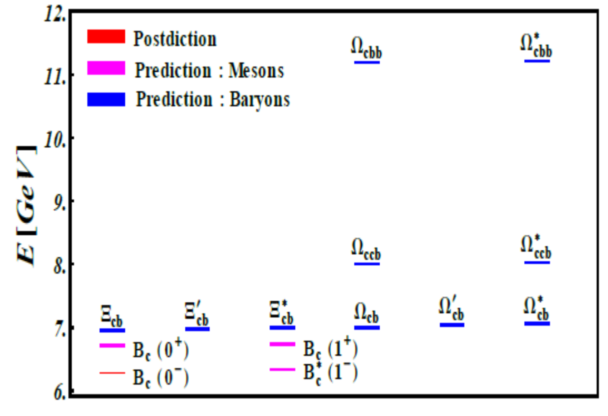
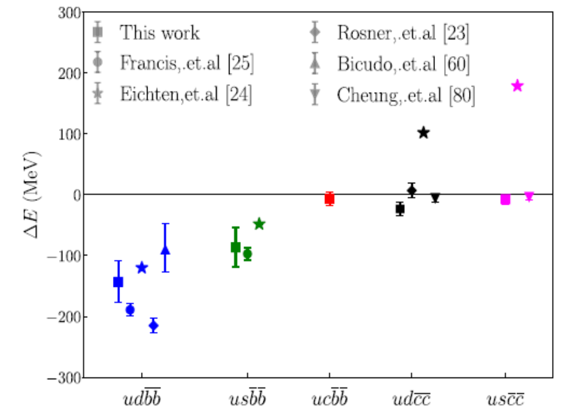
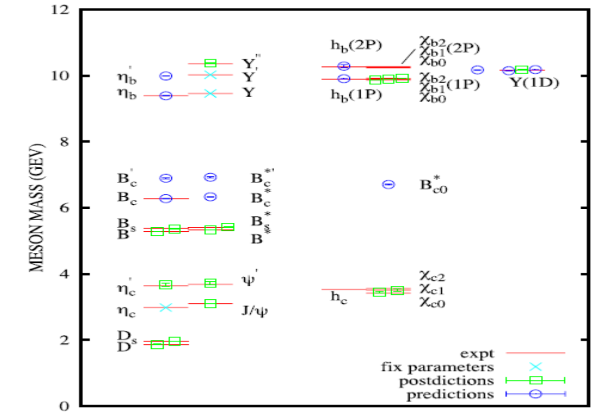
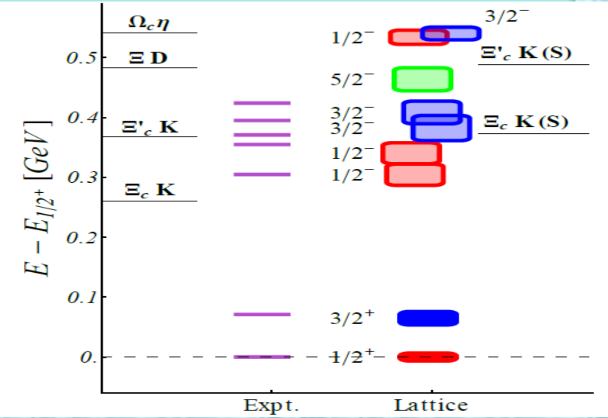
Energy spectra, decay constants and form factors of heavy flavoured hadrons from lattice QCD









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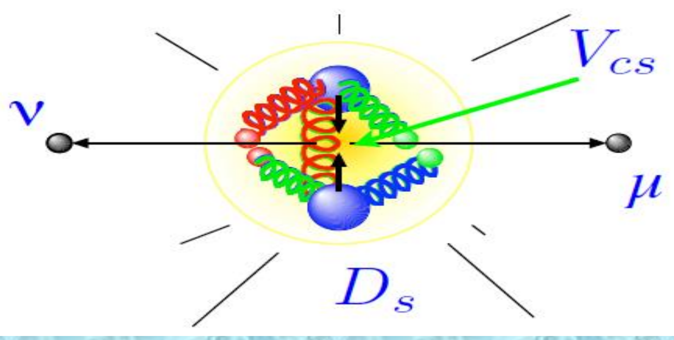
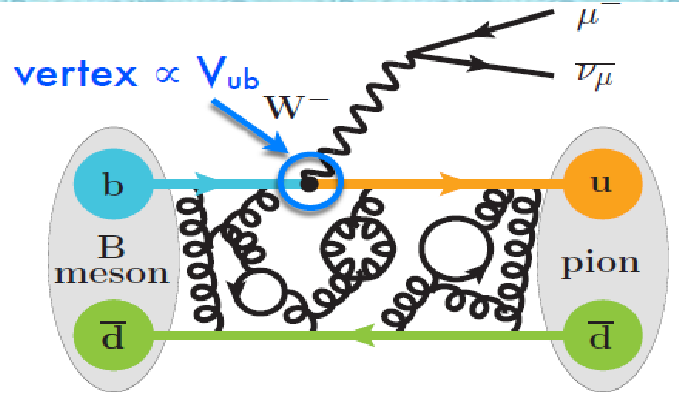
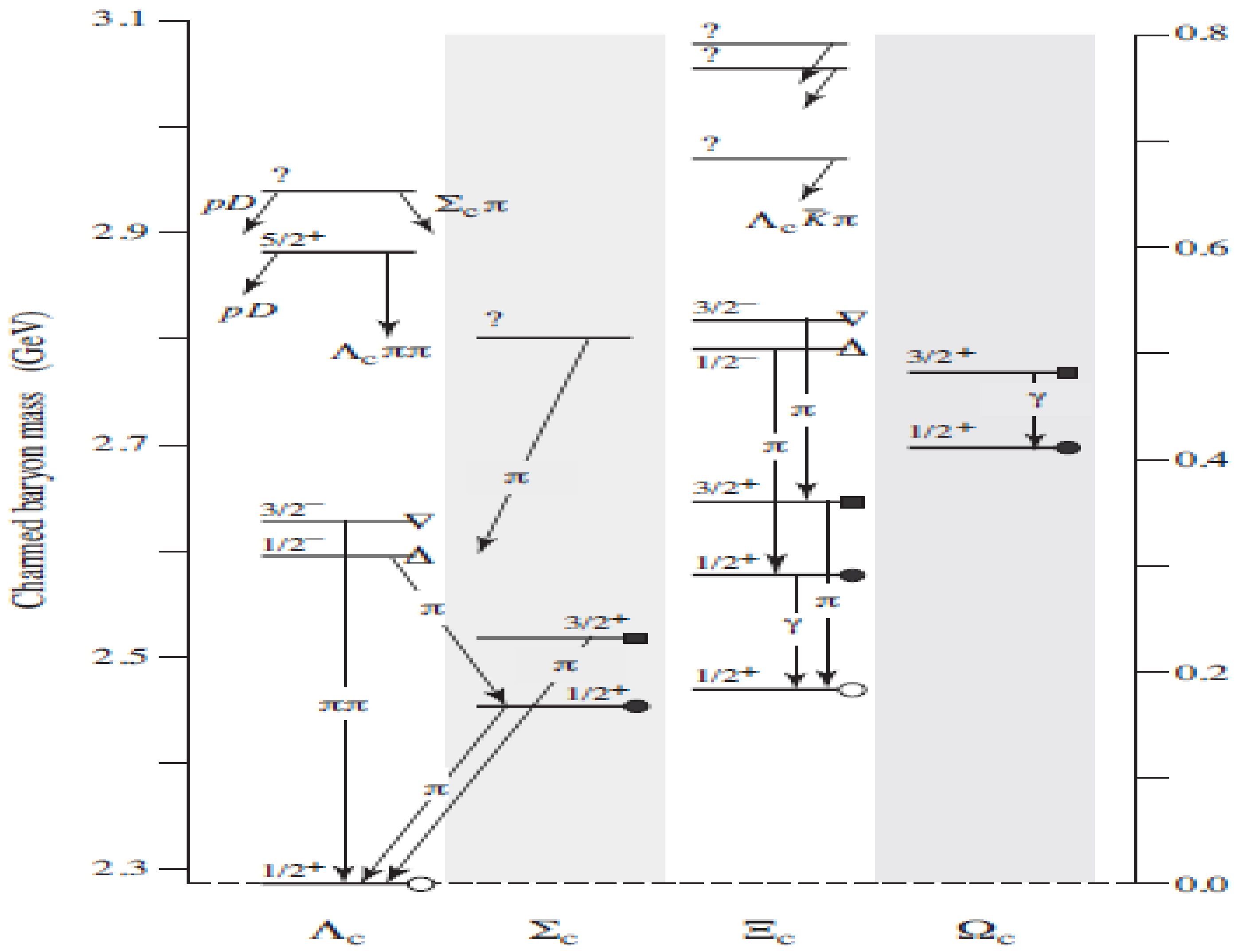


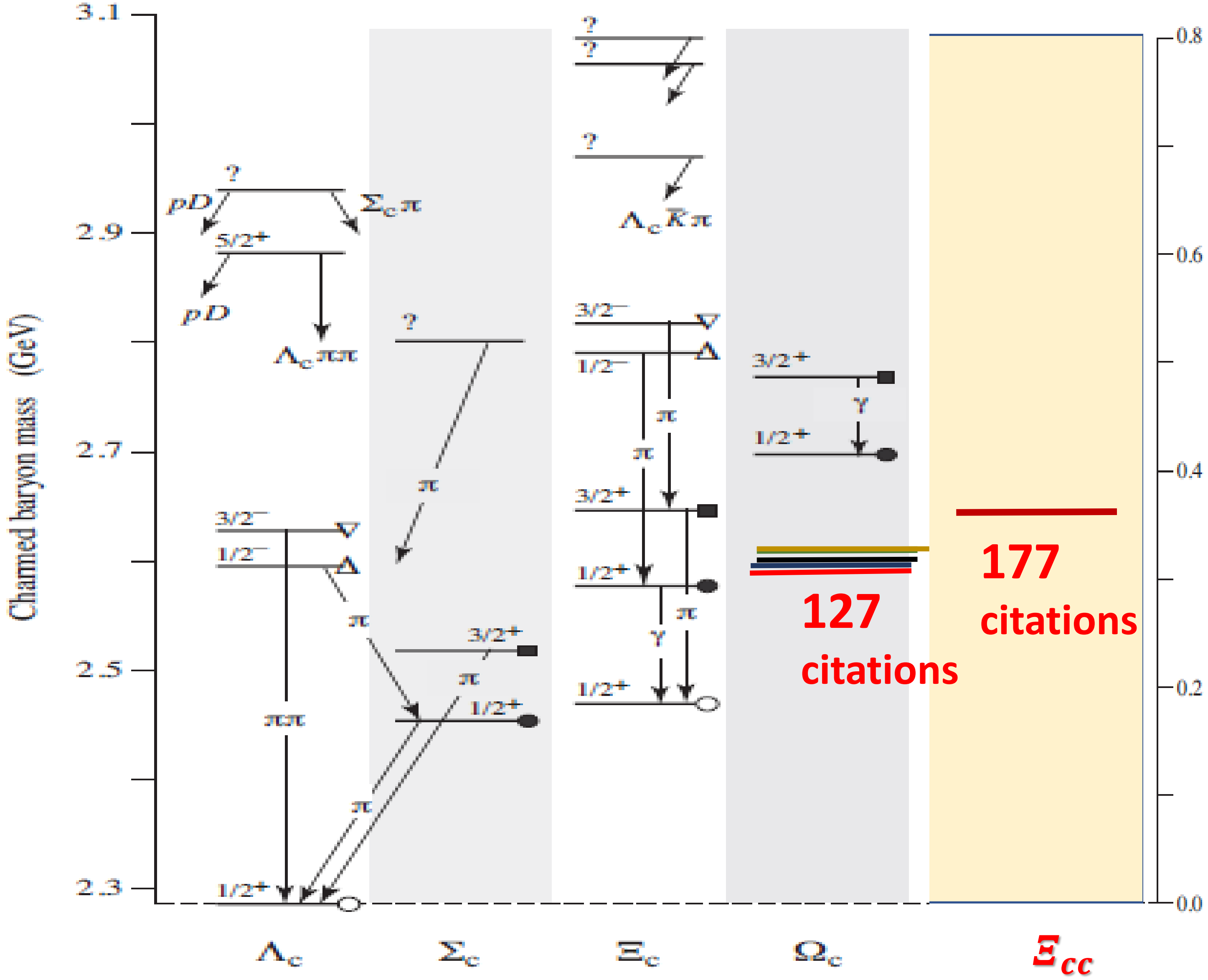
$\mathbf{V_{ud}}$ $\pi \to \ell\nu$	$\mathbf{V_{us}}$ $K \to \ell\nu$ $K \to \pi\ell\nu$	$\mathbf{V_{ub}}$ $B \to \ell\nu$ $B \to \pi\ell\nu$
$\mathbf{V_{cd}}$ $D \to \ell\nu$ $D \to \pi\ell\nu$	$\mathbf{V_{cs}}$ $D_s \to \ell\nu$ $D \to K\ell\nu$	$\mathbf{V_{cb}}$ $B \to D\ell\nu$ $B \to D^*\ell\nu$
$\mathbf{V_{td}}$ $\langle B_d \vert \bar{B}_d \rangle$ $B \to \pi\ell\ell$	$\mathbf{V_{ts}}$ $\langle B_s \vert \bar{B}_s \rangle$ $B \to K\ell\ell$	$\mathbf{V_{tb}}$



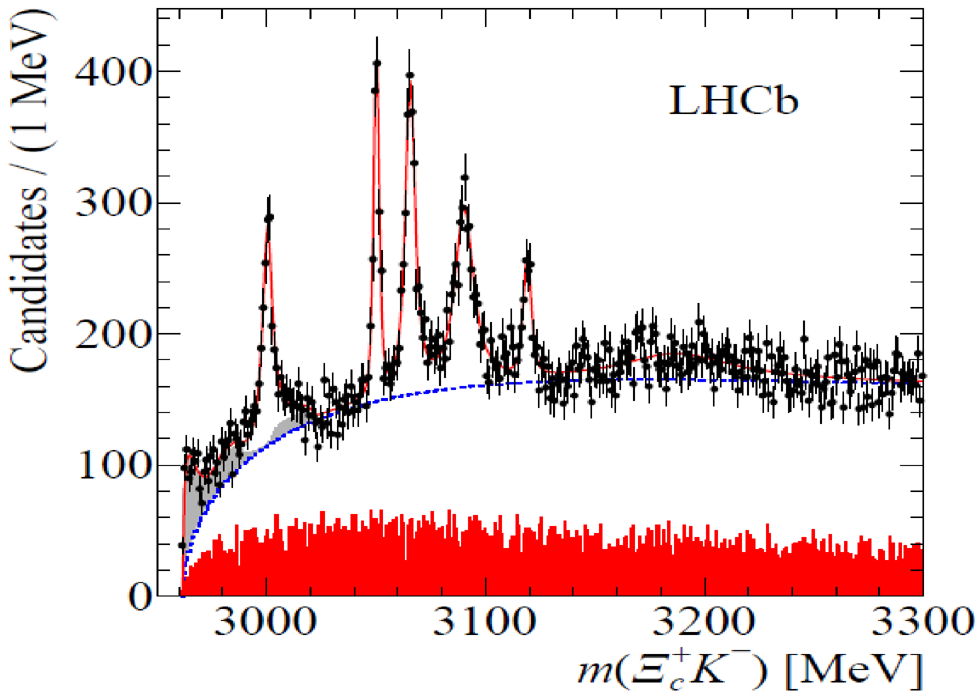
Charmed Baryons (PDG, 2016)



Charmed Baryons (2017)



Ω_c^0 Baryons



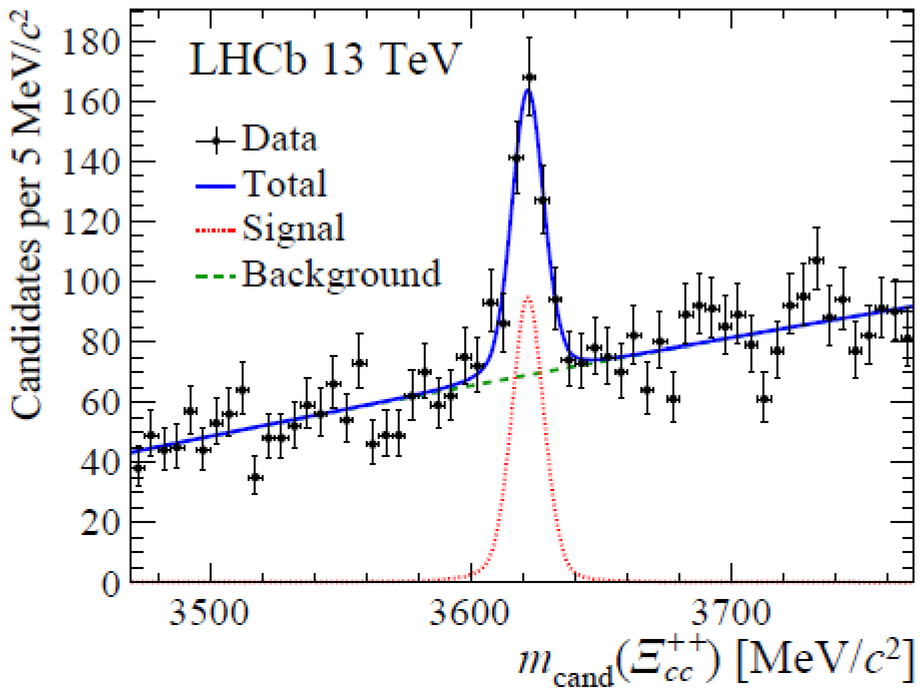
LHCb Collaboration :
Phys. Rev. Lett. 118 (2017) no.18, 182001

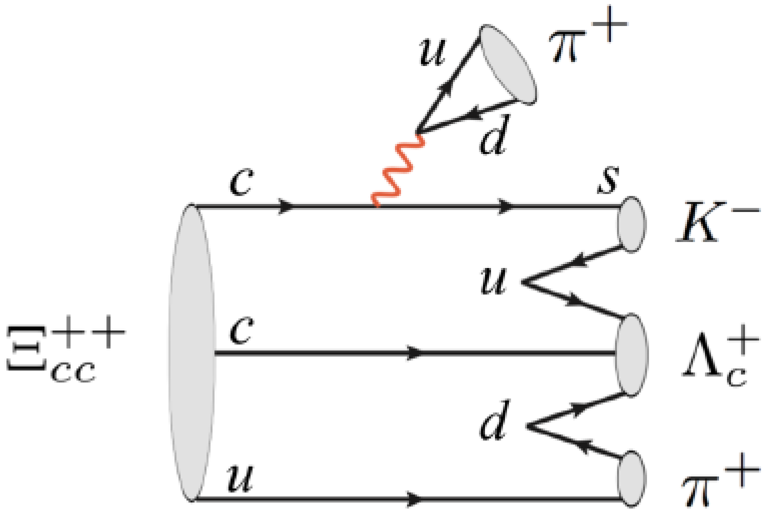
Resonance	Mass (MeV)	Γ (MeV)	Yield	N_σ
$\Omega_c(3000)^0$	$3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}$	$4.5 \pm 0.6 \pm 0.3$	$1300 \pm 100 \pm 80$	20.4
$\Omega_c(3050)^0$	$3050.2 \pm 0.1 \pm 0.1^{+0.3}_{-0.5}$	$0.8 \pm 0.2 \pm 0.1$	$970 \pm 60 \pm 20$	20.4
		< 1.2 MeV, 95% CL		
$\Omega_c(3066)^0$	$3065.6 \pm 0.1 \pm 0.3^{+0.3}_{-0.5}$	$3.5 \pm 0.4 \pm 0.2$	$1740 \pm 100 \pm 50$	23.9
$\Omega_c(3090)^0$	$3090.2 \pm 0.3 \pm 0.5^{+0.3}_{-0.5}$	$8.7 \pm 1.0 \pm 0.8$	$2000 \pm 140 \pm 130$	21.1
$\Omega_c(3119)^0$	$3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}$	$1.1 \pm 0.8 \pm 0.4$	$480 \pm 70 \pm 30$	10.4
		< 2.6 MeV, 95% CL		
$\Omega_c(3188)^0$	$3188 \pm 5 \pm 13$	$60 \pm 15 \pm 11$	$1670 \pm 450 \pm 360$	
$\Omega_c(3066)^0_{\rm fd}$			$700 \pm 40 \pm 140$	
$\Omega_c(3090)^0_{\rm fd}$			$220 \pm 60 \pm 90$	
$\Omega_c(3119)^0_{\rm fd}$			$190 \pm 70 \pm 20$	

Discovery of doubly-charmed baryon

$\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$

LHCb : Phys. Rev. Lett. 119, 112001(2018)



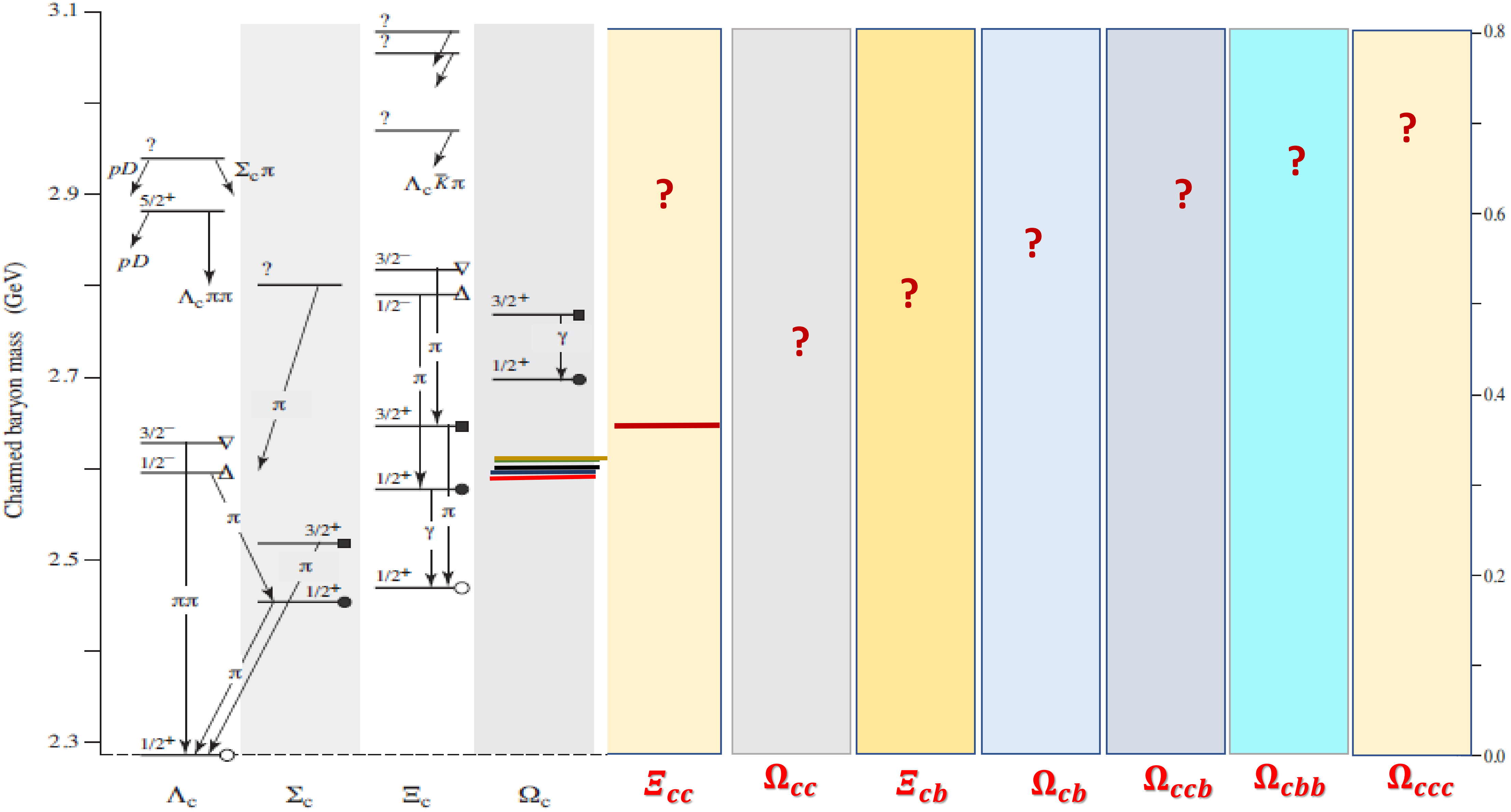


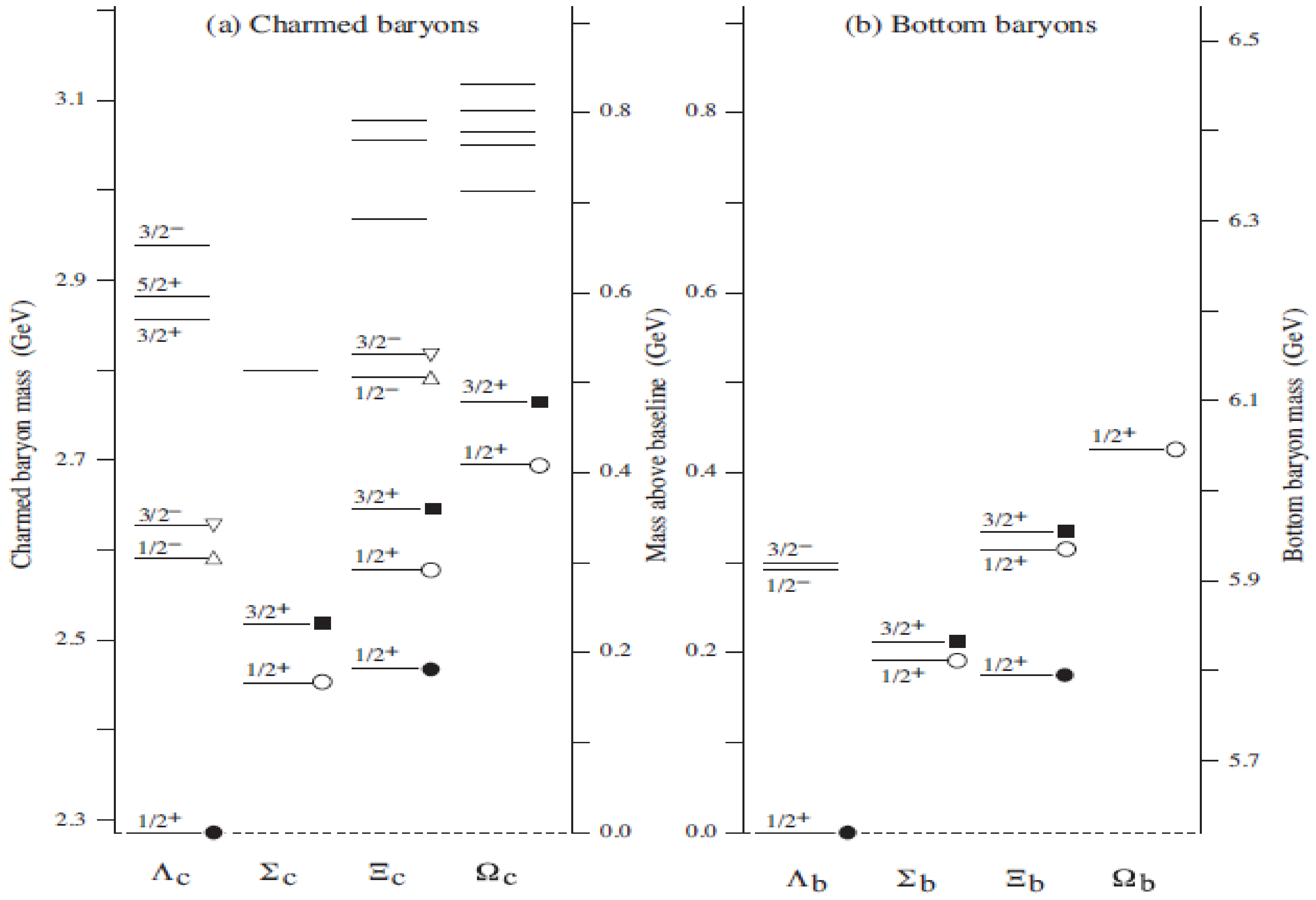
$$3621.40 \pm 0.72\,(\text{stat}) \pm 0.27\,(\text{syst}) \pm 0.14\,(\Lambda_c^+)\ \text{MeV}/c^2$$

