A Second Law Study of the Regenerators in Cryocoolers Based on Analysis of Entropy Generation

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Overview

- Optimizing regenerators using entropy generation analysis have been explored.
- Two methods are developed for the calculation of entropy generation in regenerators.
  - Pore-level CFD model
  - Semi-analytical model
- Two evaluation criteria are proposed to evaluate the performance of regenerators in terms of entropy generation.
  - Modified Bejan number
  - Performance Evaluation Factor

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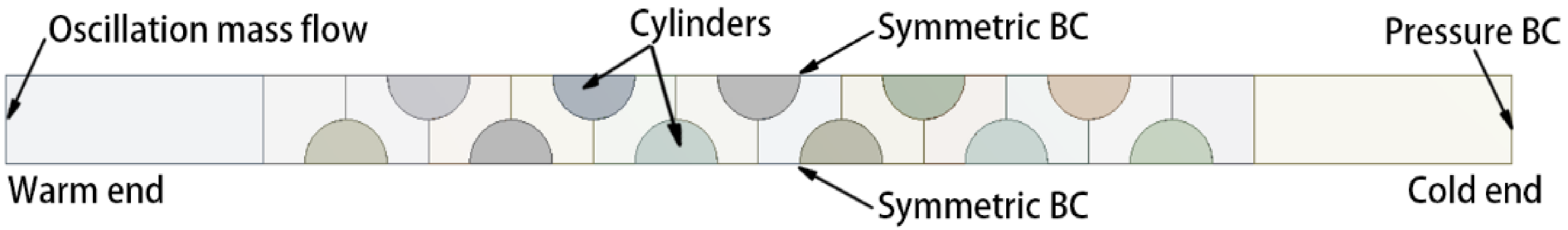
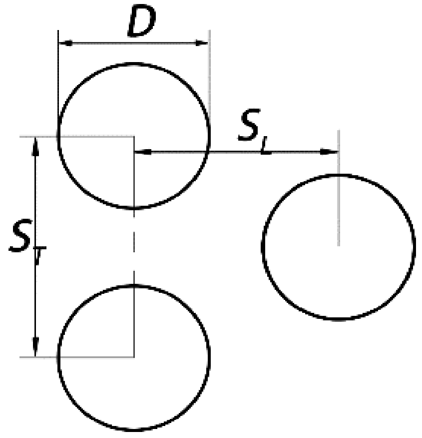
Methods to calculation S_gen

Pore-level CFD model



- 2D
- Laminar flow
- Mass flow rate amplitude varies
- D varies $S_T = 2S_L = 2D$



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PDE form of Entropy Generation

$$\dot{S}'''_{gen} = \frac{k}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{T}\left\{2\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2\right] + \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2\right\}$$

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$$\dot{S}'''_{gen,heat} \qquad \dot{S}'''_{gen,vis}$$

$$\dot{S}'''_{gen,cond} + \dot{S}'''_{gen,conv}$$

PDE form of Entropy Generation

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- Can be implemented in CFD easily.
- CFD can be time consuming.
- Cannot be directly used in volume averaging method and design calculation.
- Therefore, a semi-analytical form was derived.

Semi-analytical Form – Assumptions & Approximations

- Real gas effect is negligible;
- The porous material in the control volume is represented with a uniform temperature T_w;
- The temperature and properties of the fluid inside the control volume are uniform;
- The difference of temperature between solid and fluid is small compared with the absolute temperature of the fluid;
- Convective heat exchange rate is uniformly distributed on the solid-fluid interface.

