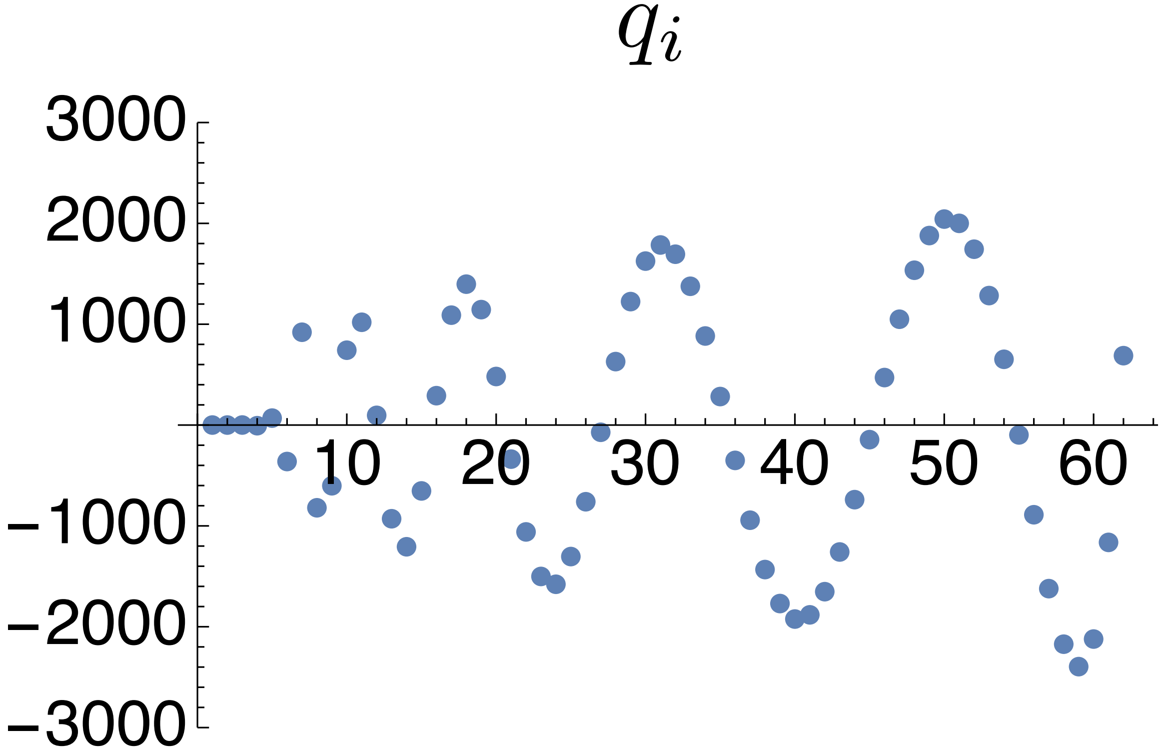

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