Heavy quark systems in QGP: results from lattice QCD

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Heavy quark probes of plasma:

- ▶ Quarkonia yield: dissociation and QGP formation.
- ▶ Flow of heavy-light mesons
- ▶ Flow of quarkonia
- ▶ R_{AA} of hard heavy mesons
- ▶ . . ..

Lattice inputs:

- ▶ Quarkonia in QGP: dissociation?
- ▶ Diffusion of heavy quarks?

Vector current correlators?

Simple: calculate thermal vector current correlators on lattice!
Take a meson decay through a certain current, e.g., $J/\psi \to \mu^+ \mu^-$

$$\mathcal{M} = e^2 \, \bar{u}(p)\gamma_\mu v(-p) \, \frac{1}{q^2} \, \langle H|J_\mu|0 \rangle$$

$$\sigma(\omega) = \frac{\alpha^2}{3\pi^3 q^2} \, n_B(q^0)\rho_{V,\mu}^\mu(\omega)$$

$$\rho_H(\omega, \vec{p}) = \int d^4x \, e^{iq.x} \, \langle [J_H(\vec{x}, t), J_H(\vec{0}, 0)] \rangle_T$$
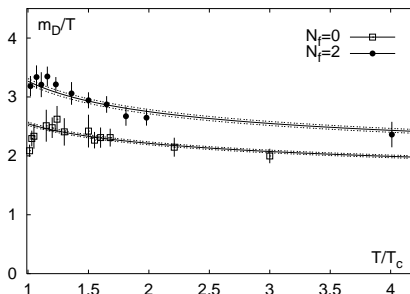
Theoretically most concrete: look at correlator of $\bar{Q}(\vec{x})\gamma_i Q(\vec{x})$.
Thermal width.

Perturbation theory?

HTL perturbation theory:
separation of scales $T \gg m_D$, integrate out T.
Doesn't always work in the temperaturs of interest.

Kaczmarek & Zantow, PRD 71 (2005) 114510.

Note that impressive results have been obtained for some thermodynamic quantities.

Kajantie et al.; Vuorinen et al.; Haque et al.; ....

We can calculate the 2-pt Matsubara correlators.
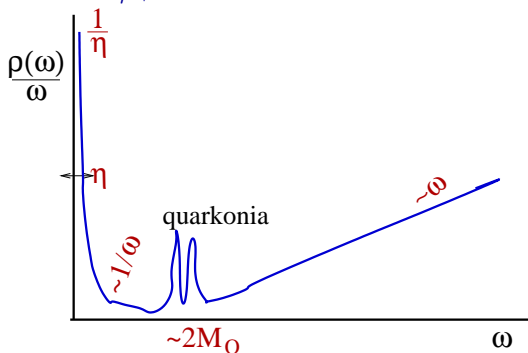Cannot we get the spectral function directly from the correlator?
e.g., $\rho_{\bar{q}\gamma_i q}(\omega)$

$$G_{\bar{q}\gamma_i q}(\tau, \vec{p}) = \int_0^\infty d\omega \, \rho_{\bar{q}\gamma_i q}(\omega, \vec{p}) \, \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

Direct inversion of this equation very difficult: numerically unstable.

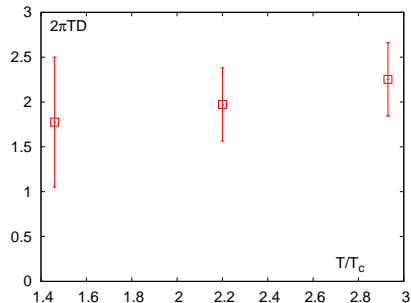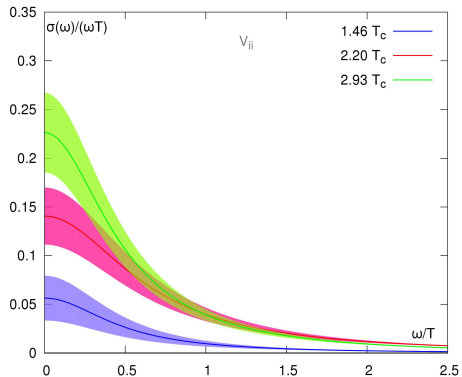# Structure of vector current spectral function

$$\frac{\rho_{jj}(\omega)}{\omega}\Big|_{\omega\to 0} \sim \chi \frac{T}{\pi M}\frac{\eta}{\eta^2+\omega^2}$$

