$B \to \pi, K, \bar{D}$ and $B \to \rho, K^*, \bar{D}^*$ Form Factors from B-Meson Light-Cone Sum Rules

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Based on 1811.00983
with D. Van Dyk and A. Kokulu

Towards the Ultimate
Precision in Flavour Physics
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What's new?

- update of the previous calculation for the $B \to \pi, K, D$ and $B \to \rho, K^*, D^*$ form factors, using B-meson Light-Cone Sum Rules [Khodjamirian et al. '06 + '08]

What's new?

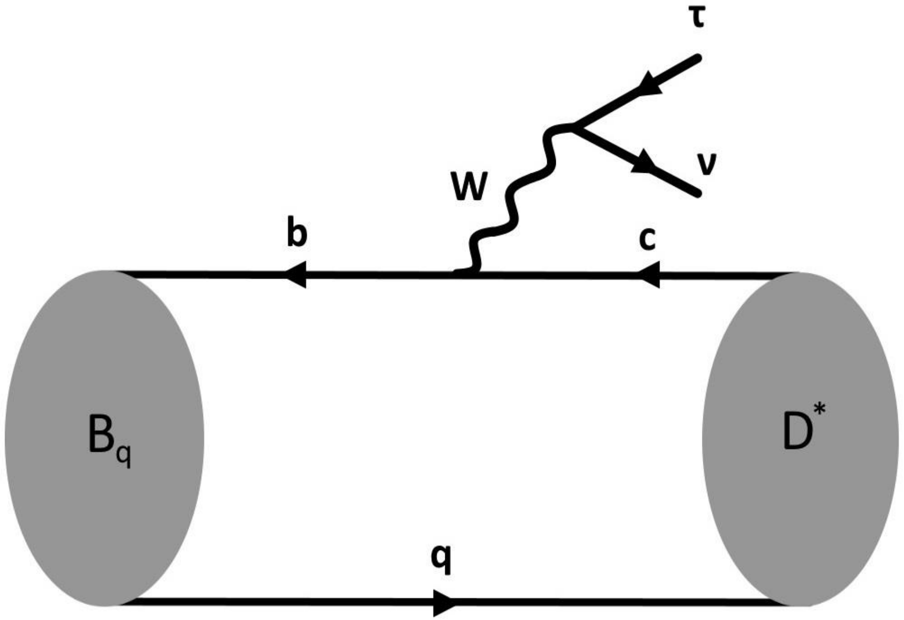
- update of the previous calculation for the $B \to \pi, K, D$ and $B \to \rho, K^*, D^*$ form factors, using B-meson Light-Cone Sum Rules [Khodjamirian et al. '06 + '08]
- inclusion of new $1/m_b$ corrections
- shift down of 10% - 30% the form factors values, comparing with the previous calculation
- prediction of $R(D^*)$ using only theoretical inputs for the first time
  and without using HQET relations for the *charm* quark
  (no experimental inputs)
- ...

Introduction

The importance of form factors in flavour physics

The $B\to P$ and $B\to V$ form factors (FFs) are needed to

- **predict decay amplitudes**, such as $B\to\{P,\ V\}l\bar{l}$ or $B\to\{P,\ V\}l\nu$
- extract $|V_{CKM}|$ matrix elements from branching ratios
- **test the Standard Model and constrain new physics contributions** [Jung/Straub '18]
- …



Anomalies in semileptonic B decays

FFs are a crucial inputs for accurate predictions of observables in various semileptonic B-meson decays

$B \to D^{(*)}$ and $B \to K^{(*)}$ form factors are particularly important in the context of the *b* anomalies

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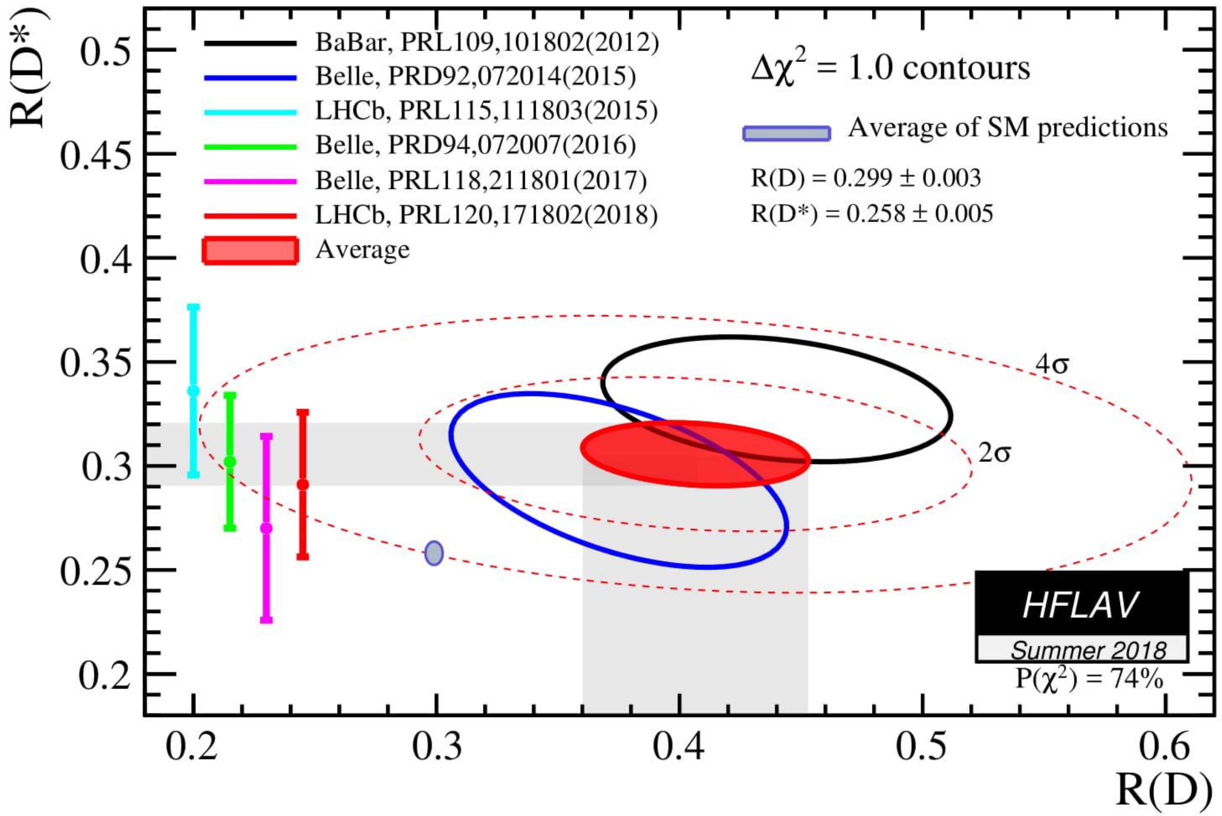
$$R_{D^{(*)}} = \frac{BR(B \to D^{(*)}\tau^+\nu)}{BR(B \to D^{(*)}\mu^+\nu)}$$

combination of experimental results (from LHCb, Belle and BaBar) leads to a 4σ deviation from the SM prediction