Life time : $0.256\,^{+0.024}_{-0.022}\,(\text{stat}) \pm 0.014\,(\text{syst})$ ps.

arXiv:1806.02744

Charmed Baryons (2017)





(a) Charmed baryons
(b) Bottom baryons
Charmed baryon mass (GeV)
Mass above baseline (GeV)
Bottom baryon mass (GeV)
3.1
2.9
2.7
2.5
2.3
0.8
0.6
0.4
0.2
0.0
6.5
6.3
6.1
5.9
5.7
3/2−
5/2+
3/2+
1/2−
1/2+
Λ_c
Σ_c
Ξ_c
Ω_c
Λ_b
Σ_b
Ξ_b
Ω_b

Heavy Hadron Energy Spectra:
What we know and do not know

- Ground states of heavy hadrons are known except $\boldsymbol{bc}$ baryons
- Excited states of quarkonia are well-known
- Excited states of other heavy hadrons are not so well-known compared to their light quark counterparts.
- Not many $\boldsymbol{b}$ baryons are known
- Excited state of $\boldsymbol{B_s}$ mesons are not well determined
- Except $\boldsymbol{B_c(1S, 2S)}$, $\boldsymbol{B_c}$ states are unknown
- A large number of unusual heavy hadrons have been discovered in last 15 years!

Theory

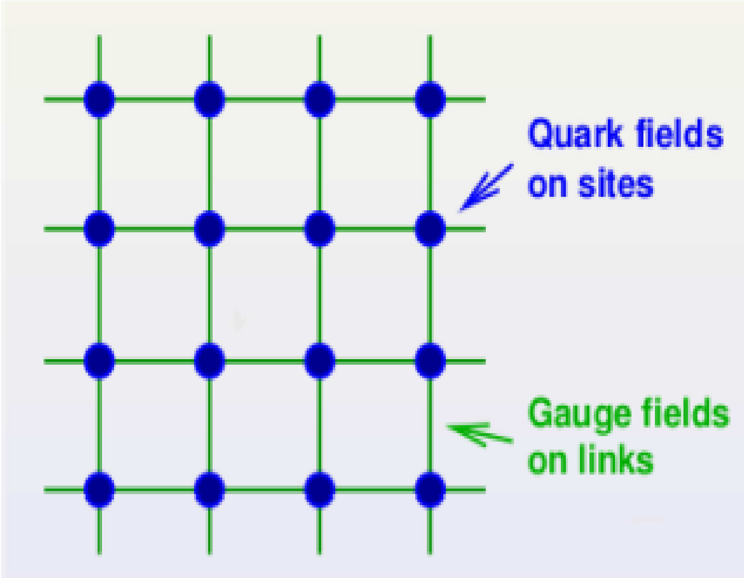
- **QCD** : Theory of strong Interactions
  - So far, we are incapable in solving the full theory analytically in the non-perturbative regime
  - pQCD
- Define theory in 4D Euclidean space-time grid and solve numerically

  : **Lattice QCD** (systematics needs to be in good control, the study of excited states with multi-hadron scatterings has made impressive progress)
- **Models :**
  - Motivated from QCD or low energy limits of QCD
    - Use chiral Lagrangian rather than full QCD Lagrangian, HQET, NRQCD etc.
      - Spin independent confining interaction (linear or HO)
      - Spin dependent hyperfine interactions: $V_{HF} \propto \sum_{i>j} (\vec{\sigma}\lambda_a)_i(\vec{\sigma}\lambda_a)_j$
      - Spin-orbit and tensor interactions $V_{HF} \propto \sum_{i>j} V(\vec{r}_{ij})\lambda_i^F.\lambda_j^F\vec{\sigma}_i.\vec{\sigma}_j$
      - Flavour dependent short range quark force
      - Flavour symmetric spin-spin interaction

LQCD : Euclidean formulation of QCD on hypercubic lattice where lattice spacing provides the ultraviolet cut-off

LQCD : A non-perturbative, gauge invariant regulator for the QCD path integrals.

- Quark fields lives on sites
- Gauge fields lives on links
- Lattice spacing : UV cut off
- Lattice size : IR cut off



$$<\hat{O}> = \frac{\int DU \{\det D\}^{n_f} O[U, D^{-1}] e^{-S_g[U]}}{\int DU \{\det D\}^{n_f} e^{-S_g[U]}} = \prod_n \int dU_n \frac{1}{Z} \{\det D(U)\}^{n_f} e^{-S_g[U]} O[U, D^{-1}]$$

Discretization $\Rightarrow$ Finite number of degrees of freedom

$\Rightarrow$ Infinte dimensional path integrals $\rightarrow$ finite dimensional integrals.

Employ Monte Carlo importance sampling methods on Euclidean metric for numerical studies.

$$\langle O \rangle = \frac{1}{N} \sum_{i=1}^{N} O\{U_i\} + \Delta O, \qquad \Delta O \propto \frac{1}{\sqrt{N}} \xrightarrow{N\to\infty} 0$$

LQCD

Inputs :

Lattice spacing (coupling) : from known masses : say M_Ω
or static quark potential
heavy meson spectrum etc.

Quark mass : $u/d \leftarrow m_\pi$

$s \leftarrow m_k$ or $m_{\bar{s}s}$ or m_Ξ

$c \leftarrow 1/4(3J/\psi + \eta_c)$

$b \leftarrow 1/4(\Upsilon_b + \eta_b)$

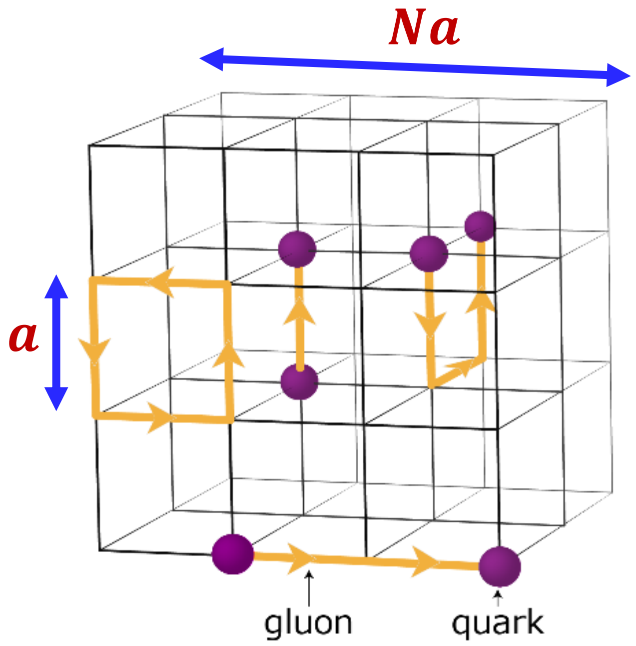
Continuum limit : $a \to 0, Na$ fixed

Infinite volume : $Na \to \infty$

Lattice Spacing : $a = 0.15 - 0.04$ fermi

$Na = 1.5 - 6$ fermi

$m_\pi L \sim 3 - 5$

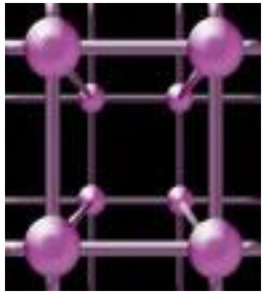


Theoretical developments, Algorithms

Hadron spectroscopy

Hadron Structure

Vacuum structure and confinement



Weak decays and matrix elements

Nonzero temperature and density

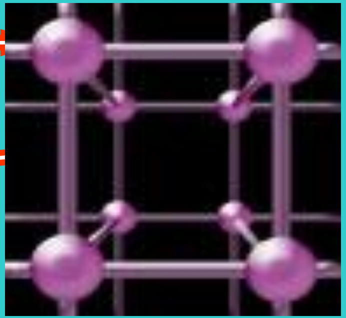
Physics Beyond the Standard Model, Supersymmetry

Nuclear Physics

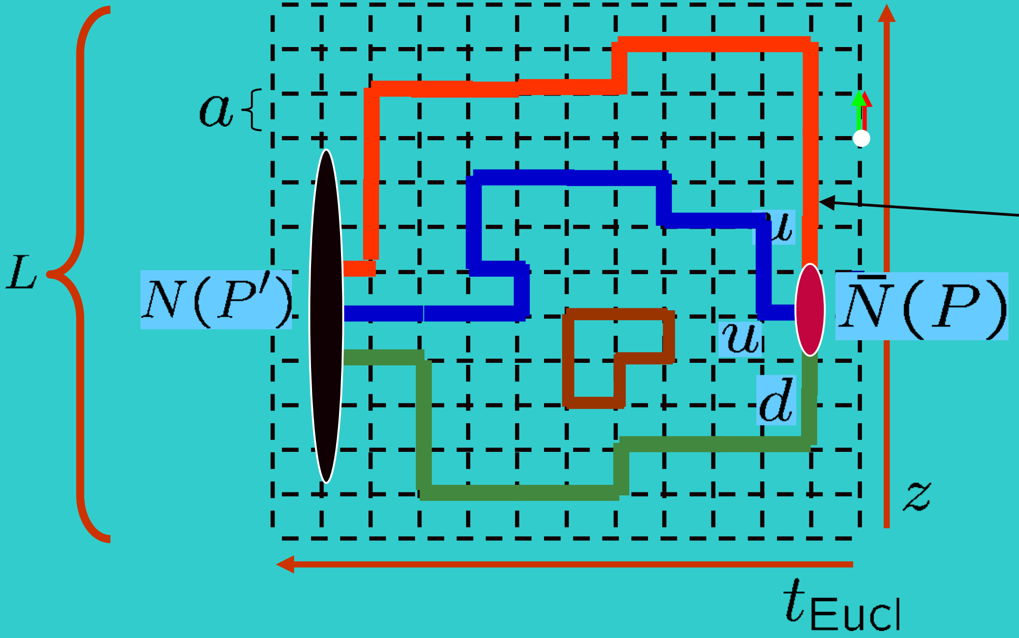
Standard model parameters

Quark
(on Lattice sites)

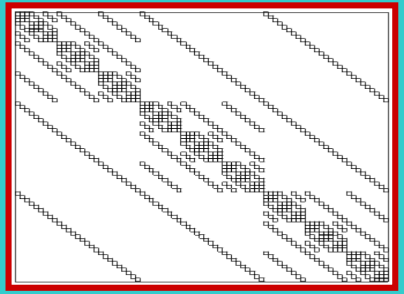
Gluon
(on Links)

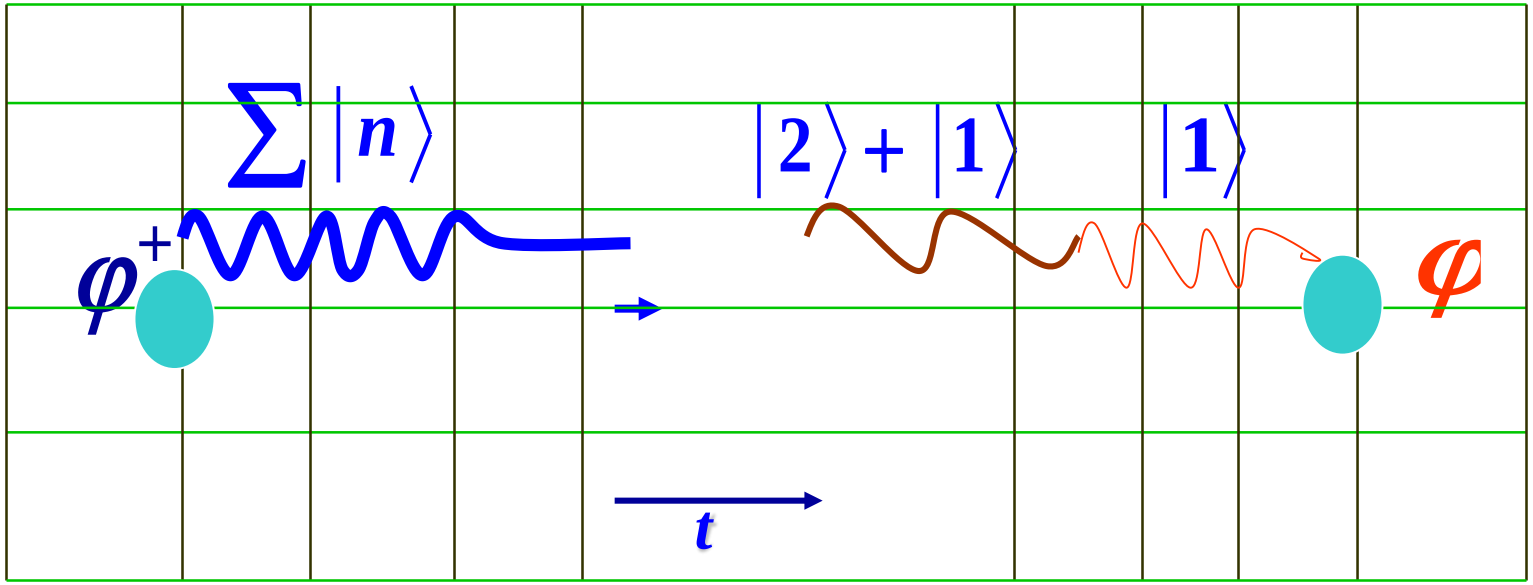


Quark Jungle Gym



quark propagators :
Inverse of very large matrix of space-time, spin and color





$$\varphi(t) = e^{Ht}\varphi(0)e^{-Ht}$$

$$G(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} \langle 0|\varphi(x)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$= \sum_n W_n e^{-E_p^n(t-t_0)} \xrightarrow[t\to\infty]{} W_1 e^{-E_1^n(t-t_0)}$$

Analysis (Extraction of Mass)

$$G(\tau) = \sum_{i=1}^{N} W_i\, e^{-m_i\tau} \underset{\tau\to\infty}{\approx} W_1\, e^{-m_1\tau}$$

Effective mass :

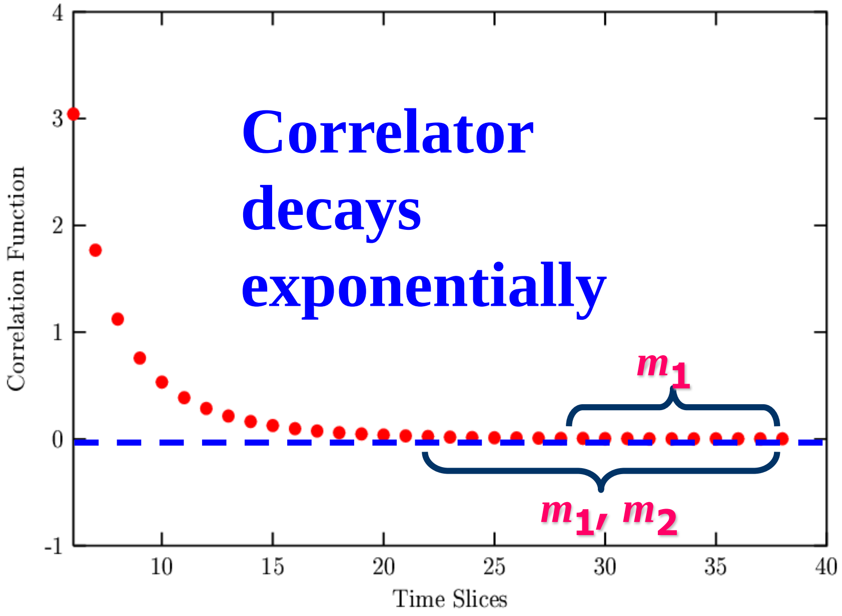
$$\frac{G(\tau)}{G(\tau+1)} = e^{-m_1\tau + m_1(\tau+1)}$$

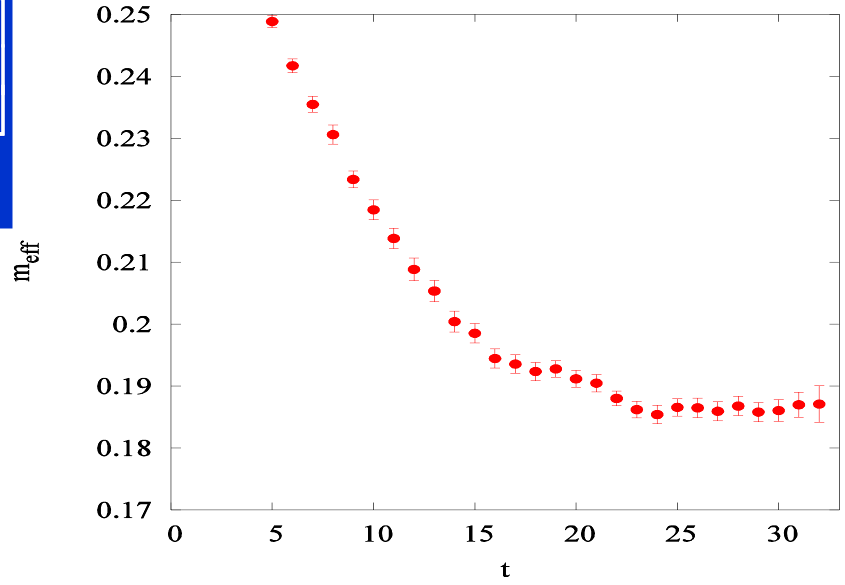
$$m(\tau) = \ln\left[\frac{G(\tau)}{G(\tau+1)}\right]$$

$$\chi^2 = \sum_{i=1}^{N}\left[\frac{f(t_i) - \langle G(t_i)\rangle}{\varepsilon(t_i)}\right]$$

$$e^{-mt} = e^{-(ma)(t/a)}$$

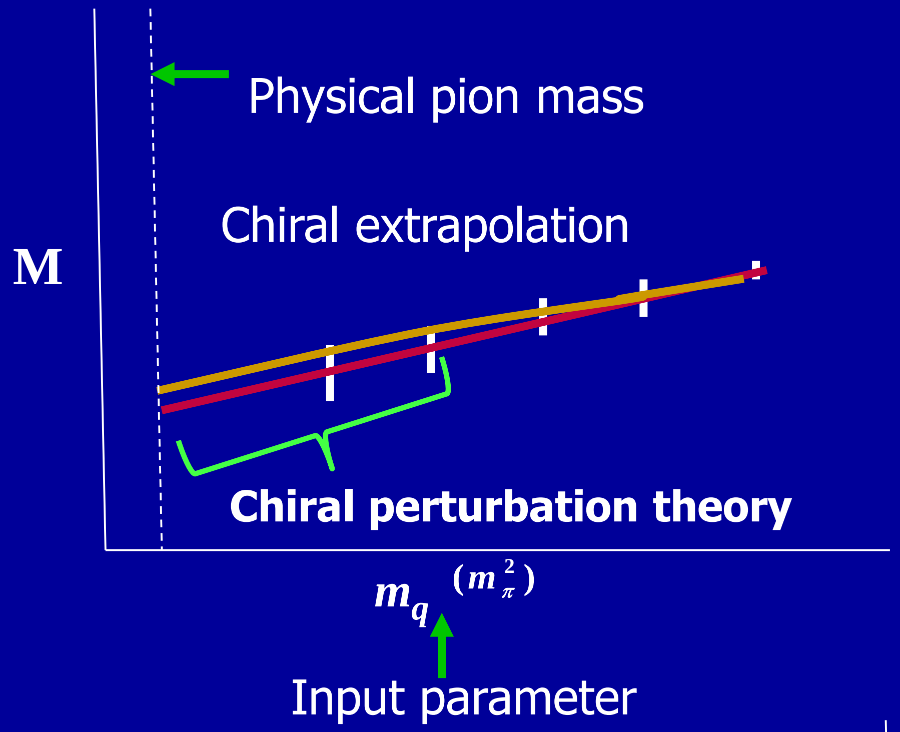
dimensionless mass   integer timeslices

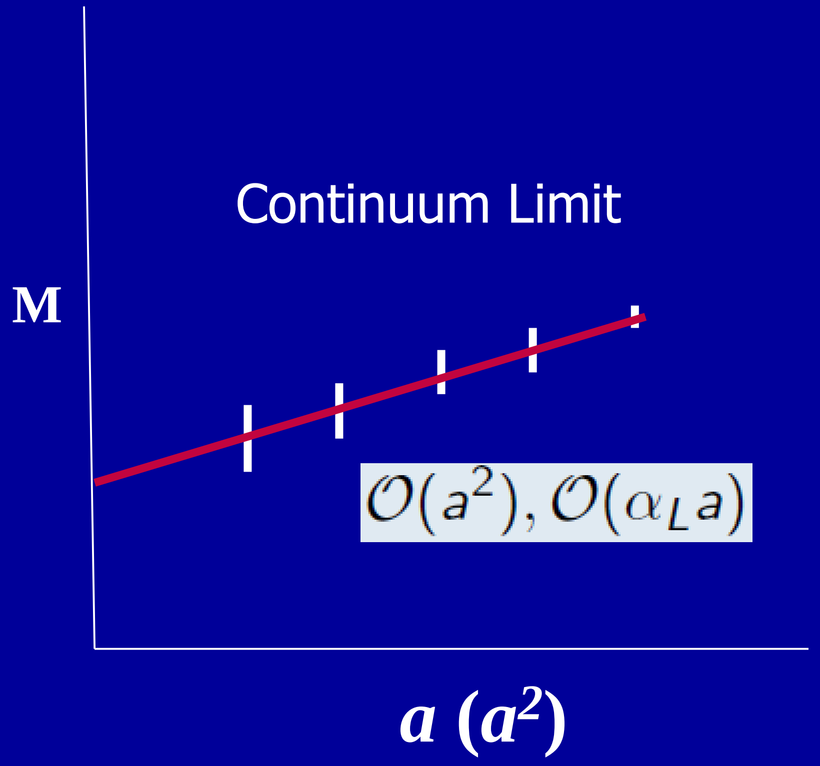


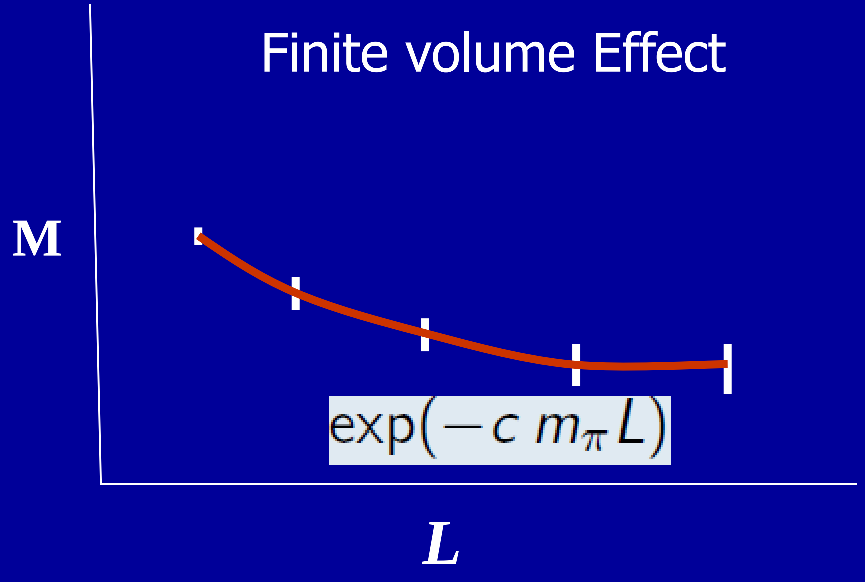


Determine *a* by measuring some physical quantity and compare that to expt, like parameter tuning in any renormalized field theory

