

Sensitivity analysis on miniaturize pulse tube boundary layer losses

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**Introduction**

- Miniaturized cryocoolers are key components of small satellites (SmallSats).
- Operating at high frequency is essential condition for small size cryocoolers. Compression and expansion sweeps volumes; moving mechanisms as well as other components need to be miniaturized.
- Miniaturization in pulse tubes that work at high frequency leads to high boundary layer losses and a drastic reduction of thermal efficiency.
- In a previous work, it was shown that boundary layer losses increase as diameter of a pulse tube decreases from meso-scale 10 mm down to 1 mm at 300Hz, and this leads to very low thermodynamic efficiency at 1 mm diameter.
- In this follow-up study, CFD is utilized for sensitivity analysis on pulse tubes to investigate the effect of geometric parameters and operating conditions on boundary layer losses.
- The effects of bounding temperatures, pulse tube aspect ratio, and frequency are investigated.
- The results provide the threshold condition beyond which miniature pulse tube can no longer be used at high frequency cryocoolers.

**Physical System**

- **Computational Domain**
  - Cold IX wall
  - Pulse tube wall
  - Hot IX wall

  ![Figure 1. Computational domain with boundaries.](image)

- **Model parameters**

<table>
<thead>
<tr>
<th>Working gas</th>
<th>Helium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>Varies between 175Hz-300 Hz (simulon)</td>
</tr>
<tr>
<td>Cold end temperature ((T_{\text{cold}}))</td>
<td>Varies between 80K-190K at constant (T_{\text{cold}}=300K). Varies between 40K-60K at constant (T_{\text{cold}}=80K).</td>
</tr>
<tr>
<td>Hot end temperature ((T_{\text{hot}}))</td>
<td>Varies between 300K-390K at constant (T_{\text{hot}}=300K). Varies between 80K-60K at constant (T_{\text{hot}}=40K).</td>
</tr>
<tr>
<td>Mean pressure ((P_{\text{mean}}))</td>
<td>1.54 - 1.59 MPa (derived after running simulation, not specified explicitly)</td>
</tr>
<tr>
<td>Cold and hot mass flow rate phase angle ((\phi))</td>
<td>30° cold end leading</td>
</tr>
<tr>
<td>Pulse tube diameter ((D))</td>
<td>1.47 mm</td>
</tr>
<tr>
<td>Pulse tube length ((L_{\text{PT}}))</td>
<td>Derived based on aspect ratio</td>
</tr>
<tr>
<td>Pulse tube aspect ratio ((L_{\text{PT}}/D))</td>
<td>Varies between 6 - 14</td>
</tr>
<tr>
<td>Heat exchanger diameters ((D))</td>
<td></td>
</tr>
<tr>
<td>Heat exchanger lengths ((L_{\text{Hx}}))</td>
<td>0.34m</td>
</tr>
<tr>
<td>Heat exchanger viscous resistance</td>
<td>7.43 x 10^{-4} m^2</td>
</tr>
<tr>
<td>Heat exchanger inertial resistance</td>
<td>8.147 m</td>
</tr>
<tr>
<td>Heat exchanger porosity</td>
<td>0.68</td>
</tr>
</tbody>
</table>

**Set of Equations**

- **continuity, momentum and energy equations**
  
  \[
  \frac{\partial u_i}{\partial x_i} = 0
  \]
  
  \[
  \frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \frac{\rho}{\rho} \frac{\partial u_i}{\partial x_j} \right] + \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} \right)
  \]
  
  \[
  \rho c_v \left( \frac{\partial T}{\partial t} + \frac{\partial (u_i u_j T)}{\partial x_j} \right) = \frac{\partial}{\partial x_j} \left( \frac{\partial T}{\partial x_j} \right)
  \]

- **Mass flow rate normalization**
  
  \[
  dV = \frac{dV_0}{\sqrt{\gamma}} = \frac{dV_0}{\sqrt{2}} \approx 2\pi \frac{D^2}{8} \frac{D}{L}
  \]
  
  \[
  dM_0 = \frac{dV_0}{R} \frac{P_{\text{mean}}}{\sqrt{\gamma}} \frac{dM}{\sqrt{\gamma}}
  \]
  
  \[
  m_{\text{wall}} = \sqrt{2} \frac{dM_0}{dV_0} = \frac{\sqrt{2}}{R} \frac{P_{\text{mean}}}{\sqrt{\gamma}} \frac{dM}{dV_0} = \sqrt{2} \frac{dM_0}{dV_0} = \frac{dV_0}{\sqrt{\gamma}} \frac{dM}{dV_0} \]

- **Efficiency definition**
  
  \[
  \eta = \frac{< H >}{< PV >}
  \]
  
  \[
  < H > = \int_0^{\text{cycle}} \left( h + \frac{1}{2} | \vec{v} |^2 \right) 2\pi r dx dt
  \]
  
  \[
  < PV > = \int_0^{\text{cycle}} R_T \frac{P_{\text{mean}}}{\sqrt{\gamma}} m_{\text{wall}} \cos \theta
  \]

**Periodic Steady State**

- **Figure 2.** Convergence of results for cases with \(T_{\text{cold}}=300 K, T_{\text{hot}}=80 K,\) and \(f=200\) Hz, (a) the effect of number of cycles on convergence of total enthalpy flux at the middle of the pulse tube, (b) total enthalpy flux throughout the length of the pulse tube at steady periodic conditions.

**Results**

- D= 1 mm
- D= 4 mm
- D= 7 mm

  ![Figure 3. Temperature contours (K) at 1/4 cycle for \(f=300\) Hz with a pulse tube aspect ratio of 10, (a) \(T_{\text{cold}}=80 K\) and \(T_{\text{hot}}=300 K\), (b) \(T_{\text{cold}}=40 K\) and \(T_{\text{hot}}=80 K\).](image)

  ![Figure 4. Standard deviation of temperature over the cross-section of the pulse tube for D= 1 mm, f=300 Hz, and aspect ratio of 10 at the middle of pulse tube (L/2).](image)

  ![Figure 5. Effect of different parameters on pulse tube efficiency, (a) effect of cold end temperature, (b) effect of hot end temperature, (c) effect of aspect ratio, (d) effect of frequency.](image)

**Conclusion**

- The effects of diameter, bounding (cold and warm end) temperatures, frequency and pulse tube aspect ratio on pulse tube efficiency were investigated.
- Pulse tube efficiency deteriorates drastically as the diameter shrinks from 7 mm to 1 mm, particularly for end temperatures equal to 80K and 300K, respectively.
- Miniature pulse tubes are not appropriate for the first stage of miniaturize cryocoolers that function at high frequency.
- Lowering the temperature gradient across the pulse tube by reducing the bounding temperatures as well as increasing the aspect ratio of the pulse tube improve the efficiency of the pulse tube.
- Miniature pulse tubes may be appropriate for the second stage of miniaturize cryocoolers.