



**Neutral kaon CP violation and matter interaction
in measurements of the CKM angle γ using
 $B^\pm \rightarrow DK^\pm, D \rightarrow K_S^0 \pi^+ \pi^-$ decays**

Mikkel Bjørn, Sneha Malde. arXiv:1904.01129

Towards the Ultimate Precision in Flavour Physics

University of Durham

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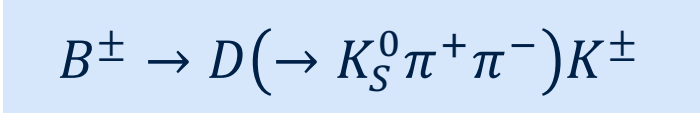


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Precision in GGSZ

Sensitivity to γ from interference between $B \rightarrow D^0 K$ and $B \rightarrow \bar{D}^0 K$ decays

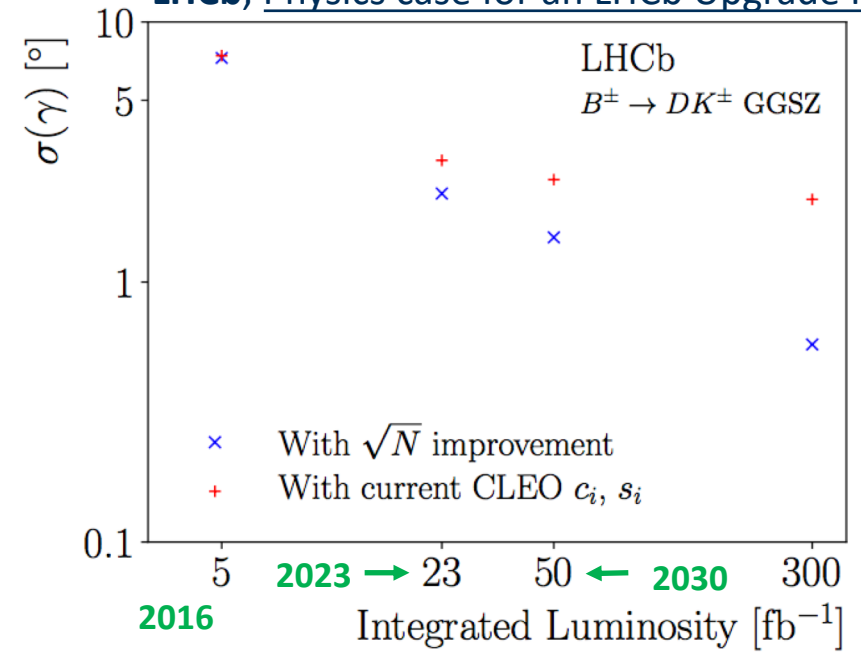
A Golden Mode at Belle II and LHCb:



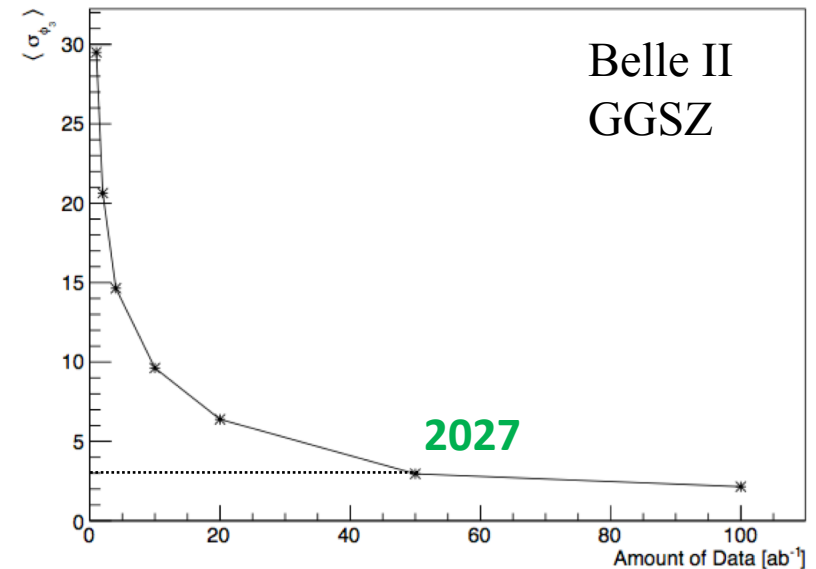
- Sensitivity to γ via Dalitz-plot-distribution analysis: the **GGSZ** method [[hep-ph/0303187](https://arxiv.org/abs/hep-ph/0303187)]

Both LHCb and Belle II will reach precision of **a few degrees** in GGSZ analyses