Control of Sytemetics







LQCD for heavy quark physics

$$L_q = \overline{\psi}(\not{D} + m)\psi \longrightarrow \overline{\psi}(\gamma \cdot \Delta + ma)\psi$$

Source of discretization error (need improved discretization method preserving continuum symmetries)

Light hadron scale : Λ_{QCD}

Heavy hadron scale : m_Q **?**

$$E = E_{a=0}(1 + A(m_Q a)^2 + B(m_Q a)^3 + \ldots)$$

***ma* << 1**

LQCD for heavy quark physics

$$ma << 1$$

- **Charm :** ma **= 1.275 GeV,**
  ma **= 0.5 → a ~ 0.075 fm**
  ma **= 0.3 → a ~ 0.046 fm**
- **Bottom :** ma **= 4.66 GeV**
  ma **= 0.5 → a = 0.021 fm**
  ma **= 0.3 → a = 0.013 fm**

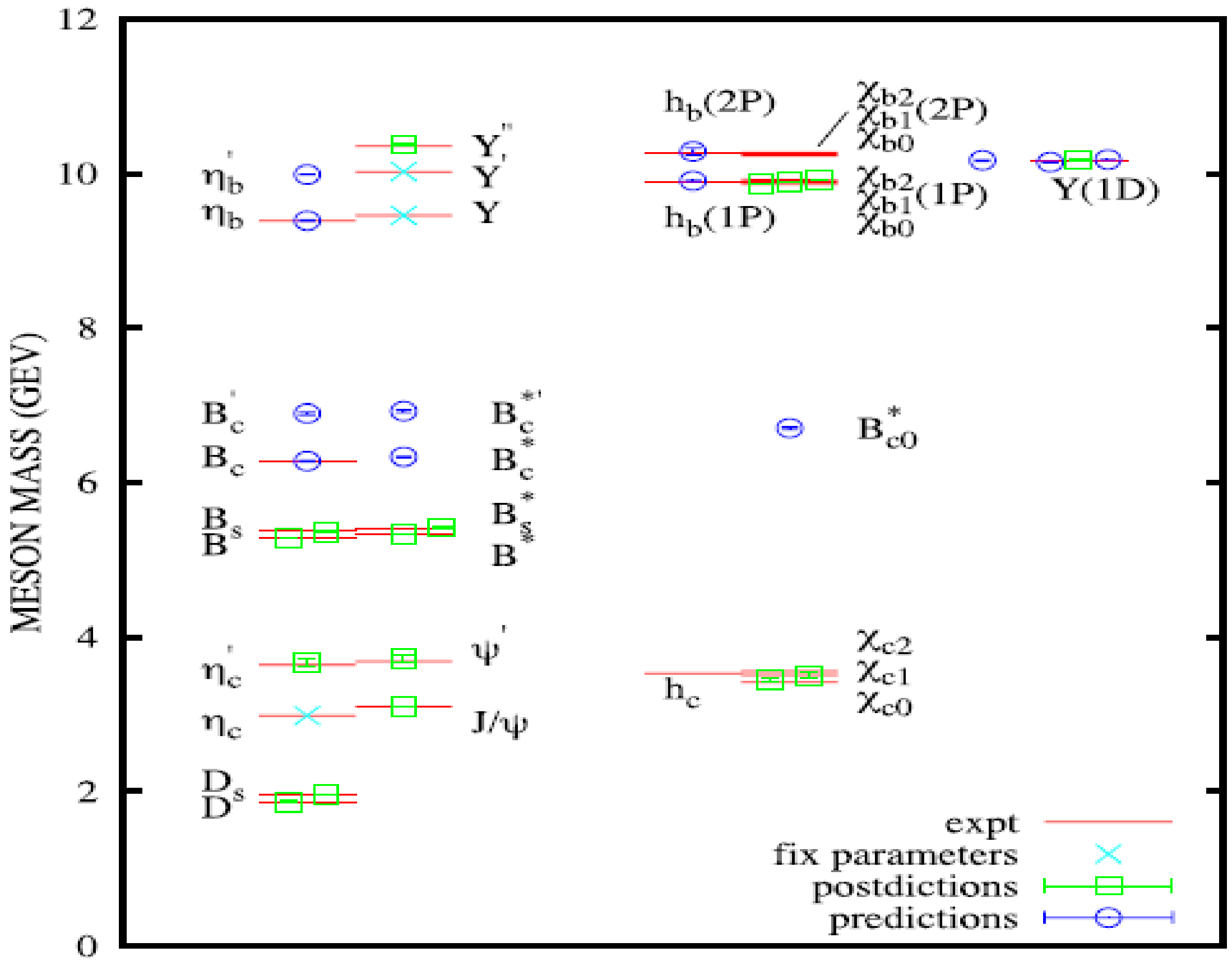
Computational cost $\propto \frac{1}{a^6}$!

Being heavy lattice correlation functions for heavy quarks decay rapidly.

Relativistic charm quark calculations are now possible.

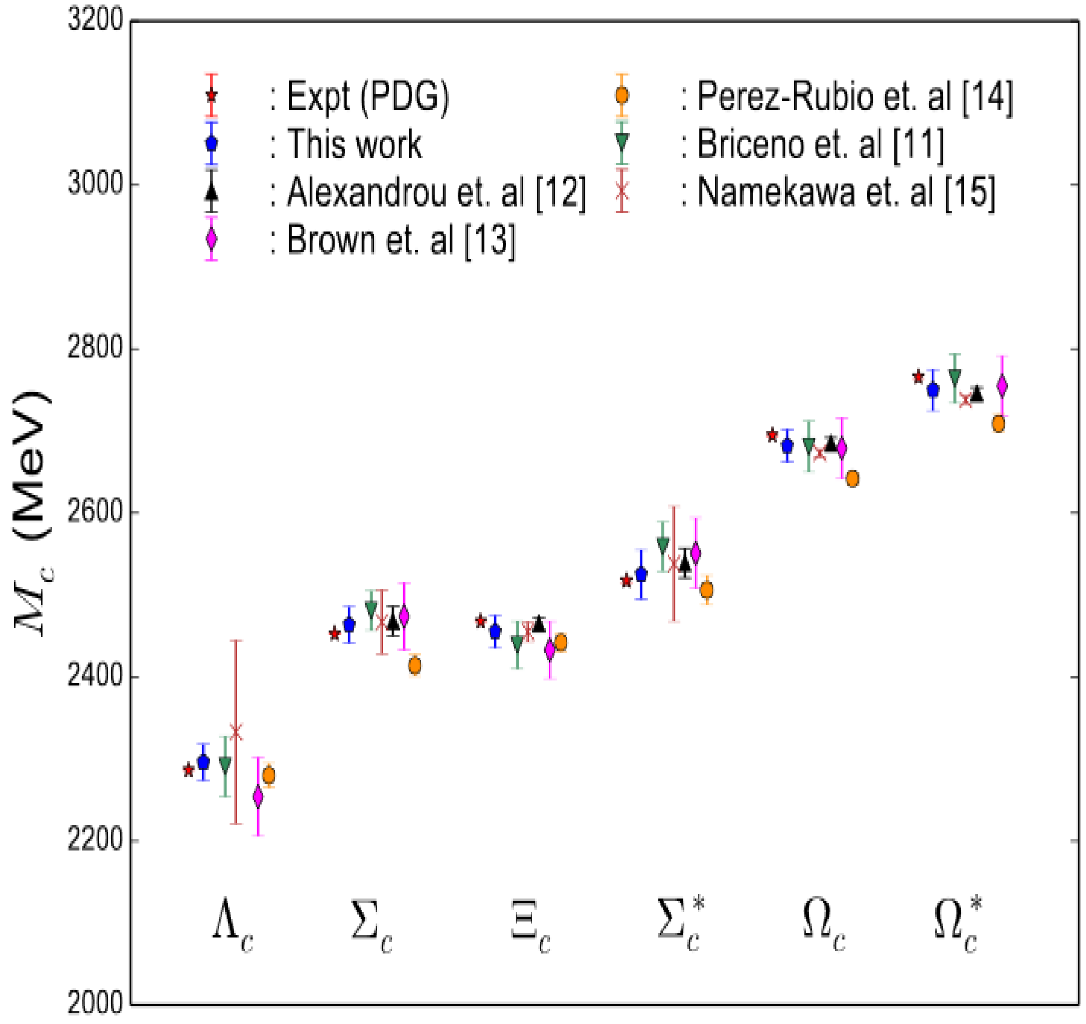
However, relativistic bottom-quark is still prohibitively costly.

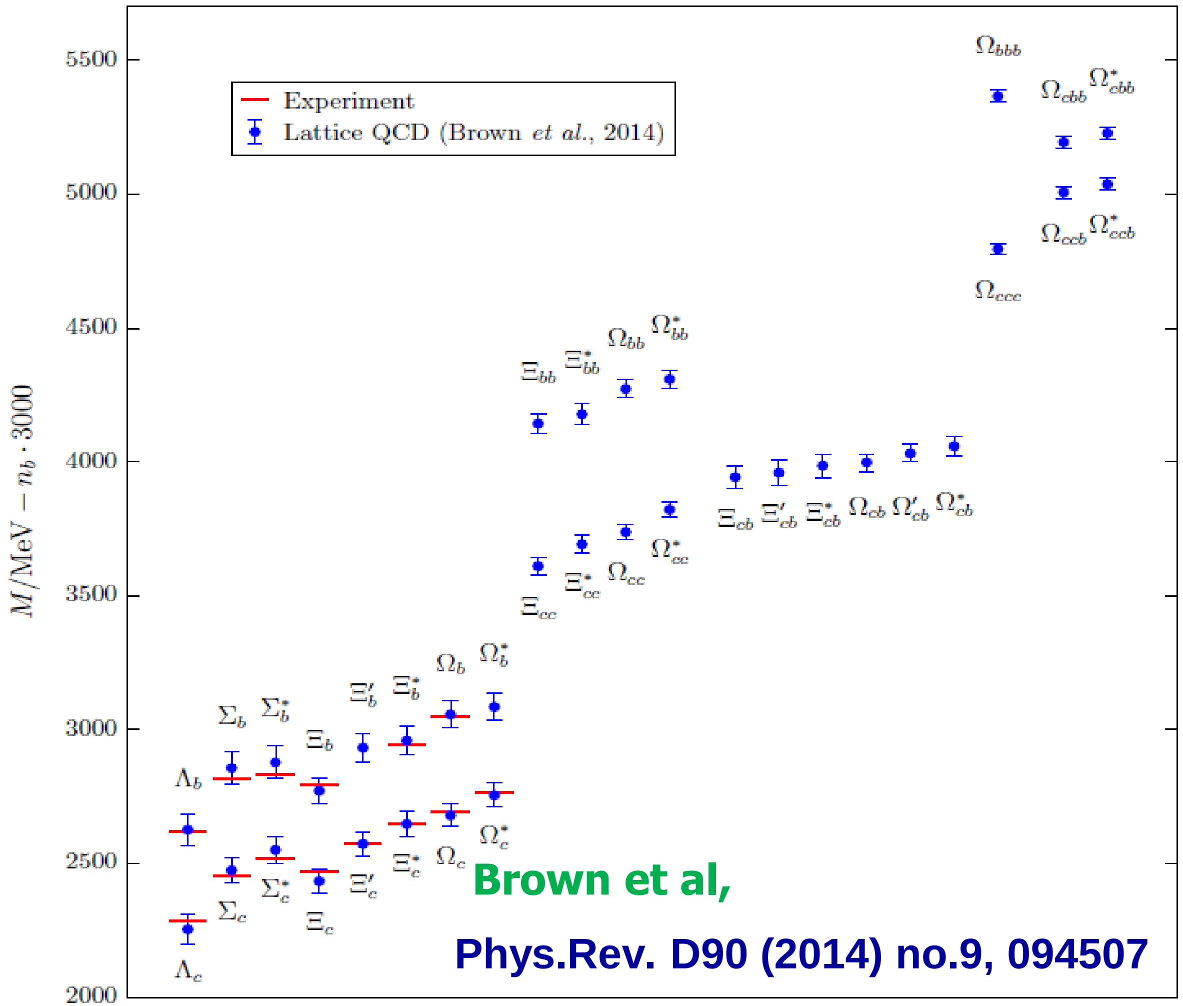
Mesons



HPQCD

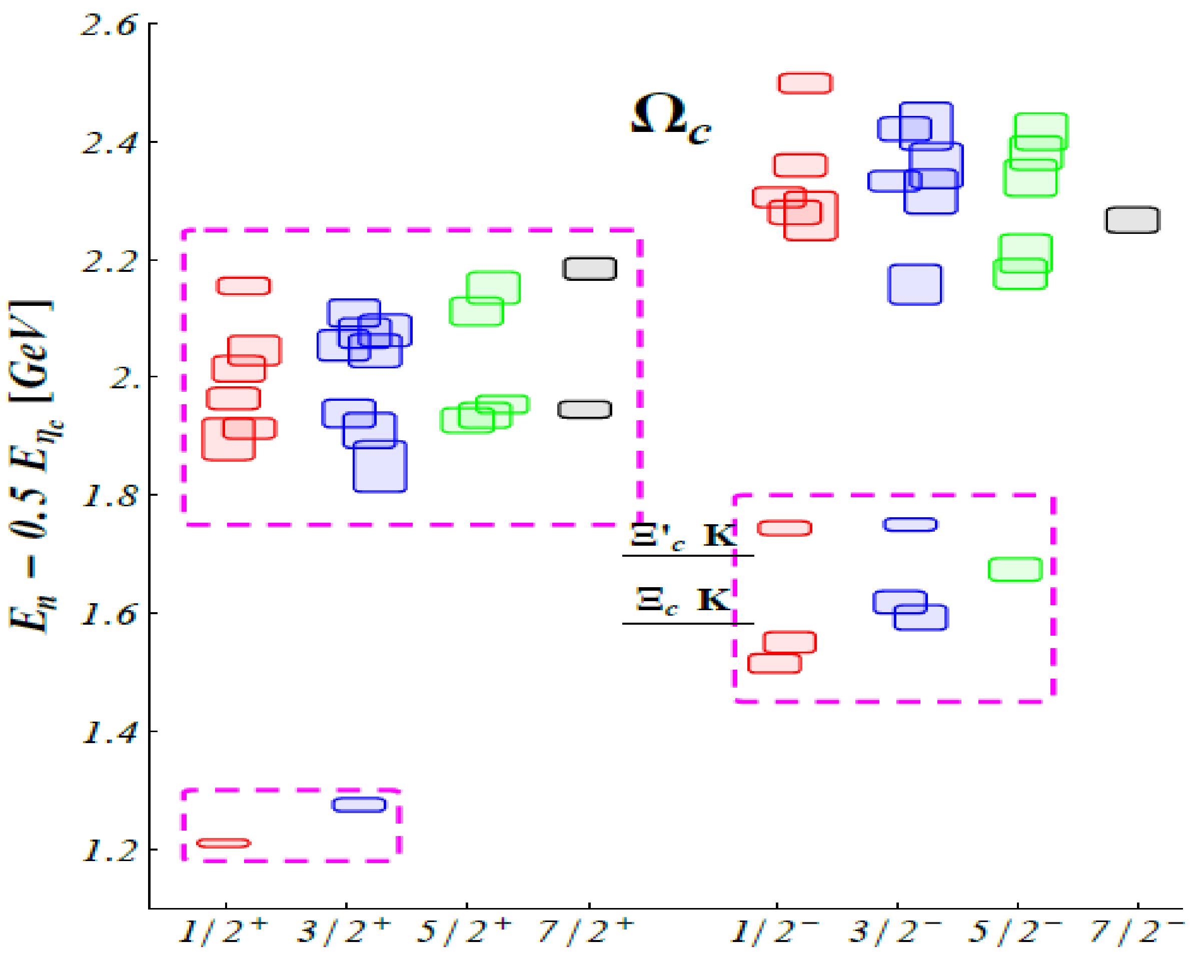
Baryons





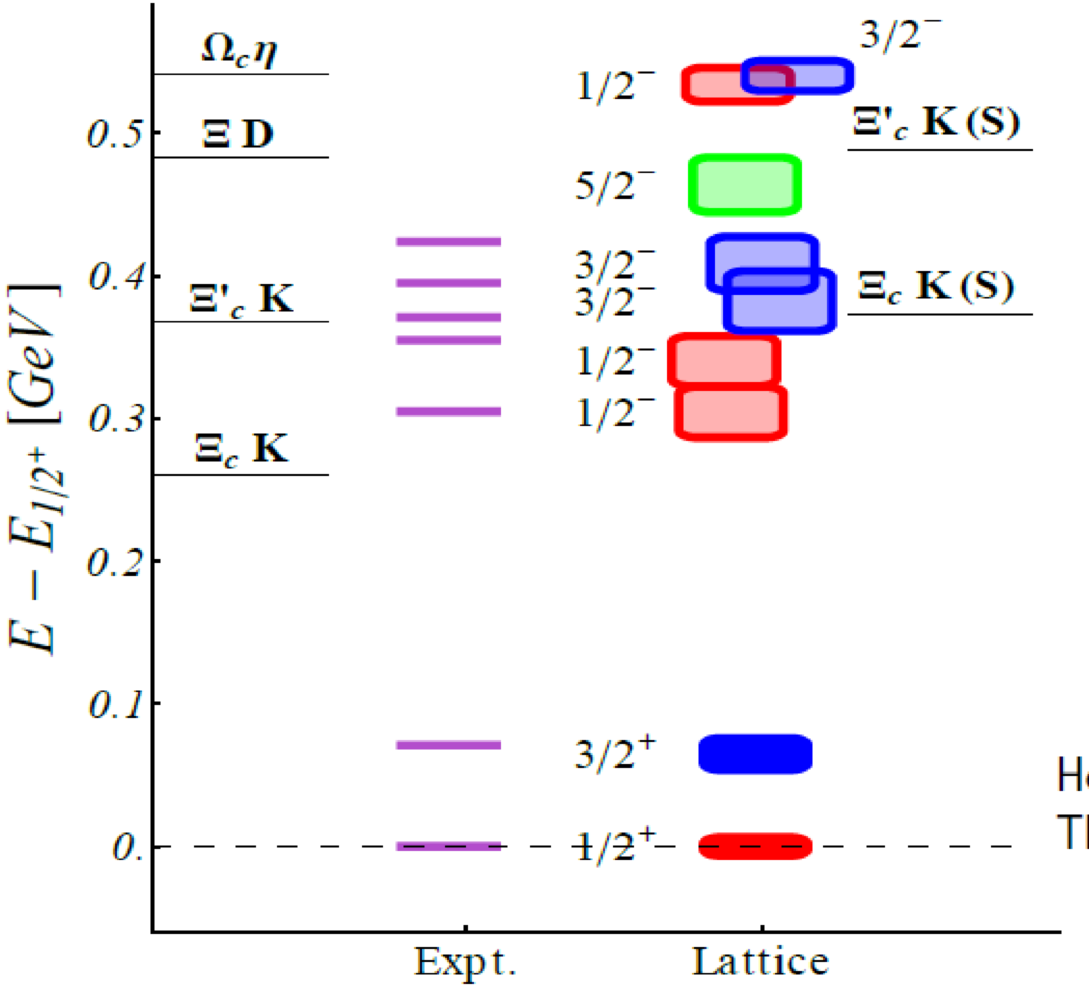
Brown et al,

Phys.Rev. D90 (2014) no.9, 094507



HSC : Padmanath et al, 1311.4806, Charm 2013 and 2015
Padmanath, TIFR thesis (2014)

Padmanath and NM : Phys. Rev. Lett. 119 (2017) no.4, 042001

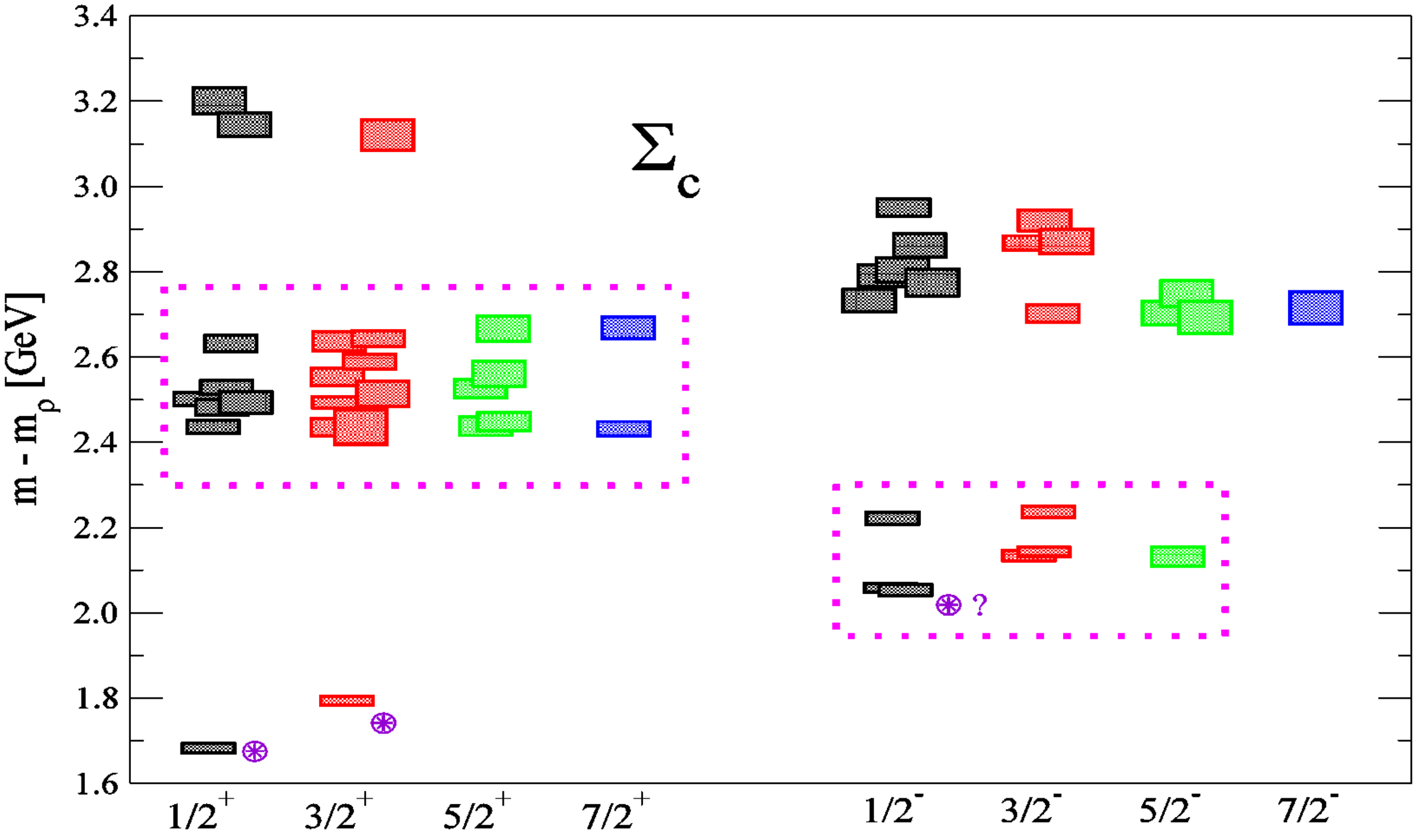


Energy Splittings (ΔE)	Experiment		Lattice	
	ΔE (MeV)	J^P (PDG)	ΔE (MeV)	J^P
$E_{\Omega_c^0} - \frac{1}{2}E_{\eta_c}$	1203(2)	$1/2^+$	1209(7)	$1/2^+$
$\Delta E_{\Omega_c^0(2770)}$	70.7(1)	$3/2^+$	65(11)	$3/2^+$
$\Delta E_{\Omega_c^0(3000)}$	305(1)	?	304(17)	$1/2^-$
$\Delta E_{\Omega_c^0(3050)}$	355(1)	?	341(18)	$1/2^-$
$\Delta E_{\Omega_c^0(3066)}$	371(1)	?	383(21)	$3/2^-$
$\Delta E_{\Omega_c^0(3090)}$	395(1)	?	409(19)	$3/2^-$
$\Delta E_{\Omega_c^0(3119)}$	422(1)	?	464(20)	$5/2^-$

Here $\Delta E^n = E^n - E^0$.
The new states correspond to the excited p-wave states.

