New analytical solution of Balitsky-Kovchegov equation in the unitarity limit

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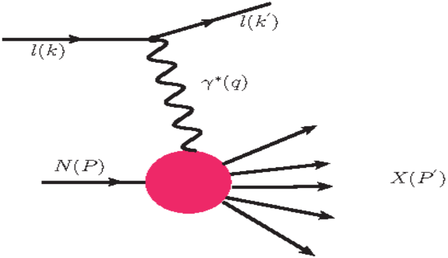
Outline

- Deep inelastic scattering
- QCD evolution landscape
- New solution of Balitsky-Kovchegove (BK) equation
- Conclusion

Deep Inelastic scattering (DIS)

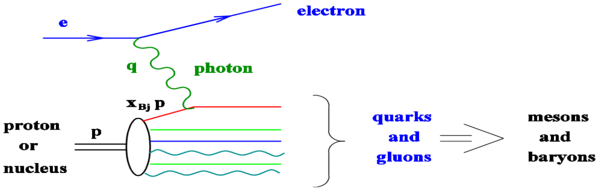
• Deep inelastic scattering is the process used to probe the insides of hadrons (particularly the baryons, such as protons and neutrons), using highly energetic leptons.

$$l(k) + N(P) \longrightarrow l(k^{'}) + X(P^{'})$$



$$\frac{d^2\sigma}{d\Omega' dE'} = \frac{1}{16\pi^2} \frac{|k^{'}|}{|k|} \frac{e^4}{q^4} \mathcal{L}_{\mu\nu} \mathcal{W}^{\mu\nu}$$

Kinematics of DIS



The squared momentum transfer Q^2: Virtuality of photon.

Large $Q^2 \Longrightarrow$ Small distance probed

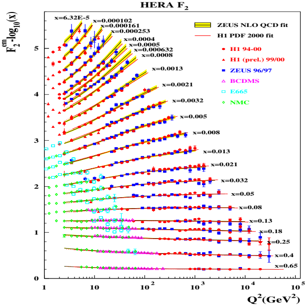
The Bjorken variable x: It determines the protons fraction momenta carried by parton.

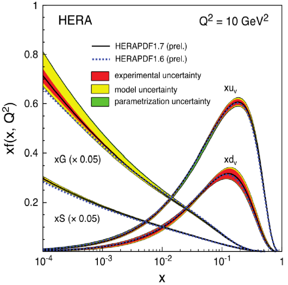
small $x \Longrightarrow$ High energies

$$\frac{d^2\sigma^{ep}}{dE' \, d\Omega} = \frac{4\alpha^2 E'^2 \cos^2(\theta/2)}{q^4 M_P \nu} \left[2\nu \tan^2(\theta/2) \; F_1(x, Q^2) + M_P \; F_2(x, Q^2) \right]$$

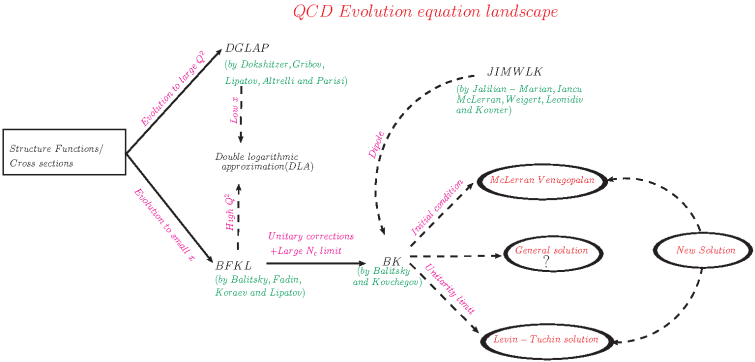
Deep Inelastic scattering (DIS)

Structure function or total cross section are not static but evolve as we increase the scale Q^2 and/or energy x.





QCD evolution equations



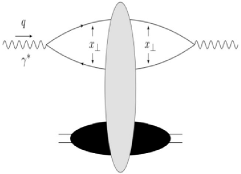
DIS at small-x

To describe DIS in the high energy limit which corresponds to small values of Bjorken-x,it is useful to perform a boost from infinite momentum frame to the so called "Dipole frame"

Dipole Picture

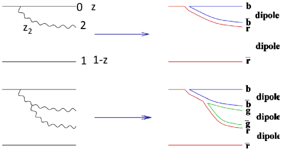
a) Decay of virtual photon into colorless dipole consisting of quark and an antiquark

b) Then interaction of this dipole with the hadron.