LHCb, Physics case for an LHCb Upgrade II



Belle II, The Belle II Physics Book



Neutral kaon CPV as a source of systematic uncertainty



↑
Neutral Kaon CPV → source of bias

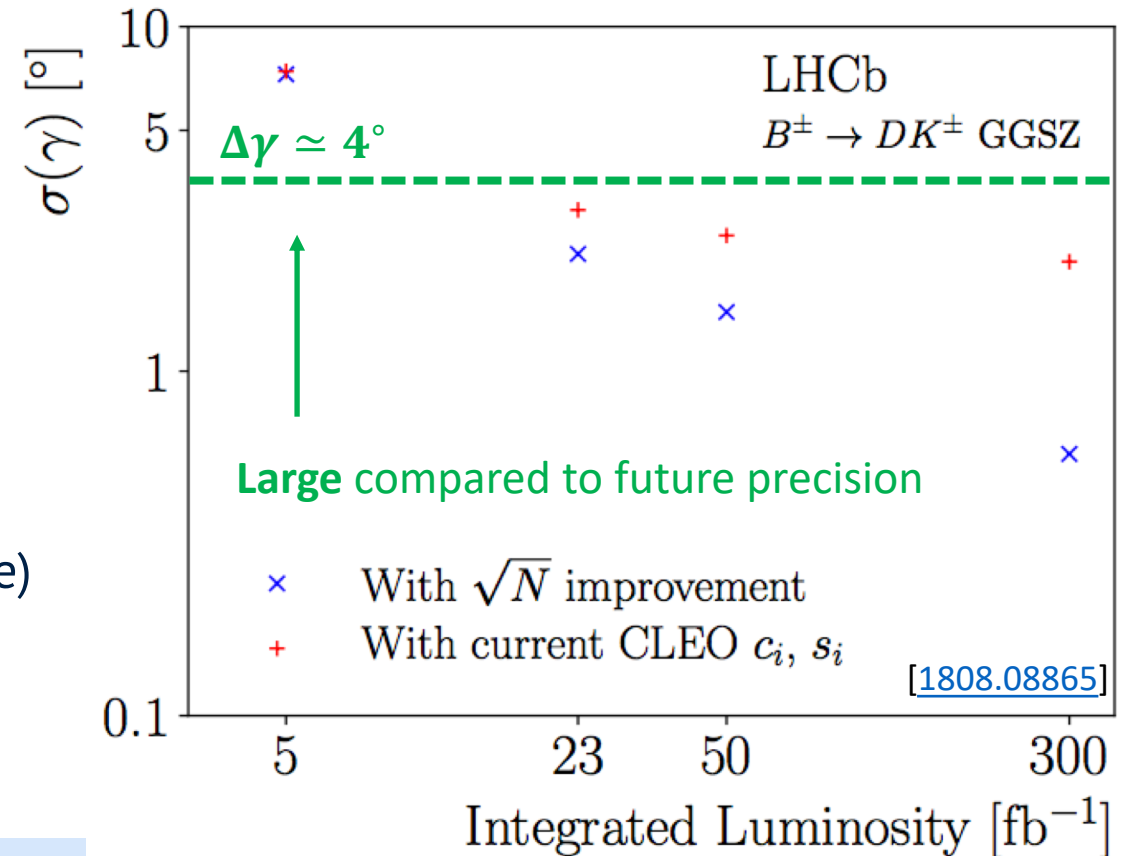
Grossman & Savastio [[1311.3575](#)] estimate impact on γ measurements

- $\Delta\gamma/\gamma = O(|\epsilon|/r_B) = \%$ level
- Valid for GLW measurements ($D \rightarrow CP$ eigenstate)
- Rough estimate: $\Delta\gamma \simeq 4^\circ$

Large compared to future precision

We present detailed further studies

- for current experimental procedure
- including kaon regeneration
- with bias estimates for LHCb and Belle II



$$|\epsilon| \simeq 2 \times 10^{-3}$$

$$r_B^{DK} \simeq 10^{-1}$$

Rich structure of $D \rightarrow K_S^0 \pi^+ \pi^-$ amplitude over phase-space

→ $D^0 - \bar{D}^0$ interference varies over Dalitz-plot

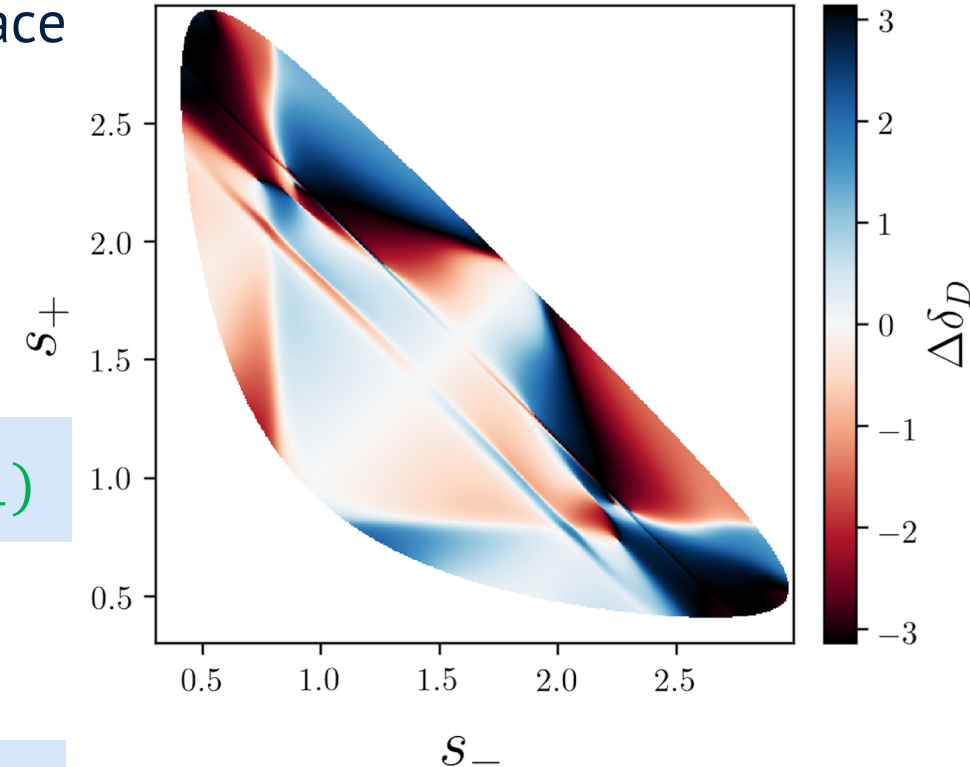
→ CP asymmetry varies over Dalitz-plot

K_S^0 is (almost) a CP eigenstate

→ $K_S^0 \pi^+ \pi^-$ is self-conjugate → $A_S^{\bar{D}}(s_-, s_+) = A_S^D(s_+, s_-)$

- $A_S^D = A(D^0 \rightarrow K_S^0 \pi^+ \pi^-) = |A_S^D| e^{i\delta_D}$
- $\Delta\delta_D = \delta_D(s_-, s_+) - \delta_D(s_+, s_-)$

$$d\Gamma^{B^\pm} \propto |A_S^D(s_{\mp}, s_{\pm})|^2 + r_B^2 |A_S^D(s_{\pm}, s_{\mp})|^2 + 2r_B |A_S^D(s_{\pm}, s_{\mp})| |A_S^D(s_{\mp}, s_{\pm})| \cos[\Delta\delta_D + \delta_B \pm \gamma]$$