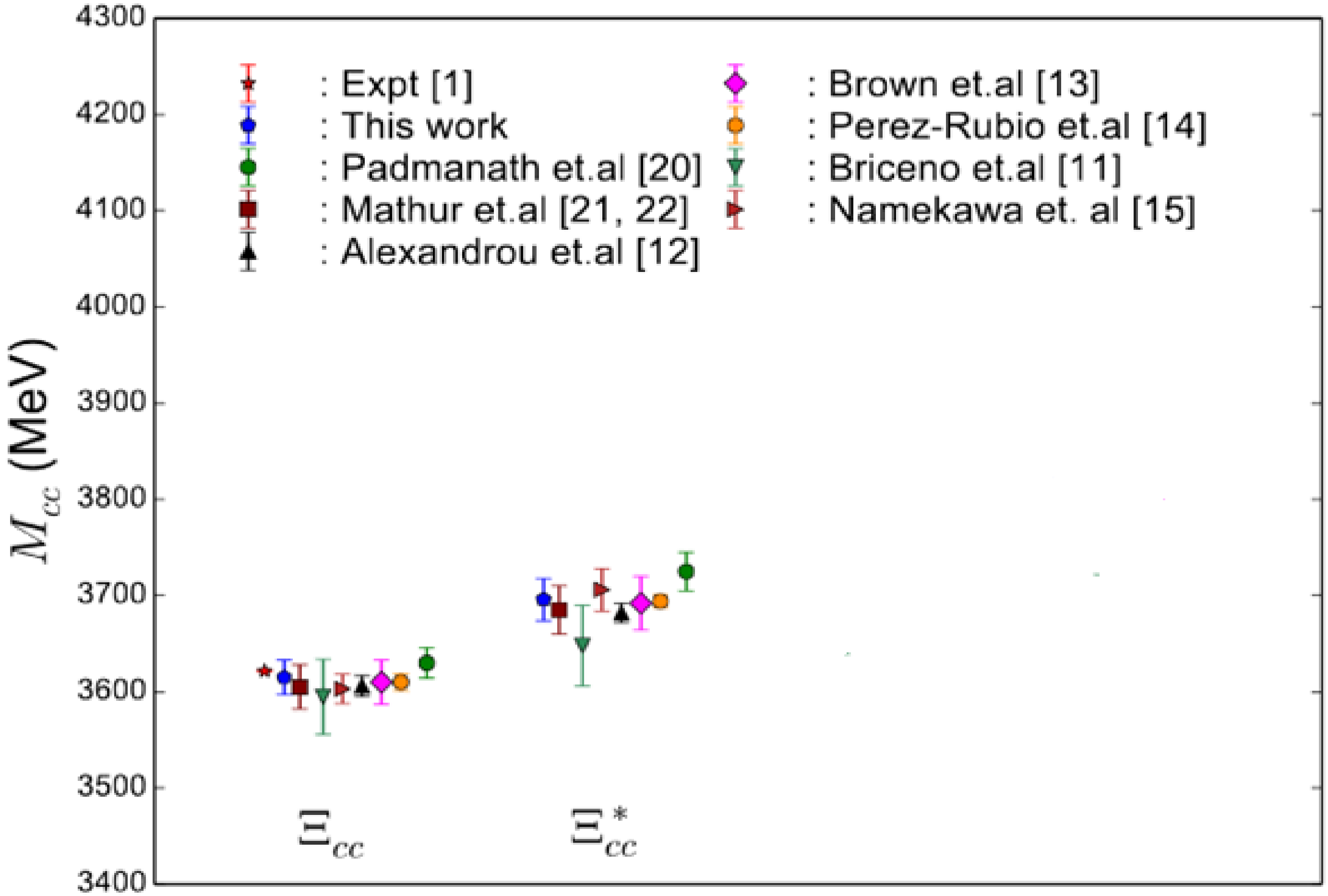
Padmanath and NM: Phys. Rev. Lett. 119 (2017) no.4, 042001

Singly Charmed baryons



HSC : Padmanath et al, 1311.4806
Padmanath, TIFR thesis 2014

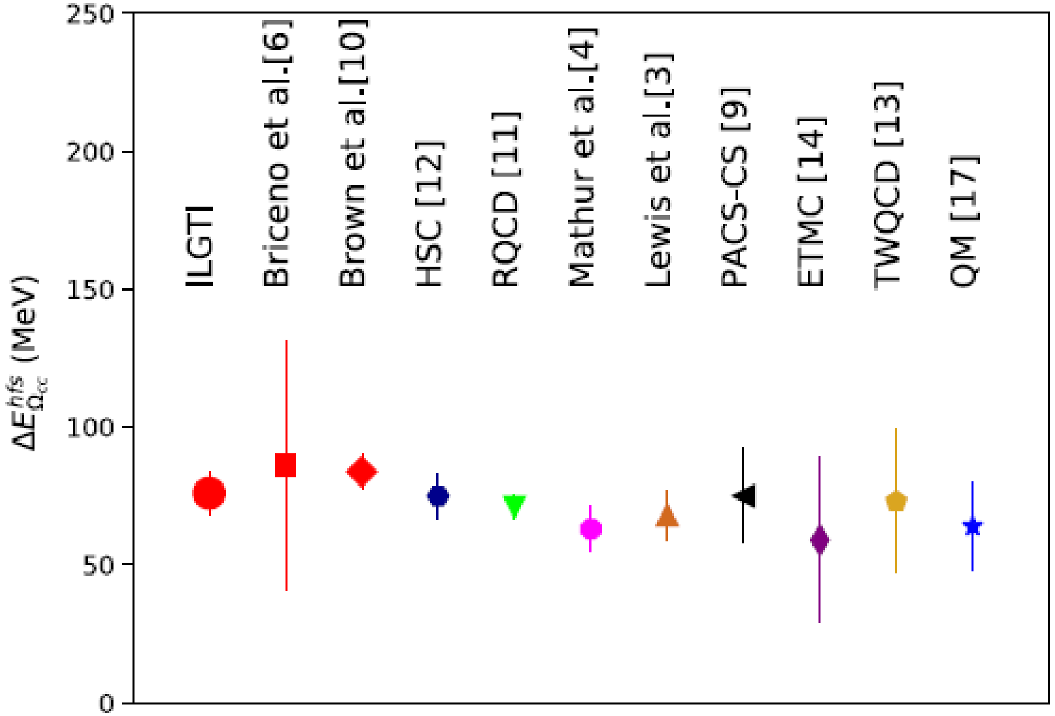
Doubly Charmed baryons

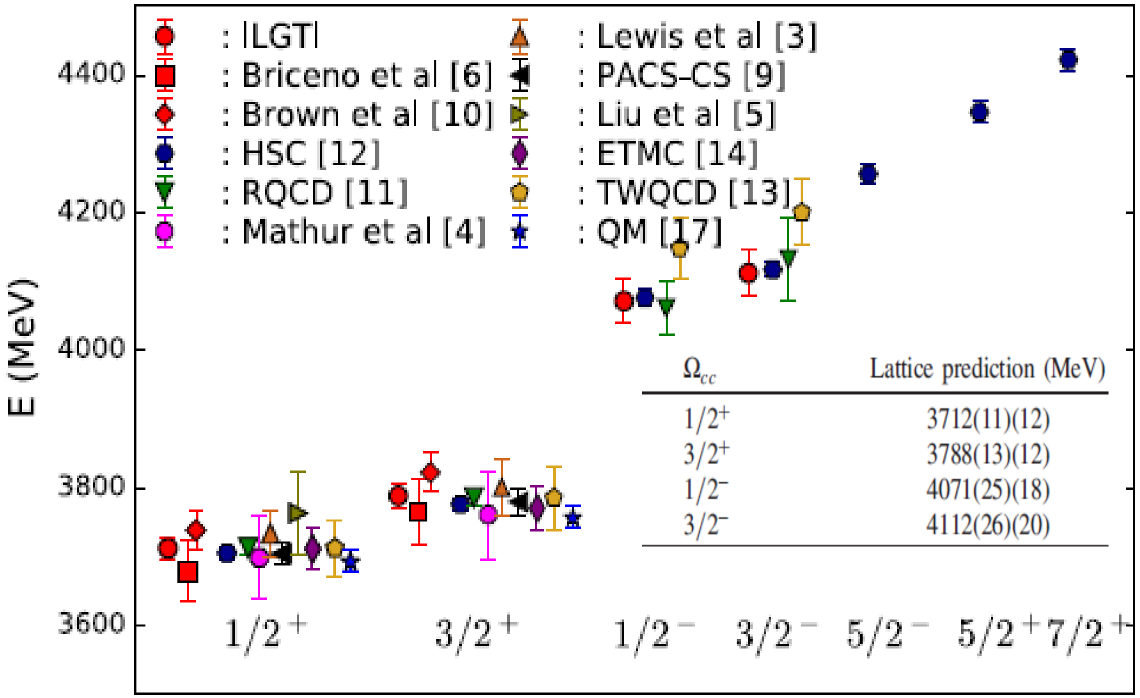


Doubly charmed-strange baryons ($\Omega_{cc}(ccs)$)

Is it the next doubly charmed baryon to be discovered?

Decay to : $\Xi^0 K^+ \pi^+ \pi^+$ and $\Omega_c \pi^+$

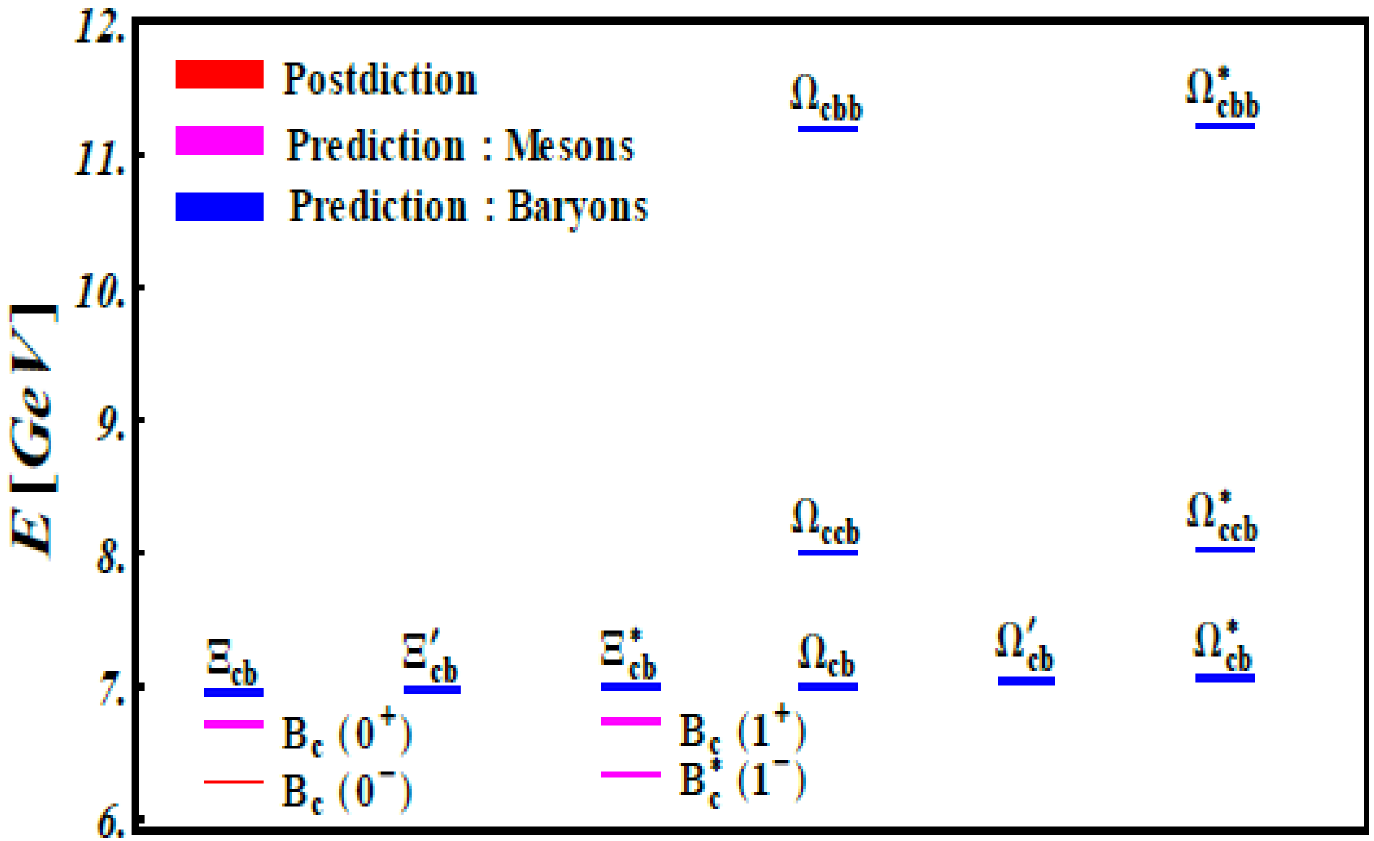




Ω_{cc}	Lattice prediction (MeV)
$1/2^+$	3712(11)(12)
$3/2^+$	3788(13)(12)
$1/2^-$	4071(25)(18)
$3/2^-$	4112(26)(20)

NM and Padmanath : **Phys. Rev. D 99, 031501 (Rapid Comm) (2019)**

bc hadrons

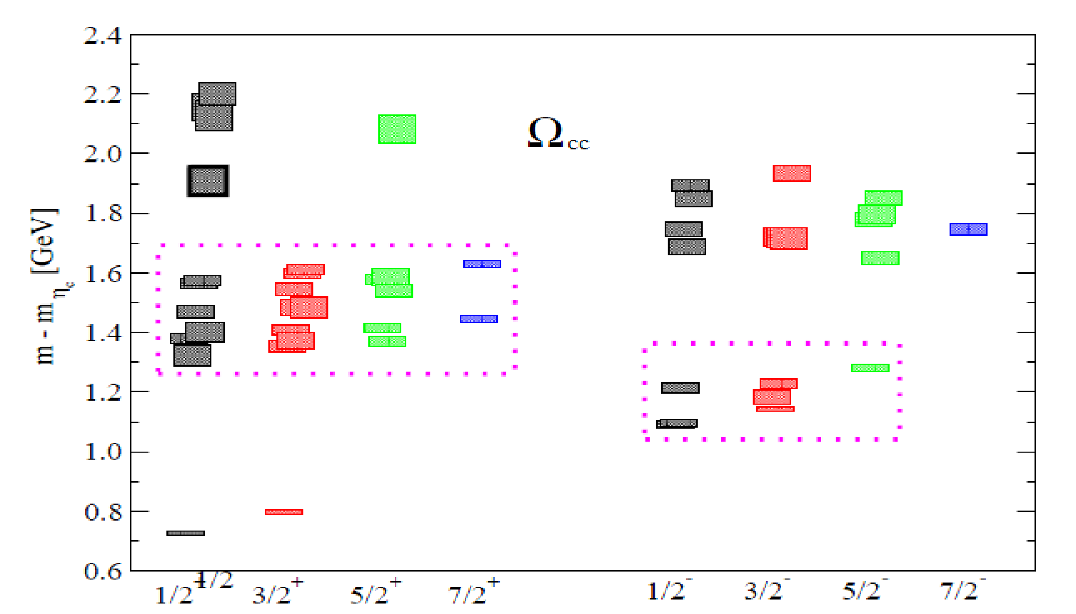


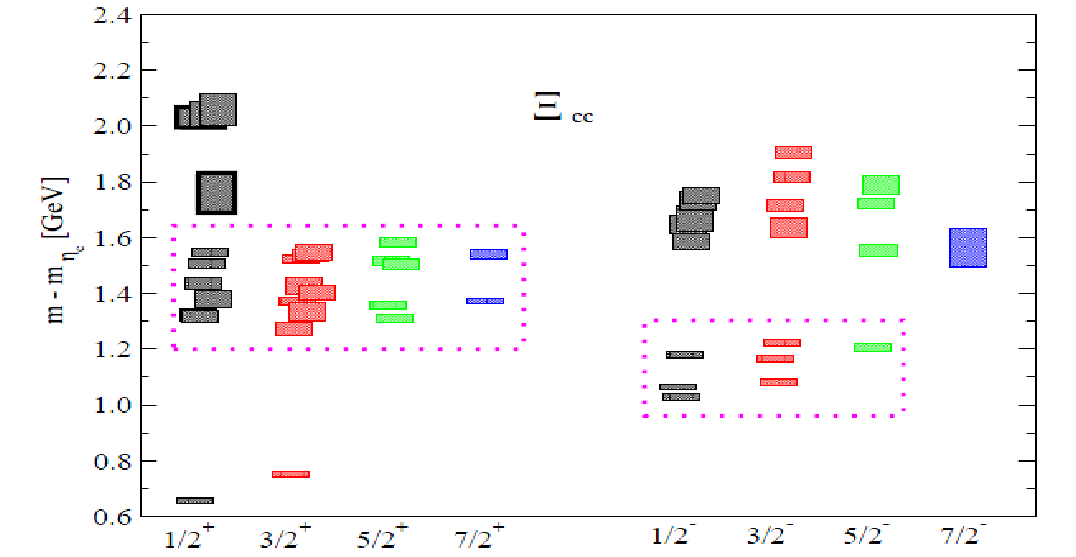
Hadrons	Lattice	Experiment
$B_c(0^-)$	6276(3)(6)	6274.9(8)
$B_c^*(1^-)$	6331(4)(6)	?
$B_c(0^+)$	6712(18)(7)	?
$B_c(1^+)$	6736(17)(7)	?
$\Xi_{cb}(cbu)(1/2^+)$	6945(22)(14)	?
$\Xi'_{cb}(cbu)(1/2^+)$	6966(23)(14)	?
$\Xi^*_{cb}(cbu)(3/2^+)$	6989(24)(14)	?
$\Omega_{cb}(cbs)(1/2^+)$	6994(15)(13)	?
$\Omega'_{cb}(cbs)(1/2^+)$	7045(16)(13)	?
$\Omega^*_{cb}(cbs)(3/2^+)$	7056(17)(13)	?
$\Omega_{ccb}(1/2^+)$	8005(6)(11)	?
$\Omega^*_{ccb}(3/2^+)$	8026(7)(11)	?
$\Omega_{cbb}(1/2^+)$	11194(5)(12)	?
$\Omega^*_{cbb}(3/2^+)$	11211(6)(12)	?

Mesons($\bar{q}_1 q_2$)	Baryons ($[q_1q_2q_3](J^P)$)		
	$J^P \equiv 1/2^+$	$1/2^+$	$3/2^+$
$B_c(\bar{b}c)(0^-)$	$\Xi_{cb}[cbu]$	$\Xi'_{cb}[cbu]$	$\Xi^*_{cb}[cbu]$
$B_c^*(\bar{b}c)(1^-)$	$\Omega_{cb}[cbs]$	$\Omega'_{cb}[cbs]$	$\Omega^*_{cb}[cbs]$
$B_c(\bar{b}c)(0^+)$	$\Omega_{ccb}[ccb]$		$\Omega^*_{ccb}[ccb]$
$B_c(\bar{b}c)(1^+)$	$\Omega_{cbb}[bbc]$		$\Omega^*_{cbb}[bbc]$

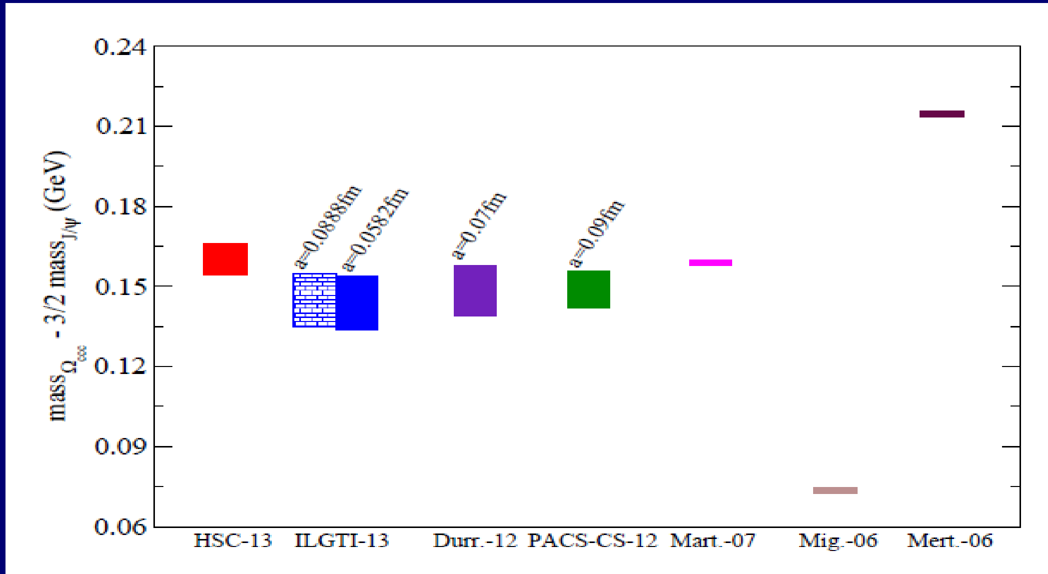
NM, Padmanath and Mondal :
Phys. Rev. Lett. 121, 202002 (2018)

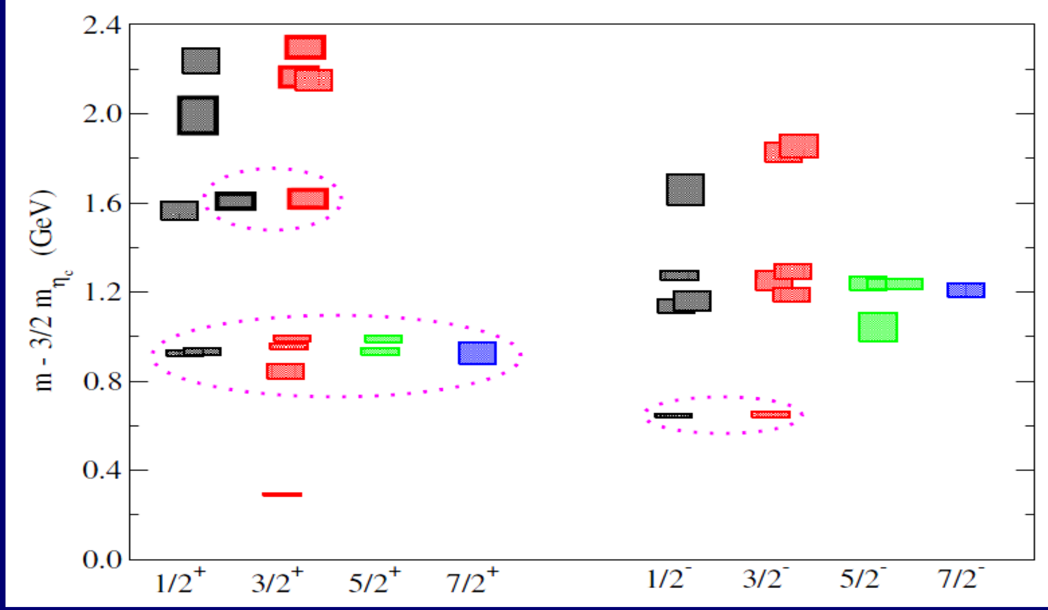
DOUBLY CHARM





TRIPLY CHARM

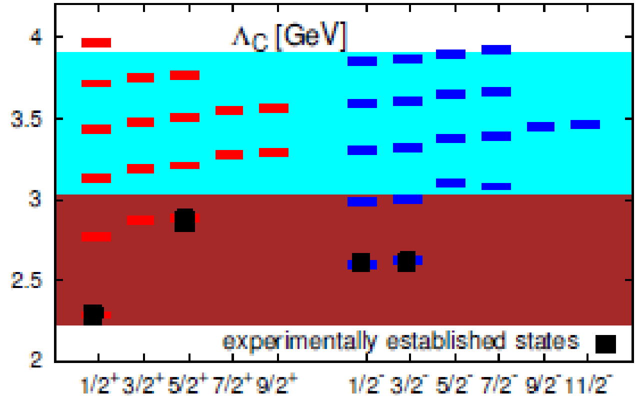




HSC : Padmanath, NM et al, 1311.4354

NM and Padmanath et al, arXiv:1311.4806

Ebert *et al.*, PRD, **84**, 014025, 2011



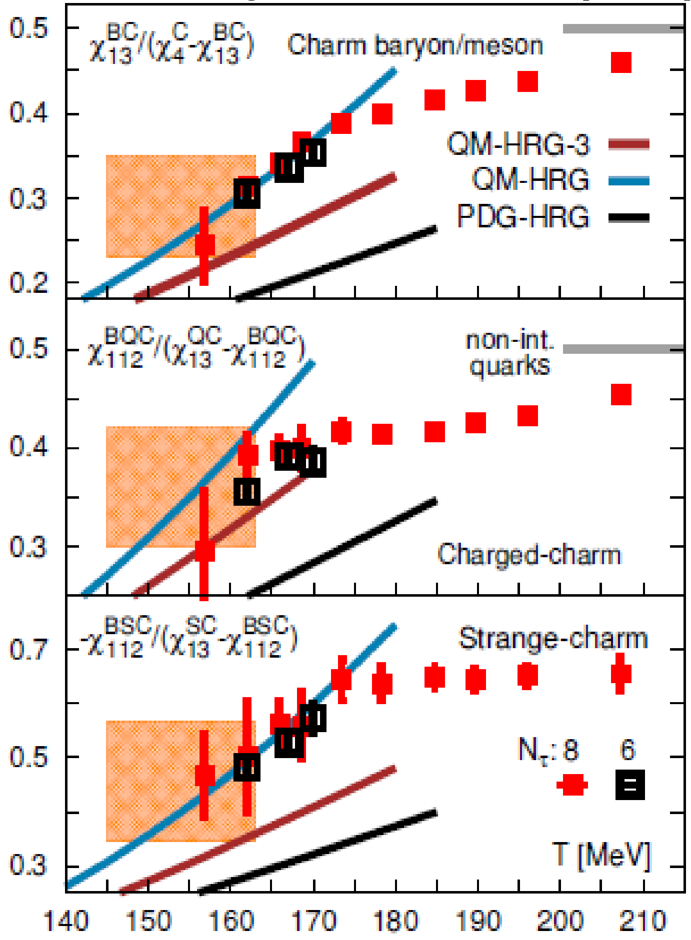
- Charm hadron pressure (HRG) :

$$P(\hat{\mu}_C, \hat{\mu}_B) = P_M cosh(\hat{\mu}_C) + P_{B,C=1} cosh(\hat{\mu}_C + \hat{\mu}_B)$$

$$\chi_{kl}^{BC} = \frac{\partial^{(k+l)}[P(\hat{\mu}_C, \hat{\mu}_B)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_C^l}$$