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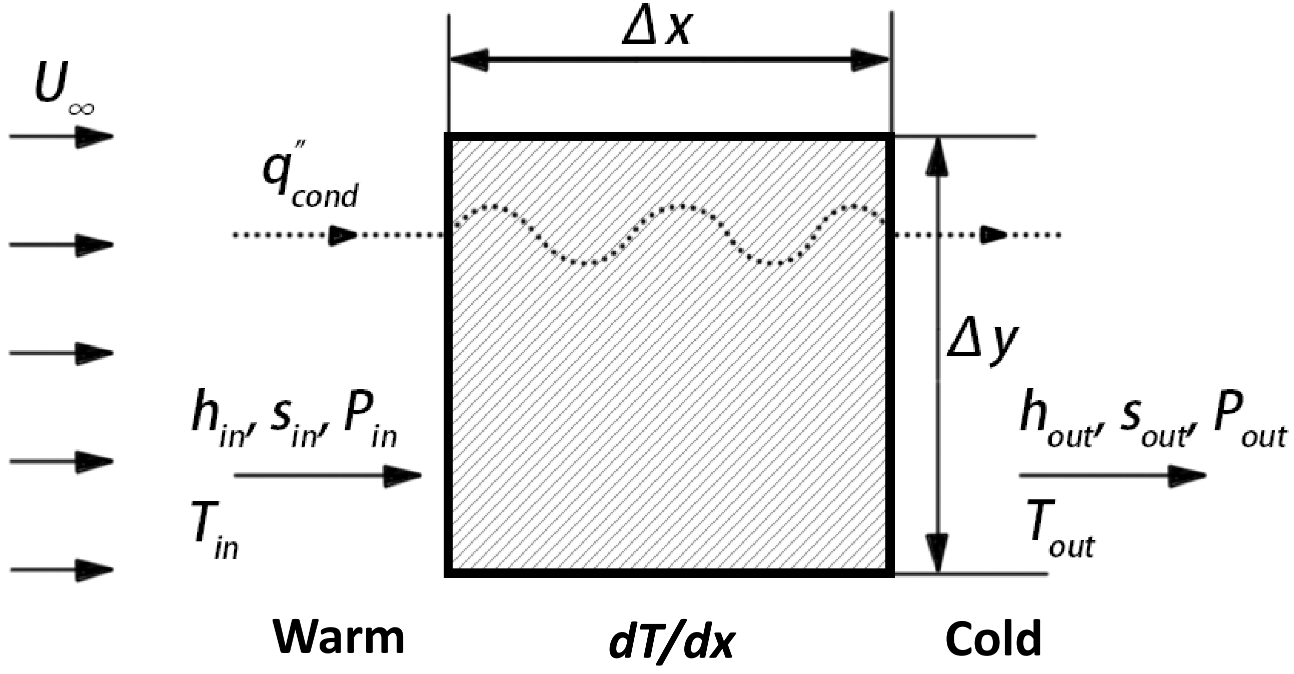
Semi-analytical Form of Entropy Generation in Regenerators

- A control volume in a regenerator



$$\left\{ \begin{array}{l} \dot{S}'''_{gen,cond} \\ \\ \dot{S}'''_{gen,flow} = \dot{S}'''_{gen,conv} + \dot{S}'''_{gen,vis} \end{array} \right.$$

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Entropy Generation due to Conduction

Conductive heat flux: $q''_{cond} = k_{eff}\frac{dT}{dx}$

$$T_{out} - T_{in} = \Delta x\frac{dT}{dx} \ \& \ T_{out} \approx T_{in} \approx \bar{T}_f$$

$$\dot{S}'''_{gen,cond} \approx k_{eff}\left(\frac{dT}{dx}\right)^2 \Big/ \bar{T}_f^{\,2}$$

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Entropy Generation due to Flow

1st law: $$\dot{m}h_{in} + \iint_A q''_A dA - \dot{m}h_{out} = 0$$

2nd law: $$\dot{S}_{gen,flow} = \dot{m}s_{out} - \dot{m}s_{in} - \iint_A \frac{q''_A}{T_w} dA$$

Tds relationship: $$h_{out} - h_{in} \approx \overline{T_f}(s_{out} - s_{in}) + \frac{1}{\overline{\rho_f}}(P_{out} - P_{in})$$

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$$\dot{S}_{gen,flow} = \underbrace{\left(\frac{1}{\overline{T}_f} - \frac{1}{T_w}\right) \iint_A q'' \, dA}_{\dot{S}'''_{gen,cond}} - \underbrace{\frac{\dot{m}}{\overline{T}_f \, \overline{\rho_f}} (P_{out} - P_{in})}_{\dot{S}'''_{gen,vis}}$$

