Direct *CP*-violation in *D*-meson decays from QCD light-cone sum rules

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based on A. K. and A. Petrov, PLB 774 (2017) 235 (1706.07780 [hep-ph])

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□ Direct CP asymmetry in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ decays

- definition:

$$a_{CP}^{dir}(f) = \frac{\Gamma(D^0 \to f) - \Gamma(\bar{D}^0 \to \bar{f})}{\Gamma(D^0 \to f) + \Gamma(\bar{D}^0 \to \bar{f})} = \frac{|A(D^0 \to f)|^2 - |A(\bar{D}^0 \to \bar{f})|^2}{|A(D^0 \to f)|^2 + |A(\bar{D}^0 \to \bar{f})|^2}$$

$f = \pi^+\pi^-,\ K^+K^-$

- if $A(D^0 \to f)$ can be separated into at least two different parts,

$$A(D^0 \to f) = A_f^{(1)} e^{i\delta_1} e^{i\phi_1} + A_f^{(2)} e^{i\delta_2} e^{i\phi_2},$$

$\phi_1 \neq \phi_2$ - the weak phases (odd under CP),
$\delta_1 \neq \delta_2$ - the strong phases (even under CP):

$$A(\bar{D}^0 \to \bar{f}) = A_f^{(1)} e^{i\delta_1} e^{-i\phi_1} + A_f^{(2)} e^{i\delta_2} e^{-i\phi_2},$$

- the CP-violating asymmetry:

$$a_{\rm CP}^{\rm dir}(f) \sim \frac{A_f^{(1)}}{A_f^{(2)}} \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2).$$

- a quantitative estimate and/or upper bound of $a_{\rm CP}^{\rm dir}(f)$ in SM ?

□ Single Cabibbo-suppressed (SCS) decays

- effective Hamiltonian: $O_{i\geq 3}$ with $c_i \ll c_{1,2}$ are neglected

$$H_{eff} = \frac{G_F}{\sqrt{2}}\Big\{\lambda_d\big(c_1\ O_1^d + c_2\ O_2^d\big) + \lambda_s\big(c_1\ O_1^s + c_2\ O_2^s\big)\Big\}\,,$$

$$O_1^d = (\bar{u}\Gamma_\mu d)\,(\bar{d}\Gamma^\mu c)\ ,\ O_2^d = (\bar{d}\Gamma_\mu d)\,(\bar{u}\Gamma^\mu c) \underset{d\to s}{\Longrightarrow}\ O_1^s\,,\ O_2^s$$

- hereafter using a compact notation:

$$\mathcal{O}^D \equiv \frac{G_F}{\sqrt{2}} \sum_{i=1,2} c_i O_i^D\,, \quad (D = d, s)\,.$$

- CKM unitarity in SM:

$$\sum_{D=d,s,b} \lambda_D = 0\,, \quad \text{or} \quad \lambda_d = -(\lambda_s + \lambda_b)\,.$$

$$\lambda_D = V_{uD}V_{cD}^*,\ \ (D = d, s, b)\,, \qquad \lambda_b \ll \lambda_{s,d},\ \ \mathrm{Im}(\lambda_b) \neq 0$$

□ Decomposition of decay amplitudes

- separating the contributions of $O^d_{1,2}$ and $O^s_{1,2}$ operators

$$A(D^0 \to \pi^+\pi^-) = \lambda_d \langle \pi^+\pi^- | \mathcal{O}^d | D^0 \rangle + \lambda_s \langle \pi^+\pi^- | \mathcal{O}^s | D^0 \rangle \,,$$
$$A(D^0 \to K^+K^-) = \lambda_s \langle K^+K^- | \mathcal{O}^s | D^0 \rangle + \lambda_d \langle K^+K^- | \mathcal{O}^d | D^0 \rangle \,,$$