Definition of the form factors

FFs parametrize exclusive local hadronic matrix elements

$$\langle P(k)|\,\bar{q}_v\gamma^\mu b\,|B(k+q)\rangle = 2k^\mu\, f_+^{B\to P} + q^\mu\left[f_+^{B\to P} + f_-^{B\to P}\right]$$

$$\langle V(k,\eta)|\,\bar{q}\gamma^\mu b\,|B(k+q)\rangle = \epsilon^{\mu\nu\rho\sigma}\eta_\nu^* p_\rho k_\sigma \frac{2V^{B\to V}}{M_B + M_V}$$

FFs are functions of q^2, where q^2 is the dilepton mass squared

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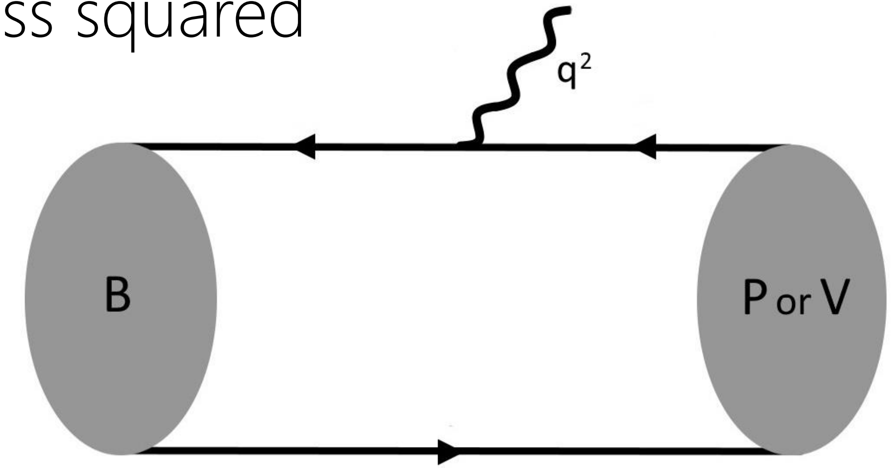
3 independent *B* to pseudoscalar (*P*) FFs
7 independent *B* to vector (*V*) FFs

We consider here the final states *P* = π, *K*, *D*
and *V* = ρ, *K**, *D**



Our approach
to the calculation

Methods to compute FFs

QCD perturbation theory breaks down at low energies

non-perturbative techniques are needed to FFs

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quark-hadron duality approximation

universal B-meson matrix elements

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Lattice QCD

numerical evaluation of correlators in a finite and discrete space-time

effective at high q^2

Light-cone Sum Rules in a nutshell 1

LCSRs are used to determine FFs from a correlation function $\Pi(k, q)$

$$\Pi(k,q) = i \int \mathrm{d}^4 x e^{ik\cdot x} \langle 0|\mathcal{T}\{J_{\mathrm{int}}(x), J_{\mathrm{weak}}(0)\}|B(q+k)\rangle \quad \text{with } \boldsymbol{x^2 \simeq 0}$$

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two ways to compute the correlator

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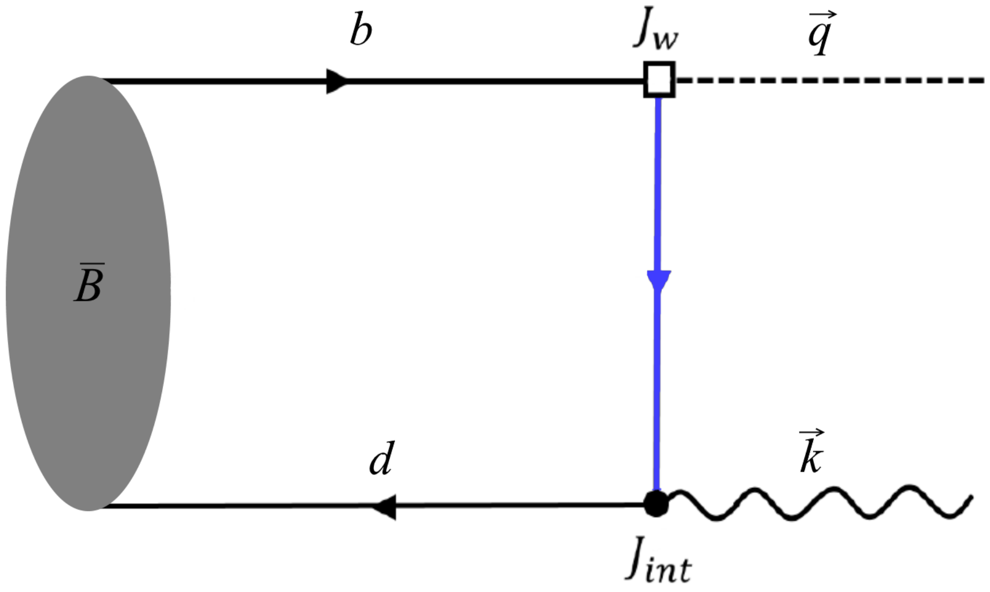
Hadronic representation for positive k^2



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OPE representation for large negative k^2 and low q^2

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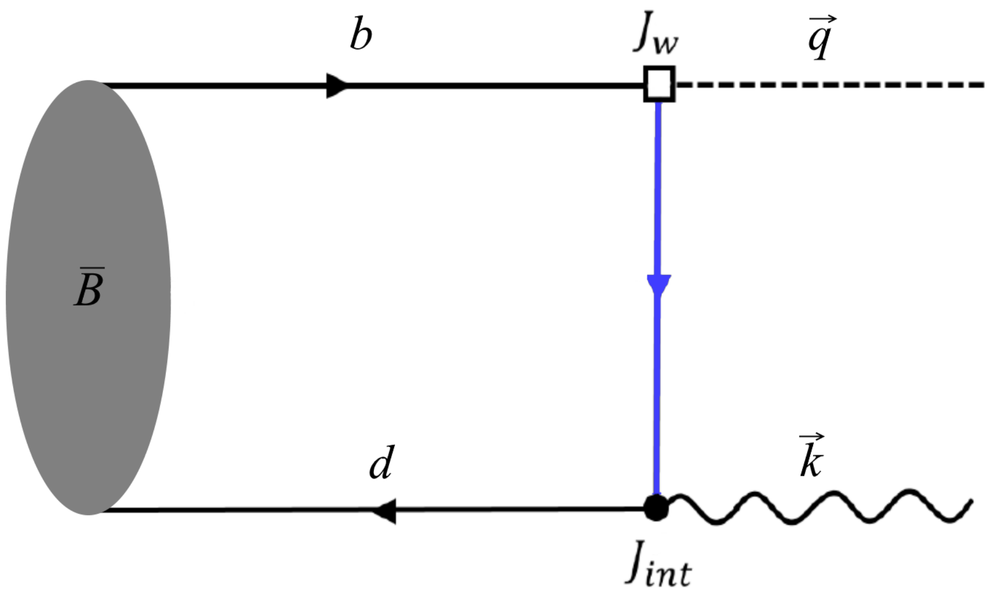


