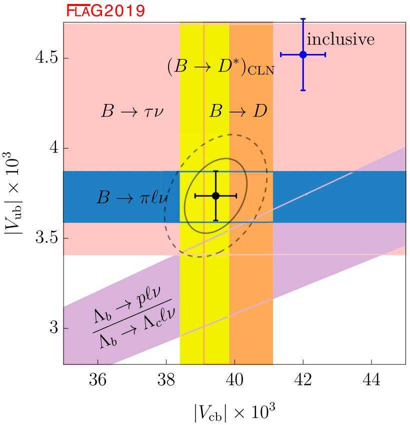
Towards the ultimate precision in V_{cb} and V_{ub}

Keri Vos

DFG FOR 1873

UNIVERSITÄT SIEGEN



$|V_{cb}|$

- Exclusive $B \to D^{(*)}\ell\bar{\nu}$
- Inclusive $B \to X_c\ell\nu$

$|V_{ub}|$

- Exclusive $B \to \pi\ell\nu (B \to \tau\nu)$
- Inclusive $B \to X_u\ell\nu$

$|V_{ub}|/|V_{cb}|$

- First determination in baryons

$$\frac{\mathcal{B}(\Lambda_b \to p\ell\nu)}{\mathcal{B}(\Lambda_b \to \Lambda_c\ell\nu)} = (1.5 \pm 0.1)\left|\frac{V_{ub}}{V_{cb}}\right|^2$$

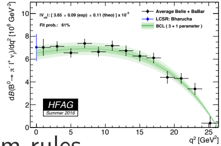
LHCb'18; Detmold, Lehner, Meinel'15

Exclusive V_{ub}

Exclusive $B \to \pi \ell \nu$

Bourrely, Caprini, Lellouch, PRD79, 013008 (2009); Bharucha, JHEP 1205 (2012)

$$\frac{d\mathcal{B}(B \to \pi \ell \nu)}{dq^2} = \frac{G_F^2 \tau_B}{24\pi^3} |V_{ub}|^2 p_\pi^3 |f_+^{B\pi}(q^2)|^2$$



- Only one form factor required
- Combined inputs from Lattice QCD (BCL) and QCD sum rules

$$|V_{ub}|_{\text{excl}} = (3.70 \pm 0.16) \times 10^{-3}$$ PDG'18

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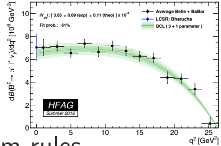

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Other probes:

- $B_s \to K \ell \nu$
  - Form factors available in QCD sum rules Khodjamirian, Rusov, JHEP 08 (2017) 112
  - on the Lattice Fermilab/MILC [1901.02561]
- Pure leptonic $B \to \tau \nu$

Exclusive V_{cb}

Caprini, Lellouch, Neubert (1998); Boyd, Grinstein, Lebed (1997)

$B \to D^* \ell \nu$

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^5}{48\pi^3} |V_{cb}|^2 (w^2 - 1)^{1/2} P(w) (\eta_{\rm ew} F(w))^2 \text{ with } w = v_B \cdot v_D$$

- Form factor $F(w = 1)$ computed on the lattice
- Extrapolation to zero-recoil point necessary
- Different parameterization; BGL and CLN
- CLN relies on HQET relations between form factors to reduce parameters
- CLN has limited flexibility of the form factor slope
- Recent preliminary data from Belle allows for analysis using BGL Belle [1702.01521]

Grinstein, Kobach, PLB 771 359 (2017)
Bigi, Gambino, Schacht, PLB 769 441 (2017).

HFLAV'17

$$|V_{cb}|_{\text{excl, CLN}} = (39.2 \pm 0.7) \times 10^{-3} \quad |V_{cb}|_{\text{excl, BGL},D^*} = (41.7 \pm 2.0) \times 10^{-3}$$

Healey, Turczyk, Gambino PLB 763 (2016) 60

$$|V_{cb}|_{\text{incl}} = (42.0 \pm 0.6) \times 10^{-3}$$

FNAL/MILC,HPQCD

$B \to D\ell\nu$

- Lattice form factors available at different kinematical points
- Latest Belle analysis uses both BGL and CLN Belle [1510.03657]
- BGL results closer to inclusive than CLN