- replacing $\lambda_d = -(\lambda_s + \lambda_b)$
- "penguin" type amplitudes - the main object of our interest

$$\mathcal{P}^s_{\pi\pi} = \langle \pi^+\pi^- | \mathcal{O}^s | D^0 \rangle \,, \quad \mathcal{P}^d_{KK} = \langle K^+K^- | \mathcal{O}^d | D^0 \rangle \,,$$

"penguin" indicates that the operator contains a quark-antiquark pair not belonging to the valence content of final state,

Decomposition of decay amplitudes

- separating the $O(\lambda_b)$ contribution with CP-phase

$$A(D^0 \to \pi^+\pi^-) = -\lambda_s \mathcal{A}_{\pi\pi}\left\{1 + \frac{\lambda_b}{\lambda_s}\Big(1 + r_\pi \exp(i\delta_\pi)\Big)\right\},$$
$$A(D^0 \to K^+K^-) = \lambda_s \mathcal{A}_{KK}\left\{1 - \frac{\lambda_b}{\lambda_s} r_K \exp(i\delta_K)\right\},$$

with the notation:

$$\mathcal{A}_{\pi\pi} = \langle \pi^+\pi^- | \mathcal{O}^d | D^0 \rangle - \langle \pi^+\pi^- | \mathcal{O}^s | D^0 \rangle ,$$
$$\mathcal{A}_{KK} = \langle K^+K^- | \mathcal{O}^s | D^0 \rangle - \langle K^+K^- | \mathcal{O}^d | D^0 \rangle ,$$

$$r_\pi = \left|\frac{\mathcal{P}^s_{\pi\pi}}{\mathcal{A}_{\pi\pi}}\right| , \quad r_K = \left|\frac{\mathcal{P}^d_{KK}}{\mathcal{A}_{KK}}\right| , \; \delta_{\pi(K)} = arg[\mathcal{P}^{s(d)}_{\pi\pi(KK)}] - arg[\mathcal{A}_{\pi\pi(KK)}]$$

- to a good approximation

$$-\lambda_s \mathcal{A}_{\pi\pi} \simeq A(D^0 \to \pi^+\pi^-) , \quad \lambda_s \mathcal{A}_{KK} \simeq A(D^0 \to K^+K^-)$$

□ Direct *CP*-asymmetry

- In terms of the parameters entering the decomposition:

$$a_{CP}^{dir}(K^+K^-) = \frac{-2r_b r_K \sin\delta_K \sin\gamma}{1 - 2r_b r_K \cos\gamma\cos\delta_K + r_b^2 r_K^2}\,,$$

$$a_{CP}^{dir}(\pi^+\pi^-) = \frac{2r_b r_\pi \sin\delta_\pi \sin\gamma}{1 + 2r_b \cos\gamma(1 + r_\pi\cos\delta_\pi) + r_b^2(1 + 2r_\pi\cos\delta_\pi + r_\pi^2)}$$

- the CKM elements involved:

$$\frac{\lambda_b}{\lambda_s} \equiv r_b e^{-i\gamma}, \quad r_b = \left|\frac{V_{ub}V_{cb}^*}{V_{us}V_{cs}^*}\right|.$$

- a "clean" observable (after time-integration)

$$\begin{aligned}\Delta a_{CP}^{dir} &= a_{CP}^{dir}(K^+K^-) - a_{CP}^{dir}(\pi^+\pi^-)\\ &= -2r_b\sin\gamma(r_K\sin\delta_K + r_\pi\sin\delta_\pi) + O(r_b^2)\,.\end{aligned}$$

- a QCD-based calculation of $\mathcal{P}_{\pi\pi}^s$ and $\mathcal{P}_{KK}^d$
- combined with $\mathcal{A}_{\pi\pi}$ and $\mathcal{A}_{KK}$ extracted from experiment
  $\Rightarrow$ an estimate of r_π and r_K

□ Calculation of the "penguin" hadronic matrix element

- The method employing QCD Light-Cone Sum Rules (LCSRs) used earlier for the $B \to \pi\pi$ decays:

  AK, Nucl. Phys. B **605** (2001) 558 [hep-ph/0012271];

  AK, T. Mannel and B. Melic, Phys. Lett. B **571** (2003) 75 [hep-ph/0304179];

  AK, T. Mannel, M. Melcher and B. Melic, Phys. Rev. D **72** (2005) 094012 [hep-ph/0509049].