Entropy Generation due to Flow

$$\dot{S}_{gen,flow} = \left(\frac{1}{\overline{\overline{T_f}}} - \frac{1}{T_w}\right) \iint_A q''\, dA - \frac{\dot{m}}{\overline{\overline{T_f}}\ \overline{\rho_f}} (P_{out} - P_{in})$$

$$\iint_A q''\, dA = \overline{h_{conv}} A (T_w - \overline{\overline{T_f}})$$

$$\iint_A q''\, dA = \dot{m} C_p \Delta x \frac{dT}{dx}$$

$$\dot{S}'''_{gen,flow} = \frac{\left(U_\infty \overline{\rho_f}\, C_p \frac{dT}{dx}\right)^2}{\overline{h_{conv}}\, \overline{\overline{T_f}}^{\,2} \Omega} + \frac{U_\infty \overline{\rho_f}}{\overline{\overline{T_f}}} \frac{dP}{dx}$$

where $\overline{h_{conv}}$ is the average convective heat transfer coefficient, and Ω is the wetted solid surface area per unit volume

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Entropy Generation due to Flow

Instantaneous: $\dot{S}'''_{gen,flow} = \dfrac{\left(U_\infty \overline{\rho_f}\, C_p \dfrac{dT}{dx}\right)^2}{\overline{h_{conv}}\,\overline{T_f}^{\,2}\,\Omega} + \dfrac{U_\infty \overline{\rho_f}}{\overline{T_f}} \dfrac{dP}{dx}$

Cycle-averaging: $< \dot{S}'''_{gen,flow} >= f \displaystyle\int_0^{1/f} \left[\dfrac{\left(U_\infty \overline{\rho_f}\, C_p \dfrac{dT}{dx}\right)^2}{\overline{h_{conv}}\,\overline{T_f}^{\,2}\,\Omega} + \dfrac{U_\infty \overline{\rho_f}}{\overline{T_f}} \dfrac{dP}{dx} \right] dt$

Assumption: U_∞ *and* $\frac{dP}{dx}$ are sinusoidal, $\overline{h_{conv}}$ is constant

$$< \dot{S}'''_{gen,flow} >= \frac{\left(U_{\infty,max} \overline{\rho_f}\, C_p \dfrac{dT}{dx}\right)^2}{2\overline{h_{conv}}\,\overline{T_f}^{\,2}\,\Omega} + \frac{U_{\infty,max} \overline{\rho_f}}{2\overline{T_f}} \left(\frac{dP}{dx}\right)_{max}$$

Entropy Generation due to Flow

Adjusted: $$< \dot{S}'''_{gen,flow} >= \alpha \frac{\left(U_{\infty,max}\overline{\rho_f}\, C_p \frac{dT}{dx}\right)^2}{2\overline{h_{conv}}\,\overline{T_f}^{\,2}\,\Omega} + \beta \frac{U_{\infty,max}\overline{\rho_f}}{2\overline{T_f}} \frac{dP}{dx}_{max}$$

Empirical adjustment using CFD results: $\alpha = 1$ and $\beta = 2$

Components of Entropy Generation in Regenerators

Conduction: $$< \dot{S}'''_{gen,cond} > = k_{eff} \left(\frac{dT}{dx}\right)^2 \Big/ \bar{T_f}^2$$

Convection: $$< \dot{S}'''_{gen,conv} >= \alpha \frac{\left(U_{\infty,max}\overline{\rho_f}\, C_p \frac{dT}{dx}\right)^2}{2\overline{h_{conv}}\bar{T_f}^2 \Omega}$$

Viscous dissipation: $$< \dot{S}'''_{gen,vis} >= \beta \frac{U_{\infty,max}\overline{\rho_f}}{2\bar{T_f}} \frac{dP}{dx}_{max}$$

- These can be calculated using empirical correlations.