Muellers dipole model

To include the quantum evolution in a dipole amplitude one has to use the approach developed by A. H. Mueller in '93-'94 .
Emission of a small-x gluon taken in the large-N_c limit would split the original color dipole into two. Gluon cascade becomes a dipole cascade.



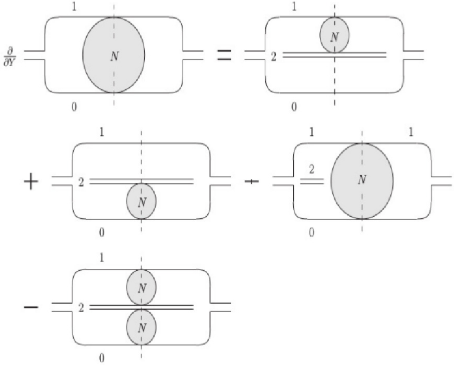
Probability of spilitting of original color dipole into two daughter dipoles

$$I_{dip} = \int d^2 x_2 \frac{x_{10}^2}{x_{20}^2 x_{21}^2} = \int \frac{(x_1 - x_0)^2}{(x_2 - x_0)^2 (x_2 - x_1)^2} d^2 z$$

Balitsky-Kovchegove (BK) equation

The non-linear evolution in the large N_c limit is governed by the Balitsky-Kovchegov equation.

$$\frac{\partial N(x_{10};Y)}{\partial Y} = \frac{\alpha_s N_c}{2\pi^2}\int d^2z \frac{x_{10}^2}{x_{20}^2 x_{21}^2}\left[N(x_{12};Y)+N(x_{20};Y)-N(x_{10};Y)-N(x_{12};Y)N(x_{20};Y)\right]$$



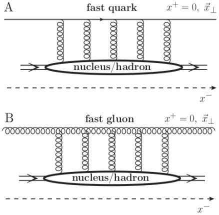
Balitsky-Kovchegove (BK) equation

Fundamental representation

$$V_{x_\perp} = \mathcal{P} \exp \frac{ig}{2} \int_{-\infty}^{\infty} dx^- A^+(x^+ = 0, x^-, x_\perp)$$

Adjoint representation

$$V_{x_\perp} = \mathcal{P} \exp \frac{ig}{2} \int_{-\infty}^{\infty} dx^- \mathcal{A}^+(x^+ = 0, x^-, x_\perp)$$



S-matrix for quark dipole on nuclear target in terms of wilson line

$$S(x_{10}, Y) = \frac{1}{N_c} \langle V(x_{1\perp}, Y) V^\dagger(x_{0\perp}, Y) \rangle$$

S-matrix for the gluon dipole in terms of wilson lines

$$S(x_{10}, Y) \;\; = \;\; \frac{1}{N_c^2 - 1} \langle U(x_{1\perp}, Y) U^\dagger(x_{0\perp}, Y) \rangle$$

S-matrix in terms of scattering amplitude

$$S(x_{\perp, Y}) \;\; = \;\; 1 - N(x_\perp, Y)$$

Balitsky-Kovchegove (BK) equation

Balitsky-Kovchegov equation in terms of S-matrix.

$$\frac{\partial S(x_{10};Y)}{\partial Y} = \frac{\alpha_s N_c}{2\pi^2}\int d^2z \frac{x_{10}^2}{x_{20}^2 x_{21}^2}\left[S(x_{20};Y)S(x_{21};Y) - S(x_{10};Y)\right]$$

In the limit when $x_{20}x_{21} > x_{10}, S(x_{20},Y)S(x_{21},Y) < S(x_{10},Y)$
Linear order BK equation

$$\frac{\partial}{\partial Y}S(x_{10},Y) = -\frac{\alpha_s N_c}{2\pi^2}\int d^2x_2 \, \frac{x_{10}^2}{x_{02}^2 x_{21}^2}\, S(x_{10},Y)$$