- reproducing reasonably well branching fractions, of ($\bar{c}c$) penguin dominated $B \to K\pi$ modes

  M.Jung, AK, B.Melic, work in progress

  AK, talk at Mainz Workshop on NL decays, January 2019
  https://indico.mitp.uni-mainz.de/event/177

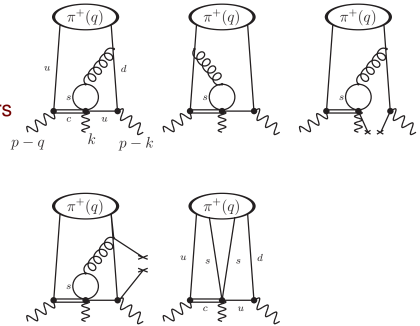
□ Calculation of the “penguin” matrix elements

- correlation function for $D \to \pi^+\pi^-$ ($\pi \to K$, $s \to d$ for $D \to K^+K^-$)

$$F_\alpha(p,q,k) = i^2 \int d^4x\, e^{-i(p-q)x} \int d^4y\, e^{-i(p-k)y} \langle 0|T\{j^{(\pi)}_{\alpha 5}(y) O^s_{1,2}(0) j^{(D)}_5(x)\}|\pi^+(q)\rangle$$

$$c_1 O^s_1 + c_2 O^s_2 = 2c_1 \widetilde{O}^s_2 + \left(\frac{c_1}{3} + c_2\right) O^s_2\,, \qquad \widetilde{O}^s_2 = \left(\bar{s}\Gamma_\mu \frac{\lambda^a}{2} s\right)\left(\bar{u}\Gamma^\mu \frac{\lambda^a}{2} c\right)$$

- we actually calculate: $\langle \pi^+\pi^- | \widetilde{O}^s_2 | D^0 \rangle$
- OPE diagrams for $D \to \pi^+\pi^-$ in terms of pion LCDAs:

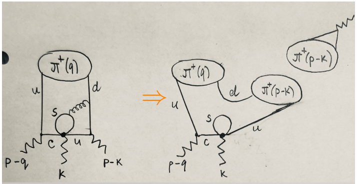


- some details:
  - LCSR’s for $D \to \pi, K$ form factors
  - finite quark masses m_c, m_s
  - $SU(3)$ not used, only isospin
  - tw 2,3 accuracy, fact. tw 5,6
  - selection of diagrams

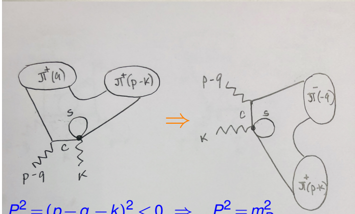
(see eariler $B \to \pi\pi$ papers)

□ Obtaining LCSR

- step 1: Dispersion relation in the pion channel ⊕ duality

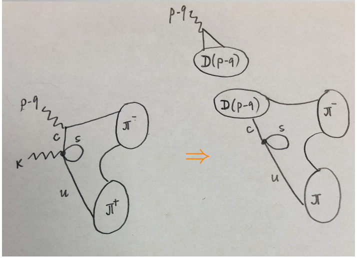


- step 2: Analytic continuation; (cf. transition from spacelike to the timelike pion form factor)