2

OPE representation for large negative k^2 and low q^2

the sum rule is obtained matching the result the two different representations of $\Pi(k, q)$ using semi-global quark-hadron duality

Hadronic calculation

for **positive k^2**

$$\Pi(k,q) = i\int \mathrm{d}^4 x e^{ik\cdot x}\langle 0|\mathcal{T}\{J_{\text{int}}(x), J_{\text{weak}}(0)\}|B(q+k)\rangle \quad \text{with } \boldsymbol{x^2 \simeq 0}$$

↓ inserting a full set of hadronic states

$$\propto \frac{\overbrace{\langle 0|J_{\text{int}}(x)|D^*\rangle}^{f_{D^*}}\overbrace{\langle D^*|J_{\text{weak}}(0)|B(q+k)\rangle}^{FF_{B\to D^*}}}{k^2 - m_{D^*}} + \text{continuum}$$

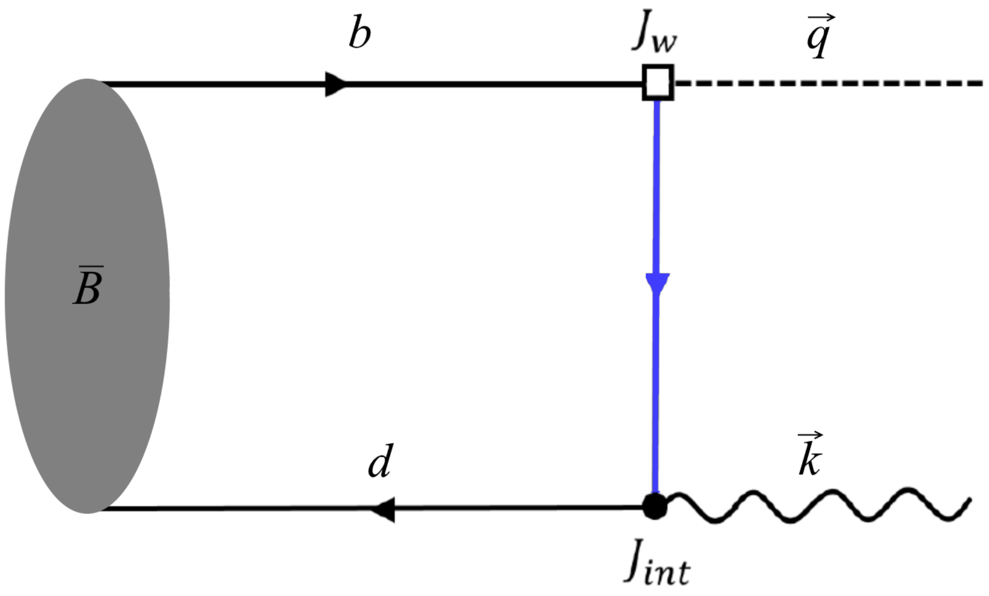
↓

hadronic dispersion relation

Light-cone Sum Rules in a nutshell 2

LCSRs are used to determine FFs from a correlation function $\Pi(k, q)$

$\Pi(k, q)$ is then expanded near the light-cone



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LCSRs are used to determine FFs from a correlation function $\Pi(k, q)$

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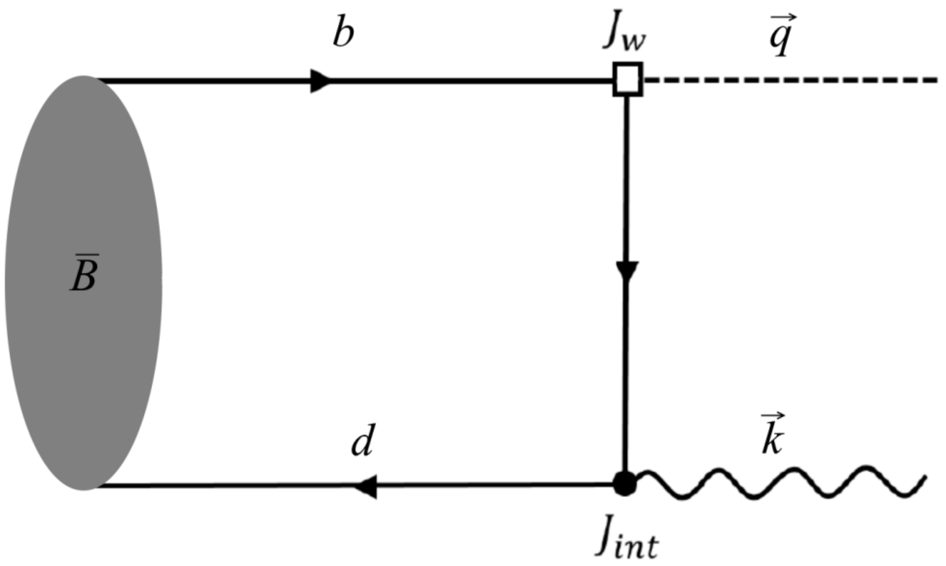
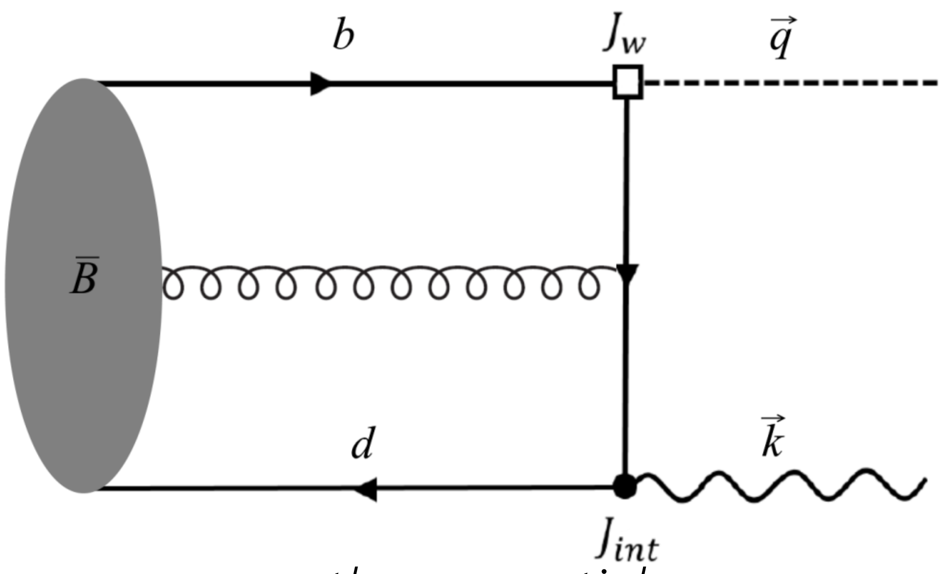
$$\Pi(k^2, q^2) = f_B m_B \int_0^\infty \mathrm{d}s \sum_{n,t} \frac{J_{n,t}(s, q^2)}{[k^2 - s]^n} \phi_t(s)$$