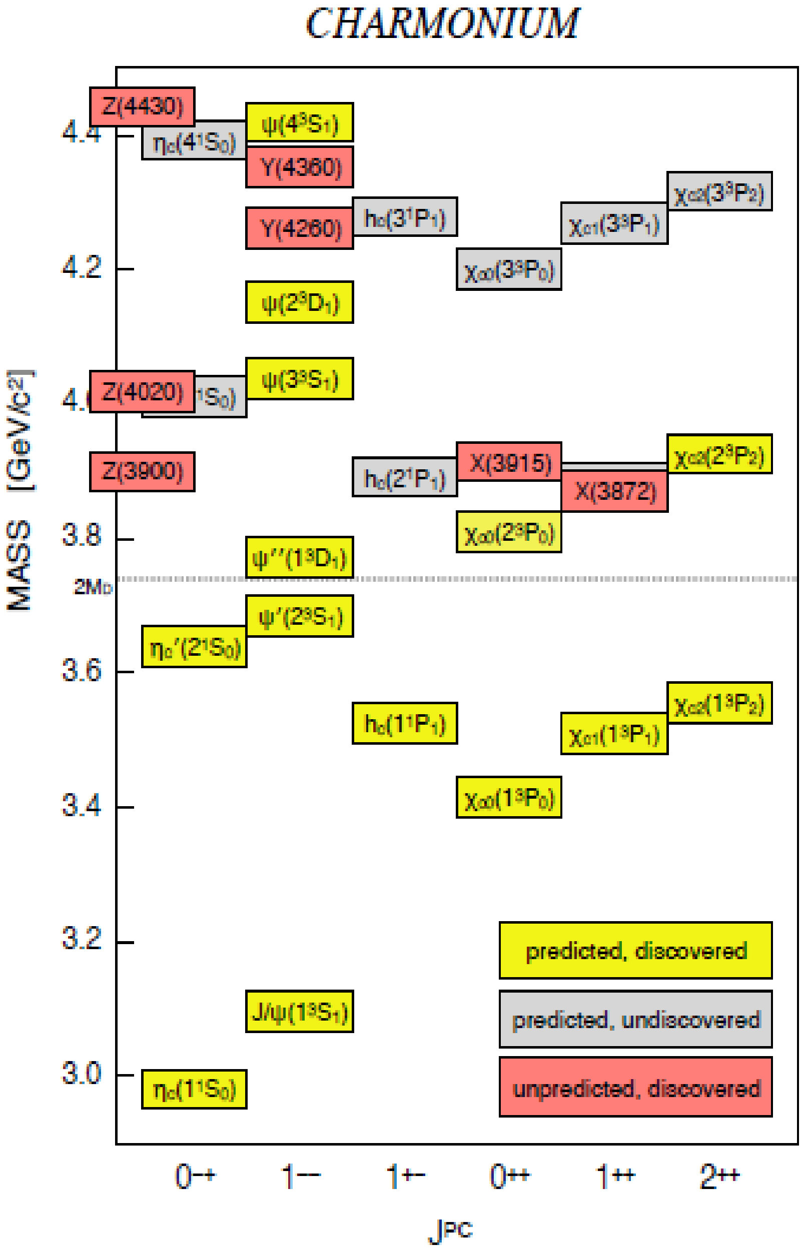
Bazavov *et al.*, PLB, **737**, 210, 2014

Phys.Rev.Lett. 113 (2014) no.7, 072001



Exotic heavy flavour states

Newly found heavy flavour exotics



Q: What are heavy quark exotica?

A: Phenomena in the heavy quark sector that do not easily fit into the naive quark model picture of mesons and baryons.

Q: Why are they interesting?

A: They can be used to explore novel phenomena in QCD:

hybrid mesons, tetraquarks, pentaquarks, molecules, hadroquarkonium, thresholds

Q: Why are they called XYZ?

A: Mostly historical reasons.

But now there are patterns:

Z: electrically charged (I = 1).

Y: $J^{PC} = 1^{--}$, made directly in e^+e^-.

X: whatever is leftover.

But there are many exceptions!

[And the PDG has now renamed them by IJ^{PC}.]

Q: How many have been found?

A: Many.

R. Mitchel@Quark confinement 2018

R. Lebed et al:

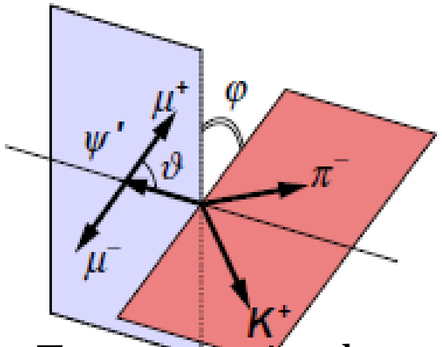
Progress in Particle and Nuclear Physics 93, 143 (2017)

Particle	$I^G J^{PC}$	Mass [MeV]	Width [MeV]	Production and Decay
$X(3823)$ ($\psi_2(1D)$)	(0^-2^{--})	3822.2 ± 1.2 [176]	< 16	$B \to KX;\ X \to \gamma\chi_{c1}$ $e^+e^- \to \pi^+\pi^- X;\ X \to \gamma\chi_{c1}$
$X(3872)$	0^+1^{++}	3871.69 ± 0.17 [176]	< 1.2	$B \to KX;\ X \to \pi^+\pi^- J/\psi$ $B \to KX;\ X \to D^{*0}\bar{D}^0$ $B \to KX;\ X \to \gamma J/\psi, \gamma\psi(2S)$ $B \to KX;\ X \to \omega J/\psi$ $B \to K\pi X;\ X \to \pi^+\pi^- J/\psi$ $e^+e^- \to \gamma X;\ X \to \pi^+\pi^- J/\psi$ pp or $p\bar{p} \to X$ + any.; $X \to \pi^+\pi^- J/\psi$
$Z_c(3900)$	1^+1^{+-}	3886.6 ± 2.4 [176]	28.1 ± 2.6	$e^+e^- \to \pi Z;\ Z \to \pi J/\psi$ $e^+e^- \to \pi Z;\ Z \to D^*\bar{D}$
$X(3915)$	0^+0^{++}	3918.4 ± 1.9 [176]	20 ± 5	$\gamma\gamma \to X;\ X \to \omega J/\psi$
$Y(3940)$				$B \to KX;\ X \to \omega J/\psi$
$Z(3930)$ ($\chi_{c2}(2P)$)	0^+2^{++}	3927.2 ± 2.6 [176]	24 ± 6	$\gamma\gamma \to Z;\ Z \to D\bar{D}$
$X(3940)$		$3942^{+7}_{-6} \pm 6$ [41]	$37^{+26}_{-15} \pm 8$	$e^+e^- \to J/\psi + X;\ X \to D\bar{D}^*$
$Y(4008)$	1^{--}	$3891 \pm 41 \pm 12$ [23]	$255 \pm 40 \pm 14$	$e^+e^- \to Y;\ Y \to \pi^+\pi^- J/\psi$
$Z_c(4020)$	$1^+?^{?-}$	4024.1 ± 1.9 [176]	13 ± 5	$e^+e^- \to \pi Z;\ Z \to \pi h_c$ $e^+e^- \to \pi Z;\ Z \to D^*\bar{D}^*$
$Z_1(4050)$	$1^-?^{?+}$	$4051 \pm 14^{+20}_{-41}$ [133]	82^{+21+47}_{-17-22}	$B \to KZ;\ Z \to \pi^\pm\chi_{c1}$
$Z_c(4055)$	$1^+?^{?-}$	$4054 \pm 3 \pm 1$ [148]	$45 \pm 11 \pm 6$	$e^+e^- \to \pi^\mp Z;\ Z \to \pi^\pm\psi(2S)$
$Y(4140)$	0^+1^{++}	$4146.5 \pm 4.5^{+4.6}_{-2.8}$ [125]	$83 \pm 21^{+21}_{-14}$	$B \to KY;\ Y \to \phi J/\psi$ pp or $p\bar{p} \to Y$ + any.; $Y \to \phi J/\psi$
$X(4160)$		$4156^{+25}_{-20} \pm 15$ [41]	$139^{+111}_{-61} \pm 21$	$e^+e^- \to J/\psi + X;\ X \to D^*\bar{D}^*$
$Z_c(4200)$	1^+1^{+-}	4196^{+31+17}_{-29-13} [46]	$370^{+70+70}_{-70-132}$	$B \to KZ;\ Z \to \pi^\pm J/\psi$
$Y(4230)$	0^-1^{--}	$4230 \pm 8 \pm 6$ [149]	$38 \pm 12 \pm 2$	$e^+e^- \to Y;\ Y \to \omega\chi_{c0}$
$Z_c(4240)$	1^+0^{--}	$4239 \pm 18^{+45}_{-10}$ [138]	$220 \pm 47^{+108}_{-74}$	$B \to KZ;\ Z \to \pi^\pm\psi(2S)$
$Z_2(4250)$	$1^-?^{?+}$	$4248^{+44+180}_{-29-35}$ [133]	$177^{+54+316}_{-39-61}$	$B \to KZ;\ Z \to \pi^\pm\chi_{c1}$
$Y(4260)$	0^-1^{--}	4251 ± 9 [176]	120 ± 12	$e^+e^- \to Y;\ Y \to \pi\pi J/\psi$
$Y(4274)$	0^+1^{++}	$4273.3 \pm 8.3^{+17.2}_{-3.6}$ [125]	$52 \pm 11^{+8}_{-11}$	$B \to KY;\ Y \to \phi J/\psi$
$X(4350)$	$0^+?^{?+}$	$4350.6^{+4.6}_{-5.1} \pm 0.7$ [170]	$13^{+18}_{-9} \pm 4$	$\gamma\gamma \to X;\ X \to \phi J/\psi$
$Y(4360)$	1^{--}	4346 ± 6 [176]	102 ± 10	$e^+e^- \to Y;\ Y \to \pi^+\pi^-\psi(2S)$
$Z_c(4430)$	1^+1^{+-}	4478^{+15}_{-18} [176]	181 ± 31	$B \to KZ;\ Z \to \pi^\pm J/\psi$ $B \to KZ;\ Z \to \pi^\pm\psi(2S)$
$X(4500)$	0^+0^{++}	$4506 \pm 11^{+12}_{-15}$ [125]	$92 \pm 21^{+21}_{-20}$	$B \to KX;\ X \to \phi J/\psi$
$X(4630)$	1^{--}	4634^{+8+5}_{-7-8} [150]	92^{+40+10}_{-24-21}	$e^+e^- \to X;\ X \to \Lambda_c\bar{\Lambda}_c$
$Y(4660)$	1^{--}	4643 ± 9 [176]	72 ± 11	$e^+e^- \to Y;\ Y \to \pi^+\pi^-\psi(2S)$
$X(4700)$	0^+0^{++}	$4704 \pm 10^{+14}_{-24}$ [125]	$120 \pm 31^{+42}_{-33}$	$B \to KX;\ X \to \phi J/\psi$
$P_c(4380)$		$4380 \pm 8 \pm 29$ [35]	$205 \pm 18 \pm 86$	$\Lambda_b \to KP_c;\ P_c \to pJ/\psi$
$P_c(4450)$		$4449.8 \pm 1.7 \pm 2.5$ [35]	$39 \pm 5 \pm 19$	$\Lambda_b \to KP_c;\ P_c \to pJ/\psi$
$X(5568)$		$5567.8 \pm 2.9^{+0.9}_{-1.9}$ [175]	$21.9 \pm 6.4^{+5.0}_{-2.5}$	$p\bar{p} \to X$ + anything; $X \to B_s\pi^\pm$
$Z_b(10610)$	1^+1^{+-}	10607.2 ± 2.0 [176]	18.4 ± 2.4	$e^+e^- \to \pi Z;\ Z \to \pi\Upsilon(1S,2S,3S)$ $e^+e^- \to \pi Z;\ Z \to \pi h_b(1P,2P)$ $e^+e^- \to \pi Z;\ Z \to B\bar{B}^*$
$Z_b(10650)$	1^+1^{+-}	10652.2 ± 1.5 [176]	11.5 ± 2.2	$e^+e^- \to \pi Z;\ Z \to \pi\Upsilon(1S,2S,3S)$ $e^+e^- \to \pi Z;\ Z \to \pi h_b(1P,2P)$ $e^+e^- \to \pi Z;\ Z \to B^*\bar{B}^*$
$Y_b(10888)$	0^-1^{--}	10891 ± 4 [176]	54 ± 7	$e^+e^- \to Y;\ Y \to \pi\pi\Upsilon(1S,2S,3S)$ $e^+e^- \to Y;\ Y \to \pi\pi h_b(1P,2P)$

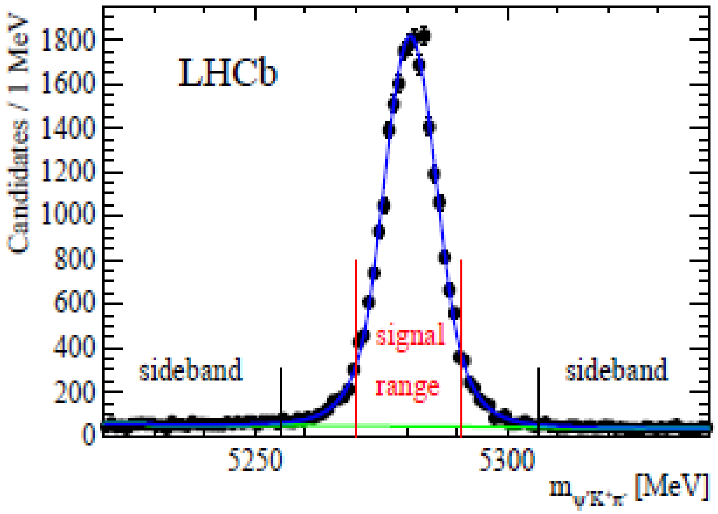
$Z(4430)$ in $B \to \psi(2S)K^+\pi^-$

[PRL 112, 222002 (2014)]

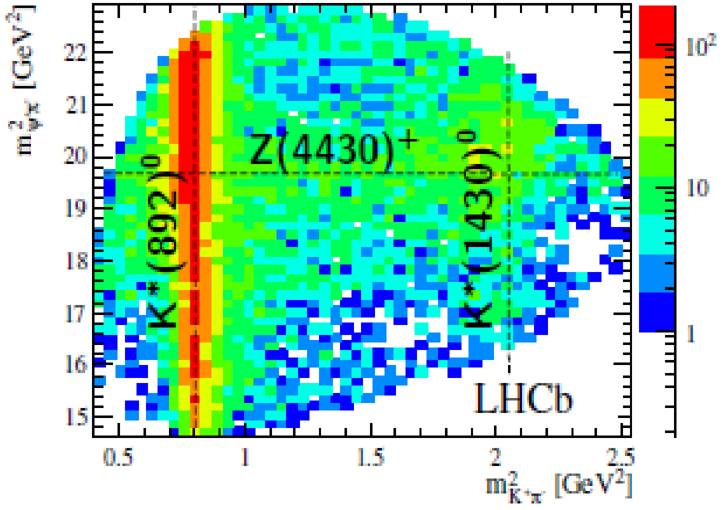
- Decay $B^0 \to \psi(2S)K^+\pi^-$
- Signal yield: 25k events
- Combinatorial background: $\sim 4\%$
- 4D amplitude analysis:
  $(m^2(K\pi),\ m^2(\psi(2S)\pi),\ \theta_{\psi'},\ \phi_{\psi'})$



Type equation here.



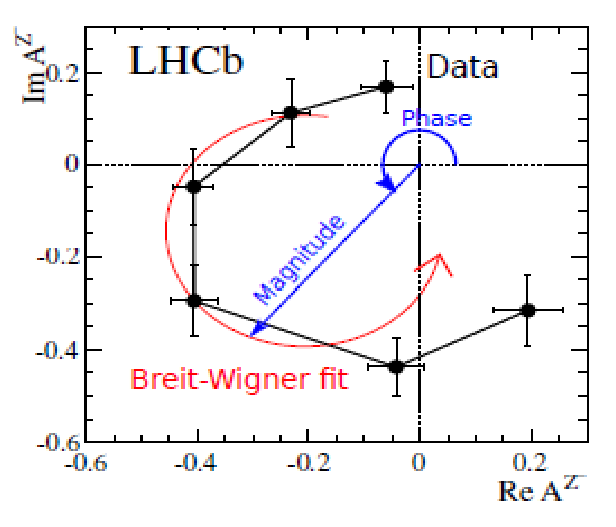
$\psi(2S)\pi \equiv c\bar{c}\bar{d}u$



Structure in $\psi(2S)\pi^-$ spectrum

Model-independent test of phase rotation

[PRL 112, 222002 (2014)]



K^* states provide reference amplitude for phase motion measurement.

Clockwise rotation: characteristic of a resonant behaviour.

How to build a stable tetraquark?

- Two heavy quarks with two light quarks
- $C_l = \bar{3}$, good light diquark

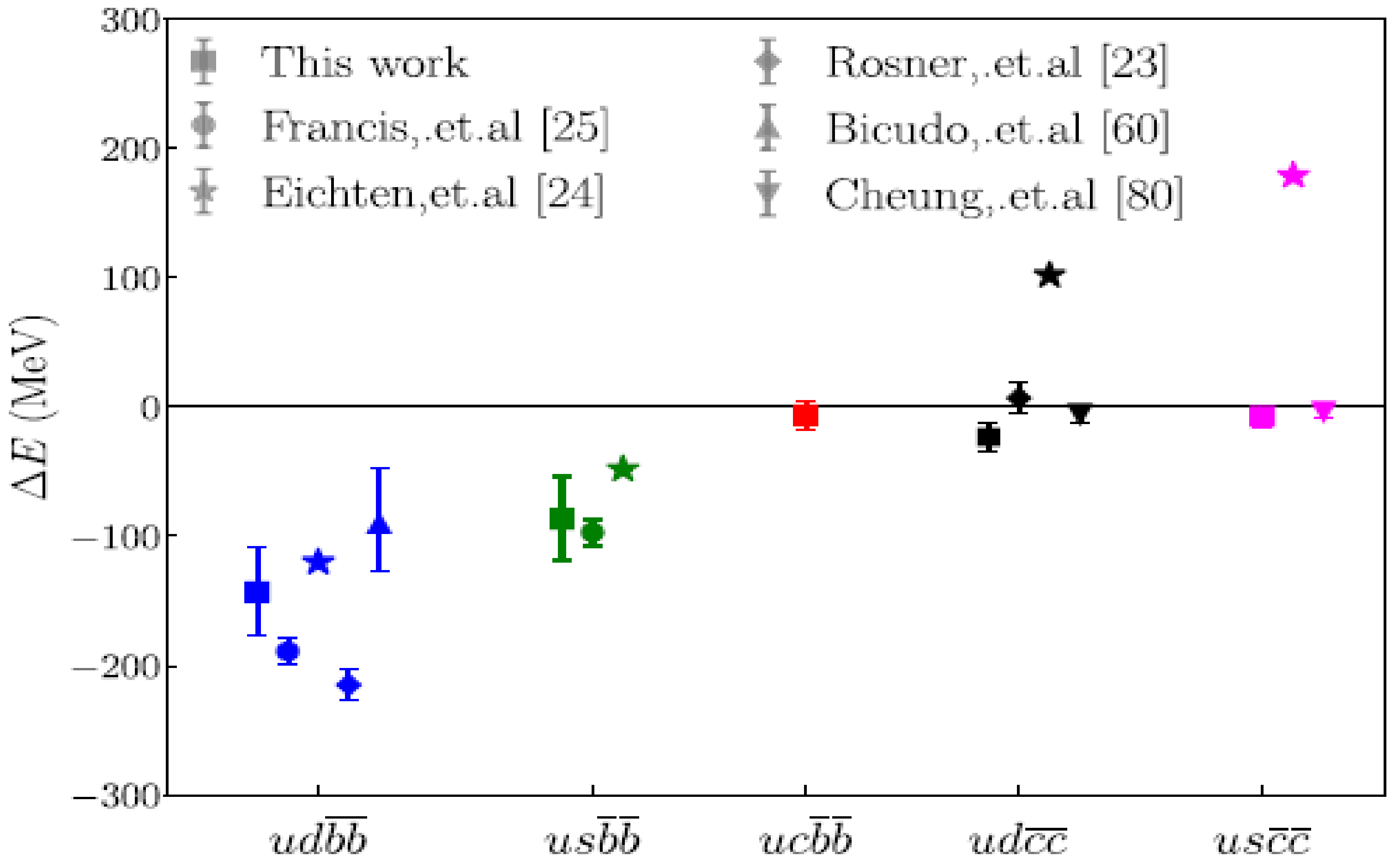
  - $F = \bar{3}, J_l = 0 \Rightarrow J_h = 1, C_h = 3, \; J^P = 1^+$

  - Spin dependent interaction $\propto 1/m_h$. For threshold $J^P = 1^+$ states, like $B^*B, B^*D, B_s^*B_s, D^*D$, this interaction will be supressed.

  - With $C_h = 3$, color Coulomb attraction, this is not present for two-meson thresholds.