Diffusion part: very Difficult to extract from lattice correlator.
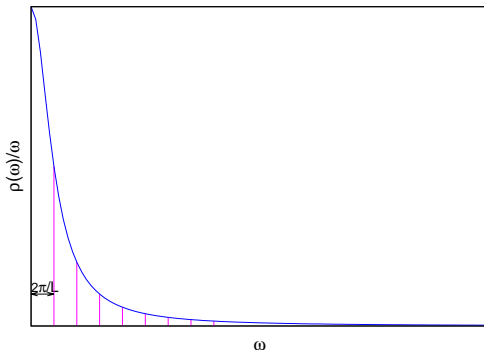
Umeda, 2007; Teaney & Petreczky, 2007

Diffusion coefficient from $\bar{c}\gamma_i c$ correlator

Ding et al., PRD 86 (2012) 014509, 1107.0311
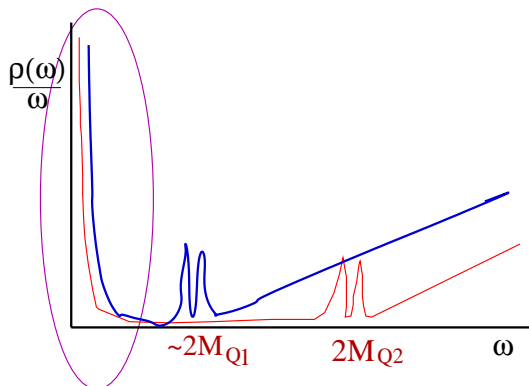
Diffusion part in lattice correlator

Finite lattice provides an infrared as well as ultraviolet cutoff.
Spectral function on lattice sum of discrete δ function peaks.

For extraction of diffusion part: $\frac{2\pi}{L} \ll \eta$

For $DT \sim 1/\pi \qquad \Rightarrow \qquad LT \gg 2M/T \sim 7$ for charm at 1.5 T_c
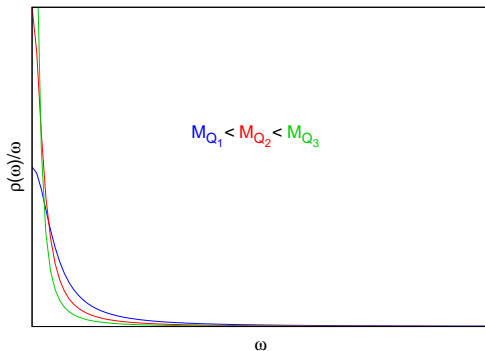
Suggests going to large m_Q

Extracting the diffusion part

$\rho(\omega)/\omega$

$\Rightarrow$ spectral function for $\frac{dJ_i}{dt}$

$$\omega^2 \rho_V(\omega) \;=\; \int dt \; e^{i\omega(t-t')} \int dx \; \frac{1}{2}\langle \dot{J}_i(\mathbf{x}, t)\, \dot{J}_i(\mathbf{0}, t')\rangle$$

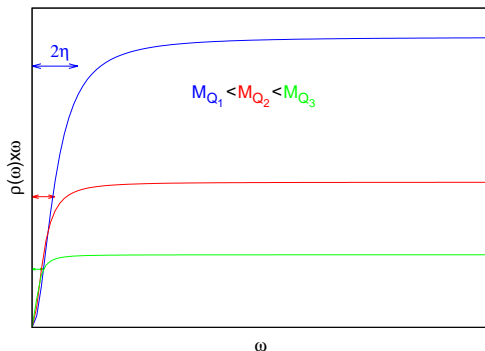# Extracting the diffusion part

$\rho(\omega)/\omega$ $x\omega^2$

$\Rightarrow$ spectral function for $\frac{dJ_i}{dt}$

$$\omega^2 \rho_V(\omega) \;=\; \int dt\; e^{i\omega(t-t')} \int dx\; \frac{1}{2}\langle \dot{J}_i(\mathbf{x},t)\, \dot{J}_i(\mathbf{0},t')\rangle$$

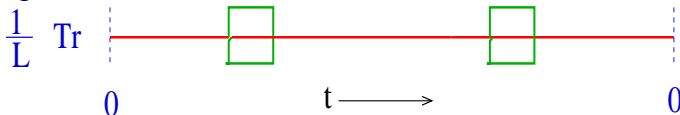# Force-force correlator

J_i : velocity operator for heavy quark

spectral function of $M_Q \dot{J}_i$ $\Leftarrow$ force-force correlator.