two-particle



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- compute $J_{n,t}$ from a **perturbative** hard scattering kernel
- B-meson **Light-Cone Distribution Amplitudes** (LCDAs) ϕ_t are necessary **non-perturbative inputs**

- both 2pt and 3pt B-LCDAs are organized in a twist expansion (**twist = dimension – spin**)
- higher twist contributions are powers of $1/m_b$ suppressed

The Sum Rule

matching of the Hadronic calculation with the OPE

apply **Borel transformation** and **quark-hadron duality** (removes continuum contribution and the tail of the OPE)

SUM RULE

$$FF_{B\to D^*}(q^2) = \frac{f_B m_B}{f_{D^*}} \int_0^{s_0} ds\, e^{\frac{m_{D^*} - s}{M^2}} \sum_{n,t} J_{n,t}(s, q^2)\, \phi_t(s)$$

s_0 is an effective threshold parameter

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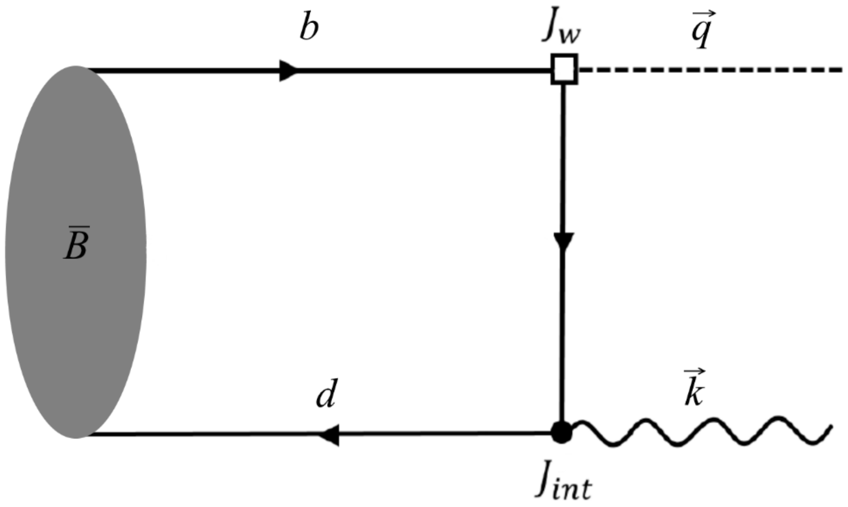
method already applied to $B \to \{P, V\}$ transitions up to **twist 3** [Khodjamirian et al. '06 + '08]

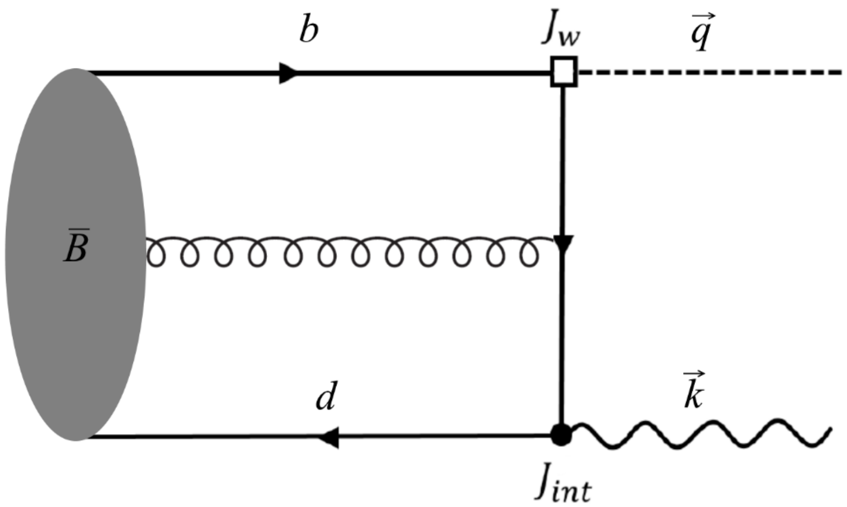
new models and higher twist LCDAs triggered our revisiting of the sum rules [Braun/Ji/Manashov '17]

More about Sum Rules

- **expansion** of the propagator **near the light-cone** gives two-particle (2pt) and three-particle (3pt) contributions, organized in a twist expansion
- higher twist contributions are powers of $1/m_b$ suppressed
- we present new twist 4 corrections to the $B \to \{P, V\}$ LCSRs (previous calculation was up to twist 3)



two-particle



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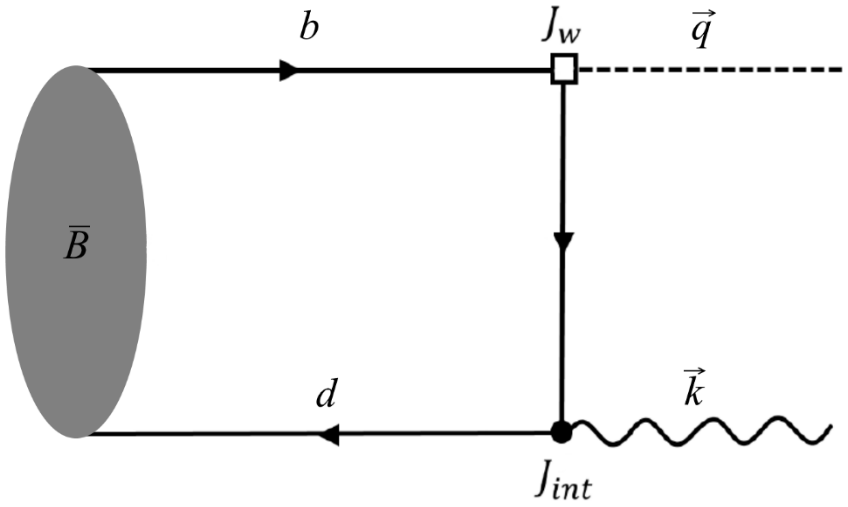
- B-LCDAs of twists > 4 are order $1/m_b^2$, therefore not considered here
- $O(\alpha_s)$ corrections are not (yet) included



