

# M1Po2D-06 [49]: Superconducting state on a square lattice: a case of linear electron-phonon interaction

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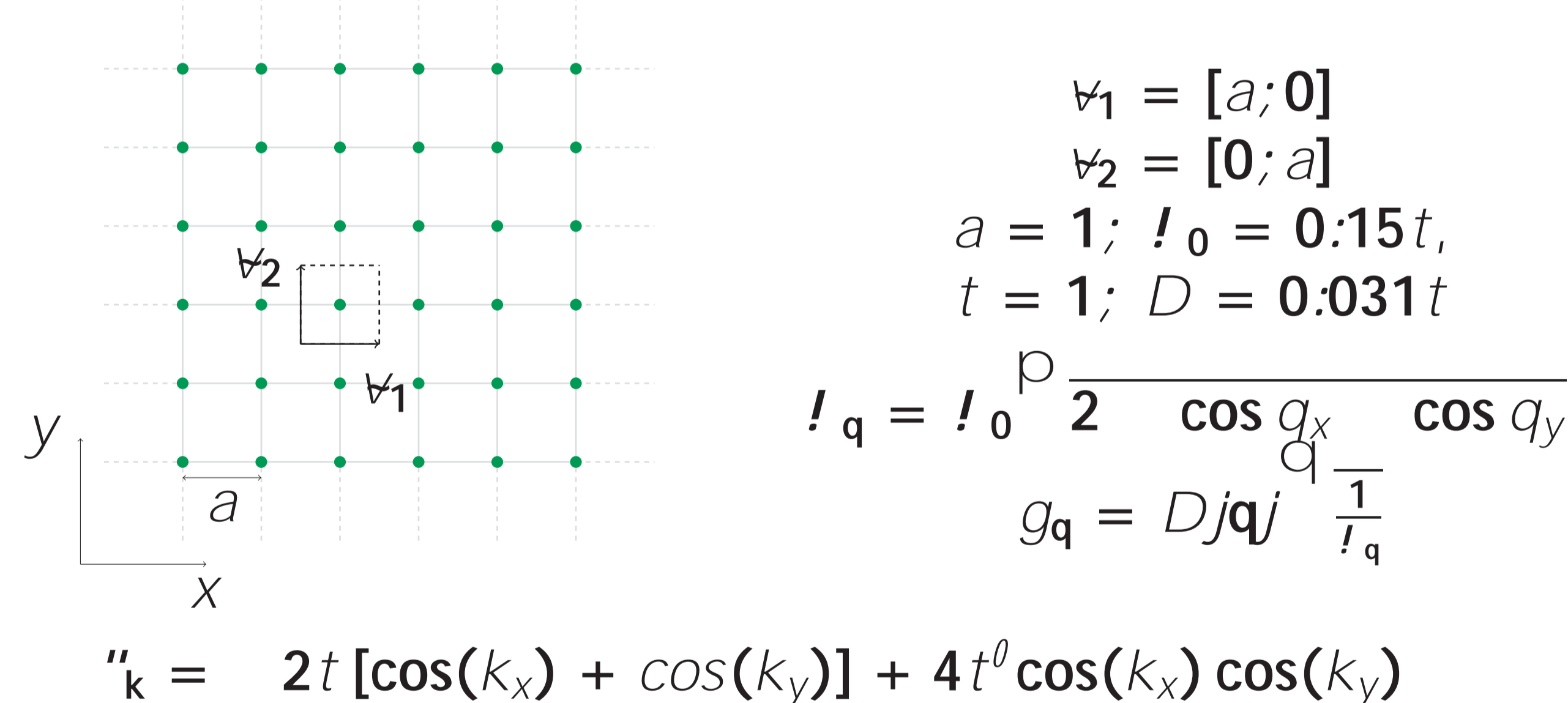
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## ABSTRACT

In the framework of the Eliashberg formalism we have analyzed the tendency to create phonon-induced superconducting state on a square lattice. We have shown unbalanced superconducting state cannot create. However, for unbalanced parameter value smaller than  $\gamma_c < 0.42$  electron-phonon interaction may induce non-classical superconducting state. The thermodynamic functions of this phase, the more deviate from the predictions of the BCS mean-field theory, the higher the value of the parameter. Our results suggest that the key to the phonon induction of a superconducting state on a square lattice is the existence of additional interaction (mechanism), not necessarily purely electronic origin, which will force the appropriate degree of unbalancing of the system.

