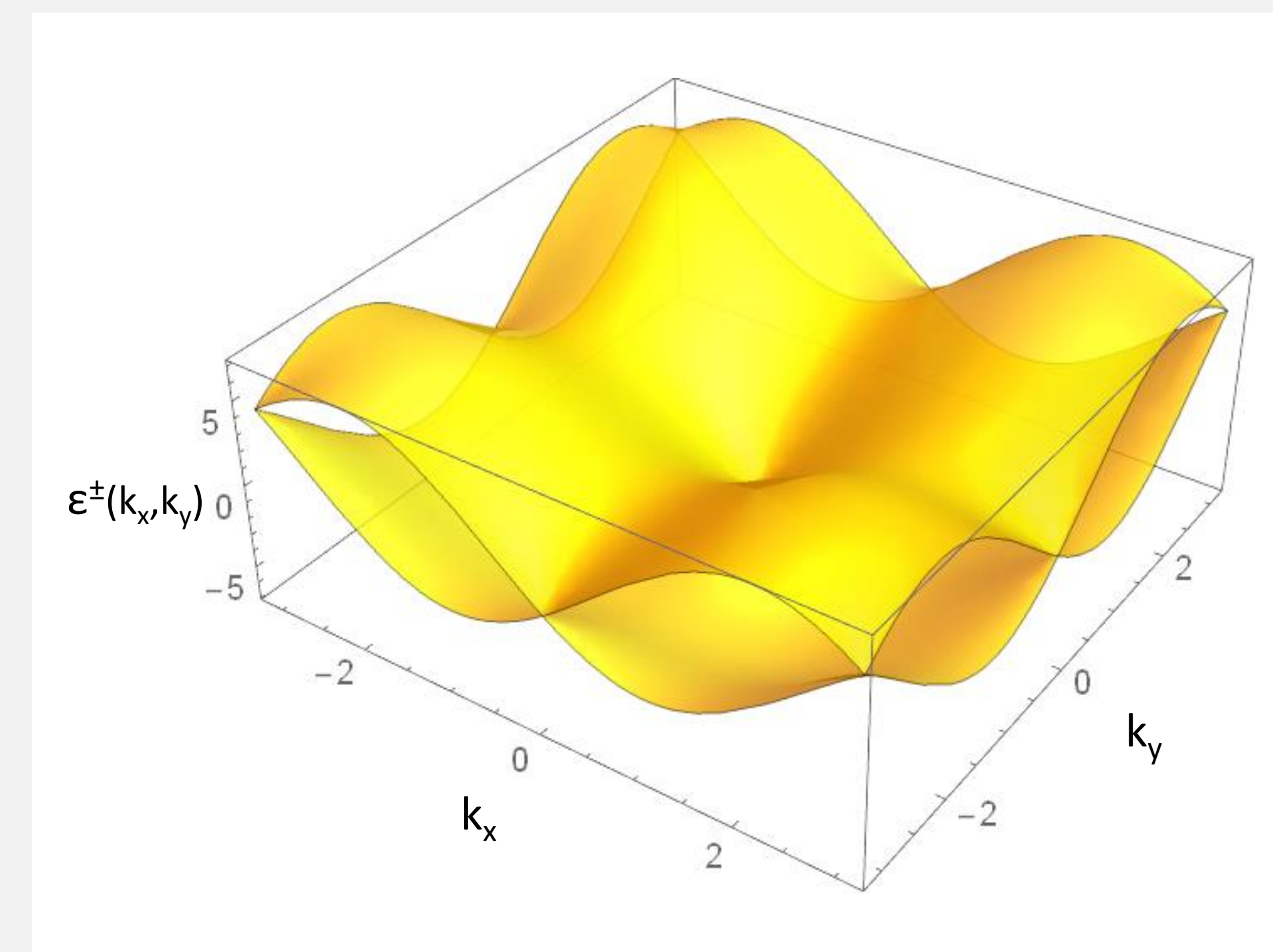


## Introduction

The analysis of superconducting properties in the presence of spin-orbit coupling is the subject of many scientific papers published in recent years. The understanding of the problem is insufficient due to the fact that the theoretical results have been obtained using too simple models. In the most advanced works on phonon-induced superconducting state, formed in the presence of spin-orbit coupling, the spin-orbit interaction is included only in the Eliashberg function, while as the additional interaction also changes the very form of Eliashberg equations.

The aim of the research is to derive full thermodynamic equations for phonon-induced superconducting state in the presence of Rashba type spin-orbit coupling.

## Electronic dispersion relation



Due to the presence of spin-orbit interaction, the spin degeneration disappears and the electronic dispersion relation is divided into two bands (Figure: the case of square lattice for  $\gamma_0 = 4t$ ).