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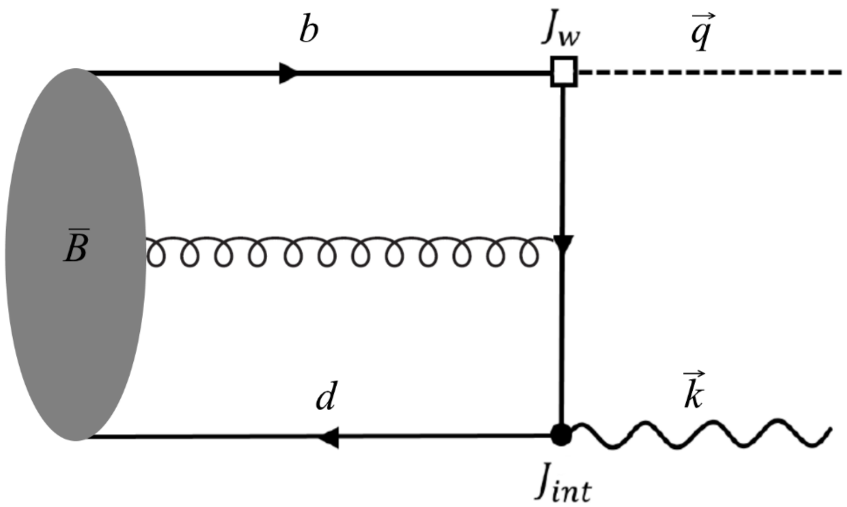

three-particle

Numerical Results

Numerical Results for $B \to D^*$ FFs

		NEW Contrib.	
$B \to D^*$ FF	2pt tw2+3	2pt tw4	3pt tw3+4
$V(q^2 = 0)$	1.02	-0.33	-0.0038
$A_0(q^2 = 0)$	0.78	-0.09	-0.0002
$A_1(q^2 = 0)$	0.73	-0.13	-0.0010
$A_{12}(q^2 = 0)$	0.22	-0.02	-0.0001

[q^2 is the dilepton mass square]

ϕ_+, ϕ_- two-particle L + NL twist contributions

g_+, g_-^{WW} new two-particle NNL twist contributions

$\phi_3, \phi_4, \psi_4, \chi_4,$ three-particle NL + NNL twist contributions

[Braun/Ji/Manashov '17]

Comparison with FKKM2008

$B \to D^*$ FF	FKKM2008 2pt tw2+3 + 3pt	GKvD2018 2pt tw2+3+4 + 3pt*
$V(q^2 = 0)$	0.96 ± 0.29	0.69 ± 0.13
$A_0(q^2 = 0)$	0.78 ± 0.22	0.67 ± 0.11
$A_1(q^2 = 0)$	0.73 ± 0.19	0.60 ± 0.09
$A_{12}(q^2 = 0)$	0.22 ± 0.07	0.21 ± 0.04

[Faller/Khodjamirian/Klein/Mannel '08]

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[Faller/Khodjamirian/Klein/Mannel '08]

- include twist 4 correction for the 2pt B-LCDAs
- for the first time results considering the full set of 3pt B-LCDAs up to twist 4
- new models for the B-LCDAs [Braun/Ji/Manashov '17]
- we use up-to-date inputs

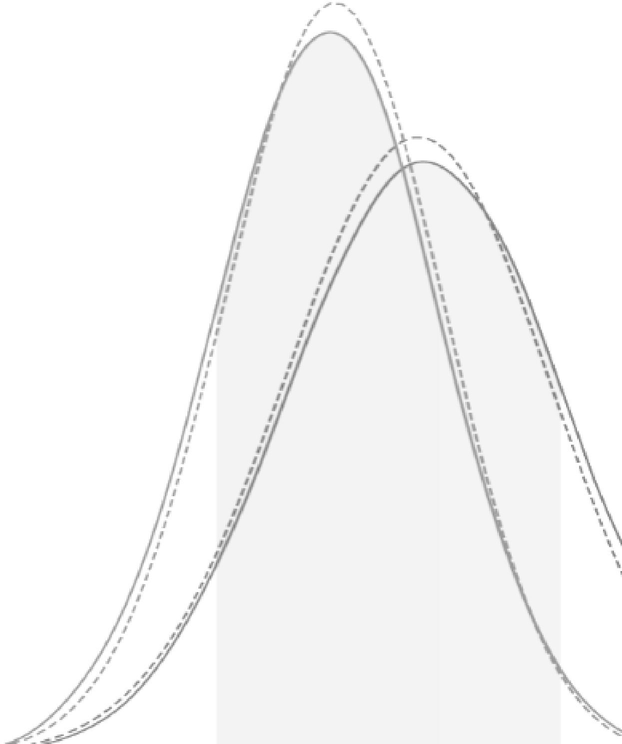
Uncertainties

- parametric uncertainties (decay constants, $\lambda_B, \lambda_H^2, \lambda_E^2$,...) → $B\to\gamma l\nu$ measurement, lattice QCD

  [Beneke/Braun/Ji/Wei '18]

- sum rule stability (dependence on the Borel parameter M^2)
- off-light cone ($O(1/m_b^2)$) corrections (estimated 5%)