Bigi, Gambino, Phys. Rev. D94 (2016) 094008

$$|V_{cb}|_{\text{excl, Global Fit},D} = (40.5 \pm 1.0) \times 10^{-3}$$

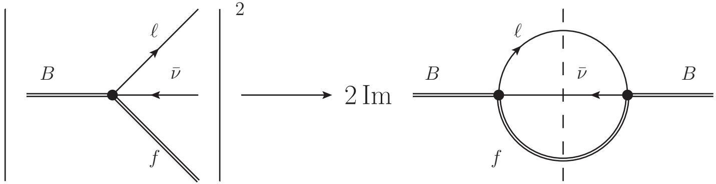
Towards the Ultimate Precision:

- Both BGL and CLN parameterization require more studies
  see e.g. Bernlochner, Ligeti, Robinson [1902.09553]; Straub, Jung, JHEP 1901 (2019) 009
  - Crucial to understand difference between CLN/BGL better
  - Requires higher-order corrections in CLN
- Lattice $B \to D^*$ away from zero recoil in progress
  Aviles-Casco, DeTar, Du, El-Khadra, Kronfeld, Laiho, Van de Water [1710.09817]
- More data necessary

Stay Tunded

Inclusive B decays

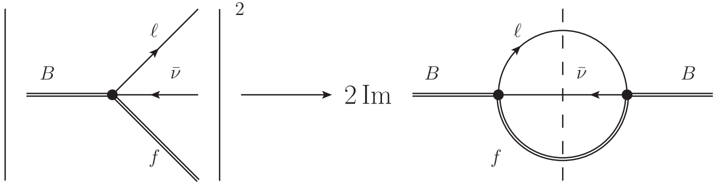
Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, · · ·



Optical Theorem

$$\sum_X |\langle X|\mathcal{H}_{eff}|B(v)\rangle|^2 = 2\ \text{Im} \int d^4x e^{-iq\cdot x} \langle B(v)| T\left\{\mathcal{H}_{eff}(x), \mathcal{H}^\dagger_{eff}(0)\right\} |B(v)\rangle$$

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, · · ·



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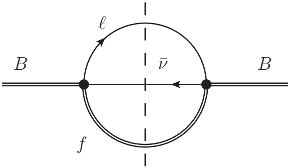
$$= 2\ \text{Im} \int d^4x\, e^{-im_b v\cdot x}\langle B(v)| T\left\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\right\}|B(v)\rangle$$

Heavy Quark Expansion

- Field-redefinition of the heavy field $Q(x) = exp(-im(v\cdot x))Q_v(x)$
- Split the momentum $p_b = m_b v + k$, expand in $k \sim iD\ Q_v$

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, $\cdots$

Operator Product Expansion (OPE)

$2\,\mathrm{Im}$ 

$$= \sum_{n,i} \frac{C_i^{(n)}(\mu,\alpha_s)}{m_b^i} \langle B|\mathcal{O}_i^{(n)}|B\rangle_\mu$$

- $\mathcal{C}_i(\mu)$: short distance, perturbative coefficients
- $\mathcal{O}_i$: operators of dimension $n+3$, contain chains of covariant derivatives
- $\langle B|\mathcal{O}_i|B\rangle_\mu$: non-perturbative forward matrix elements of local operators

Non-perturbative matrix elements

Γ_i are power series in $\mathcal{O}(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 \cdots$$

- Γ_0: decay of the free quark (partonic contributions), $\Gamma_1 = 0$
- Γ_2: μ_π^2 kinetic term and the μ_G^2 chromomagnetic moment

$$2M_B\mu_\pi^2 = -\left\langle B|\bar{b}_v iD_\mu iD^\mu b_v|B\right\rangle$$
$$2M_B\mu_G^2 = \left\langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\right\rangle$$

- Γ_3: ρ_D^3 Darwin term and ρ_{LS}^3 spin-orbit term

$$2M_B\rho_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu,[ivD,iD^\mu]\right]b_v|B\right\rangle$$
$$2M_B\rho_{LS}^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left\{iD_\mu,\left[ivD,iD_\nu\right]\right\}(-i\sigma^{\mu\nu})b_v|B\right\rangle$$

- Γ_4: 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- Γ_5: 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Inclusive V_{cb}

Gambino, Schwanda, PRD 89 (2014) 014022; Alberti, Gambino et al, PRL 114 (2015) 061802

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \frac{\mu_\pi^2}{m_b^2}\left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right)\right.$$
$$\left. + \frac{\mu_G^2}{m_b^2}\left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3}\Gamma^{(D,0)} + \mathcal{O}\left(\frac{1}{m_b^4}\right) \ldots \right)$$

- Both hadronic matrix elements and $|V_{cb}|$ extracted from moments of differential rates and total rate
- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109
- How to include power corrections?