Model-independent GGSZ measurements in practice

Total normalisation **independent** of (x, y)
and between B^+ and B^-
→ measurement unaffected by overall
production/detection asymmetry

Measured observables are:
 $x_{\pm} + iy_{\pm} = r_B \exp[i(\delta_B \pm \gamma)]$
Better fit stability than directly fitting γ

$$N_{\pm i}^{B^-} = h^- \{ K_{\pm i} + (x_-^2 + y_-^2) K_{\mp i} + 2\sqrt{K_i K_{-i}} (x_- c_{\pm i} + y_- s_{\pm i}) \}$$

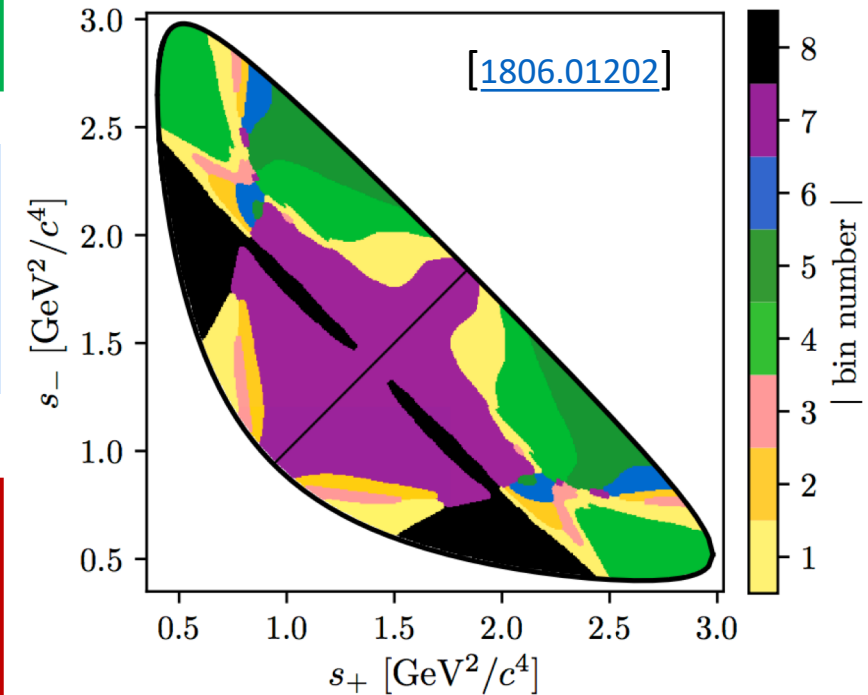
$$N_{\pm i}^{B^+} = h^+ \{ K_{\mp i} + (x_+^2 + y_+^2) K_{\pm i} + 2\sqrt{K_i K_{-i}} (x_+ c_{\pm i} - y_+ s_{\pm i}) \}$$

F_i : Fractional yield of flavour tagged D^0
into bin i

Measured in **control-channel with flavour
tagged D^0 & \bar{D}^0**

c_i/s_i : Strong phase difference of
 $D^0 - \bar{D}^0$ decays

External input from CLEO-c/BES III
measurement [[1010.2817](#)]



Neutral Kaon CP Violation

CP violation in the kaon sector changes GGSZ equations $\rightarrow A^D(s_-, s_+) \neq A^{\bar{D}}(s_+, s_-)$!

$$d\Gamma \propto |\psi_S(t, s_-, s_+) + \epsilon \cdot \psi_L(t, s_-, s_+)|^2$$

K_S^0 is not an exact CP eigenstate

$$A_S^D(s_-, s_+) \propto A_S^{\bar{D}}(s_+, s_-) + 2\epsilon A_2^{\bar{D}}(s_+, s_-)$$

Contribution from $K_S^0 - K_L^0$ interference

$$A_L^D(s_-, s_+) \propto -A_L^{\bar{D}}(s_+, s_-) - 2\epsilon A_1^{\bar{D}}(s_+, s_-)$$

Corrected yields simple to calculate in terms of $A_{1/2}^D = A(D^0 \rightarrow K_{1/2}\pi^+\pi^-)$

$$A_1^D(s_-, s_+) = A_1^{\bar{D}}(s_+, s_-)$$

$$A_S^D \propto A_1^D - \epsilon A_2^D$$

$$A_2^D(s_-, s_+) = -A_2^{\bar{D}}(s_+, s_-)$$

$$A_L^D \propto A_2^D - \epsilon A_1^D$$

$$\frac{A(K_L^0 \rightarrow \pi^+\pi^-)}{A(K_S^0 \rightarrow \pi^+\pi^-)} = \epsilon,$$

$$\epsilon \simeq (2.2 \times 10^{-3}) e^{0.24\pi i}$$

$$\widehat{CP} K_1 = K_1 \quad \widehat{CP} K_2 = -K_2$$

$$i\partial_t\psi = \mathcal{H}_{vac} \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} + \begin{pmatrix} \chi & 0 \\ 0 & \bar{\chi} \end{pmatrix}_{matter} \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}$$

χ ($\bar{\chi}$) is proportional to K^0 (\bar{K}^0) forward scattering amplitude in material

Kaon regeneration in matter introduces further dependence on $A(D \rightarrow K_L^0 \pi^+ \pi^-)$:

Non-zero $K_L^0 \leftrightarrow K_S^0$ transition amplitudes in matter

To lowest order in $r_\chi = \frac{1}{2} \frac{\chi - \bar{\chi}}{(m_L - m_S) + i/2(\Gamma_L - \Gamma_S)}$ [[Z.Phys.C.72.543](#)]

$$\psi_S(t) = e^{-i/2(\chi + \bar{\chi})t} \left((\psi_S^0 - r_\chi \psi_L^0) e^{i\lambda_S t} + r_\chi \psi_L^0 e^{i\lambda_L t} \right)$$