## Theoretical model

### Hamiltonian

The Hamiltonian used is the sum of the following Hamiltonians:

$$H^{(1)} = \sum_{k\sigma} \bar{\epsilon}_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_q \omega_q b_q^\dagger b_q$$

$$H^{(2)} = \sum_k \gamma(\mathbf{k}) c_{k\uparrow}^\dagger c_{k\downarrow} + \sum_k \gamma^*(\mathbf{k}) c_{k\downarrow}^\dagger c_{k\uparrow}$$

$$H^{(3)} = \sum_{kq\sigma} g_{k,k+q} c_{k+q\sigma}^\dagger c_{k\sigma} \phi_q$$

$$\epsilon_k = -2t [\cos(k_x) + \cos(k_y)] + 4t' \cos(k_x) \cos(k_y)$$

$$\omega_q = \omega_0 \sqrt{2 - \cos q_x - \cos q_y}$$

$$\gamma(k) = \gamma_0 [\sin k_y + i \sin k_x]$$

$$g_{k,k+q} = g_0 |q| \sqrt{\frac{1}{\omega_q}}$$

### Nambu Spinors

The standard Nambu spinors have been extended to four-elements operators:

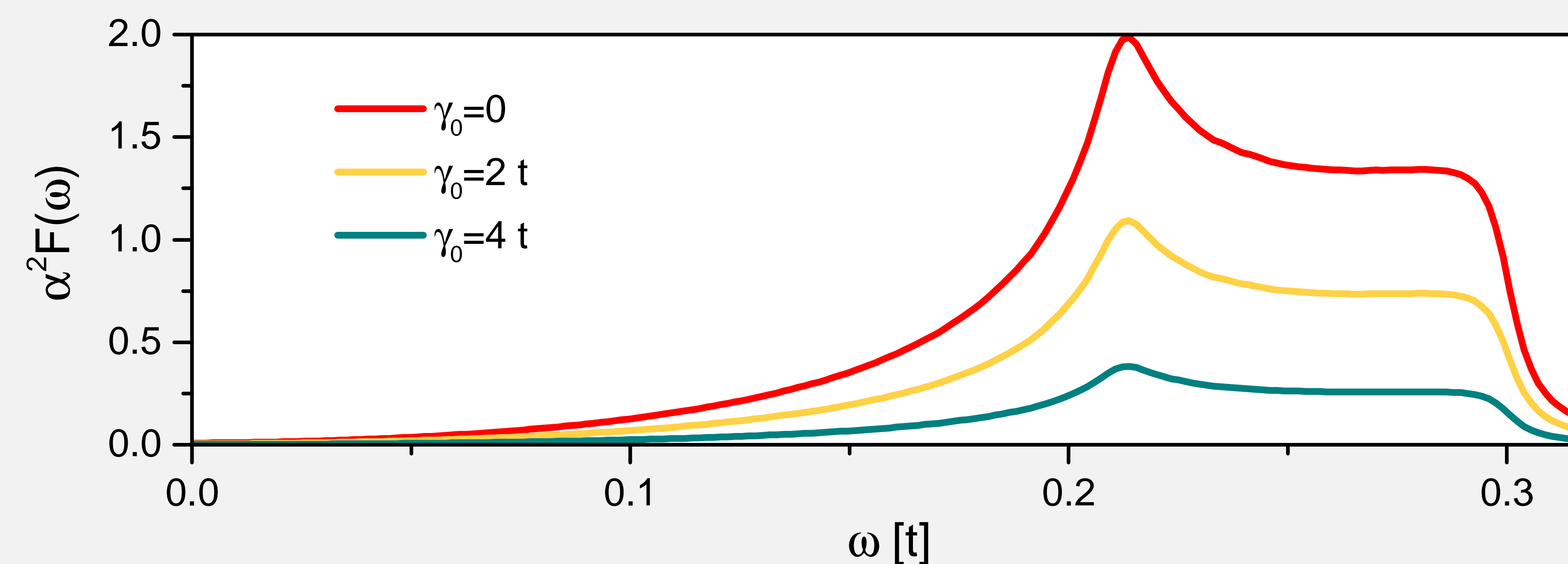
$$\Psi_k = \begin{pmatrix} c_{k\uparrow} \\ c_{-k\downarrow}^\dagger \\ c_{k\downarrow} \\ c_{-k\uparrow}^\dagger \end{pmatrix}, \quad \Psi_k^\dagger = (c_{k\uparrow}^\dagger \quad c_{-k\downarrow} \quad c_{k\downarrow}^\dagger \quad c_{-k\uparrow})$$

Using the above spinors a matrix Green's function with the following structure has been defined:

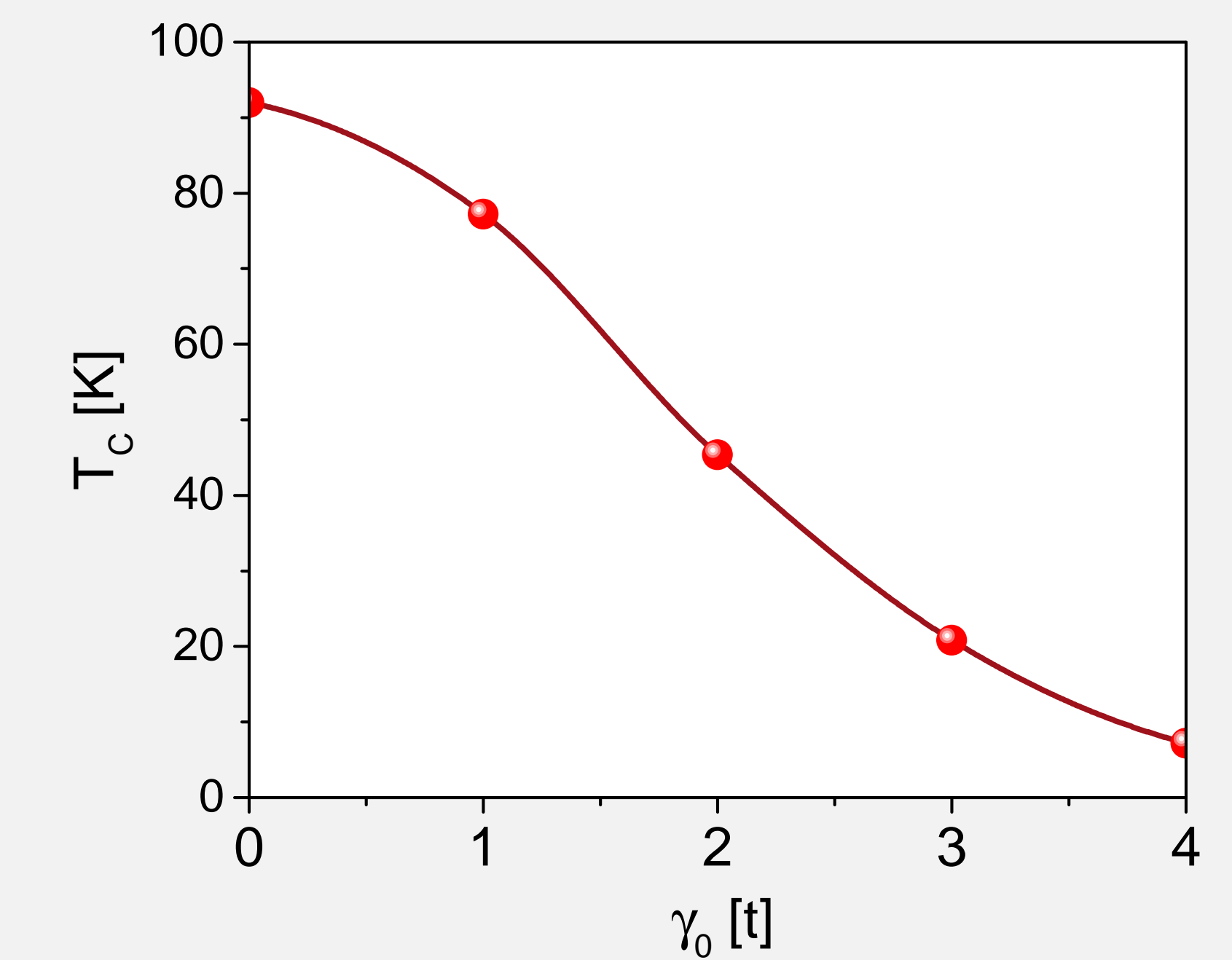
$$G_k(i\omega_n) \rightarrow \begin{pmatrix} N & Sc^s & SO & Sc^t \\ Sc^s & N & Sc^t & SO \\ \overline{SO} & \overline{Sc}^t & \overline{N} & \overline{Sc}^s \\ \overline{Sc}^t & \overline{SO} & \overline{Sc}^s & \overline{N} \end{pmatrix}$$

## Matrix self-energy and Eliashberg function

$$M_k(i\omega_n) = \left[ \Gamma(\mathbf{k}) (T_4^+ + T_4^-) + \Gamma_z(\mathbf{k}) (T_3^+ - T_3^-) \right] G_k(i\omega_n) G_{0k}^{-1}(i\omega_n) - \frac{1}{\beta} \sum_{q\omega_m} g_{k-q,k} g_{k,k-q} G_{k-q}(i\omega_m) \left[ (T_4^+ + T_4^-) X_k(i\omega_n) - (T_3^+ + T_3^-) Y_k(i\omega_n) \right] \langle\langle \phi_q | \phi_{-q} \rangle\rangle_{i(\omega_n - \omega_m)}$$



## The effect of spin-orbit coupling on critical temperature



## Conclusion

- The extension of the matrix Green's function to 4x4 is crucial when analyzing additional effects in the superconducting phase
- Obtaining a generalized formalism is possible without the use of additional approximations (only Wick and Migdal's theorem, that are normally used in the derivation of classical Eliashberg equations)

## References

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