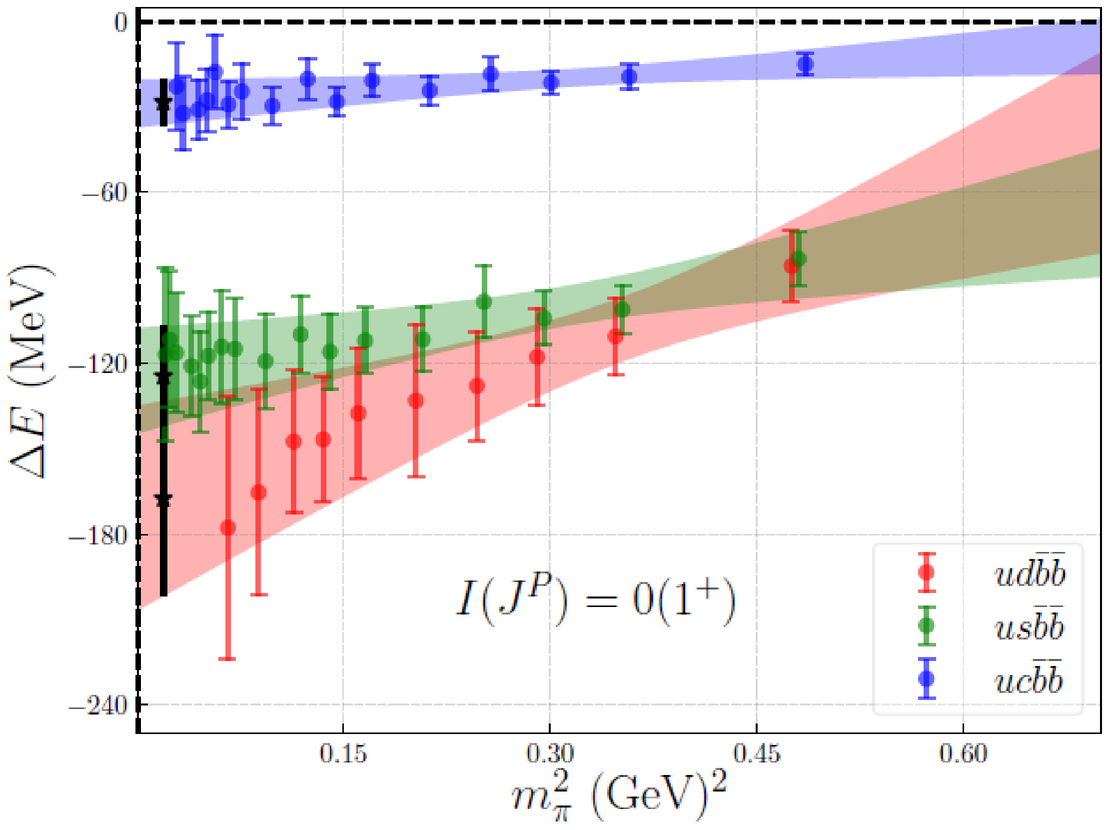
Possible states : $\bar{b}\bar{b}ud, \bar{b}\bar{b}us, \bar{b}\bar{b}uc, \bar{b}\bar{b}sc,$
$\bar{b}\bar{c}ud, \bar{b}\bar{c}us \;\; etc.$

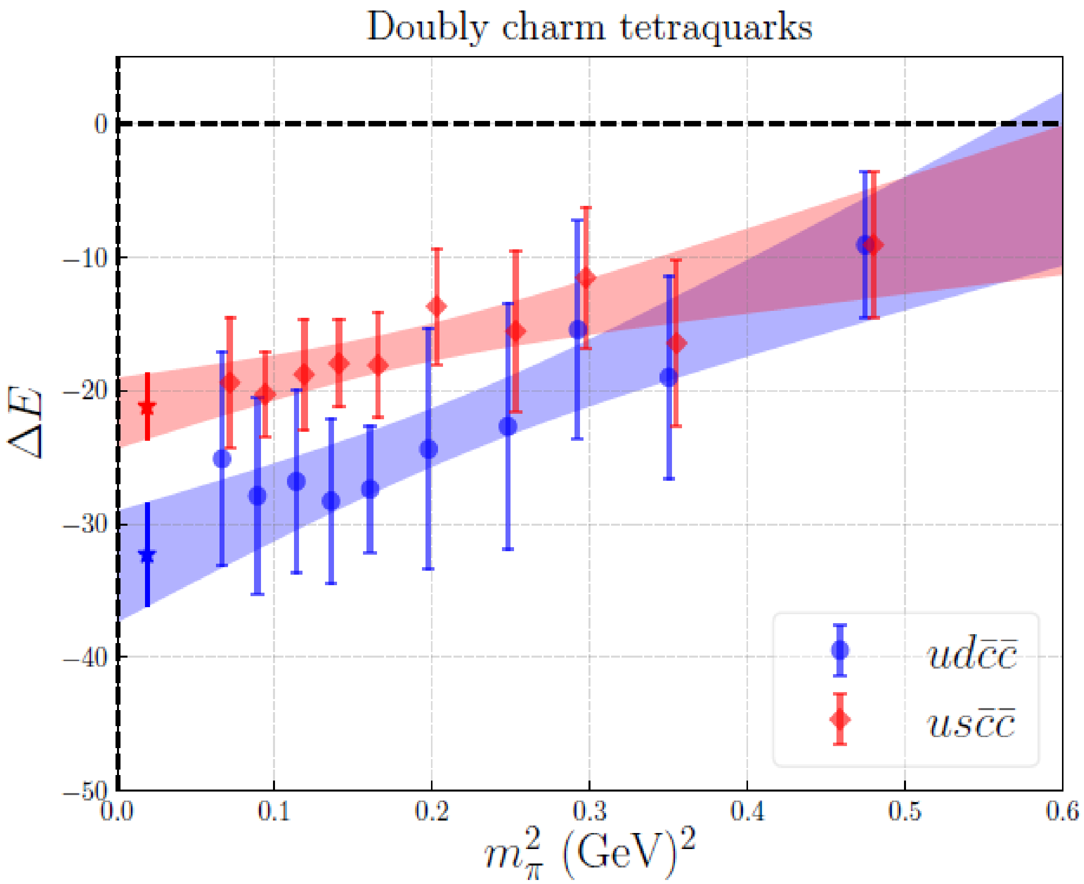
Possible heavy tetraquarks

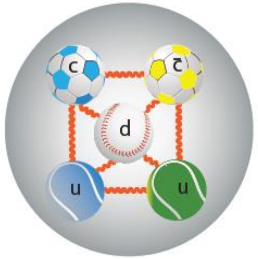


Junnarkar, NM and Padmanath:
Phys. Rev. D99, 034507 (2019)

Quark mass dependence of the binding energy

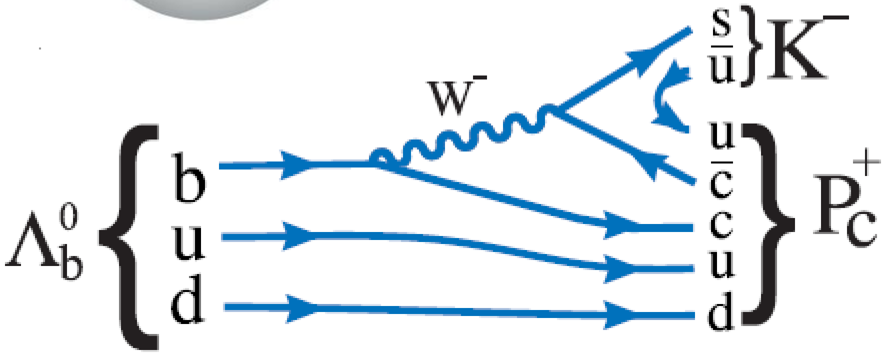




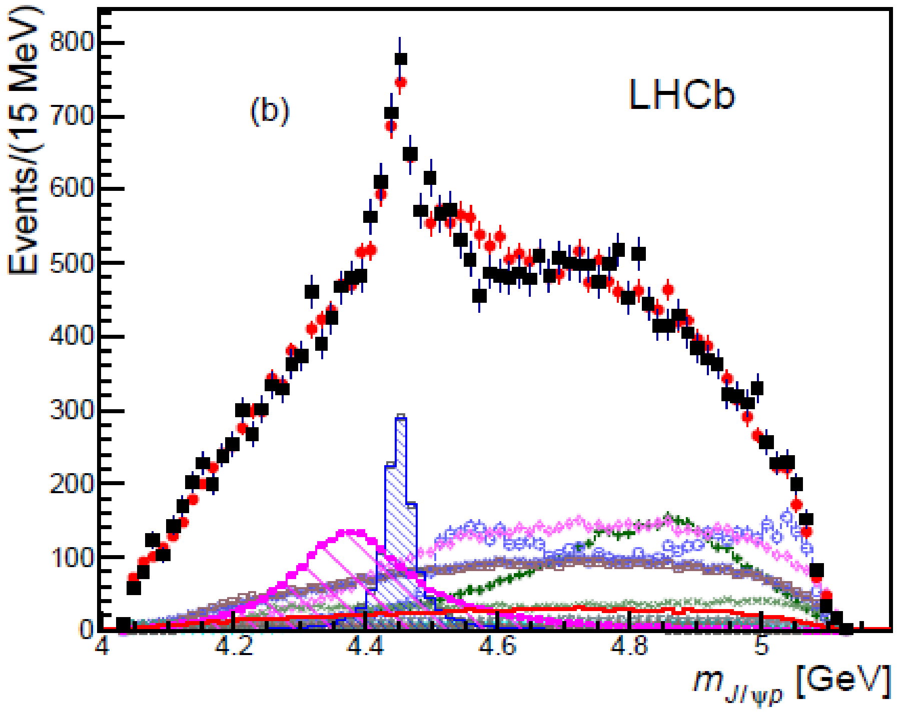


Pentaquarks

LHCb : Phys. Rev. Lett. 117, 082003 (2016)



Mass (MeV)	Width (MeV)	Fit fraction (%)
4380±8±29	205±18±86	8.4±0.7±4.2
4449.8±1.7±2.5	39±5±19	4.1±0.5±1.1



Quantum numbers (J^P) : $[3/2^{-}$ and $5/2^{+}]$

or $[3/2^{+}$ and $5/2^{-}]$

Weak interaction and Lattice QCD

- Lattice QCD is needed
  - to interpret flavour physics data
  - to extract the values of CKM matrix elements

- Most extensions of the Standard Model contain new CP- violating phases, new quark flavour-changing interactions
  - ➔ New Physics effects expected in the quark flavour sector

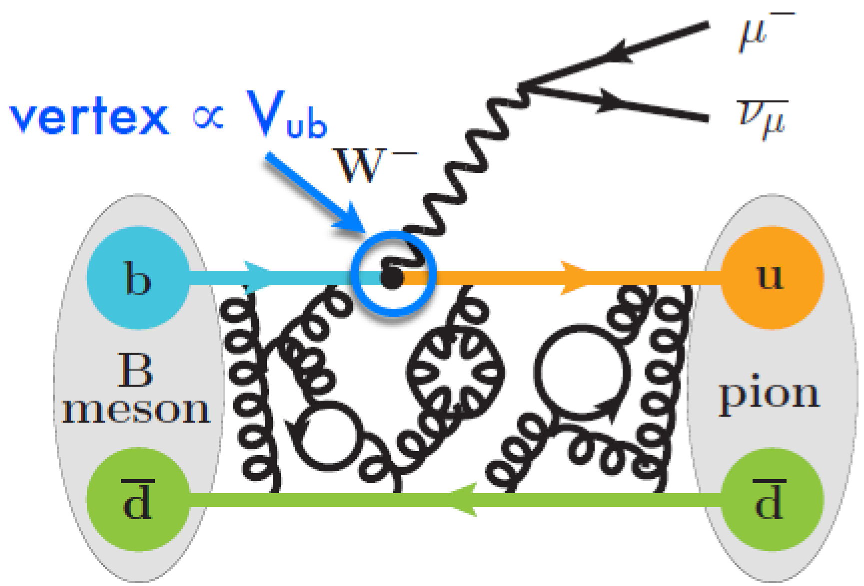
- To describe weak interaction involving quarks, one must include effects of confining quarks into hadrons.

- Typically most non-perturbative QCD effects get absorbed into hadronic matrix elements such as decay constants, form factors and bag parameters

- So far, Lattice QCD is the best tool to calculate non-perturbative QCD effects with **controlled systematics**.

Using LQCD we can calculate two, three and four point functions with control systematics

Weak matrix elements



$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2},\ \frac{d\Gamma(B\to D^{(*)}\ell\nu)}{dw},\ldots$$

Compute nonperturbative QCD parameters (decay constants, form factors, B-parameters,...) numerically with LATTICE QCD

@Van de Water

(Experiment) = (known) x (CKM factors) × (Hadronic Matrix Element)

CKM matrix elements and lattice calculations

$$\begin{pmatrix} \begin{matrix}\mathbf{V_{ud}}\\ \pi\to\ell\nu\\ \end{matrix} & \begin{matrix}\mathbf{V_{us}}\\ K\to\ell\nu\\ K\to\pi\ell\nu\end{matrix} & \begin{matrix}\mathbf{V_{ub}}\\ B\to\ell\nu\\ B\to\pi\ell\nu\end{matrix} \\ \begin{matrix}\mathbf{V_{cd}}\\ D\to\ell\nu\\ D\to\pi\ell\nu\end{matrix} & \begin{matrix}\mathbf{V_{cs}}\\ D_s\to\ell\nu\\ D\to K\ell\nu\end{matrix} & \begin{matrix}\mathbf{V_{cb}}\\ B\to D\ell\nu\\ B\to D^*\ell\nu\end{matrix} \\ \begin{matrix}\mathbf{V_{td}}\\ \langle B_d|\bar{B}_d\rangle\\ B\to\pi\ell\ell\end{matrix} & \begin{matrix}\mathbf{V_{ts}}\\ \langle B_s|\bar{B}_s\rangle\\ B\to K\ell\ell\end{matrix} & \begin{matrix}\mathbf{V_{tb}}\\ \\ \end{matrix} \end{pmatrix}$$

"Gold plated" **processes on the lattice → CKM matrix elements**

- **One hadron in the initial state and zero or one hadron in the final state**
- **Stable hadrons (that is narrow or far from threshold → easier to study on lattice)**
- **Chiral extrapolation is controllable**

Decay constants from Lattice QCD

In SM :

$$\Gamma(H \to \ell\nu) = \frac{M_H}{8\pi} f_H^2 \left|G_F V_{Qq}^* m_\ell\right|^2 \left(1 - \frac{m_\ell^2}{M_H^2}\right)^2 ,$$

Pseudoscalar to vacuum matrix element
of the axial current $\Longrightarrow$ pseudoscalar decay constant

$$\langle 0|\mathcal{A}^\mu|H(p)\rangle = ip^\mu f_H ,$$

$$\langle 0|\mathcal{A}^\mu|H(p)\rangle \, (M_H)^{-1/2} = i(p^\mu/M_H)\phi_H$$

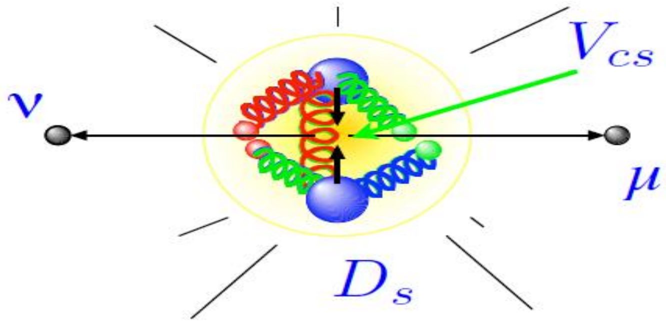
$$f_H \;=\; \phi_H/\sqrt{M_H}$$

H	$\mathcal{A}^\mu$	V
D	$\bar{d}\gamma^\mu\gamma^5 c$	V_{cd}^*
D_s	$\bar{s}\gamma^\mu\gamma^5 c$	V_{cs}^*
B	$\bar{b}\gamma^\mu\gamma^5 u$	V_{ub}
B_s	$\bar{b}\gamma^\mu\gamma^5 s$	—

Renormalization constant (to match with continuum physics) :

$$Z_{A^\mu} A^\mu \doteq \mathcal{A}^\mu + \mathrm{O}\left(\alpha_s a \Lambda f_i(m_Q a)\right) + \mathrm{O}\left(a^2 \Lambda^2 f_j(m_Q a)\right)$$

Leptonic decay constants



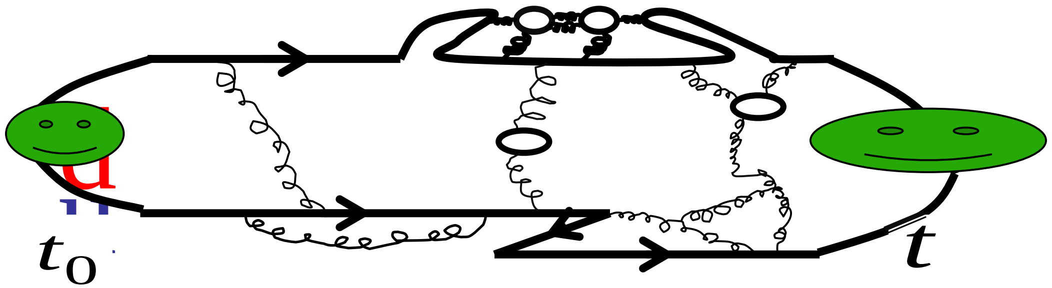
Need to calculate two point correlation functions :

$$\varphi(t) = e^{Ht}\varphi(0)e^{-Ht}$$

$$G(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} \langle 0|\varphi(x)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$= \sum_n e^{-E_p^n(t-t_0)} \left|\langle 0|\varphi(x_0)|n,\vec{p}\rangle\right|^2$$

$$= \sum_n W_n e^{-E_p^n(t-t_0)} \xrightarrow[t\to\infty]{} W_1 e^{-E_1^n(t-t_0)}$$



Two point function

Two and three point functions

$$C_2^{\eta_c}(t) = \sum_i \left(A_{\eta_c}^i\right)^2 e^{-E_{\eta_c}^i t}$$

$$C_2^{B_c}(\tau) = \sum_i \left(A_{B_c}^i\right)^2 e^{-E_{B_c}^i \tau}$$

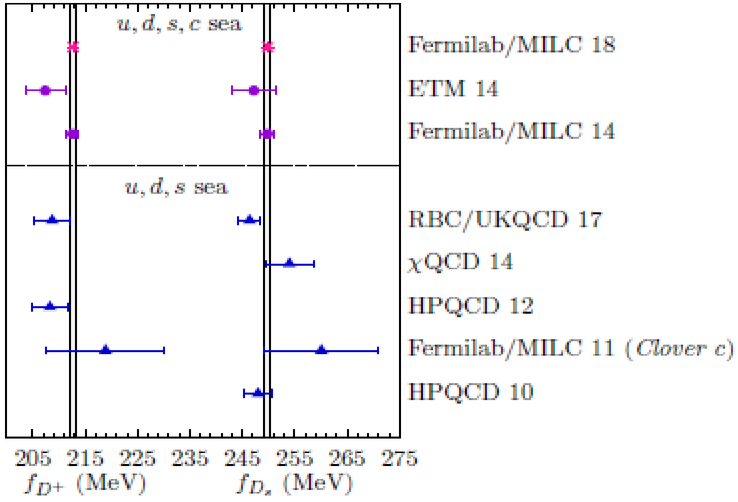
$$C_{3,m}^{B_c\to\eta_c}(t,\tau) = \sum_{i,j} A_{\eta_c}^i \varphi^m A_{B_c}^j e^{-E_{\eta_c}^i t} e^{-E_{B_c}^i \tau}$$

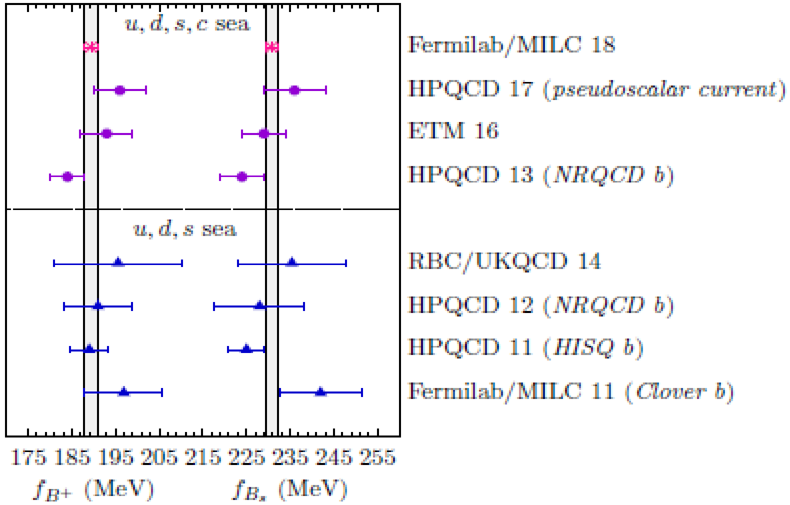
φ^m: Can be obtained by

- fitting these two and three point function simultaneously
- constructing appropriate ratios

Meson Decay Constants

$$
\begin{aligned}
f_{D^0} &= 211.6(0.3)_{\rm stat}(0.5)_{\rm syst}(0.2)_{f_\pi,{\rm PDG}}[0.2]_{\rm EM-scheme}\ {\rm MeV},\\
f_{D^+} &= 212.7(0.3)_{\rm stat}(0.4)_{\rm syst}(0.2)_{f_\pi,{\rm PDG}}[0.2]_{\rm EM-scheme}\ {\rm MeV},\\
f_{D_s} &= 249.9(0.3)_{\rm stat}(0.2)_{\rm syst}(0.2)_{f_\pi,{\rm PDG}}[0.2]_{\rm EM-scheme}\ {\rm MeV},\\
f_{B^+} &= 189.4(0.8)_{\rm stat}(1.1)_{\rm syst}(0.3)_{f_\pi,{\rm PDG}}[0.1]_{\rm EM-scheme}\ {\rm MeV},\\
f_{B^0} &= 190.5(0.8)_{\rm stat}(1.0)_{\rm syst}(0.3)_{f_\pi,{\rm PDG}}[0.1]_{\rm EM-scheme}\ {\rm MeV},\\
f_{B_s} &= 230.7(0.8)_{\rm stat}(1.0)_{\rm syst}(0.2)_{f_\pi,{\rm PDG}}[0.2]_{\rm EM-scheme}\ {\rm MeV}.
\end{aligned}
$$

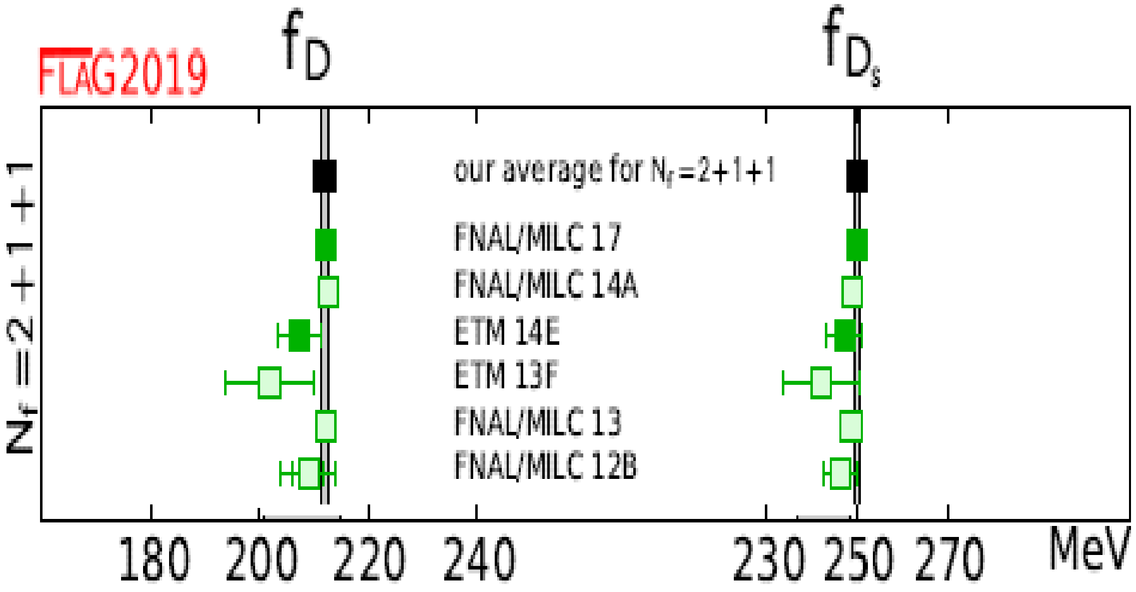


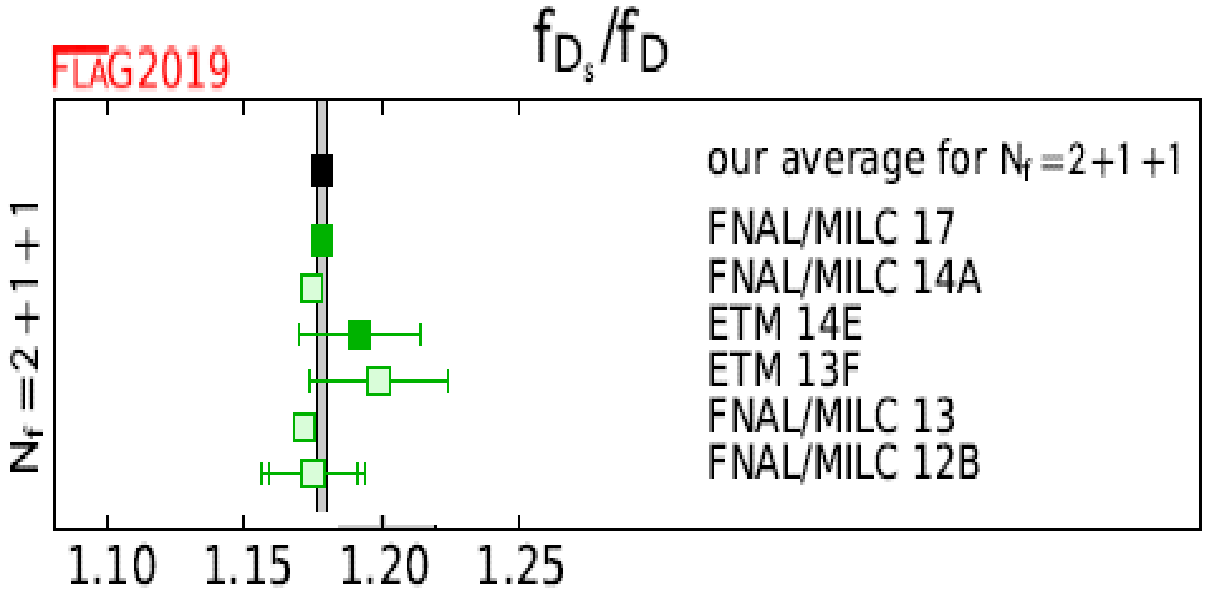


MILC : PRD98 (2018) no.7, 074512, arXiv:1810.00250

Decay Constants

FLAG 2019: arXiv:1902.08191v2



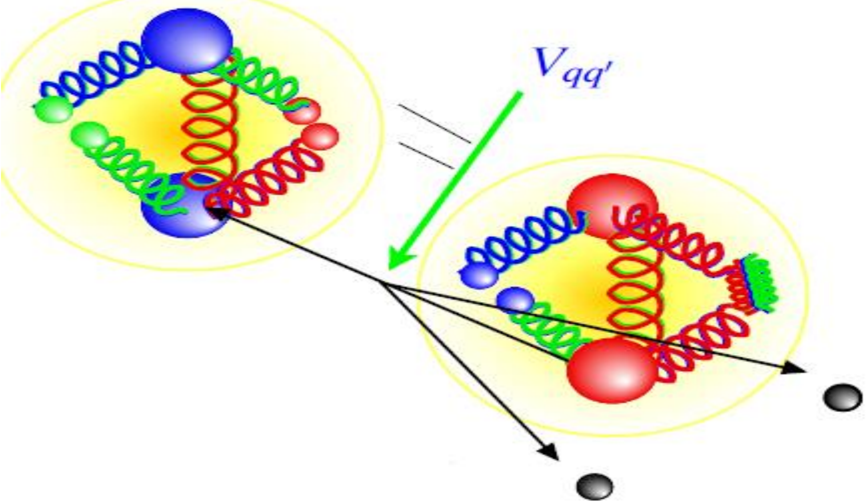


$$N_f = 2+1+1: \qquad f_D = 212.0(0.7)\ \mathrm{MeV}$$
$$N_f = 2+1+1: \qquad f_{D_s} = 249.9(0.5)\ \mathrm{MeV}$$
$$N_f = 2+1+1: \qquad \frac{f_{D_s}}{f_D} = 1.1783(0.0016)$$