## A square lattice



## The unbalance parameter

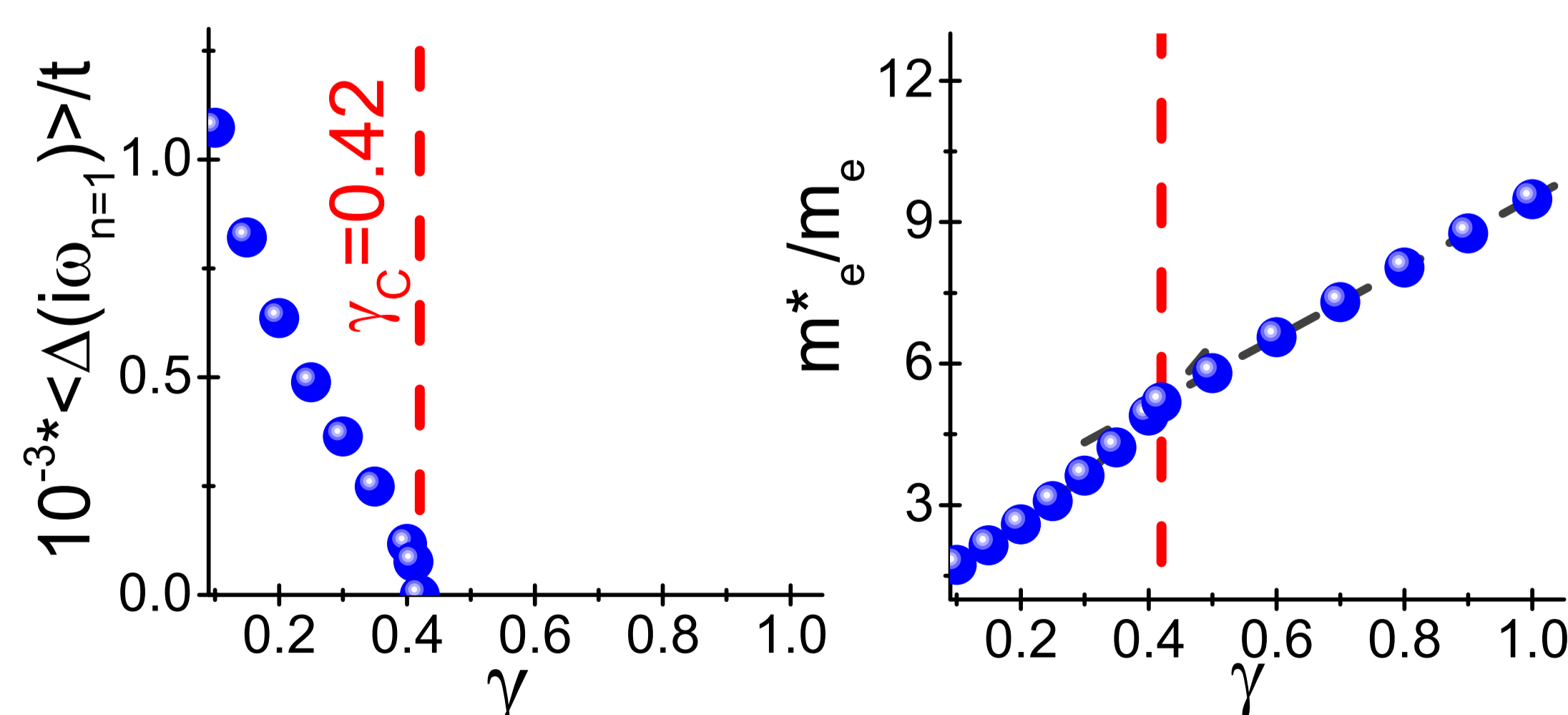


Figure 1: (left) The dependence of the average value of the order parameter on  $\gamma$ . (right) The results concerning the effective mass of the electron.