$P^2=(p-q-k)^2<0 \;\Rightarrow\; P^2=m_D^2$

- step 3: Dispersion relation in the D channel ⊕ duality



Numerical estimates

- LCSR input: quark masses, pion, kaon DAs, Borel scales, effective thresholds from the LCSR calculation of $D \to \pi$, $D \to K$ and pion form factor
- hadronic matrix elements calculated from the sum rules

$$\langle \pi^+\pi^- | \widetilde{\mathcal{Q}}_2^s | D^0 \rangle = (9.50 \pm 1.13) \times 10^{-3} \exp[i(-97.5^o \pm 11.6)]\, \mathrm{GeV}^3\,,$$
$$\langle K^+K^- | \widetilde{\mathcal{Q}}_2^d | D^0 \rangle = (13.9 \pm 2.70) \times 10^{-3} \exp[i(-71.6^o \pm 29.5)]\, \mathrm{GeV}^3\,,$$

- converting into an estimate of the penguin amplitude:

$$\mathcal{P}^s_{\pi\pi} \simeq 2c_1 \frac{G_F}{\sqrt{2}} \langle \pi^+\pi^- | \widetilde{O}_2^s | D^0 \rangle, \quad \mathcal{P}^d_{KK} \simeq 2c_1 \frac{G_F}{\sqrt{2}} \langle K^+K^- | \widetilde{O}_2^d | D^0 \rangle$$

$$|\mathcal{P}^s_{\pi\pi}| = (1.96 \pm 0.23) \times 10^{-7} \mathrm{GeV}\,,$$
$$|\mathcal{P}^d_{KK}| = (2.86 \pm 0.56) \times 10^{-7} \mathrm{GeV}\,,$$

the uncertainties are only parametrical !

- using experimentally measured branching fractions [PDG]:

$$r_\pi = \frac{|\mathcal{P}^s_{\pi\pi}|}{|\mathcal{A}_{\pi\pi}|} = 0.093 \pm 0.011\,, \qquad r_K = \frac{|\mathcal{P}^d_{KK}|}{|\mathcal{A}_{KK}|} = 0.075 \pm 0.015\,.$$

Results for direct CP asymmetry

- CKM averages yield $r_b\, sin\gamma = 0.64 \times 10^{-3}$,
  $|V_{ub}| = 0.00357$, $|V_{cb}| = 0.0411$, $|V_{us}| = 0.22506$, $|V_{cs}| = 0.97351$, $\gamma = 73.2^o$
- the difference of asymmetries: (indirect asymmetries largely cancel)

$$\Delta a^{dir}_{CP} = -2 r_b \sin\gamma (r_K \sin\delta_K + r_\pi \sin\delta_\pi)$$

- the resulting upper limits: (independent of strong phases)

$$\left|a^{dir}_{CP}(\pi^-\pi^+)\right| < 0.012 \pm 0.001\%, \quad \left|a^{dir}_{CP}(K^-K^+)\right| < 0.009 \pm 0.002\%$$

$$\left|\Delta a^{dir}_{CP}\right| < 0.020 \pm 0.003\% \, .$$

- Assuming that δ_π and δ_K are given by the calculated phases of $\mathcal{P}^s_{\pi\pi}$ and $\mathcal{P}^d_{KK}$:

$$a^{dir}_{CP}(\pi^-\pi^+) = -0.011 \pm 0.001\%, \; a^{dir}_{CP}(K^-K^+) = 0.009 \pm 0.002\%,$$

$$\Delta a^{dir}_{CP} = 0.020 \pm 0.003\% \, .$$

- The two most recent LHCb collaboration results:

$\Delta a^{dir}_{CP} = (-0.10 \pm 0.08 \pm 0.03)$ % R. Aaij *et al.* [LHCb Collaboration], PRL 116,191601 (2016)

$\Delta a^{dir}_{CP} = (-0.154 \pm 0.029)$% R. Aaij *et al.* [LHCb Collaboration], arXiv:1903.08726 [hep-ex] (2019)

□ How accurate is our estimate ? Can it be improved?