For LHCb and Belle II:

$$\langle r_\chi \rangle \simeq 10^{-3}$$

The A1 and A2 amplitude relations

The amplitudes $A_1 = A(D^0 \rightarrow K_1 \pi^+ \pi^-)$ and $A_2 = A(D^0 \rightarrow K_2 \pi^+ \pi^-)$ are related under the assumption that they are well described by isobar-like models [\[1010.2817\]](#)

$$A_2^D(s_-, s_+) = -A_1^D(s_-, s_+) + r_A \Delta A(s_-, s_+), \quad r_A \simeq \tan^2 \theta_C$$

$$A_1^D = \alpha^0 + \sum_{CF} \alpha_i A(K^{*-} \pi^+) + \sum_{DCS} \alpha_j A(K^{*+} \pi^-) + \sum_{\pi\pi} \alpha_k A(K_1 \pi^+ \pi^-)$$

→ \bar{K}^0 : relative “-”
→ K^0 : relative “+”
→ \bar{K}^0 & K^0 : but K^0 coupling is DCS

$$A_2^D = \alpha^{0'} - \sum_{CF} \alpha_i A(K^{*-} \pi^+) + \sum_{DCS} \alpha_j A(K^{*+} \pi^-) + \sum_{\pi\pi} -(1 - 2r_k e^{i\delta_k}) \alpha_k A(K_1 \pi^+ \pi^-)$$

$$K^0 \propto K_1 + K_2$$

$$\bar{K}^0 \propto K_1 - K_2$$

Where the DCS amplitudes satisfy : $\alpha_{DCS}/\alpha_{CF} \simeq r_k \simeq \tan^2 \theta_C \simeq 0.05$

$$N_{\pm i}^{B^-} = h^- (1 + \Gamma_S/\Gamma_L |\epsilon + r_\chi|^2 + \Delta h) \{ \hat{K}_{\pm i} + (x_-^2 + y_-^2) \hat{K}_{\mp i} + 2\sqrt{\hat{K}_i \hat{K}_{-i}} (x_- \hat{c}_{\pm i} + y_- \hat{s}_{\pm i}) + \mathcal{O}(r\epsilon) \}$$

$$N_{\pm i}^{B^+} = h^+ (1 + \Gamma_S/\Gamma_L |\epsilon + r_\chi|^2 - \Delta h) \{ \hat{K}_{\pm i} + (x_-^2 + y_-^2) \hat{K}_{\mp i} + 2\sqrt{\hat{K}_i \hat{K}_{-i}} (x_- \hat{c}_{\pm i} + y_- \hat{s}_{\pm i}) + \mathcal{O}(r\epsilon) \}$$

Dalitz-plot distribution is only changed at $\mathcal{O}(r\epsilon)$

→ **expected impact on GGSZ measurements is $\mathcal{O}(r\epsilon/r_B) = \text{permille level}$**

→ need to look at higher order terms to assess bias

$$\mathcal{O}(r\epsilon) =$$

$$\mathcal{O}(r_A\epsilon) + \mathcal{O}(r_A r_\chi)$$

$$+ (r_B\epsilon) + \mathcal{O}(r_B r_\chi)$$

Well known overall yield asymmetry [eg. [1110.3790](#)]

- Here shown for infinite time-acceptance
- And including contribution from material interaction

$$\Delta h(\epsilon, r_\chi) = 2\text{Re}[\epsilon + r_\chi] \left(1 - 2 \frac{\Gamma_S}{\Gamma_S + \Gamma_L} \frac{1 + \mu \cdot \text{Im}[\epsilon + r_\chi]/\text{Re}[\epsilon + r_\chi]}{1 + \mu^2} \right) + \mathcal{O}(r\epsilon), \quad \mu = \frac{2\Delta m}{\Gamma_S + \Gamma_L}$$

Bias on γ measurements from the integrated-yield asymmetry

$$A = \frac{N^- - N^+}{N^- + N^+} = \frac{2 \sum c_i \sqrt{K_i K_{-i}} r_B \sin \delta_B \sin \gamma + \Delta h(\epsilon, r_\chi)}{1 + r_B^2 + r_B \cos \delta_B \cos \gamma}$$

$K_S^0 \pi^+ \pi^-$ is not a CP eigenstate

Non-trivial phase $\Delta\delta_D(s_-, s_+)$

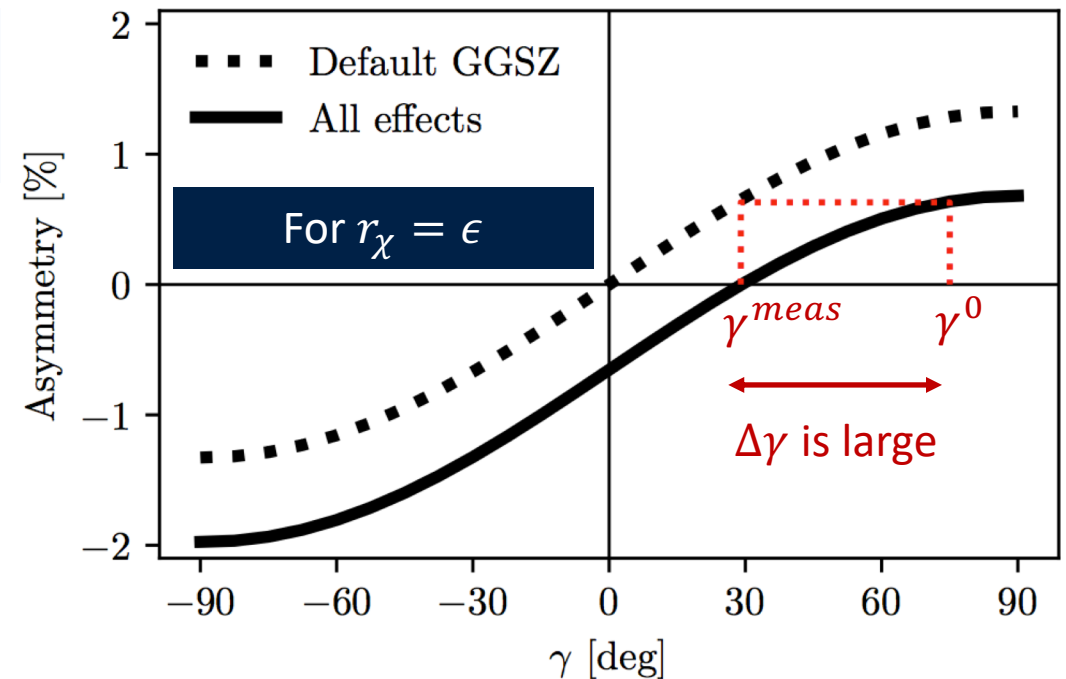
$$\rightarrow \sum c_i \sqrt{K_i K_{-i}} \simeq 0.1 \ll 1$$