$$\rho_F(\omega) = M_Q^2 \int dt \, e^{i\omega(t-t')} \int dx \, \frac{1}{2} \langle \dot{J}_i(\mathbf{x}, t) \, \dot{J}_i(\mathbf{0}, t') \rangle$$

In leading order of NRQCD,

$$M \dot{J}_i = \phi^\dagger E_i \phi - \theta^\dagger E_i \theta$$

leading to an $E - E$ correlator:

Caron-Huot, Laine & Moore, 0901.1195

Langevin description of heavy quark in plasma:

Svetitsky '88; Moore & Teaney '05; Rapp & van Hees '05; Mustafa '05

For thermal heavy quark, $M \gg T$, $\quad p \sim \sqrt{MT}$
Takes $\mathcal{O}(M/T)$ hard collisions to change momentum by $\mathcal{O}(1)$
Scattering with thermal quarks: uncorrelated momentum kicks

$$\frac{dp_i}{dt} = \xi_i(t) - \eta p_i, \qquad \langle \xi_i(t)\xi_j(t') \rangle = \kappa \delta_{ij}\delta(t - t')$$

Static limit of ρ_F: color electric field correlator; Fluctuation dissipation theorem $\Rightarrow$

$$\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega)$$

Caron-Huot, Laine & Moore, 0901.1195

Nonperturbative determination of κ

- Can calculate $\dot{J}_i(t)$ correlator on lattice.
  In static limit: lattice discretized $E_c E_c$ correlator.
  Can extract κ from such a correlator.

- Temperatures $1 - 3T_c$, lattices with $a^{-1} = 20 - 32\,T$
  $LT = 2 - 4$: finite volume effects small.

  Banerjee, Datta, Gavai, Majumdar, 2012 and in progress

- A detailed study at 1.5 T_c, continuum extrapolation with
  $a^{-1} \to 48\,T$

  Francis, Kaczmarek, Laine, Neuhaus, Ohno, 2015
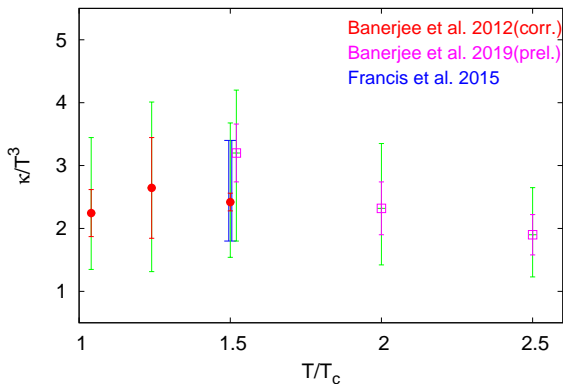
Results
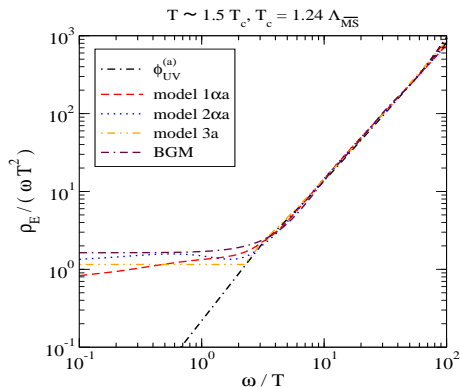
Francis et al., 2015
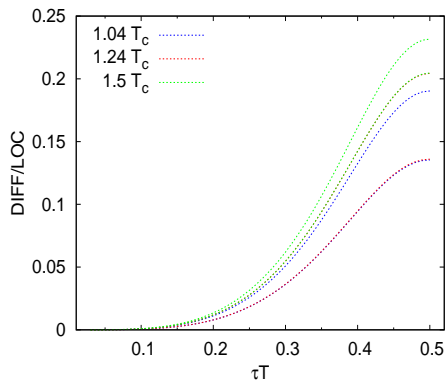
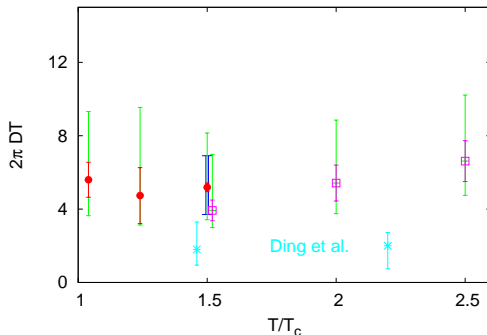
Banerjee et al., 2012

Results for κ

(Here our analysis kept similar to 2012.)