## THEORETICAL MODEL

$$\psi_{\mathbf{k}}(i!_n) = \frac{1}{N} \sum_{m\mathbf{q}} K_{\mathbf{q}}(i!_n - i!_m) \frac{\psi_{\mathbf{k}+\mathbf{q}}(i!_m)}{D_{\mathbf{k}+\mathbf{q}}(i!_m)}; \quad (1)$$

$$Z_{\mathbf{k}}(i!_n) = 1 + \frac{1}{N} \sum_{m\mathbf{q}} \frac{I_{\mathbf{m}}}{I_{\mathbf{n}}} K_{\mathbf{q}}(i!_n - i!_m) \frac{Z_{\mathbf{k}+\mathbf{q}}(i!_m)}{D_{\mathbf{k}+\mathbf{q}}(i!_m)}; \quad (2)$$

$\psi_{\mathbf{k}}(i!_n)$  - the function of the order parameter;  
 $Z_{\mathbf{k}}(i!_n)$  - the wave function renormalisation factor;  
 $K_{\mathbf{q}}(i!_n - i!_m) = 2g_{\mathbf{q}}^2 \frac{I_{\mathbf{q}}}{(I_{\mathbf{n}} - I_{\mathbf{m}})^2 + I_{\mathbf{q}}^2}$  - the pairing kernel of the electron-phonon interaction;  
 $D_{\mathbf{k}}(i!_n) = (I_{\mathbf{n}} Z_{\mathbf{k}}(i!_n))^2 + \epsilon_{\mathbf{k}}^2 + \psi_{\mathbf{k}}^2(i!_n);$   
 $I_{\mathbf{q}} = I_0$  - the unbalance parameter.

## Complete solutions of Eliashberg's equations

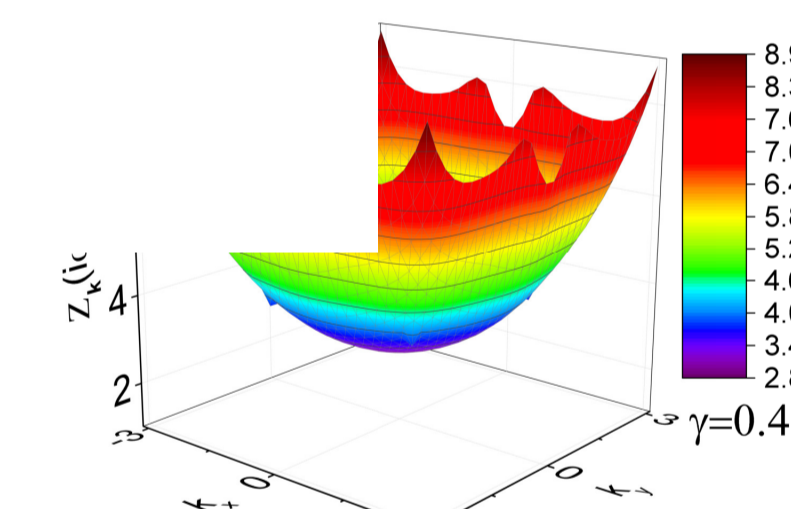


Figure 2: The dependence of the wave function renormalising factor and of the order parameter function on the wave vector for selected values of the unbalance parameter.