$$\rightarrow \Delta\gamma = O(|\epsilon| / (0.1 \times r_B)) = \text{10's of degrees}$$

Corrections are possible, but

- $\Delta h(\epsilon, r_\chi)$ depends significantly on experimental time acceptance [[1110.3790](#)]
- $\Delta h(\epsilon, r_\chi)$ depends on experiment material budget and geometry
- Terms of $O(r_A r_\chi)$ and $O(r_A \epsilon)$ lead to % level biases and must be included

Not an issue in current experiments: overall asymmetry is not used to determine γ



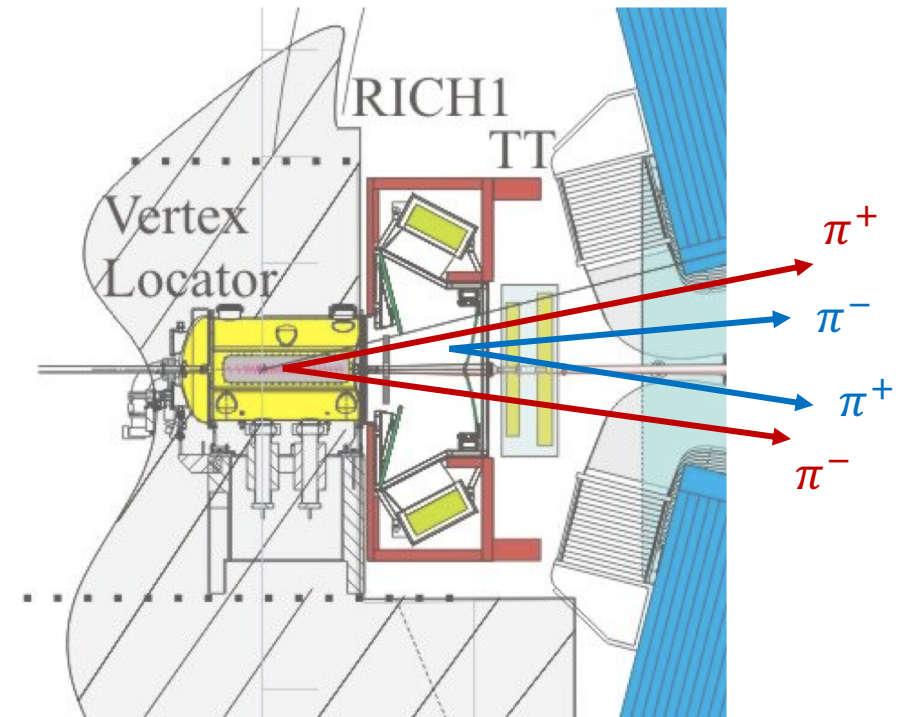
To assess impact on actual GGSZ measurements

- Higher orders of r_A, r_B, r_χ , and ϵ needed
- Realistic time-acceptance and material-interaction assumptions

Numerical studies performed for

- The Belle II detector
- The **LL** and **DD** data categories at LHCb

All orders of r, ϵ included in numerical evaluations



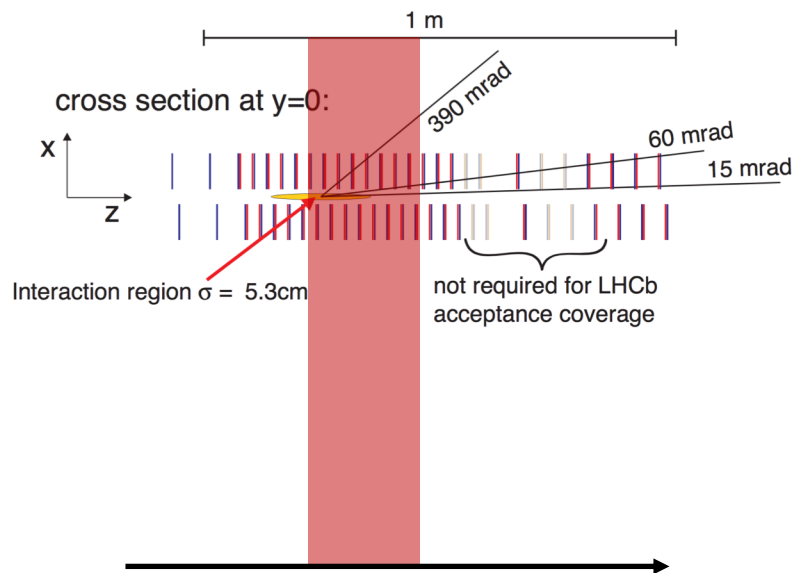
LL: both kaon decay product tracks reconstructed in VELO and downstream