Results for D

Einstein relation $D = \frac{2T^2}{\kappa}$


DT near-flat in this range.

Value in right ballpark for explaining experimental data.

Quarkonia from lattice

- First studies: charmonia with relativistic charm, Bayesian analysis for ρ.
- 1P states dissolve early, 1S states can survive in QGP.
  - Gradual dissociation through thermal broadening for $T > 1.5\,T_c$.

    Datta, Karsch, Petreczky, Wetzorke, 2004.
  - Sharp peak till 1.7 T_c, then sharp dissolution.

    Asakawa & Hatsuda, 2004.
- Correlators consistent with non-dissolution of 1P states.

  Umeda, 2007.
- 1S correlators consistent with dissolution of state, with threshold enhancement.

  Mocsy & Petreczky, 2007-2009.
- 1S states dissolve by 1.5 T_c.

  Ding et al., 2012.
- At T=350 MeV, 1S $\bar{c}\gamma_5 c$ correlator can be described perturbatively with thermally modified m_Q.

  Burnier et al., 1709.07612.
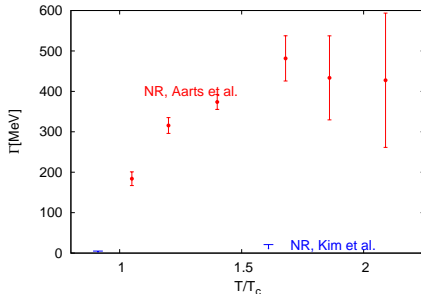
Quarkonia correlators using NRQCD

For $M \gg T$: can use NRQCD. Writing $\omega = 2M_Q + \omega'$, correlators from NRQCD hamiltonian sensitive only to physics around $\omega \sim 2M_Q$.
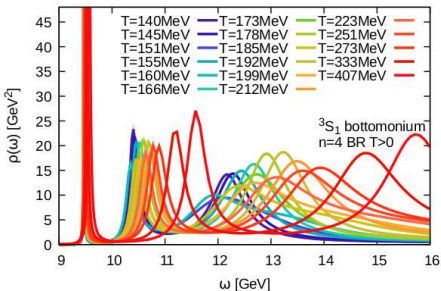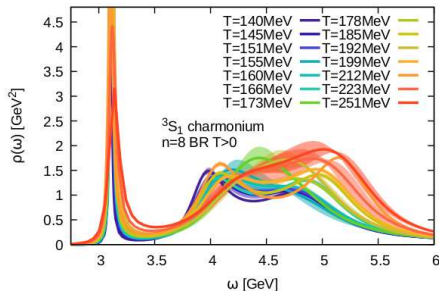
Kim, Petreczky, Rothkopf, 1808.08781

$T_c \sim 150$ MeV. J/ψ studied till $\sim 1.7 T_c$, $\Upsilon(1S)$ till $\sim 2.7 T_c$.

Bottomonia from NRQCD
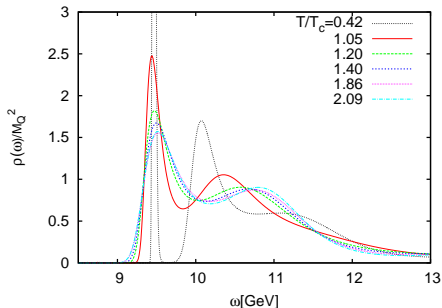
Aarts et al., JHEP 1111 (2011) 103; JHEP 1312 (2013) 064.

$m_l \sim m_s$ (i.e., $m_\pi \sim 680$ MeV).
$\Upsilon(1S)$ survive till $> 2T_c$. But large width, in contrast with Kim et al. (1409.3630).

"Potential" at finite T

Can we write down a "thermal potential" that can be used to study the properties of Bottomonia, e.g., the dilepton peak?
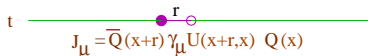
t ———————•——r——○———————
$J_\mu = \overline{Q}(x+r)\,\gamma_\mu U(x+r,x)\,Q(x)$

$$C(t) = \int d^3x \langle J_\mu(\vec{x}, t) J_\mu(\vec{0}, 0) \rangle_T$$

Leads to a time-like Wilson loop at large M.
Define $V(r, T)$ through

t=0 ———————•——○———————
r'
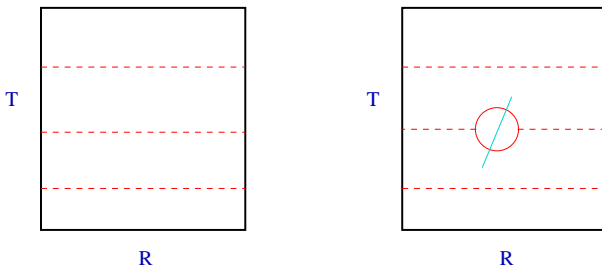
$$\lim_{t \to \infty} i\partial_t C(r, t) = V(r) C(r, t)$$