} → inclusion higher twists



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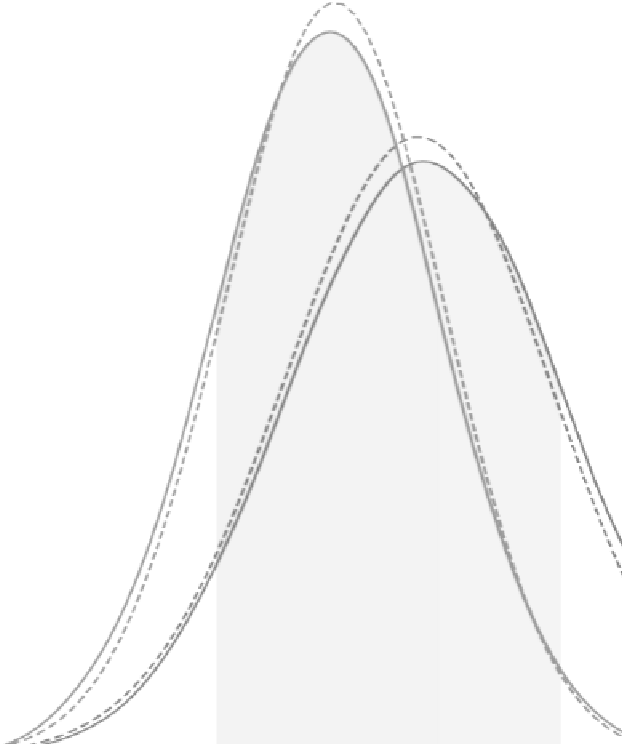
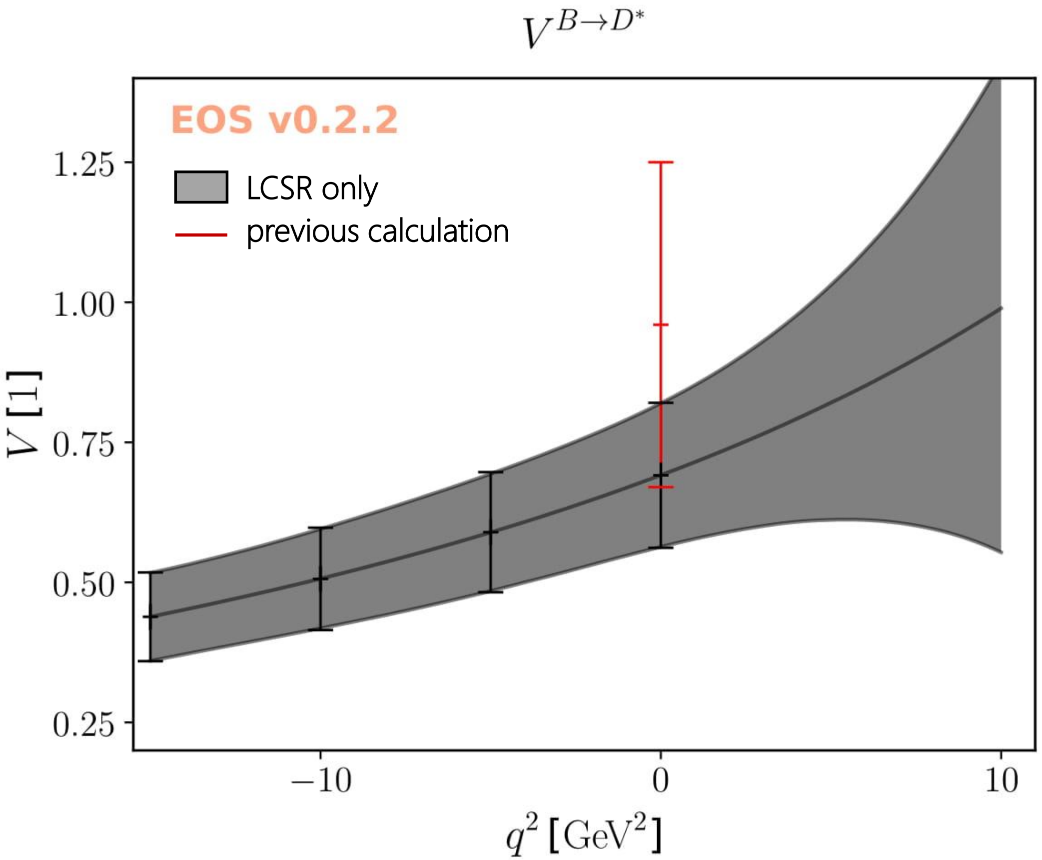
not included in our analysis

- $\boldsymbol{\alpha_s}$ corrections (10%?)? [Wang/Shen '15]
- **model dependence** of the *B*-LCDAs? [Beneke/Braun/Ji/Wei '18]
- **semi-global quark-hadron duality** approximation? (Borel transformation)



Extrapolation to large q^2 $B \to D^*$ FFs



We fit our results to the BSZ2015 parametrization to extrapolate the FFs values in the whole spectrum

[Bharrucha/Straub/Zwicky '15]

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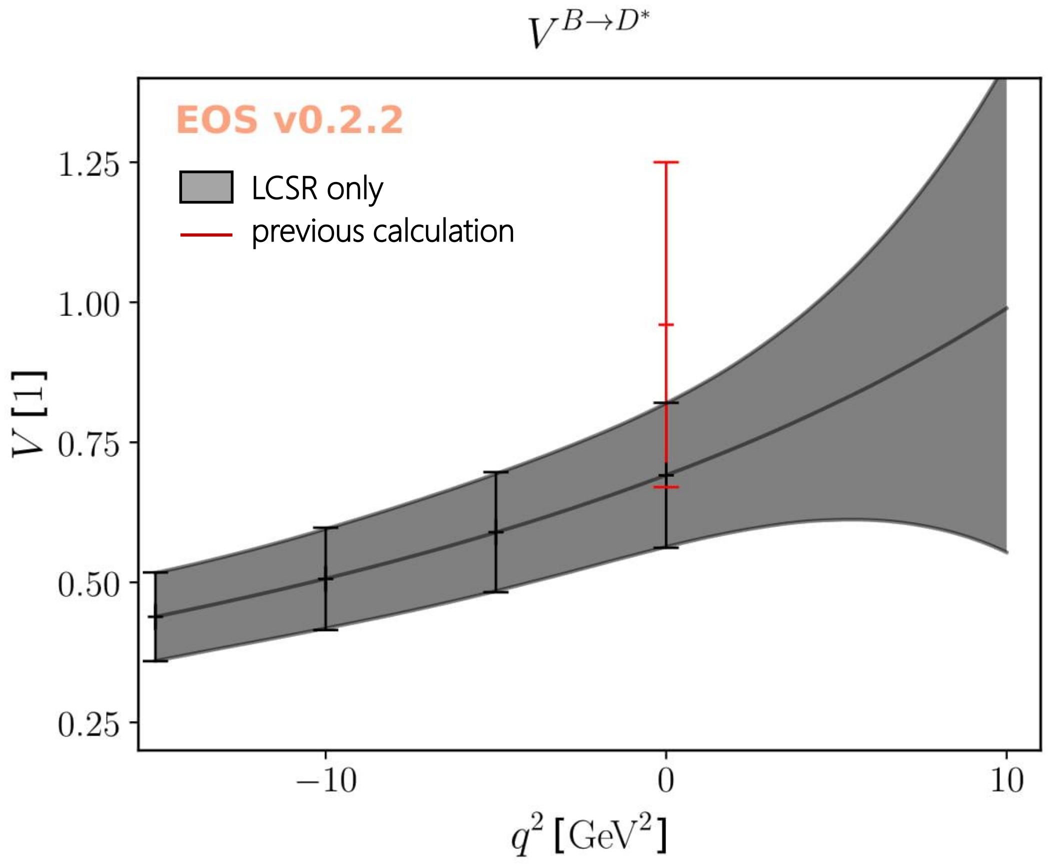