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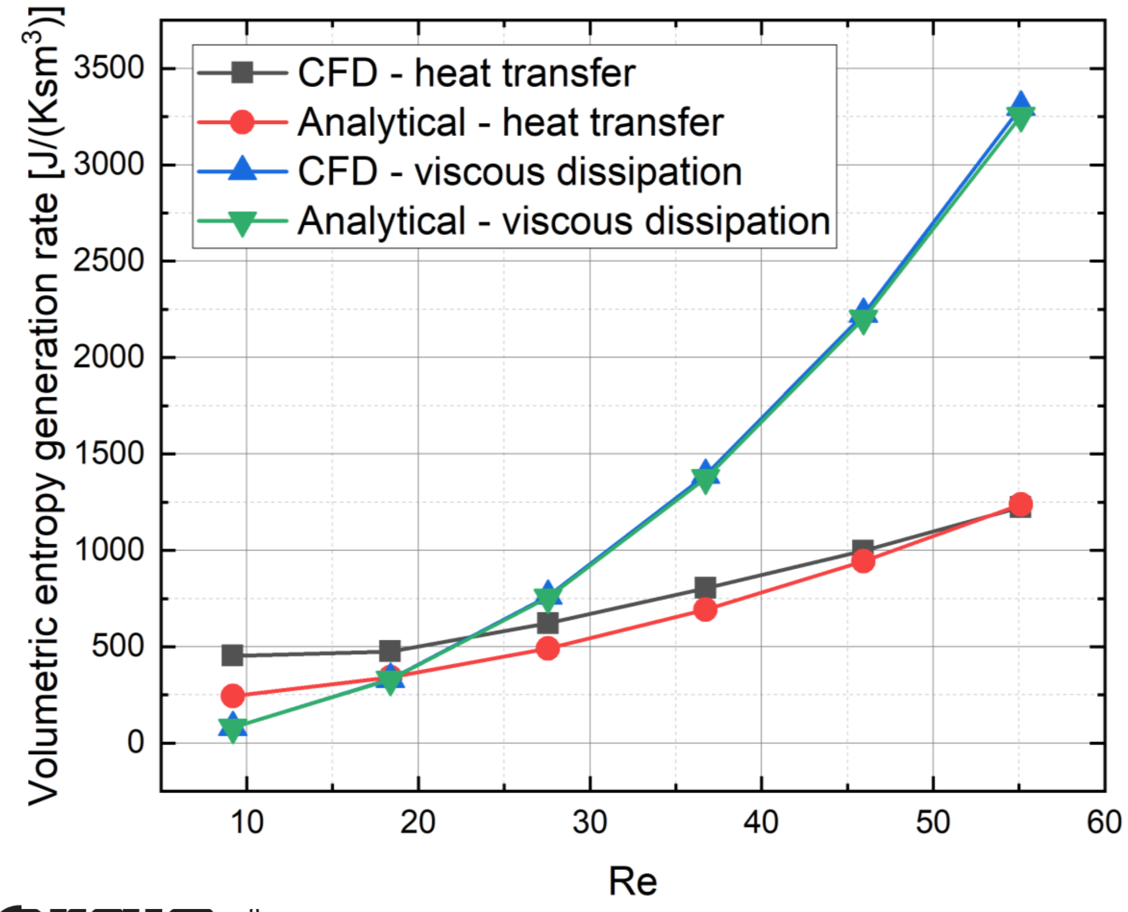