DD: kaon decay product tracks only reconstructed downstream

(Simplified) Experimental time-acceptance

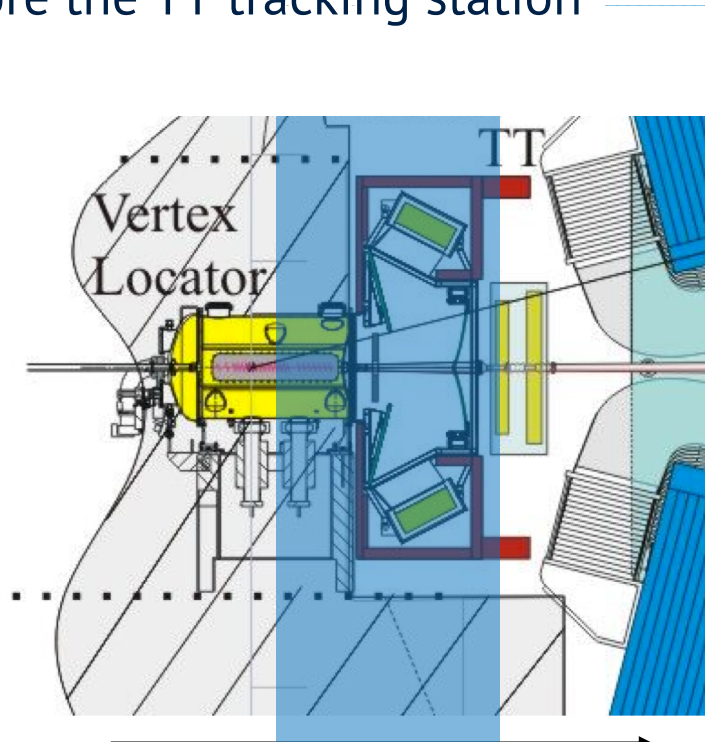
Time-acceptance ($\tau_{\text{decay}} \in [\tau_1, \tau_2]$) for kaon with $p = (p_z, p_T)$ determined by geometry

- **LL LHCb/Belle II**: require kaon decay products to cross at least 3 vertex detector segments
- **DD LHCb**: require kaon decay before the TT tracking station



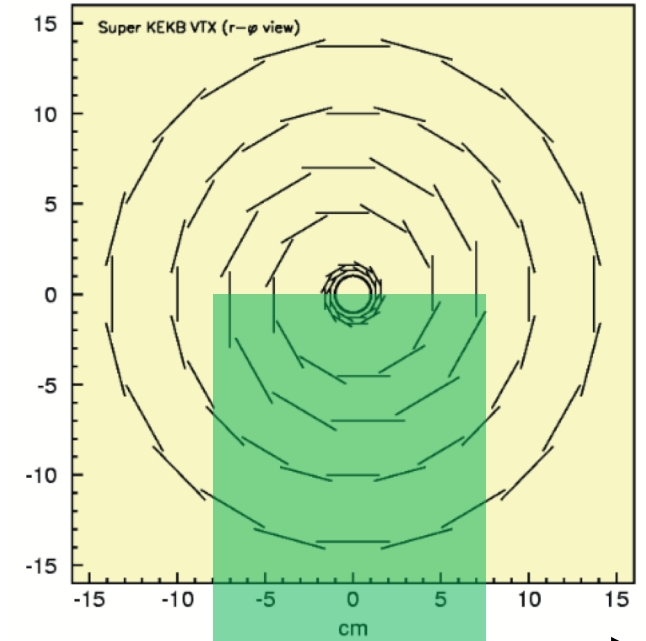
$$z_{LL} \in [0, 280] \text{ mm}$$

$$\langle \tau_{LL} \rangle \simeq 0.1 \tau_{K_S^0}$$



$$z_{DD} \in [280, 2350] \text{ mm}$$

$$\langle \tau_{DD} \rangle \simeq 0.8 \tau_{K_S^0}$$



$$r_{Belle II} \in [0, 80] \text{ mm}$$

$$\langle \tau_{Belle II} \rangle \simeq 0.7 \tau_{K_S^0}$$

Calculation procedure and input requirements

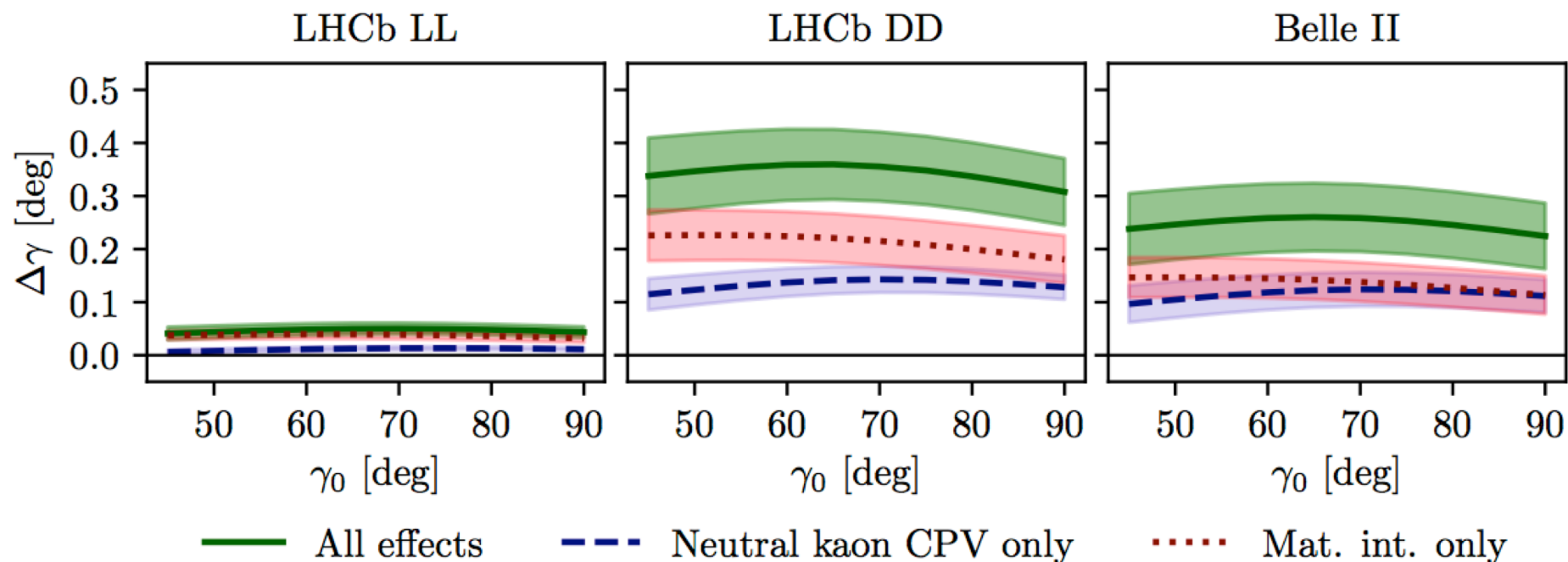
1. Model for $A_1(s_-, s_+)$ used to get $A_2(s_-, s_+)$ and then $A_{S(L)}^{D(\bar{D})}(s_-, s_+)$
2. $\psi_{S/L}^\pm(t, s_-, s_+)$ calculated keeping all orders of $r_{A,B,\chi}$ and ϵ
 - For inputs $(\gamma^0, r_B^0, \delta_B^0)$
3. N_i^\pm from integrating $d\Gamma^\pm$ over
 - experimental time-acceptance
 - Dalitz-plot bins
4. Fit (x_\pm, y_\pm) with default GGSZ eq.
5. Interpret (x_\pm, y_\pm) in terms of (γ, r_B, δ_B)
 $\rightarrow \Delta\gamma = \gamma - \gamma^0$