## The average solutions over the wave vector

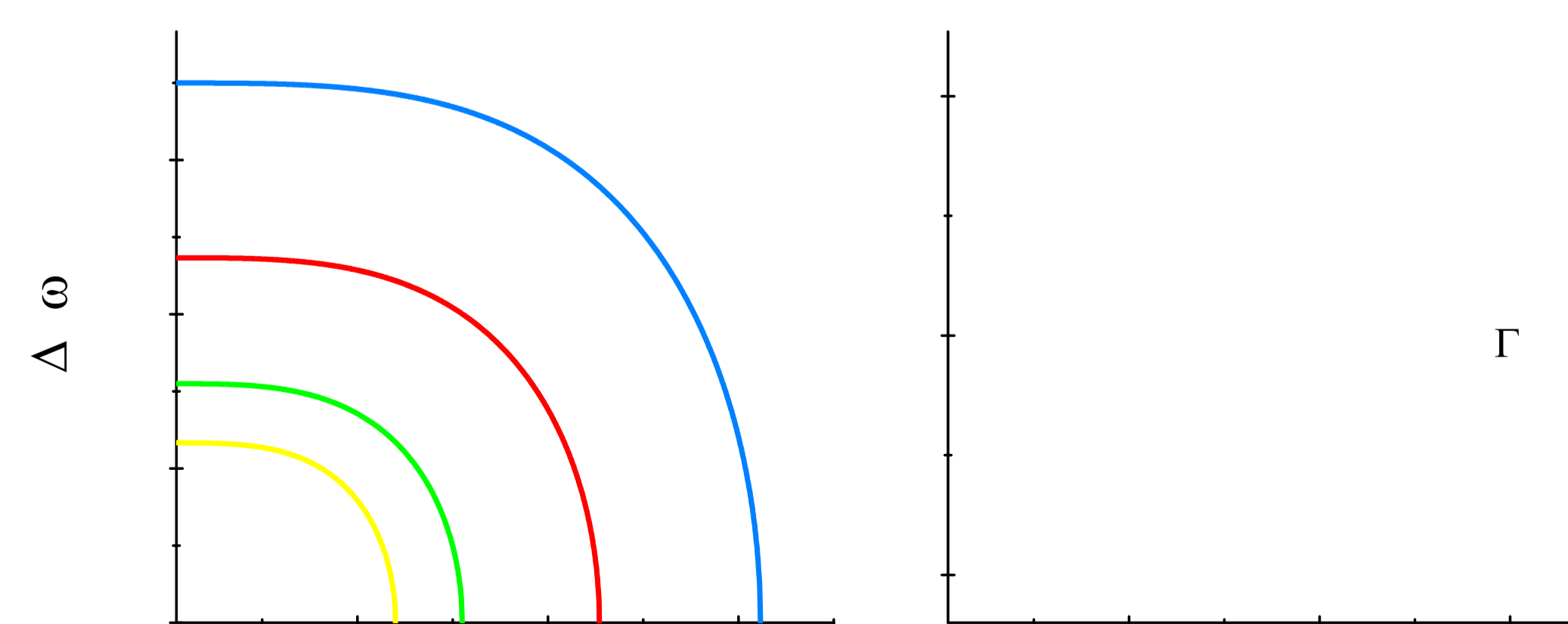


Figure 3: (a) The influence of temperature on the averaged value of the order parameter. (b) The temperature dependence of the effective electron mass.