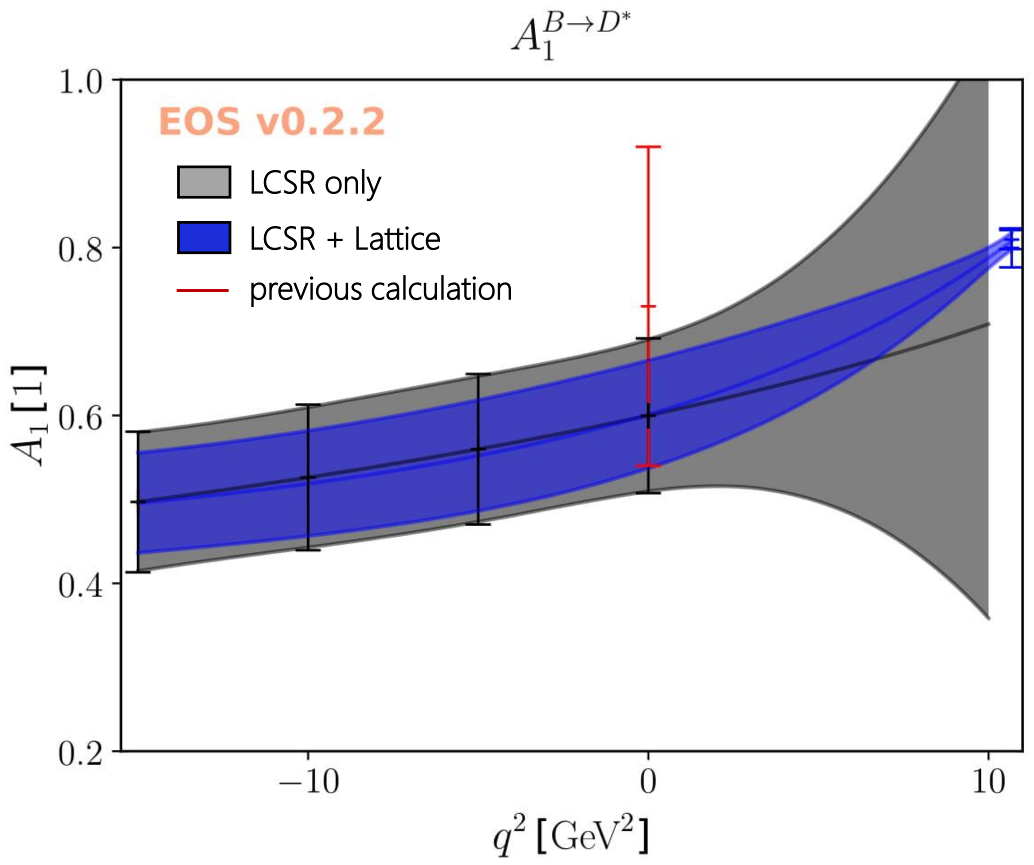

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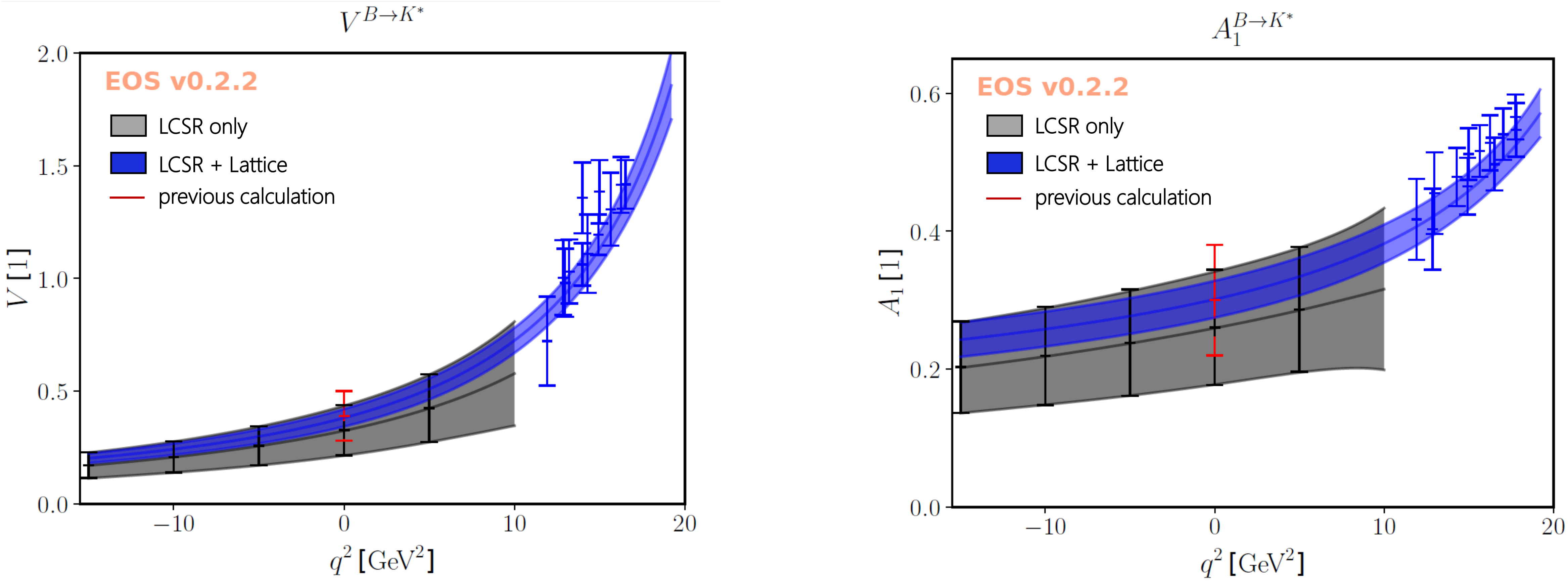
[Bharrucha/Straub/Zwicky '15]

Only A_1 is presently known from Lattice QCD at q^2_{max}
the other 6 FFs are given for the first time at different q^2 points

Extrapolation to large q^2 $B \to K^*$ FFs



B→*K** more lattice data available at different q^2 values

fits show good agreement between lattice and LCSRs calculations (p values close to one)

Our results and fits

- analytical expressions for all our sum rules ($B\to\pi, K, D$ and $B\to \rho, K^*, D^*$ transitions)