A calculation with the scale separation $T \gg \alpha M \gg gT$ leads to the potential

$$V(r) = -\frac{4}{3}\alpha_s \frac{e^{-m_D r}}{r} + i\frac{8}{3}\alpha_s T \int_0^\infty dz \frac{z}{(z^2+1)} \frac{\sin zr}{zr}$$

Laine, Philipsen, Romatschke, Tassler, JHEP 0703 (2007) 054.

What is it good for?

- The potential captures the thermal effect at leading order in $1/M$, but resums all orders in α.

- The imaginary part is associated with decay width., and leads to the widening of the spectral function.
- The physics captured is that of Landau damping.
- The perturbative calculation assumes the scale separation

$$T \gg \frac{1}{r} \gg m_D \gg \Lambda_{QCD}$$

Issues with potential

- The potential can be calculated non-perturbatively by analytical continuation of the Wilson loop calculated on lattice.

$$W(R, t) \; = \; \mathcal{Z} \; \int d\omega \; e^{-\omega \tau} \; \rho(R, \omega)$$

Rothkopf et al., 1108.1579; Burnier et al., 1509.07366; Petreczky & Weber, 2018.

- In the literature, usually the potential is calculated from Coulomb gauge fixed Wilson lines.
- To connect to the point current correlator one needs potential from Wilson loops.
- Also, the extraction of the potential usually involves assuming a form of the spectral function/ giving it as input in a bayesian analysis.

Potential from anisotropic lattices

▶ A model-independent extraction of the real part of the potential, V_r, from thermal Wilson loop has been suggested.

Dibyendu Bala, poster.
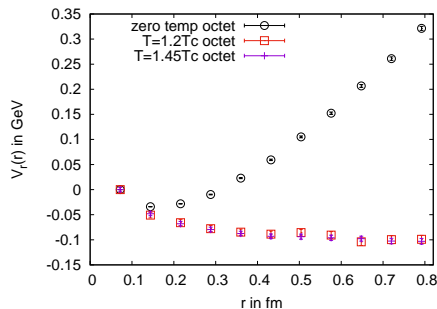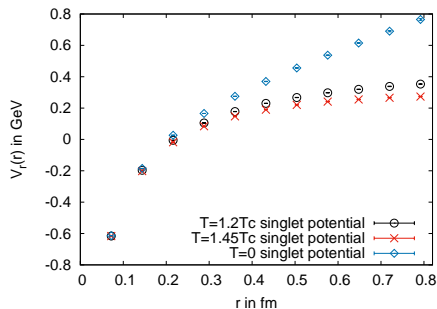
▶ The gauge invariant definition of the octet potential V_r^O involves an adjoint gluonic operator, here B_z. The potential is not expected to depend on the particulars of the operator.

▶ V_r^O is shown modulo r-independent additive const. $C(T)$.

More on potential

- The thermal octet potential is required for study of large $p_\perp$ quarkonia.

  Sharma & Vitev, 2009.

- The singlet V_r above T_c is screened, but does not fit the Yukawa form.

- For both singlet and octet, V_r is close to the corresponding free energy.

- The scale separation $T > 1/r$ not valid for bottomonia for temperatures of interest:

$$\frac{1}{r} > T \sim m_D$$

For temperature scales $T \ll 1/r$, the important physics for decay is thermal gluon scattering.

Brambilla, Petreczky, Vairo, PRD 2008

Thermal gluon scattering
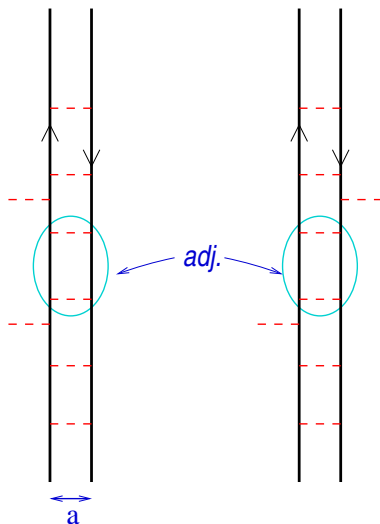
- ▶ Scattering with external gluons will cluster into insertion of a color electric operator.