New solution of Balitsky-Kovchegove (BK) equation

We have revisited the derivation of this very integral which is central to the dipole studies.

$$I_{dip} = \int \frac{x_{10}^2}{x_{20}^2 x_{21}^2} d^2 x_2$$

Here $d^2 x_2$ is equal to

$$2\pi x_{02} x_{12} \int_0^\infty dk k J_0(k x_{10}) J_0(k x_{20}) J_0(k x_{21}) dx_{20} dx_{12}$$

Where $J_0(z)$ is the Bessel function of the first kind.
So the above integral takes the form

$$I_{dip} = 2\pi x_{10}^2 \int_0^\infty dk k J_0(k x_{10}) \int_0^\infty \frac{dx_{20}}{x_{20}} J_0(k x_{20}) \int_0^\infty \frac{dx_{21}}{x_{21}} J_0(k x_{21})$$

Continued.....

In the dipole model studies one usually introduce lower cutoff into the x_{20} and x_{21} integrals.

$$\int_0^\infty \frac{dx}{x} J_0(kx) \Rightarrow \int_\rho^\infty \frac{dx}{x} J_0(kx) = \ln \frac{2}{k\rho} - \gamma + \mathcal{O}(\rho)$$

1) We have considered $O(\rho)$ and all other higher order terms that have been ignored earlier,

$$\int_\rho^\infty \frac{dx}{x} J_0(kx) = \ln \frac{2}{k\rho} - \gamma + \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{2^{2m}(m!)^2} \frac{1}{2m} k^{2m} \rho^{2m}$$

2) In the earlier studies cutoff ρ usually identified with inverse saturation momentum $\frac{1}{Q_s}$ as

$$x_{20}, x_{21} \geq \rho = \frac{1}{Q_s}$$

Here we have adopted a regularization procedure as done earlier but assumed a general x_{10} dependent form of cutoff as,

$$x_{20}, x_{21} \geq \rho = \frac{1}{Q_s} \frac{1}{\sqrt{\lambda_1 + \lambda_2 (1/x_{10} Q_s)^2}}$$

Continued.....

And finally the dipole integral becomes:

$$I_{dip} = 2\pi \ln\left(\lambda_1 x_{10}^2 Q_s^2 + \lambda_2\right) - 2\pi Li_1\left(\frac{1}{\lambda_1 x_{10}^2 Q_s^2 + \lambda_2}\right)$$

The main result of our calculation is:

$$\boxed{S(x_\perp, Y) = \exp\left(\frac{1 + 2i\nu_0}{2\chi\left(0, \nu_0\right)}\ \mathrm{Li}_2\left[-\lambda_1 x_{10}^2 Q_s^2(Y)\right]\right)}$$

Limits to the New Solution

- McLerran Venugopalan

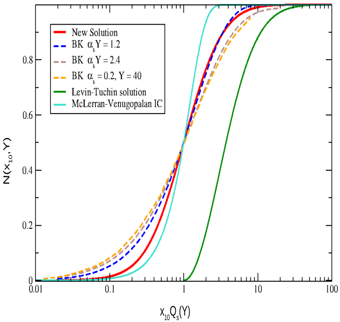
Outside Saturation region ($x_{10} Q_s \ll 1$) ,

$$S(r_\perp, Y) = \exp\left(-1.48\; x_{10}^2 Q_s^2(Y)\right)$$

- Levin-Tuchin Solution

Deep inside the saturation region ($x_{10} Q_s \gg 1$)

$$S(r_\perp, Y) = \exp\left(-\tau\; \ln^2\left[x_{10}^2 Q_s^2(Y)\right]\right)$$



Conclusion

1) We have revisited solution of a linearized form of LO BK equation were we have taken care of all the higher order terms.

2) We derived a general form of solution which reproduce both McLerran-Venugopalan initial conditions (Gaussian in τ) and Levin-Tuchin solution (Gaussian in $\ln \tau$), with τ being scaling variable, in their appropriate limits.

3) This new solution involving dilogarithm function connects both this limits smoothly and better approximates the numerical estimation of full leading order Balitsky-Kovchegov equation particularly inside saturation region.


Thank You