Belle 2018 model [[1804.06153](#)] used to represent A_1

Material parameter $\Delta\chi$ from average material budget cf. technical design reports

$\Delta\chi$ and (t_1, t_2) depend on momentum distribution, estimated using RapidSim [[1612.07489](#)]

Time acceptance (t_1, t_2) from detector geometry and p_K



$\Delta\gamma < 0.4^\circ \rightarrow \Delta\gamma/\gamma = \mathcal{O}(r\epsilon/r_B) = \mathcal{O}(\epsilon)$ as expected

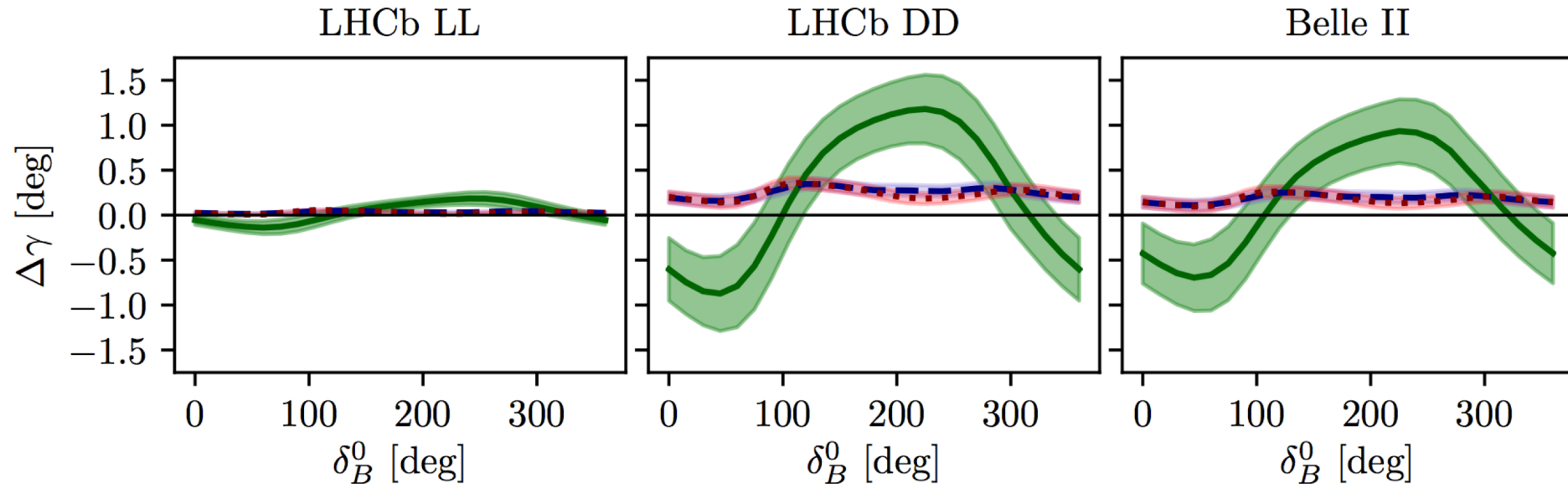
Material interaction and kaon CPV equally important

$\Delta\gamma$ small in LL LHCb: when $(t_1, t_2) < \tau_S$: K_S^0 -CPV and $K_S^0 - K_L^0$ interference approximately cancels [[1110.3790](#)]

Uncertainty bands

- Vary material density $\pm 10\%$
- Vary time-acceptance $\pm 10\%$
- 50 different (r_k, δ_k) in $A_1 \rightarrow A_2$
- Vary resolution in $p(K)$ average
- EvtGen/Belle (2010) as alternative amplitude models for A_1

Results: other B decay modes



Relevant for
 $B^\pm \rightarrow D\pi^\pm$

— $r_B^0 = 0.005$

- - - $r_B^0 = 0.1$

⋯ $r_B^0 = 0.25$

Relevant for
 $B^0 \rightarrow DK^{*0}$

$\Delta\gamma/\gamma$ large for $B^\pm \rightarrow D\pi^\pm$ because $O(r_A\epsilon/r_B^{D\pi}) \simeq 4\%$

For $r_B \in \{0.1, 0..25\}$ the $O(r_B\epsilon/r_B) = O(\epsilon)$ terms dominate bias

For **Dalits-plot based** measurements all arguments are **equally valid** for $D \rightarrow K_S^0 K^+ K^-$ and $D \rightarrow K_S^0 \pi^+ \pi^- \pi^0$
→ hence $\Delta\gamma/\gamma = O(\epsilon)$

The **global yield asymmetry from CPV in B decay** is larger in $D \rightarrow K_S^0 \pi^+ \pi^- \pi^0$ because it is dominantly CP-odd [[1710.10086](#)]
→ hence GLW-type measurements only suffer $\Delta\gamma/\gamma = O(\epsilon/r_B)$ biases