  Peskin, 1979

- ▶ E.g., the two top A^0 insertions lead to $a\partial_i A^0$

- ▶ Sum of these diagrams lead to operator insertion

$$\frac{1}{2N_c} \int dt \int_0^\infty d\tau \left\langle \vec{a}.g\vec{E}_a(t) \right.$$
$$\left. e^{-i\Delta\tau} \vec{a}.g\vec{E}_a(t-\tau) \right\rangle$$

This can be systematized by writing an effective theory for $\bar{Q}Q$.

Effective theory for $\bar{Q}Q$: pNRQCD

Binding energy of $\bar{Q}Q$ state is $\sim Mv^2 \ll p \sim Mv$, so can integrate gluon energy scale Mv. Leads to a theory non-local in r.
Decomposing the $\bar{Q}Q$ in single and octet, one gets

$$
\begin{aligned}
\mathcal{L}_{pNRQCD} &= \mathrm{Tr} \quad \left[S^\dagger \left(i\partial_0 - V_s(r) \right) S \ + \ O^\dagger \left(iD_0 - V_o(r) \right) O \right] \\
&+ V_1 \quad \mathrm{Tr} \quad \left[S^\dagger \vec{r}.g\vec{E}O \ + \ h.c. \right] + V_2 \, \mathrm{Tr} \, O^\dagger \tilde{r}.g\tilde{E}O + \mathcal{O}(\frac{1}{M})
\end{aligned}
$$

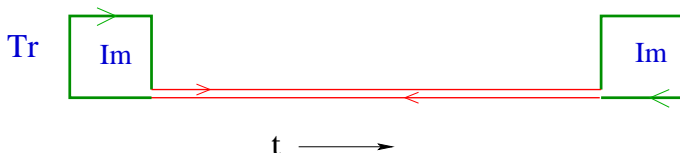Pineda & Soto (1998); Brambilla et al., RMP 77 (2005) 1423

The singlet decay width to leading order in pNRQCD is the gluodissociation width.
The thermal contribution to the SS self energy comes from $r.gE \ r.gE$ correlator.

pNRQCD at finite T

For $1/r \gg T$ one gets

$$\Sigma_s = -i\frac{1}{2N_c}\frac{\langle r^2\rangle}{2}\int_\infty^\infty dt\, e^{-i(V_o-V_s)t}\,\langle E_i^a(t)U_{ab}(t,0)E_i^b(0)\rangle_T$$

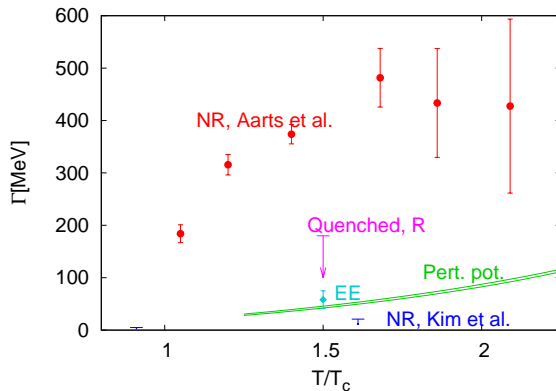Brambilla et al, PRD 78 (2008) 014017

In pert. th. $V_o - V_s \sim \mathcal{O}(\alpha)$. For a leading order estimate let us ignore this.

Note that at this order the above correlator can be related to κ.

Brambilla et al., 1612.07248

From Im $i\Sigma_s$ we can make an estimate of upsilon decay width.

Saumen Datta   Heavy quark systems in QGP: results from lattice QCD

Various estimates of $\Upsilon(1S)$ width

Summary

- Lattice QCD has given important insights into study of quarkonia in QGP; e.g., that there's more to it than Debye-screened-potential estimates.
- Quantitative predictions of, e.g., change of the spectral function with temperature remain a challenge, however.
- A combination of effective theory insights and nonperturbative lattice calculations allows us to make further progress.
- Reasonably stable estimate of the momentum diffusion of heavy quarks has been obtained.
- Progress in nonperturbative understanding of thermal potential: physics of Debye screening and Landau damping.
- A rough estimate of Υ gluodissociation width can be obtained by connecting it to the *EE* correlator.
- A more physical, open quantum system treatment of quarkonia in QGP has been initiated using pNRQCD.

  Brambilla et al., 2017, 2018. Anurag Tiwari, poster.

  Use of lattice QCD will allow going beyond pert. theory.