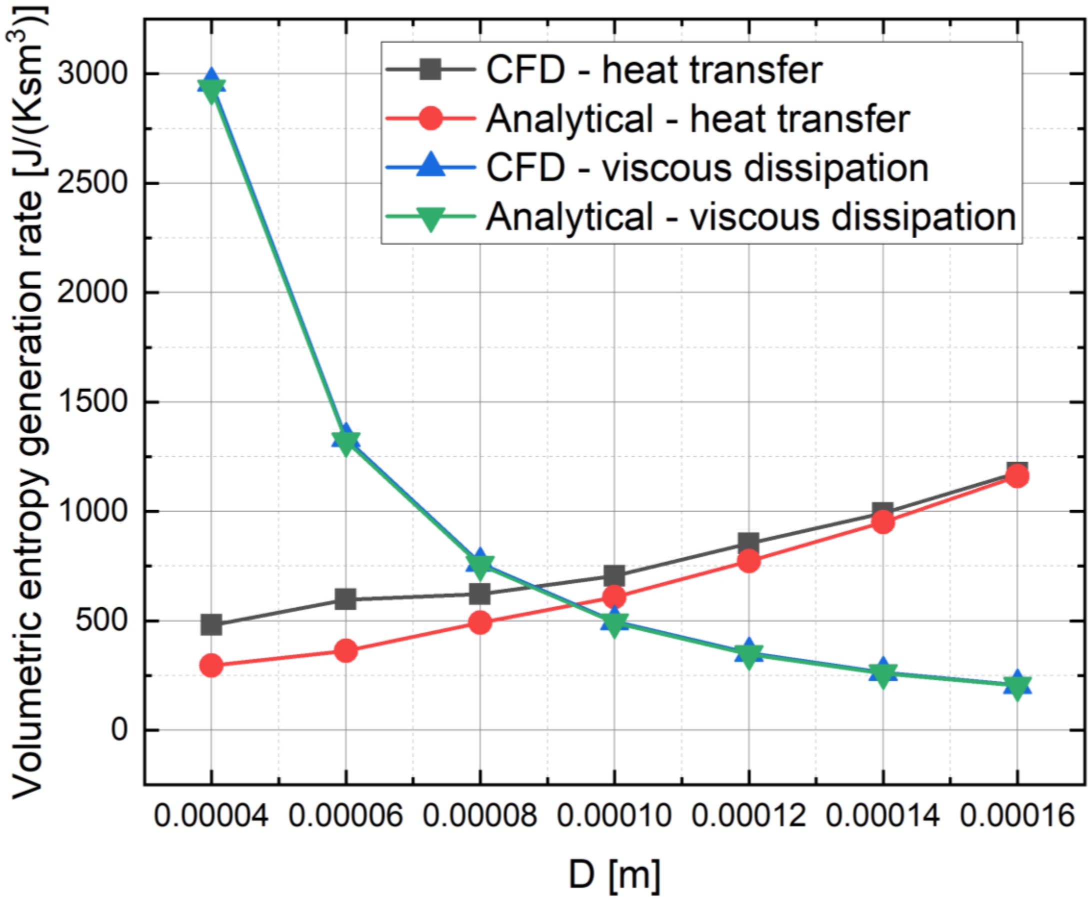
CFD vs. Analytical – Volumetric Entropy Generation Rate