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \qquad \rho_{LS}^3 = -\varepsilon\mu_G^2, \qquad \varepsilon \sim 0.4 \text{ GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- LSSA estimated as priors (60% gaussian uncertainty)
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60

$$|V_{cb}|_{\text{incl}} = (42.00 \pm 0.64) \times 10^{-3}$$

- -0.25% shift due to power corrections

Theory guidance to include power corrections

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Towards the Ultimate Precision in $|V_{cb}|$

- Include α_s corrections to for ρ_D^3 Mannel, Pivovarov [in progress]
- Full determination up to $1/m_b^4$ from data possible?

Alternative $|V_{cb}|$ determination

Reparametrization invariance

Dugan, Golden, Grinstein, Chen, Luke, Manohar, Hill, Solon, Heinonen, Mannel, KKV

- Choice of v not unique, result independent of v (Lorentz invariance)
- Reparameterization Invariant (RPI) under an infinitesimal change

$$v_\mu \to v_\mu + \delta v_\mu : \quad \delta_{RP}\, v_\mu = \delta v_\mu \quad \delta_{RP}\, iD_\mu = -m_b \delta v_\mu$$

- Reparametrization invariance links different orders in $1/m_b$
  - Gives exact relations between different orders
  - Resums towers of operators
  - Reduces the number of independent parameters

The RPI relation:

$$\delta_{\rm RP}\, C^{(n)}_{\mu_1\cdots\mu_n} = m_b \delta v^\alpha \left[C^{(n+1)}_{\alpha\mu_1\cdots\mu_n} + C^{(n+1)}_{\mu_1\alpha\mu_2\cdots\mu_n} + \cdots + C^{(n+1)}_{\mu_1\cdots\mu_n\alpha} \right]$$

Mannel, KKV, JHEP 1806 (2018) 115

Mannel, KKV, JHEP 1806 (2018) 115

- 1:
  - $2M_B\mu_3 = \langle B|\bar{b}_v b_v|B\rangle = 2M_B\left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b}\right)$
- $1/m_b^2$:
  - $2M_B\mu_G^2 = \langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\rangle$
- $1/m_b^3$:
  - $2M_B\tilde{\rho}_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu, \left[\left(ivD + \frac{(iD)^2}{2m_b}\right), iD^\mu\right]\right] b_v|B\right\rangle$
- $1/m_b^4$:
  - $2M_B r_G^4 \equiv \langle B|\bar{b}_v\,[iD_\mu, iD_\nu]\,[iD^\mu, iD^\nu]\,b_v|B\rangle \propto \langle \vec{E}^2 - \vec{B}^2\rangle$
  - $2M_B r_E^4 \equiv \langle B|\bar{b}_v\,[ivD, iD_\mu]\,[ivD, iD^\mu]\,b_v|B\rangle \propto \langle \vec{E}^2\rangle$
  - $2M_B s_B^4 \equiv \langle B|\bar{b}_v\,[iD_\mu, iD_\alpha]\,[iD^\mu, iD_\beta]\,(-i\sigma^{\alpha\beta})b_v|B\rangle \propto \langle \vec{\sigma}\cdot\vec{B}\times\vec{B}\rangle$
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  - $2M_B s_{qB}^4 \equiv \langle B|\bar{b}_v\,[iD_\mu, [iD^\mu, [iD_\alpha, iD_\beta]]]\,(-i\sigma^{\alpha\beta})b_v|B\rangle \propto \langle \Box\,\vec{\sigma}\cdot\vec{B}\rangle$.