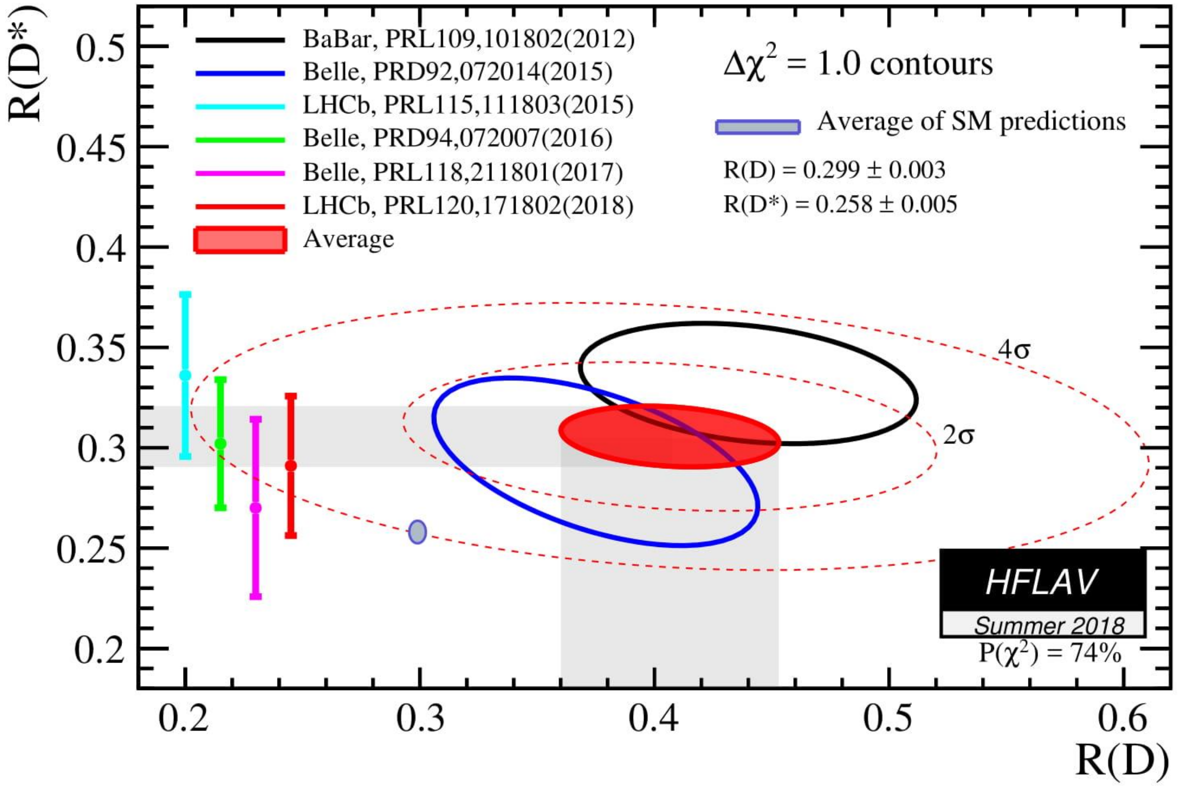
- many sum rules are given for the first time (tensor FFs)

- numerical results at different q^2 points: q^2={-15,-10,-5, 0,+5} GeV^2, for D and D* we don't consider the q^2=+5 GeV^2 point uncertainties and correlations between form factors

- uncertainties and correlations between form factors

- fit to the BSZ2015 parametrization with and without lattice points

$R(D)$ and $R(D^*)$ predictions



LCSR only

$$R(D)\Big|_{\rm SM} = 0.269 \pm 0.100$$

$$R(D^*)\Big|_{\rm SM} = 0.242 \pm 0.048$$

LCSR + Lattice

$$R(D)\Big|_{\rm SM} = 0.296 \pm 0.006$$

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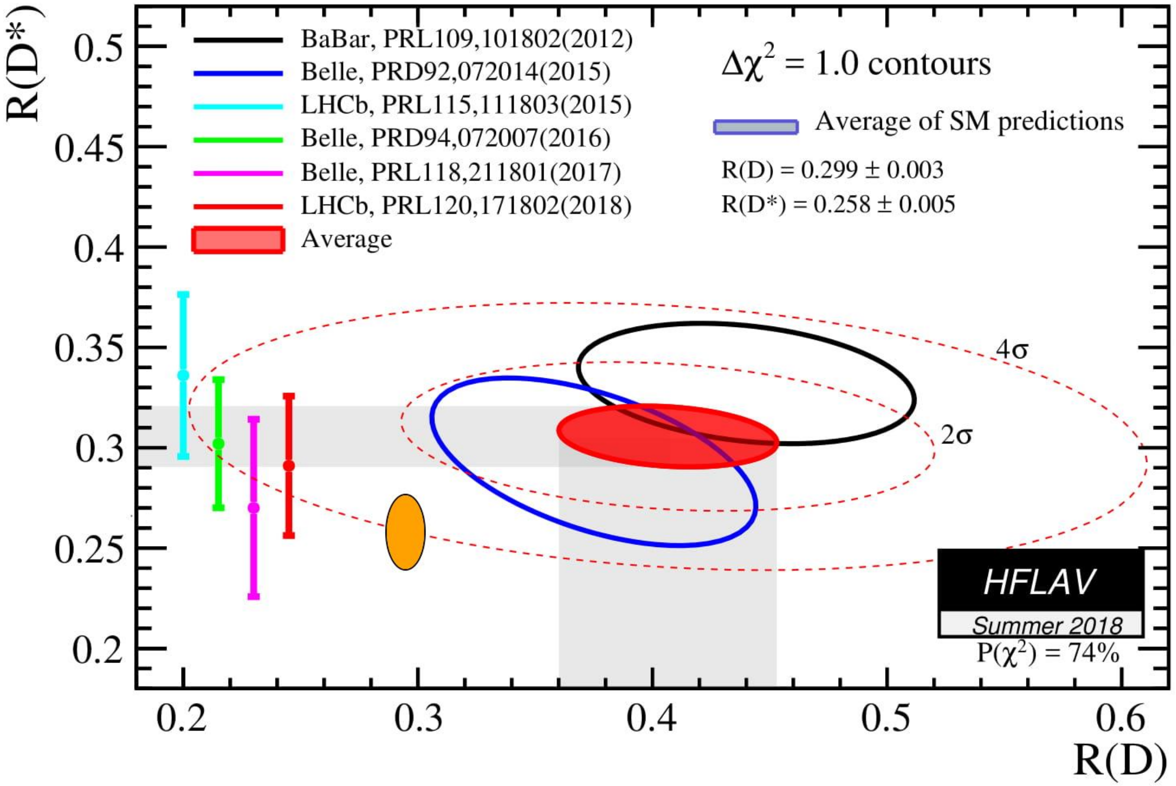

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- our numerical results, including correlations, are available as **machine readable files**
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Outlook

- impact of **radiative corrections**
- applying our framework to non-local matrix elements

Thank you for your attention!