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Methods to optimize S_gen

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Modified Bejan Number

Original definition: $$Be = \frac{< \dot{S}'''_{gen,heat} >}{< \dot{S}'''_{gen,heat} > + < \dot{S}'''_{gen,vis} >}$$

Modified definition: $$Be_{conv} = \frac{< \dot{S}'''_{gen,conv} >}{< \dot{S}'''_{gen,conv} > + < \dot{S}'''_{gen,vis} >}$$

- Entropy generation by conduction is not considered here. Therefore, it can be used when conduction is constant.
- Theoretically, $< \dot{S}'''_{gen,conv} > + < \dot{S}'''_{gen,vis} >$ can be minimized when $Be_{conv} = 0.5$.

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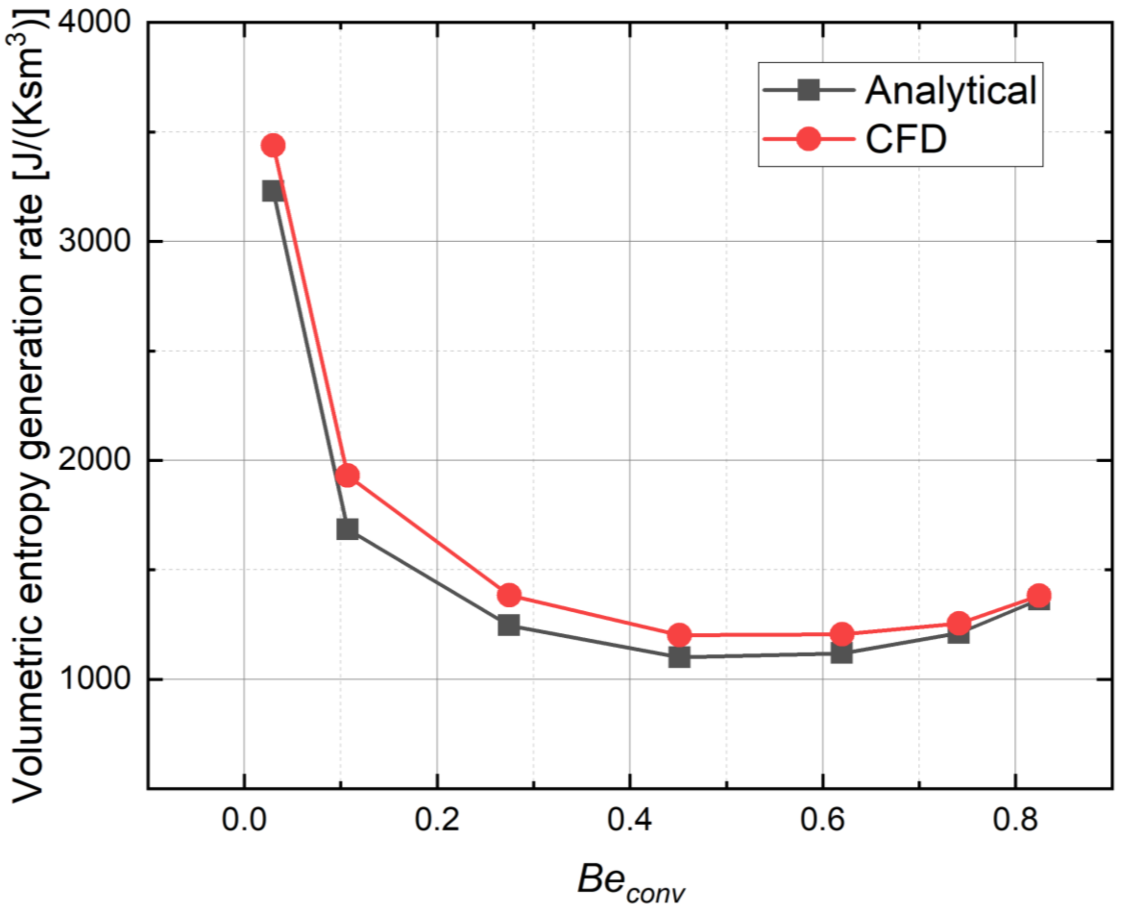
Entropy Generation Rate vs. Modified Bejan Number



- Fluid velocity amplitude: constant.
- *D:* varies.