FNAL/MILC: Phys. Rev. D98 (2018) 074512

ETM: Phys. Rev. D91 (2015) 054507

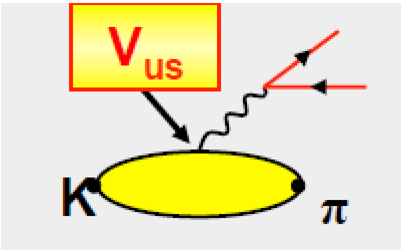
Semileptonic form factors



$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \end{pmatrix}$$

SM :

$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} = \frac{G_F^2|V_{cx}|^2}{24\pi^3}\frac{(q^2-m_\ell^2)^2\sqrt{E_P^2-m_P^2}}{q^4 m_D^2}\left[\left(1+\frac{m_\ell^2}{2q^2}\right)m_D^2(E_P^2-m_P^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_D^2-m_P^2)^2|f_0(q^2)|^2\right]$$



$$\langle P|V_\mu|D\rangle = f_+(q^2)\left(p_{D\mu}+p_{P\mu}-\frac{m_D^2-m_P^2}{q^2}q_\mu\right)+f_0(q^2)\frac{m_D^2-m_P^2}{q^2}q_\mu\,,$$

$V_\mu = \bar{x}\gamma_\mu c$

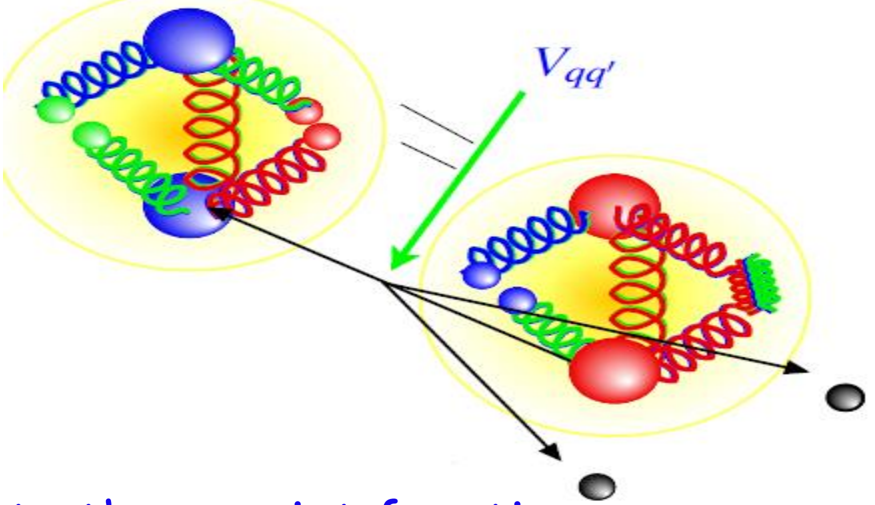
$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} = \frac{G_F^2}{24\pi^3}|\vec{p}_P|^3|V_{cx}|^2|f_+(q^2)|^2$$

$|V_{cd}|$ **and** $|V_{cs}|$

$$\langle P|T_{\mu\nu}|D\rangle = \frac{2}{m_D+m_P}\left[p_{P\mu}p_{D\nu}-p_{P\nu}p_{D\mu}\right]f_T(q^2)$$

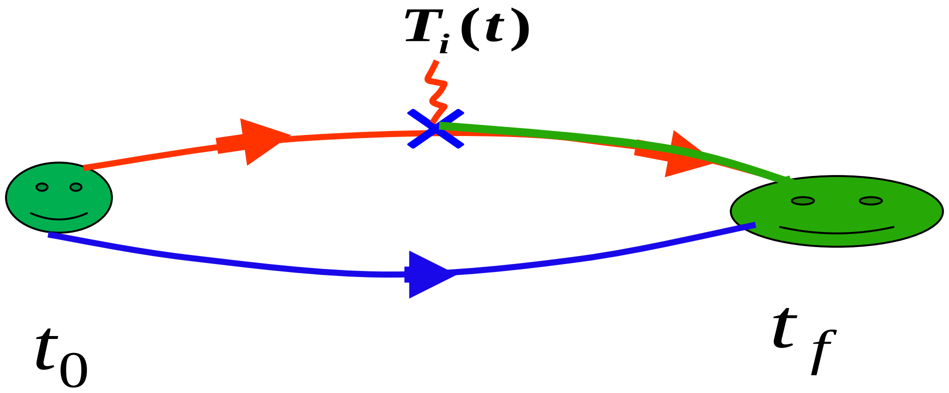
Parity even current: BSM physics

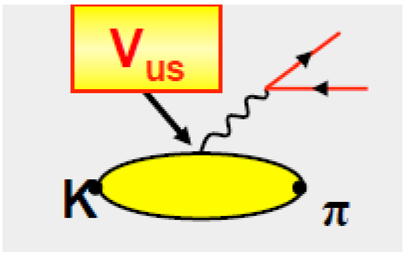
Semileptonic form factors



$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \end{pmatrix}$$

Need to calculate three point function :

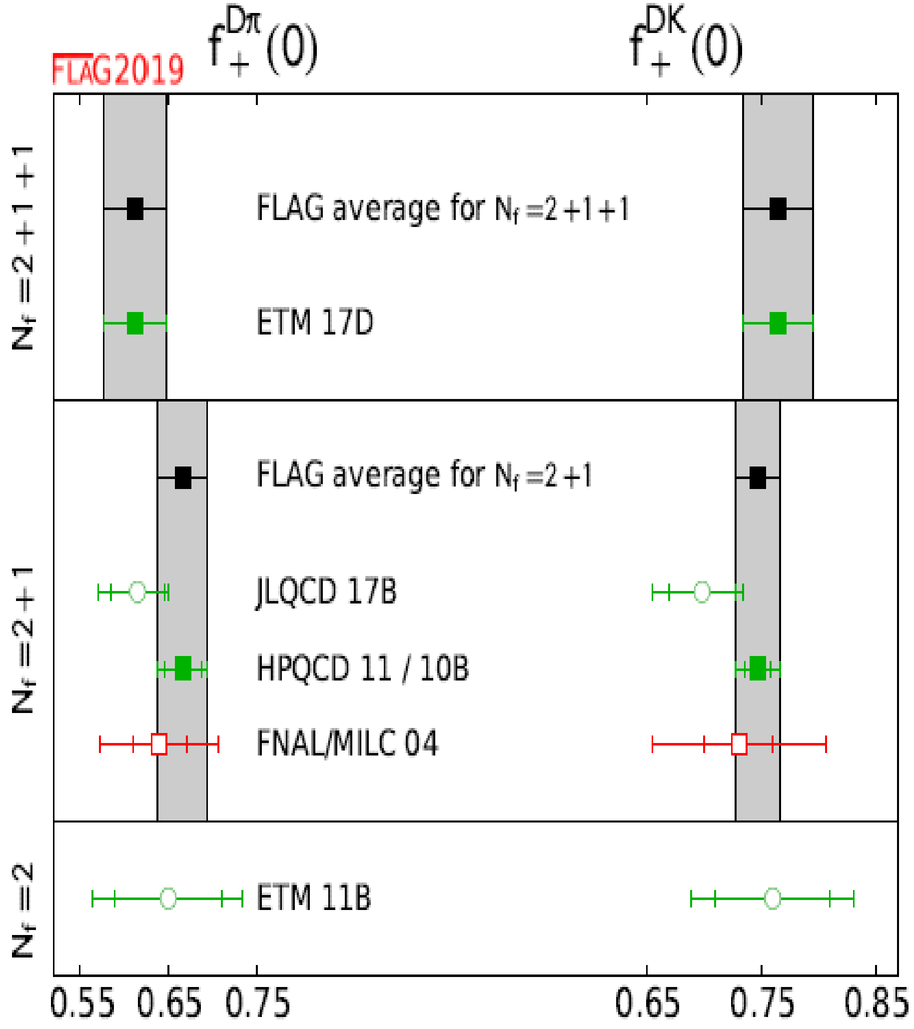




$$G^{\alpha\beta}_{PT_\mu\ P}(t_2, t_1, \vec{p}, \vec{p'}) = \sum_{\vec{x_2}, \vec{x_1}} e^{-i\vec{p}\cdot\vec{x_2}} e^{-i\vec{q}\cdot\vec{x_1}}$$

$$< 0|\,\mathrm{T}\left(\chi^\alpha(x_2)\, T_\mu\ (x_1)\, \bar{\chi}^\beta(0)\right)\,|0 >$$

Form Factors ($D_{q,s}$ semi-leptonic decay)

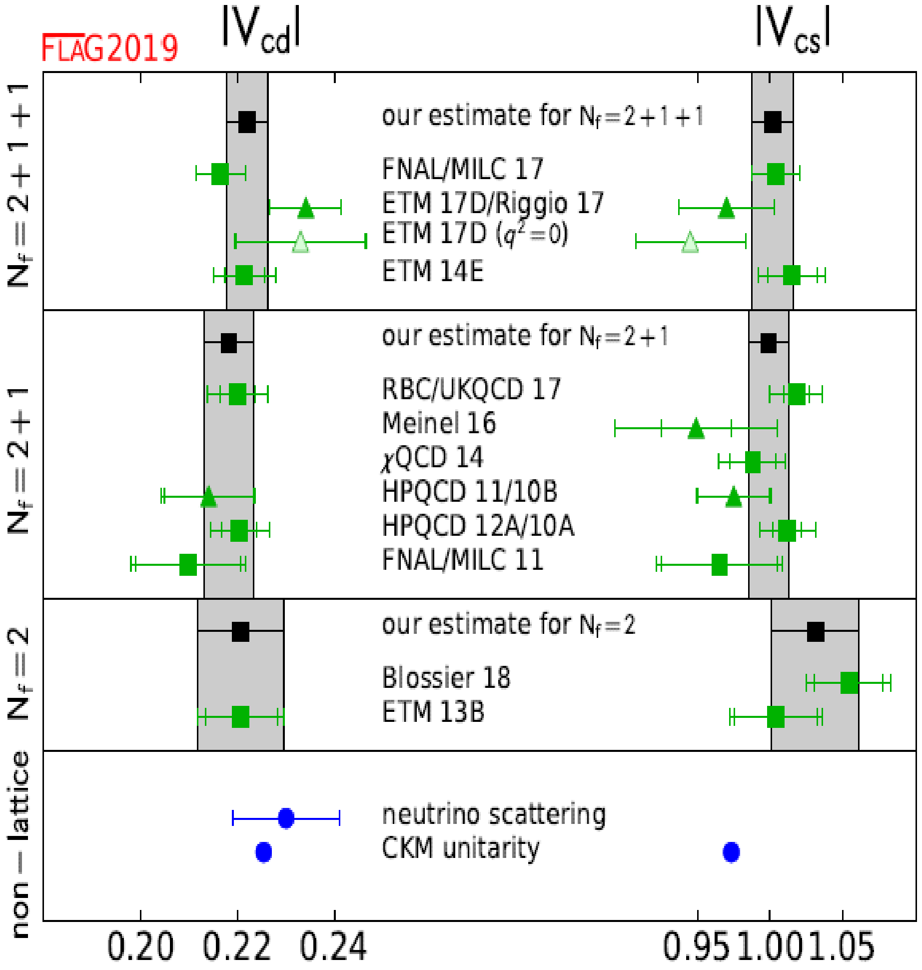


$$N_f = 2+1: \quad f_+^{D\pi}(0) = 0.666(29) \qquad f_+^{DK}(0) = 0.747(19)$$

HPQCD: Phys.Rev. D82 (2010) 114506,
Phys.Rev. D84 (2011) 114505

$$N_f = 2+1+1: \quad f_+^{D\pi}(0) = 0.612(35) \qquad f_+^{DK}(0) = 0.765(31)$$

ETM: Phys. Rev. D96 (2017) 054514



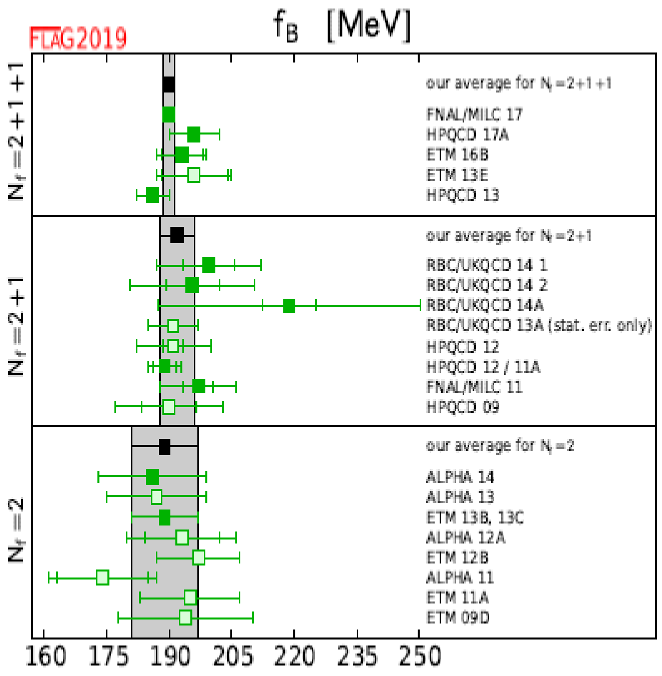
$$N_f = 2+1+1: \qquad |V_{cd}| = 0.2219(43)\,, \qquad |V_{cs}| = 1.002(14)\,,$$
$$N_f = 2+1: \qquad |V_{cd}| = 0.2182(50)\,, \qquad |V_{cs}| = 0.999(14)\,,$$
$$N_f = 2: \qquad |V_{cd}| = 0.2207(89)\,, \qquad |V_{cs}| = 1.031(30)\,,$$

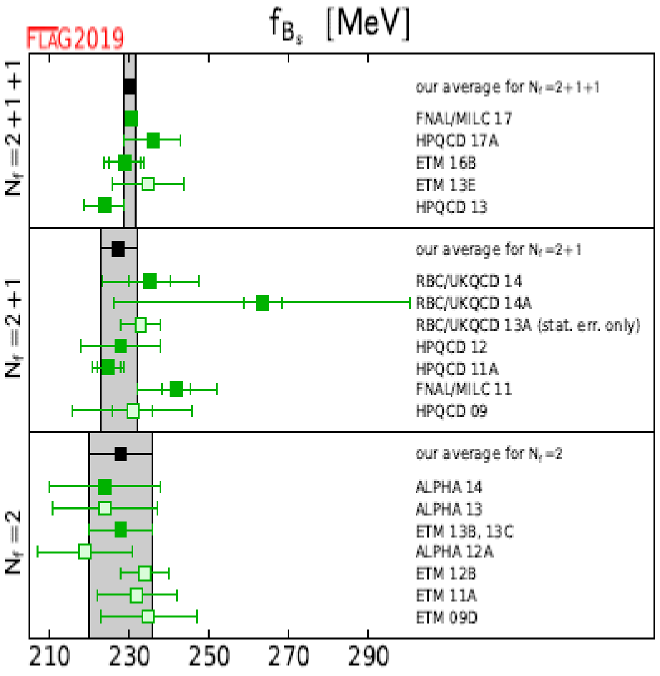
$$N_f = 2+1+1: \qquad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.05(3)\,,$$
$$N_f = 2+1: \qquad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.05(3)\,,$$
$$N_f = 2: \qquad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.11(6)\,.$$

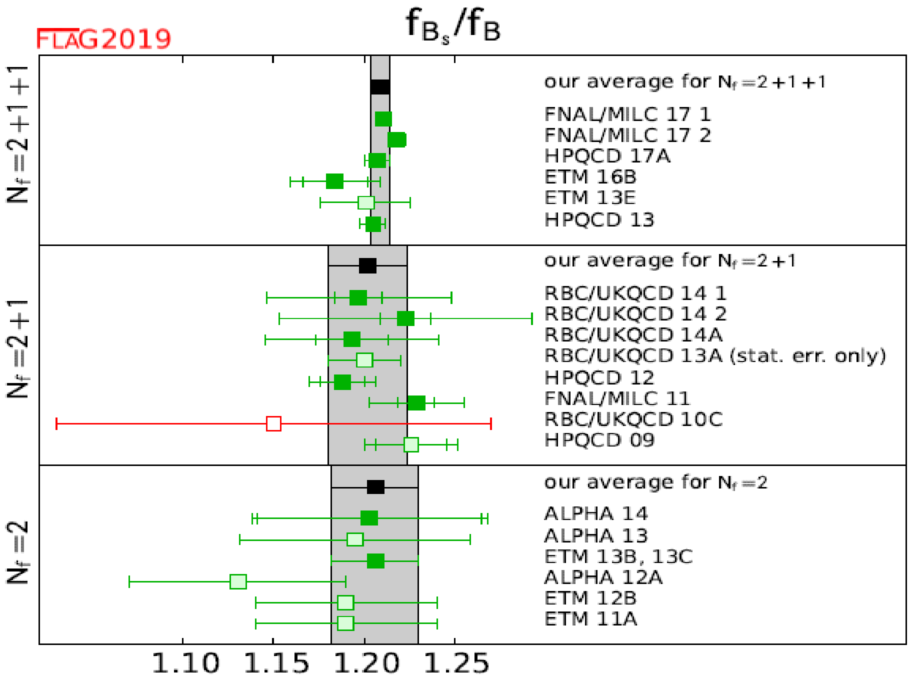
FLAG 2019: arXiv:1902.08191v2

$$\Gamma(B \to l\nu) = \frac{m_B}{8\pi} G_F^2 f_B^2 |V_{ub}|^2 m_l^2 \left(1 - \frac{m_l^2}{m_B^2}\right)^2$$

FCNC process $$B(B_q \to \ell^+\ell^-) = \tau_{B_q} \frac{G_F^2}{\pi} Y \left(\frac{\alpha}{4\pi \sin^2 \Theta_W}\right)^2 m_{B_q} f_{B_q}^2 |V_{tb}^* V_{tq}|^2 m_\ell^2 \sqrt{1 - 4\frac{m_\ell^2}{m_B^2}}$$







$$f_B = 190.0(1.3)\ \text{MeV}$$
$$f_{B_s} = 230.3(1.3)\ \text{MeV}$$
$$\frac{f_{B_s}}{f_B} = 1.209(0.005)$$

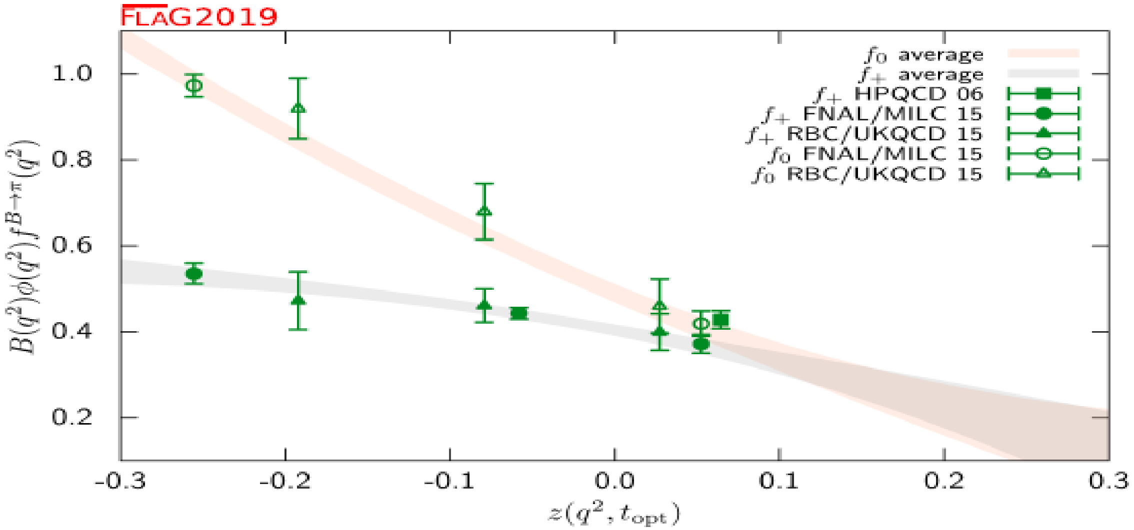
HPQCD: Phys. Rev. Lett. 110 (2013) 222003,
Phys. Rev. D97 (2018) 054509

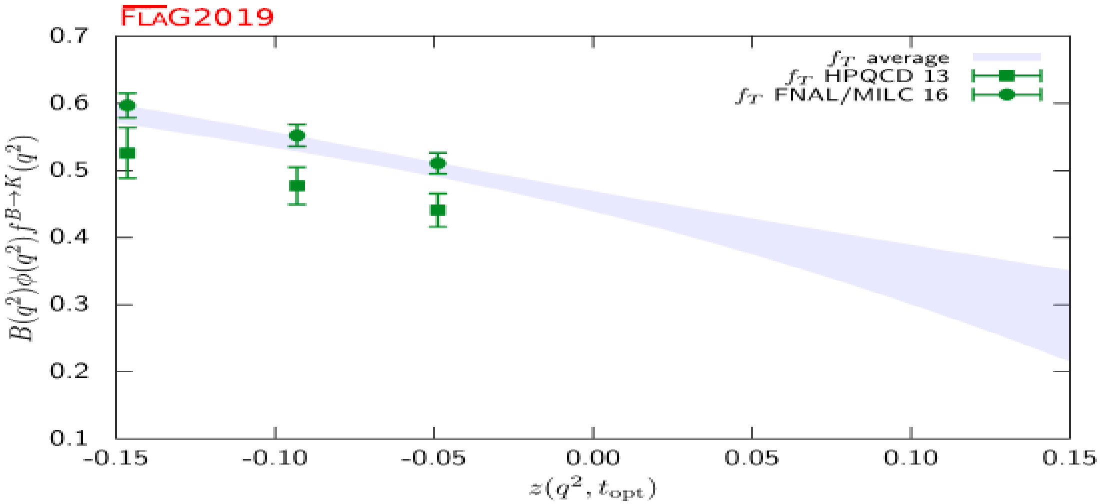
FNAL/MILC: Phys. Rev. D98 (2018) 074512

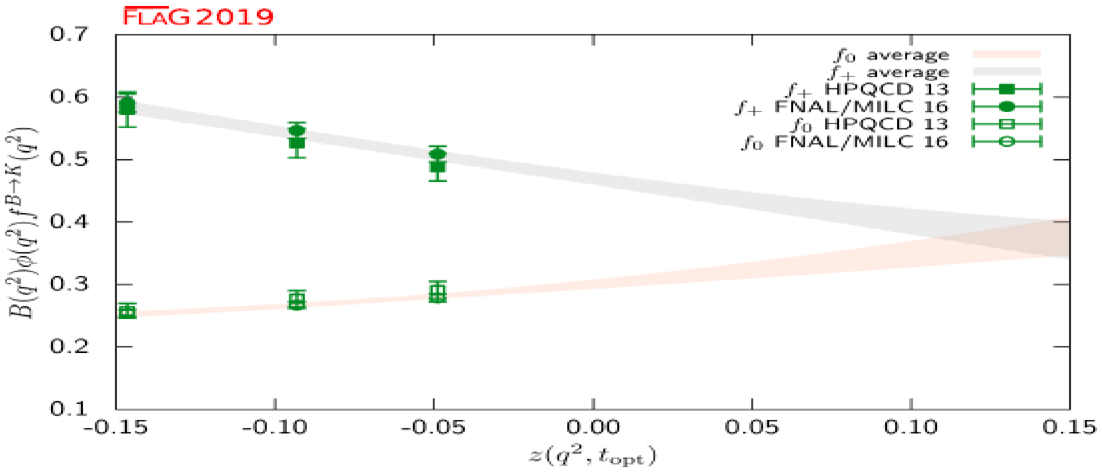
ETM: Phys. Rev. D93 (2016) 114505

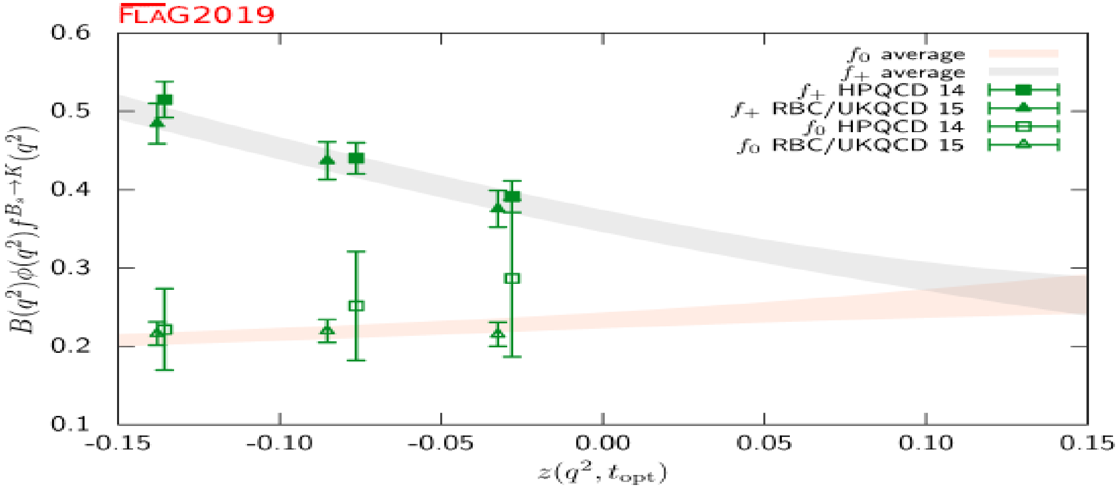
Form Factors (B_x semi-leptonic decay)

$$\frac{d\Gamma(B_{(s)}\to P\ell\nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}\frac{(q^2-m_\ell^2)^2\sqrt{E_P^2-m_P^2}}{q^4 m_{B_{(s)}}^2}\left[\left(1+\frac{m_\ell^2}{2q^2}\right)m_{B_{(s)}}^2(E_P^2-m_P^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_{B_{(s)}}^2-m_P^2)^2|f_0(q^2)|^2\right]$$