Up to $1/m_b^4$: 8 parameters (previously 13)

Alternative V_{cb} Determination

- Directly fit RPI reduced set of MEs using RPI observables

$$O = \int w(v, p_e, p_\nu) \langle \mathrm{Im}\, T(S) \rangle L(p_e, p_\nu) d\Phi_3$$

- The observable O is RPI if $\delta_{\mathrm{RP}} w(v, p_e, p_\nu) = 0$

O	$w(v, p_e, p_\nu)$	RPI
Total Rate	1	✓
Moments charged lepton energy	$(v \cdot p_e)^n$	✗
Moments hadronic invariant mass	$(M_B v - q)^{2n}$	✗
Moments leptonic invariant mass	$(q^2)^n$	✓

Fael, Mannel, KKV, JHEP 02 (2019) 177

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Fael, Mannel, KKV, JHEP 02 (2019) 177

New Method:

- $|V_{cb}|$ from Γ_{tot}, $\Delta Br(q^2_{\mathrm{cut}})$ and $\langle (q^2)^n \rangle_{\mathrm{cut}}$ up to $1/m_b^4$
- Complementary and completely data driven V_{cb} determination
- q^2 moments not (yet) available

Inclusive V_{ub}

- Experimental cuts necessary to remove charm background
- Local OPE as in $b \to c$ cannot work
- Switch to different set-up using light-cone OPE
- Introduce shape functions ($\sim$ parton DAs in DIS)
- Information on leading shape function from $B \to X_s\gamma$
- Different parameterizations of subleading shape functions BLNP, GGOU, ...

$$|V_{ub}|_{\text{incl}} = (4.5 \pm 0.3) \times 10^{-3} \quad \text{PDG'18}$$

- Inclusive determinations need to be scrutinized

Towards the Ultimate Precision:

- Extract shape functions from global fit (SIMBA and NNVub)
  Gambino, Healey, Mondino [2016]; F.Tackmann, K.Tackmann, Ligeti, Bernlochner, Stewart
- Implementing higher-order corrections in parameterizations

Weak Annihiliation

Uraltsev, Bigi, Voloshin, Mannel, TurczykLigeti, Luke, Manohar, Phys. Rev.D82 (2010) 033003
Gambino, Kamenik, Nucl.Phys.B840 (2010) 424

$$\mathcal{O}_V = (\bar{c}\gamma^\mu q)(\bar{q}\gamma_\mu c) \qquad \mathcal{O}_S = (\bar{c}q)(\bar{q}c)$$

- Starting at $\mathcal{O}(1/m_b^3)$
- Mainly contribute at end-point region
- Challenging to study non-perturbatively
- Can be obtained from D and D_s semileptonics using HQET
- Effect is $(m_b/m_c)^3$ enhanced compared to B decays
- Studies using lepton energy spectrum from CLEO-c suggests [0912.4232]
  $\sim$ few % contribution to $B \to X_u \ell\nu$
- Related by RGE to $\log(m_c/m_b)$ terms in $B \to X_c \ell\nu$ Fael, Mannel, KKV [in progress]

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Towards the Ultimate Precision:

- Very important to achieve precise $B \to X_{d,s}\ell\ell$ predictions
  Hurth, Huber, Lunghi, Jenkins, Qin, KKV [in progress]

Interesting to further explore at BESIII

Conclusion

Inclusive $|V_{cb}|$ and exclusive $|V_{ub}|$ least disputed

Towards the Ultimate Precision:

- New modes
- Perturbative corrections [in progress]
- Proliferation of non-perturbative matrix elements: q^2 moments

Inclusive $|V_{ub}|$ and exclusive $|V_{cb}|$ require more studies

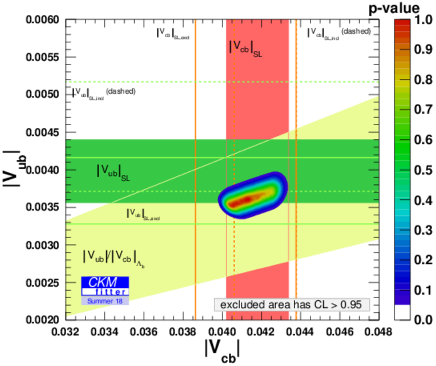
Towards the Ultimate Precision:

- Lattice form factors for $B \to D^*$ at different kinematic points
- Studies of Weak Annihilation in D and D_s semileptonics
- Implementing higer-order corrections in parameterizations

Thank you for your attention

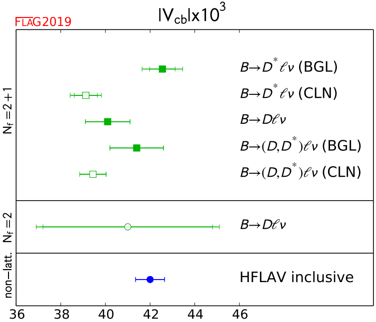
Backup

Inclusive versus Exclusive V_{cb}



- Both make use of Heavy Quark Expansion (HQE)
- Different parametrizations for exclusive (BGL and CLN)