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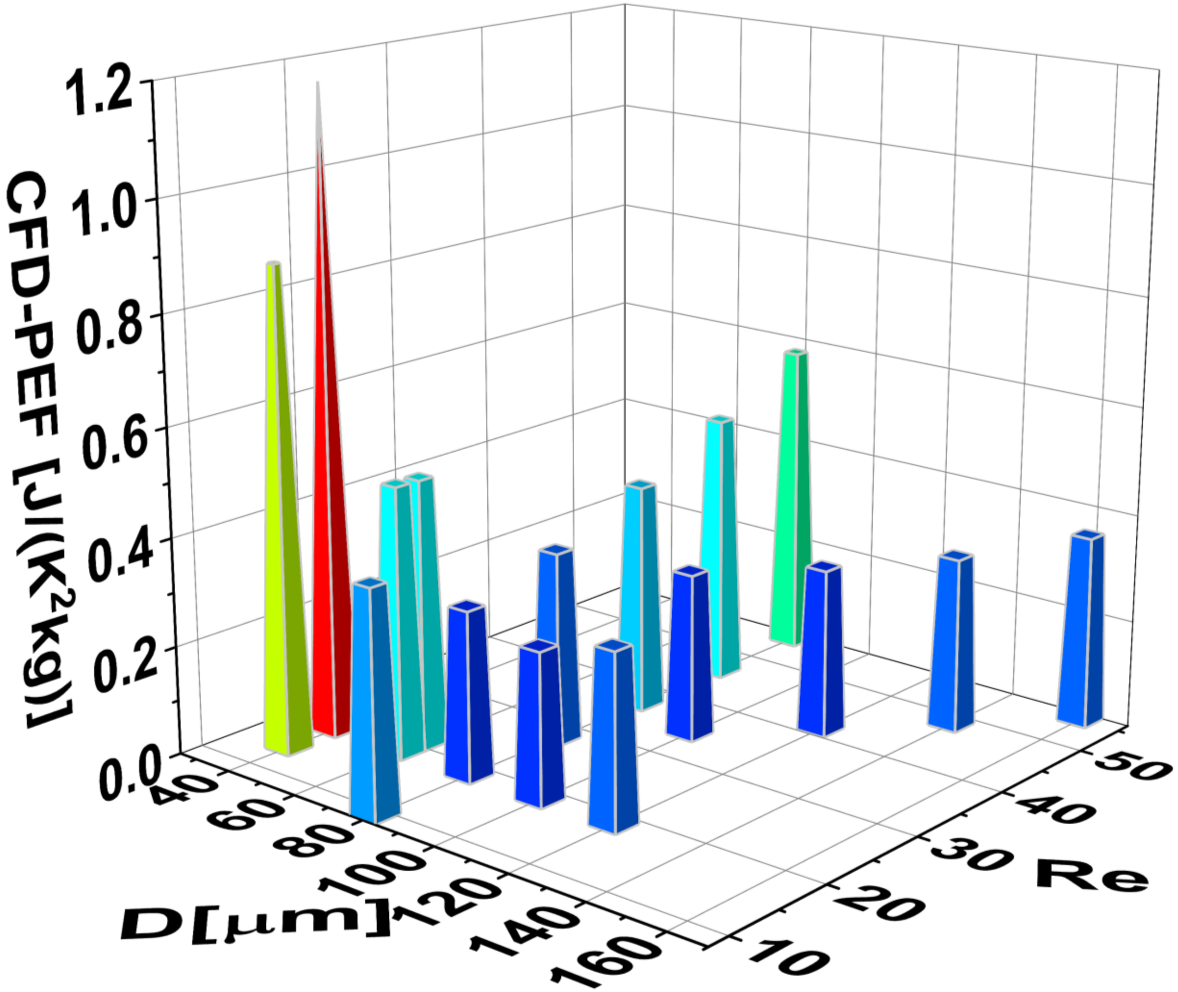
Performance Evaluation Factor

$$PEF = \frac{\dot{S}'''_{gen} V}{\dot{m}_{tot,max}(T_h - T_c)} = \frac{\dot{S}'''_{gen}}{\rho_f U_{\infty,max} \, {dT}/{dx}}$$

- *PEF* is entropy generation per unit mass flow rate amplitude per unit warm-to-cold temperature difference.
- Lower *PEF* indicates lower entropy generation and thus better performance.
- Can be used when conduction is not constant.