- we do not predict total $D \to \pi^+\pi^-, K^+K^-$ decay amplitudes in which several "topologies" contribute,
  – a difficult task, including multiloop diagrams in LCSRs

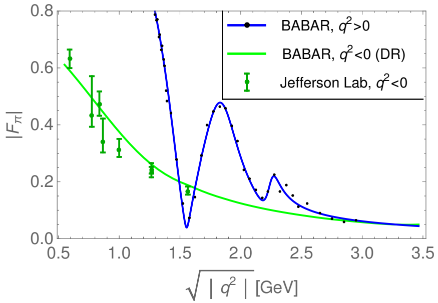
- parametric accuracy of LCSR:
  twist-4, $O(\alpha_s^2)$; neglected terms of $O(s_0^{\pi,K}/m_D^2)$;
  – improvable, needs dedicated calculation,

- systematic errors from using semilocal duality in π and D channels
  – controlled by $D \to \pi, K$ and pion form factor LCSRs

- using local duality (spacelike $\to$ timelike transition)

  – can only be estimated
  in a model-dependent way,
  e.g., with f_0 resonances
  in the vicinity of $m_D \sim 1.9$ GeV.

  how the spacelike$\to$ timelike transition
  works for the pion form factor:
  [AK, S.Cheng, A.Rusov, work in progress]



Summary

- using QCD-based tools (QCD light-cone sum rules, quark-hadron duality) it is possible to estimate hadronic matrix elements for nonleptonic charm decays
- the magnitude of direct CP-violation in $D \to \pi^+\pi^-$ and $D \to K^+K^-$ can be predicted and constrained
- future perspective: other D and charmed baryon modes, improved parametrical accuracy, model of duality violation

Backup

The light-cone sum rule

$$\langle \pi^+\pi^-|\widetilde{O}_2^s|D^0\rangle = -i\frac{\alpha_s C_F m_c^2}{8\pi^3 m_D^2 f_D}\Bigg[\int\limits_0^{s_0^\pi} ds e^{-s/M_1^2}\int\limits_{u_0^D}^1 \frac{du}{u} e^{\left(m_D^2-\frac{m_c^2}{u}\right)/M_2^2}$$

$$\times\Bigg\{P^2\int\limits_0^1 dz I(zuP^2, m_s^2)\Bigg(z(1-z)\varphi_\pi(u)$$

$$+(1-z)\frac{\mu_\pi}{2m_c}\Bigg[\left(2z+\frac{m_c^2}{uP^2}\right)u\varphi_p(u)+\frac{1}{3}\left(2z-\frac{m_c^2}{uP^2}\right)\left(\varphi_\sigma(u)-\frac{u\varphi'_\sigma(u)}{2}\right)\Bigg]\Bigg)$$

$$-\frac{\mu_\pi m_c}{4}\int\limits_0^1 dz I(-z\bar{u}m_c^2/u, m_s^2)\frac{\bar{u}^2}{u}\Bigg[\left(1+\frac{3m_c^2}{uP^2}\right)\varphi_p(1)+\left(1-\frac{5m_c^2}{uP^2}\right)\frac{\varphi'_\sigma(1)}{6}\Bigg]\Bigg\}$$

$$+\frac{2\pi^2}{3}m_c(-\langle\bar{q}q\rangle)\int\limits_{u_0^D}^1\frac{du}{u^2}e^{\left(m_D^2-\frac{m_c^2}{u}\right)/M_2^2}\Bigg\{I(uP^2, m_s^2)\Bigg(2\varphi_\pi(u)+\frac{\mu_\pi}{m_c}\Bigg[3u\varphi_p(u)$$

$$+\frac{\varphi_\sigma(u)}{3}-\frac{u\varphi'_\sigma(u)}{6}\Bigg]\Bigg)\Bigg\}\Bigg]_{P^2\to m_D^2},$$

the loop integral: $I(\ell^2, m_q^2) = \frac{1}{6}+\int\limits_0^1 dx x(1-x)\ln\left[\frac{m_q^2-x(1-x)\ell^2}{\mu^2}\right]$.