Inclusive versus Exclusive V_{cb}



- Both make use of Heavy Quark Expansion (HQE)
- Different parametrizations for exclusive (BGL and CLN)

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[\mu_3 - 2\frac{\mu_G^2}{m_b^2} + \left(\frac{34}{3} + 8\log\rho\right)\frac{\tilde{\rho}_D^3}{m_b^2} \right.$$

$$+ \frac{16}{9}(4 + 3\log\rho)\frac{r_G^4}{m_b^4} - \frac{16}{9}(1 + 3\log\rho)\frac{r_E^4}{m_b^4} - \frac{2}{3}\frac{s_B^4}{m_b^4}$$

$$\left. + \left(\frac{50}{9} + \frac{8}{3}\log\rho\right)\frac{s_E^4}{m_b^4} - \left(\frac{25}{36} + \frac{1}{3}\log\rho\right)\frac{s_{qb}^4}{m_b^4} + O\left(\rho, \frac{1}{m_b^5}\right)\right]$$

with $\rho = m_c^2/m_b^2$

RPI Observables

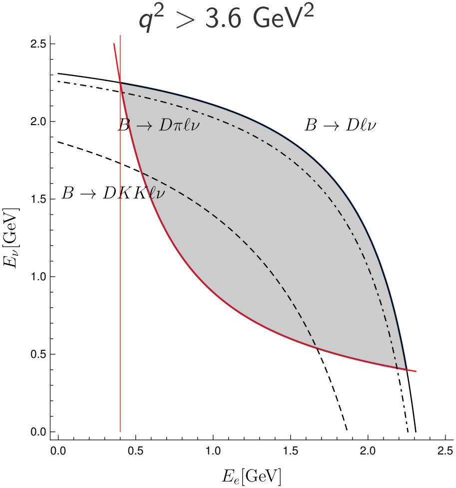
- Ratio between the rate with and without a cut

$$R^*(q^2_{\rm cut}) = \int_{q^2 > q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2} \Bigg/ \int_0 dq^2 \, \frac{d\Gamma}{dq^2}$$

- q^2 moments

$$\langle (q^2)^n \rangle_{\rm cut} = \int_{q^2 > q^2_{\rm cut}} dq^2 \, (q^2)^n \, \frac{d\Gamma}{dq^2} \Bigg/ \int_{q^2 > q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2}$$

Fael, Mannel, KKV, JHEP 02 (2019) 177


$q^2 > 3.6$ GeV^2
$q^2 > 8.4$ GeV^2
$B \to D\pi\ell\nu$
$B \to D\ell\nu$
$B \to DKK\ell\nu$
$B \to D\pi\ell\nu$
$B \to D\ell\nu$
$B \to DKK\ell\nu$
E_ν[GeV]
E_e[GeV]
E_ν[GeV]
E_e[GeV]

Total rate at tree level

Mannel, KKV, JHEP 1806 (2018) 115

$$R = \sum_{n=0}^{\infty} C^{(n)}_{\mu_1..\mu_n}(v) \otimes \bar{Q}_v (iD_{\mu_1} \cdots iD_{\mu_n}) Q_v$$

$$\delta_{\rm RP} R = 0 \;=\; \sum_{n=0}^{\infty} \left[\delta_{\rm RP} C^{(n)}_{\mu_1\cdots\mu_n}\right] \bar{Q}_v (iD^{\mu_1} \cdots iD^{\mu_n}) Q_v + \sum_{n=0}^{\infty} C^{(n)}_{\mu_1\cdots\mu_n} \left[\delta_{\rm RP} \bar{Q}_v (iD^{\mu_1} \cdots iD^{\mu_n}) Q_v\right]$$

The RPI relation:

$$\delta_{\rm RP} C^{(n)}_{\mu_1\cdots\mu_n} = m_b \delta v^\alpha \left[C^{(n+1)}_{\alpha\mu_1\cdots\mu_n} + C^{(n+1)}_{\mu_1\alpha\mu_2\cdots\mu_n} + \cdots + C^{(n+1)}_{\mu_1\cdots\mu_n\alpha}\right]$$