Comprehensive Relationship between PEF, Re and D

- CFD result



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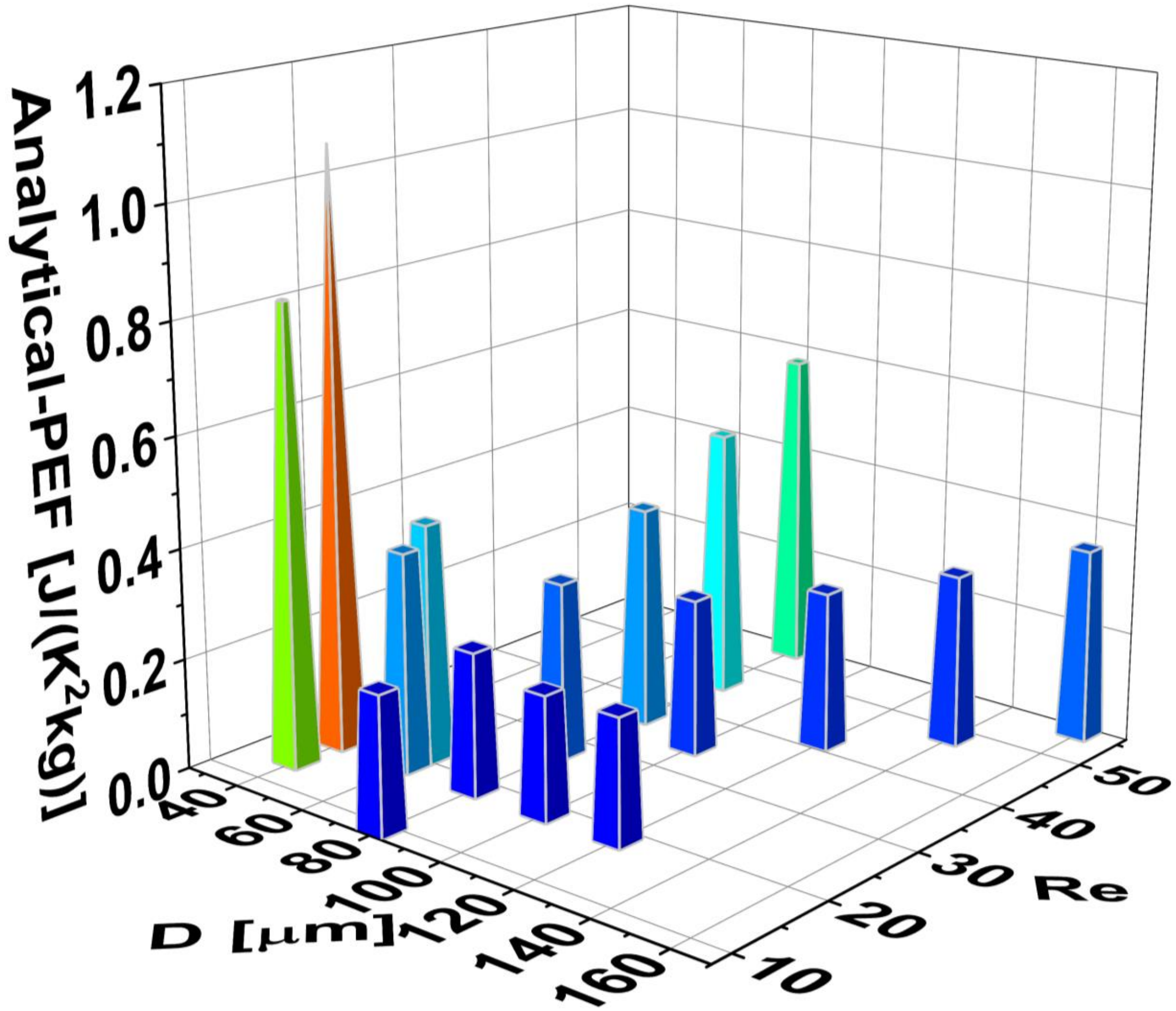
Comprehensive Relationship between PEF, Re and D

- Semi-analytical result



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Summary

- Two methods, pore-level CFD and semi-analytical calculation, were developed to calculate entropy generation.
  - Semi-analytical calculation
    - Except for very small Re or D, $< \dot{S}'''_{gen,vis} >$ predicted by the analytical approximation and CFD simulations are in very good agreement.
    - Can be used when empirical correlations of convective heat transfer and friction factor are available;
  - Pore-level CFD simulation
    - More accurate but more time-consuming.

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Summary

- Two evaluation criteria have been proposed to evaluate the performance of regenerators in terms of entropy generation.
  - Modified Bejan number
    - Can be used to optimize $< \dot{S}'''_{gen,conv} > + < \dot{S}'''_{gen,vis} >$;
    - Useful when axial conduction is constant;
  - Performance Evaluation Factor (PEF)
    - Can be used when axial conduction is not constant;
    - Can directly compared the performance of regenerators when the mass flow rate and warm-to-cold temperature difference are same;

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Thank you!

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