The studies here should be repeated for more precise estimates, with suitable **amplitude models and binning schemes** in place

Bias **corrections** following procedure outlined here are possible

- For both Dalitz-plot and global-asymmetry based measurements

But it is important that they

- Must include matter interaction
- Must include $O(r\epsilon)$ terms
- Must be made with full detector simulation

At the sub-degree level other effects become important as well

Material interaction and neutral kaon CPV of equal importance in γ measurements in $B^\pm \rightarrow D(\rightarrow K_S^0 \pi^+ \pi^-)K^\pm$ decays

Measurements from global-yield asymmetries suffer γ biases of 10's of degrees

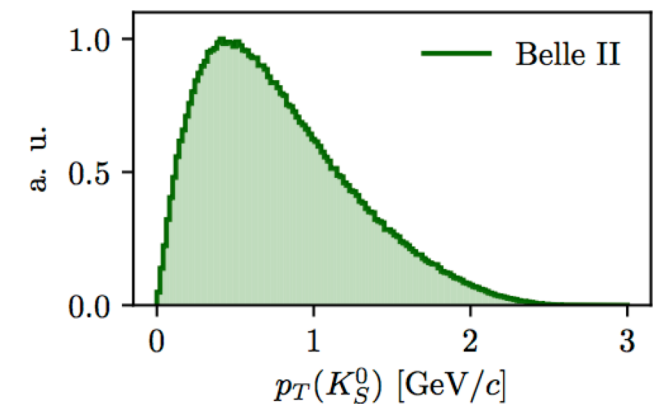
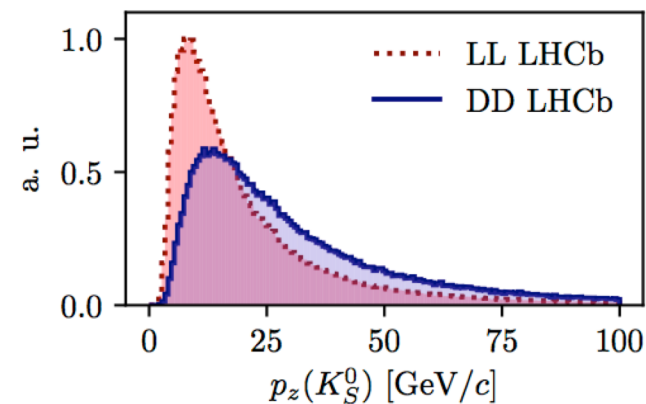
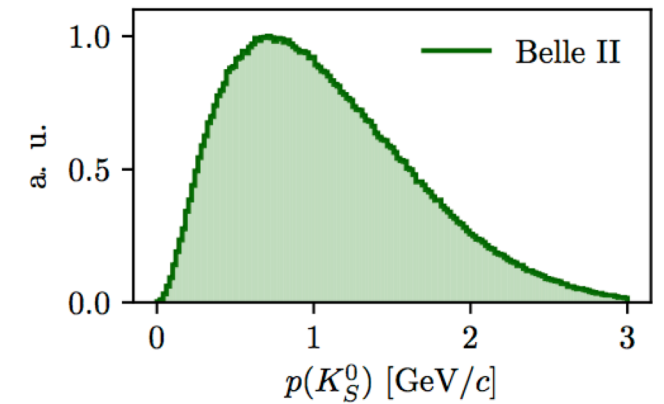
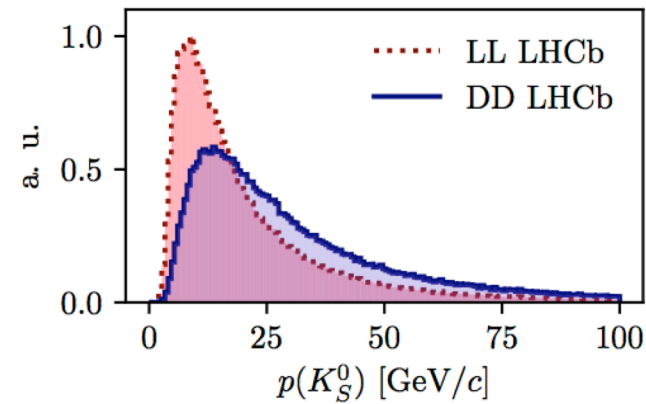
Bias in $B \rightarrow DK$ GGSZ measurements at LHCb and Belle II estimated to be **less than 0.5°**

- Can be treated as (small) systematic uncertainty in upcoming measurements

Backup Slides

Momentum distributions estimated via RapidSim (phase-space generator)

- Can generate with LHCb kinematics/acceptance
- Can decay B^\pm with $\gamma\beta = 0.28$ corresponding to Belle II (with minor tweaking)



Material interaction is governed by

$$\Delta\chi = -\frac{2\pi\mathcal{N}}{m_K}(f - \bar{f})$$

The difference in forward-scattering amplitudes of K^0 and \bar{K}^0 has been measured [[PhysRevLett.42.13](#)]

$$\left| \frac{f - \bar{f}}{p_K} \right| = 2.23 \frac{A^{0.758}}{p_K^{0.614} (\text{GeV}/c)} \text{ mb}$$

$$\arg \Delta f = -\frac{\pi}{2} (2 - 0.614)$$

Simple calculation: constant average scattering centre density \mathcal{N} used

Estimated from material budgets in TDR's

Using average momentum

- $|r_\chi|(\text{LL}) = 2.7 \times 10^{-3}$
- $|r_\chi|(\text{DD}) = 2.2 \times 10^{-3}$
- $|r_\chi|(\text{Belle II}) = 1.5 \times 10^{-3}$

Kaon momentum in lab is correlated with $m^2(\pi^+\pi^-)$

Averages taken over momentum distribution for each point in phase-space

Phase-space dependence of (τ_1, τ_2) and r_χ can affect GGSZ measurements