## Thermodynamics of superconducting state

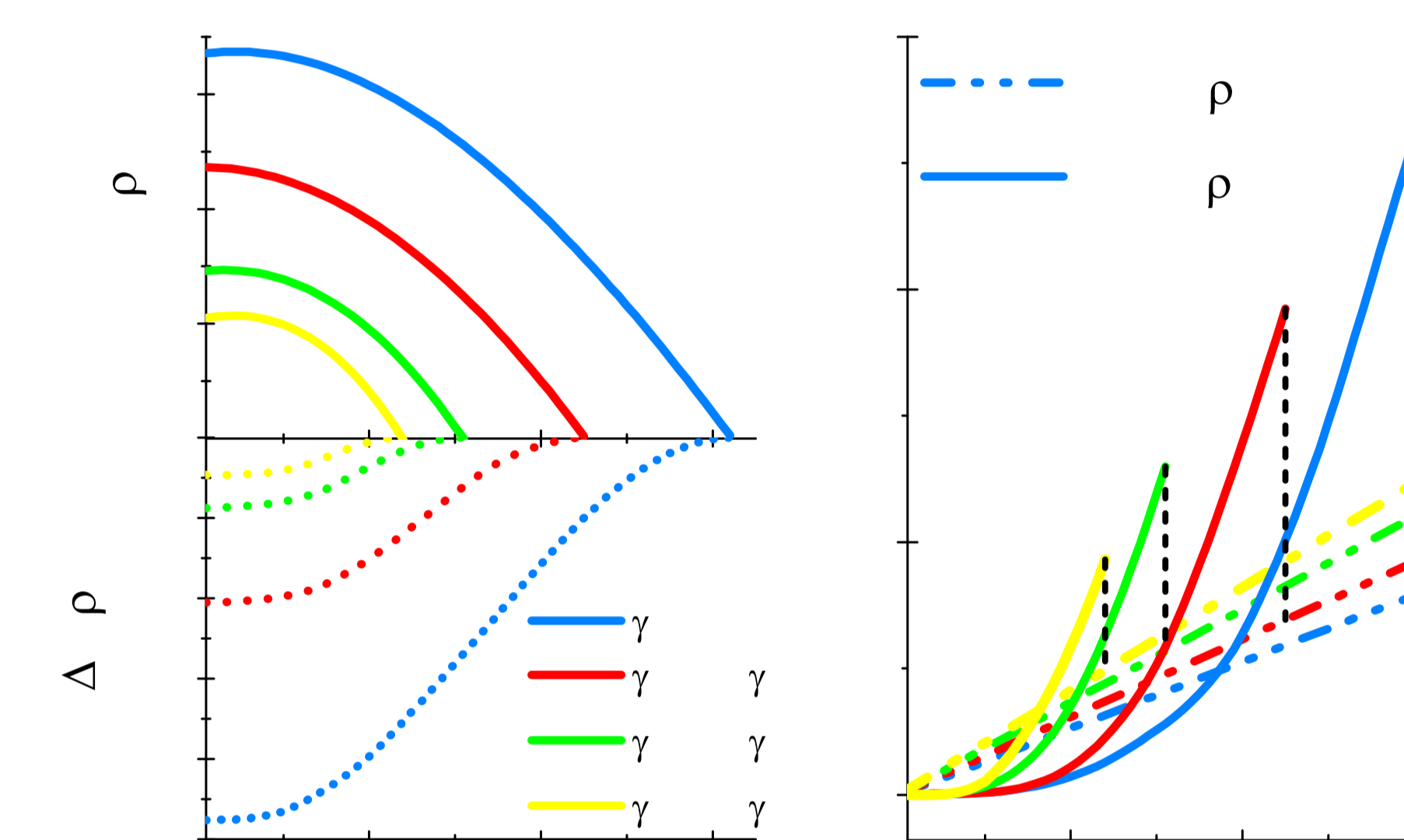


Figure 4: (a) The difference in free energy between the superconducting and the normal state (lower curve) and the thermodynamic critical field (upper curve). (b) The specific heat for the superconducting and the normal state. The black vertical line indicates the specific heat jump in the critical temperature.

## SUMMARY

We proved in the work that the balanced phonon-induced superconducting state cannot be generated on a square lattice. On the other hand, the linear electron-phonon interaction can induce the unbalanced superconducting state in cases for which the unbalance parameter takes a value less than 0.42. This effect can be observed only when the Eliashberg equations are solved in the fully self-consistent way. It should be stressed that the results presented in the work completely call into question the outcome got for the phonon-induced superconducting state on a square lattice within the isotropic approximation.

## REFERENCES

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