Neutral B-meson mixing

$$\langle \overline{B}_q^0 | H_{eff}^{\Delta B=2} | B_q^0 \rangle$$

$$H_{eff,BSM}^{\Delta B=2} = \sum_{q=d,s} \sum_{i=1}^{5} C_i Q_i^q$$

$$Q_1^q = [\bar{b}\gamma_\mu(1-\gamma_5)q]\,[\bar{b}\gamma_\mu(1-\gamma_5)q]$$

$$Q_2^q = [\bar{b}(1-\gamma_5)q]\,[\bar{b}(1-\gamma_5)q]\,,$$

$$Q_4^q = [\bar{b}(1-\gamma_5)q]\,[\bar{b}(1+\gamma_5)q]\,,$$

$$Q_3^q = \left[\bar{b}^\alpha(1-\gamma_5)q^\beta\right]\left[\bar{b}^\beta(1-\gamma_5)q^\alpha\right],$$

$$Q_5^q = \left[\bar{b}^\alpha(1-\gamma_5)q^\beta\right]\left[\bar{b}^\beta(1+\gamma_5)q^\alpha\right],$$

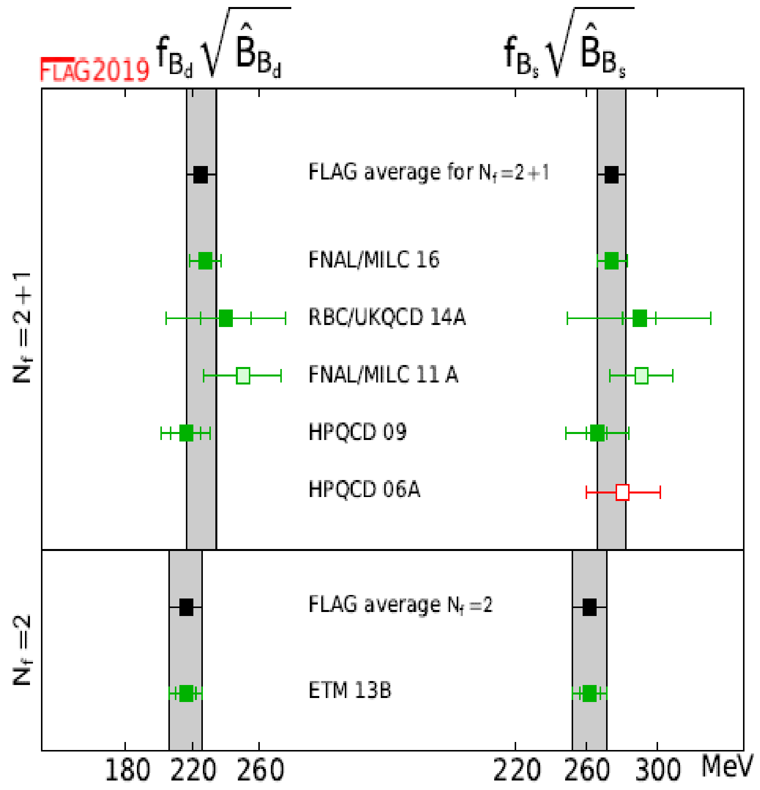
Bag Parameter: $$B_{B_q}(\mu) = \frac{\langle \overline{B}_q^0 | Q_R^q(\mu) | B_q^0 \rangle}{\frac{8}{3} f_B^2 m_B^2}$$

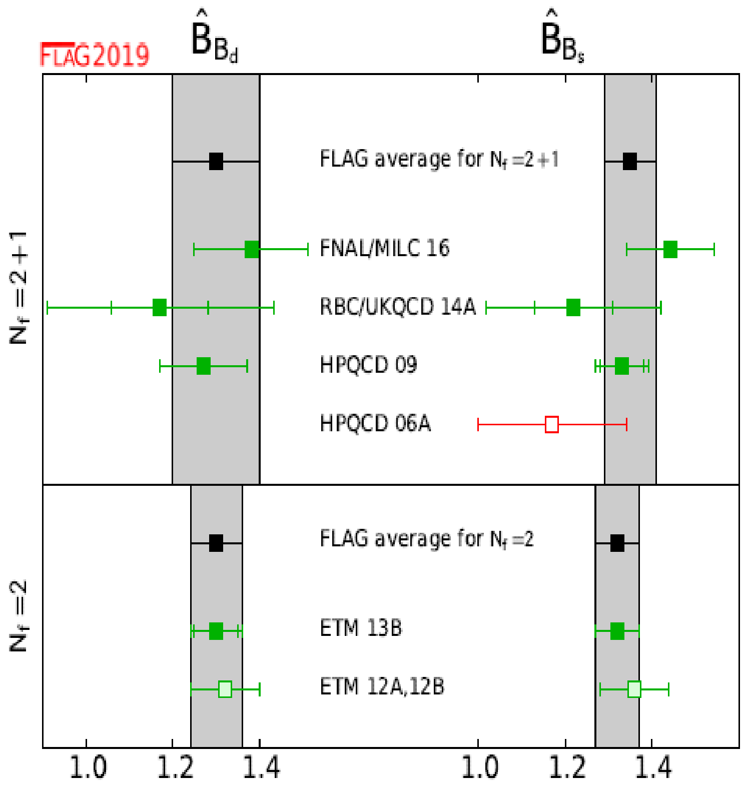
$$\hat{B}_{B_q} = \left(\frac{\bar{g}(\mu)^2}{4\pi}\right)^{-\gamma_0/(2\beta_0)} \left\{1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2}\left[\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2}\right]\right\} B_{B_q}(\mu)$$

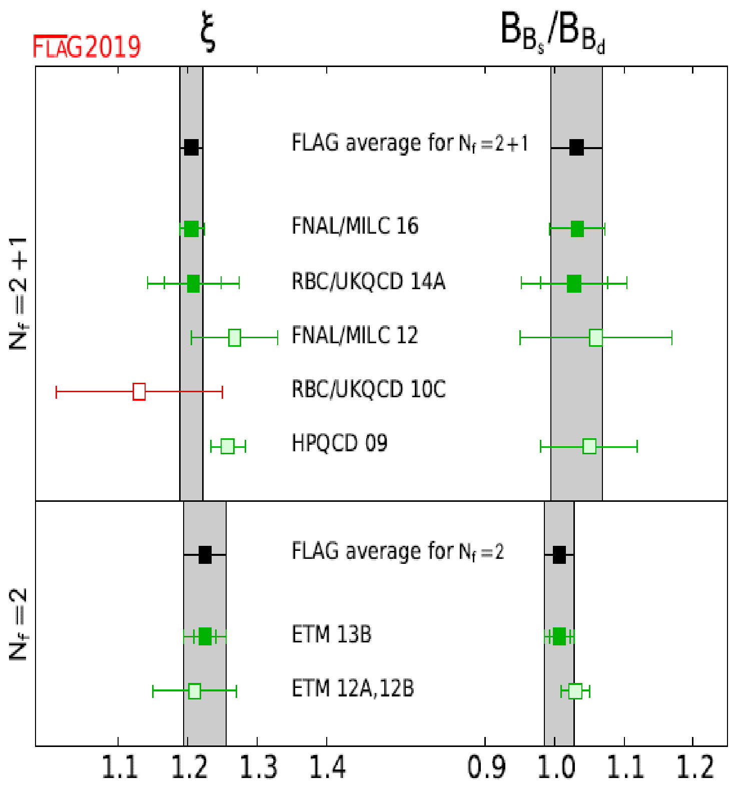
$$\Delta m_q = \frac{G_F^2 m_W^2 m_{B_q}}{6\pi^2} |\lambda_{tq}|^2 S_0(x_t)\eta_{2B} f_{B_q}^2 \hat{B}_{B_q} \qquad \lambda_{tq} = V_{tq}^* V_{tb}$$

$$\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}$$

Neutral B-meson mixing







$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 225(9)\,\text{MeV} \qquad f_{B_s}\sqrt{\hat{B}_{B_s}} = 274(8)\,\text{MeV}$$

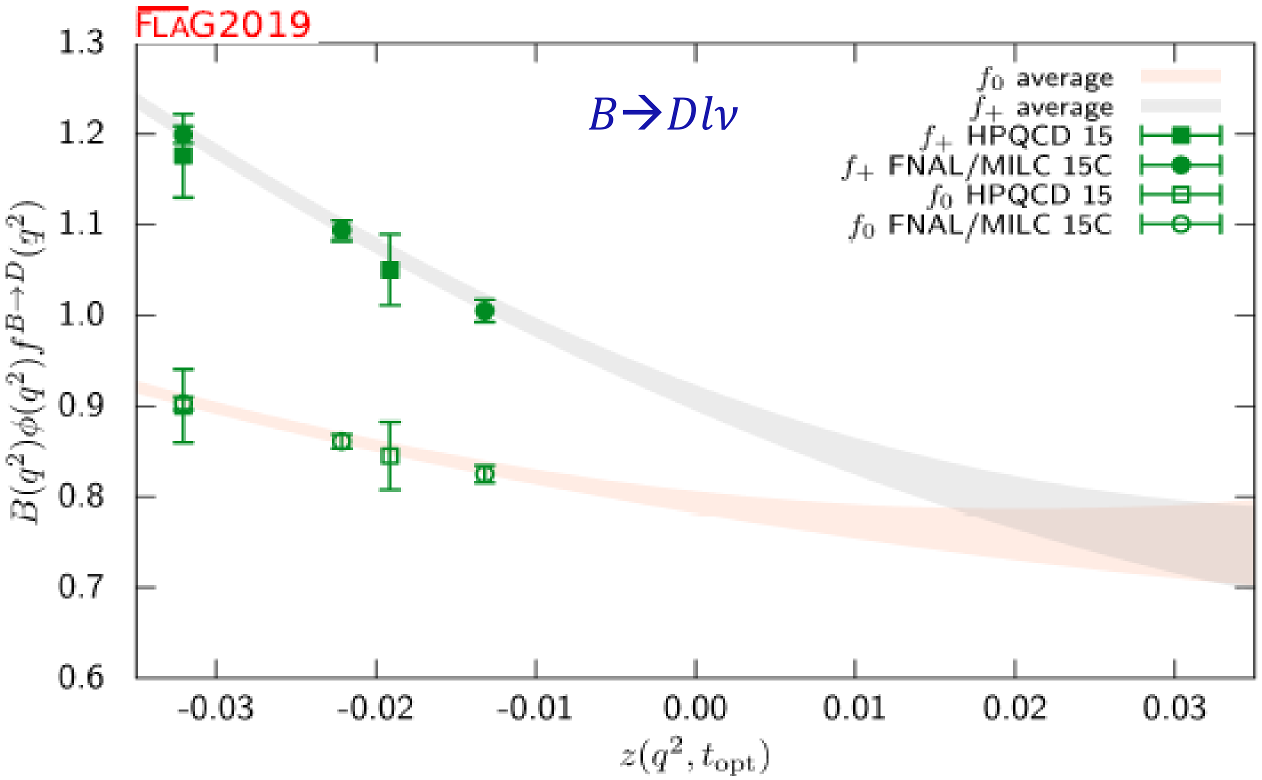
$$\hat{B}_{B_d} = 1.30(10) \qquad \hat{B}_{B_s} = 1.35(6)$$

$$\xi = 1.206(17) \qquad B_{B_s}/B_{B_d} = 1.032(38)$$

FNAL/MILC: Phys. Rev. D93 (2016) 113016,

RBC/UKQCD: Phys. Rev. D91 (2015) 114505,

HPQCD: Phys.Rev. D80 (2009) 014503,



$$R(D) = \mathcal{B}(B \to D\tau\nu)/\mathcal{B}(B \to D\ell\nu) \qquad \text{with } \ell = e, \mu$$

$$R(D) = 0.300(8) \qquad R(D_s) = 0.314(6)$$

MILC: Phys. Rev. D92 (2015) 034506,

HPQCD: Phys. Rev. D92 (2015) 054510

Rare decay:

$B_s \to \mu^+\mu^-$

LHCb, CMS: Nature 522 (2015) 68

$$f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_K^2) = 1.046(44)(15),$$
$$f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_\pi^2) = 1.054(47)(17)$$

MILC: Phys.Rev. D85 (2012) 114502

$$f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_K^2) = 1.000(62)$$
$$f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_\pi^2) = 1.006(62)$$

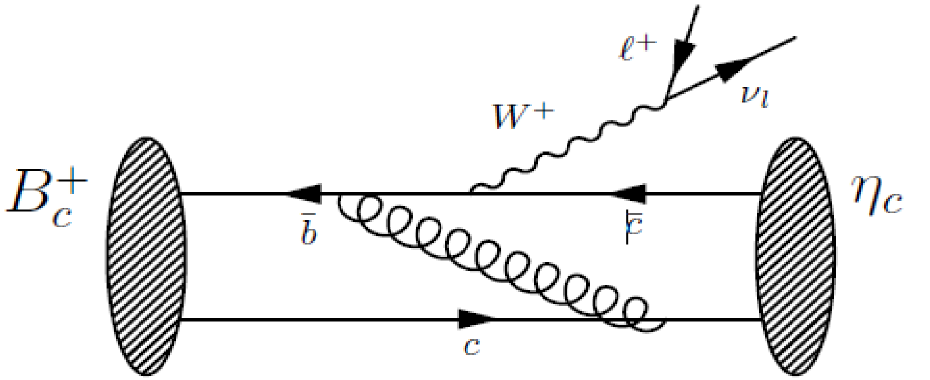
HPQCD: Phys. Rev. D95 (2017) 114506

$$\mathcal{R}(J/\psi) = \frac{\mathcal{B}(B_c^+ \to J/\psi\,\tau^+\nu_\tau)}{\mathcal{B}(B_c^+ \to J/\psi\,\mu^+\nu_\mu)} = 0.71 \pm 0.17\,(\mathrm{stat}) \pm 0.18\,(\mathrm{syst}).$$

LHCb : Phys. Rev. Lett. 120 (2018) no.12, 121801

SM : 0.25-0.28

Form factors



- $B_c \rightarrow \eta_c l\nu$

$$\langle \eta_c(p)|V^\mu|B_c(P)\rangle = f_+(q^2)\left[P^\mu + p^\mu - \frac{M^2-m^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M^2-m^2}{q^2}q^\mu$$

- $B_c \rightarrow J/\psi l\nu$

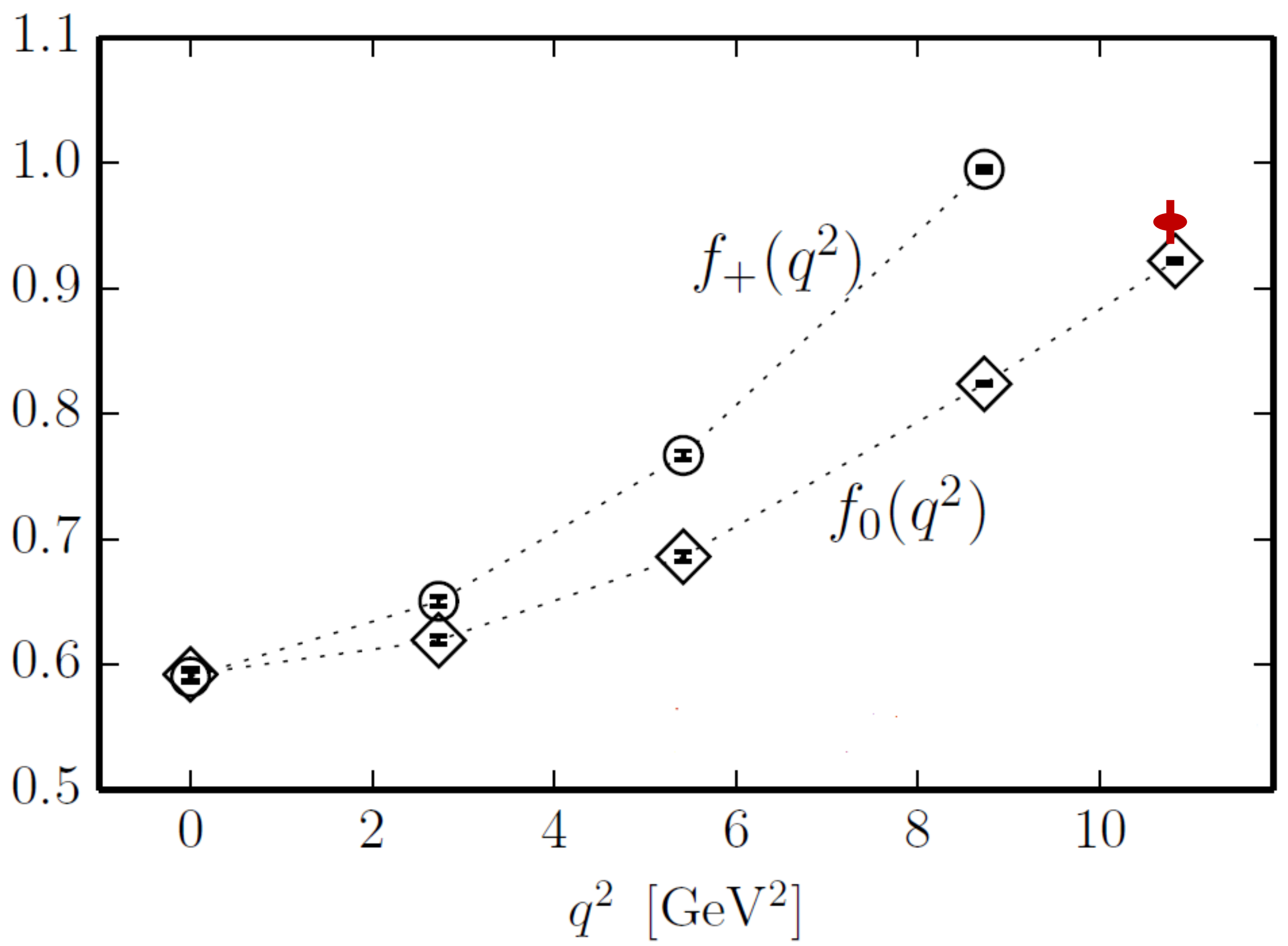
$$\langle J/\psi(p,\varepsilon)|V^\mu - A^\mu|B_c(P)\rangle = \frac{2i\varepsilon^{\mu\nu\rho\sigma}}{M+m}\varepsilon_\nu^* p_\rho P_\sigma V(q^2) - (M+m)\varepsilon^{*\mu}A_1(q^2) +$$

$$\frac{\varepsilon^*\cdot q}{M+m}(p+P)^\mu A_2(q^2) + 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_3(q^2) - 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_0(q^2)$$

$$q = P - p$$

$q^2_{\max}$ **: Outgoing hadron at rest**

$q^2 = 0$ **: Maximum recoil**



Colquhoun *et al* HPQCD : 1611.01987

A. Lytle : CKM2016

NM : Lattice 2017

Semileptonic form factors in baryon decays

$$\Lambda_c \to \Lambda \ell \nu$$

Alternate ways to get $|V_{cs}|$

$$\langle \Lambda | \bar{s} \gamma^\mu (1-\gamma_5) c | \Lambda_c \rangle$$

$$\frac{\Gamma(\Lambda_c \to \Lambda e^+ \nu_e)}{|V_{cs}|^2} = 0.2007(71)(74)\ \mathrm{ps}^{-1}\,,$$
$$\frac{\Gamma(\Lambda_c \to \Lambda \mu^+ \nu_\mu)}{|V_{cs}|^2} = 0.1945(69)(72)\ \mathrm{ps}^{-1}\,.$$

S. Meinel, Phys. Rev. Lett. 118 (2017) 082001

$$\Lambda_b \to p \ell \nu$$

Alternate ways to get $|V_{bu}|$

$$\Lambda_b \to \Lambda_c \ell \nu$$

Alternate ways to get $|V_{bc}|$

Detmold et. al.: Phys. Rev. D92 (2015) 034503,

Possibility of exclusive determination of $|V_{ub}|/|V_{cb}|$

Conclusions and Outlooks

- **There is a tremendous resurgence of interest in the study of bound states with heavy quark(s).**
  - **LQCD plays important roles for discovering many of these particles by predicting their possible energy values. Lattice QCD also plays crucial role in understanding the structures and interactions of these particles.**
  - **LQCD can provide important information about the dynamics of strong interactions at multiple scales.**

- **Lattice QCD is playing a crucial role in determining decay constants and form factors of heavy hadrons and in turn helping in precise determination of the CKM matrix elements.**

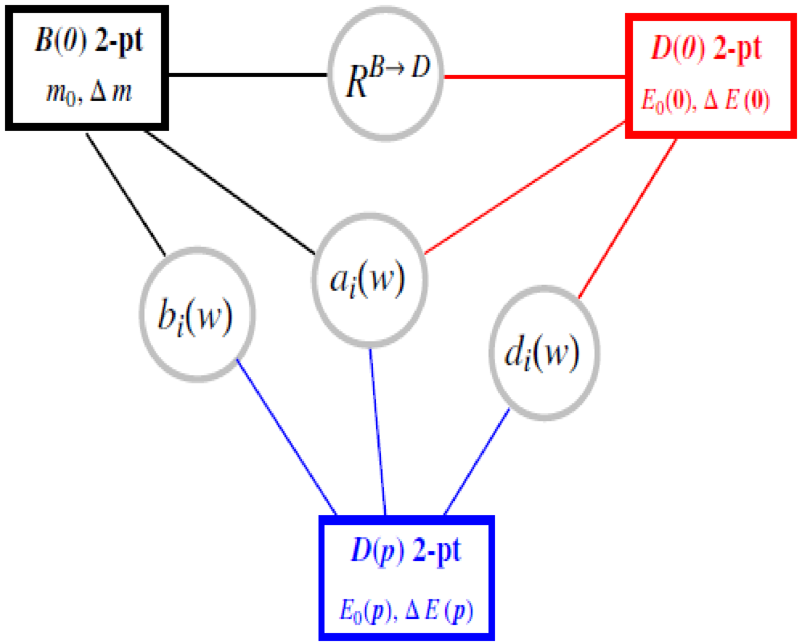
- **Heavy flavour physics is a precision tool to discover new physics. Lattice QCD calculations are absolutely necessary for this.**

$$\langle \eta_c(p')|V^\mu|B_c(p)\rangle = f_+(q^2)(p+p')^\mu + \left(f_0(q^2) - f_+(q^2)\right)\frac{M_{B_c}^2 - M_{\eta_c}^2}{q^2}(p-p')^\mu,$$

$$\frac{d\Gamma}{dq^2}(B_c^+ \to \eta_c\,\ell^+\,\nu) = \frac{\eta_{ew}^2 G_F^2 |V_{cb}|^2 M_{B_c}\sqrt{\lambda}}{192\pi^3}\left(1-\frac{m_\ell^2}{q^2}\right)^2\left[c_+ f_+(q^2)^2 + c_0 f_0(q^2)^2\right]$$

$$\lambda \equiv \lambda(q^2, M_{B_c}^2, M_{\eta_c}^2), \text{ where } \lambda(a,b,c) = a^2+b^2+c^2-2(ab+bc+ca),$$

$$c_+ = \frac{\lambda}{M_{B_c}^4}\left(1+\frac{m_\ell^2}{2q^2}\right), \quad c_0 = (1-r^2)^2\frac{3m_\ell^2}{2q^2}, \quad r = \frac{M_{\eta_c}}{M_{B_c}}.$$



Decay

- Decay into two mesons :

$$\boldsymbol{E(Q\bar{q}) + E(Q\bar{q}') - E\left(Q^2\bar{q}\bar{q}'\right) \approx \frac{2}{9} m\alpha_s^2 \left[\, 1 + O\left(m^{-1}\right)\right] > 0} \text{ (Positive)}$$

For sufficiently large m this should be bound!
No decay to two mesons

Carlson, Heller, Tjon PRD 37, 744 (1988)

- Decay into 2 baryons :

$$\begin{aligned} \boldsymbol{E(qq'q'') + E(\bar{q}''\bar{Q}\bar{Q}) - E(\bar{q}\bar{q}'\bar{Q}\bar{Q})} &= \boldsymbol{E(qq'q'') + E(\bar{q}''Q) - E(qq'Q)} \\ &\geq \boldsymbol{E}(\text{proton}) + \boldsymbol{E(\bar{q}''Q) - E(qq'Q)} \end{aligned}$$

For known charm and bottom masses right hand side is positive
Therefore, no strong decay to two baryons

Eichten, Quigg, PRL 199, 202002 (2017)

$$V_C = \sum_{i>j} \alpha_s \frac{F_i . F_j}{r_{ij}}$$

$F_i . F_j$ = – 2/3 in $\overline{3}$

Bohr radius $a = \frac{3}{m\alpha_s}$

$$E(Q^2) \approx -\frac{1}{9} m\alpha_s^2$$

$$E(Q\overline{q}) + E(Q\overline{q}') - E(Q^2\overline{q}\overline{q}') \approx \frac{2}{9} m\alpha_s^2 \,[\,1 + O(m^{-1})]$$