

Anisotropic escape mechanism and elliptic flow of bottomonia

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Based on:

P. P. Bhaduri, N. Borghini, AJ and M. Strickland, arXiv:1809.06235.

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Hydrodynamics in relativistic heavy-ion collisions

- Hydrodynamic modeling of relativistic heavy-ion collisions has been tremendously successful in describing and often predicting experimental measurements.
- Signals such as transverse flow, mass ordering, elliptic and triangular flow are attributed to the presence of a hydrodynamic phase.
- Hydrodynamics naturally leads to measurements of precisely the type and magnitude of these signals found in experiments.
- In particular, flow signatures in experimental data are usually interpreted as a fingerprint of the presence of a hydrodynamic phase during the evolution.
- The pressure gradients, generated from initial-state anisotropy, leads to anisotropic expansion and final-state elliptic flow.
- Success of hydrodynamics also indicate that the system created in relativistic heavy-ion collisions is nearly thermalized.

Flow in small systems

- The sizable flow signals that have been measured in small systems, such as those created in p-A, d-A and ^3He -A collisions, is surprising. [ALICE: PLB 719, 29; ATLAS: PRL 110, 182302; PHENIX: PRL 111, 212301; CMS: PLB 724, 213 (2013).]
- More to our surprise, high multiplicity p-p collisions also indicate collectivity. [CMS Collaboration, JHEP 09, 091 (2010).]
- The flow signals measured in these collisions may have the same origin as those in heavy-ion collisions, namely the presence of a hydrodynamic phase.
- Indeed hydrodynamics has been applied to successfully explain the collective phenomena in these small systems. [P. Bozek, PRC85, 014911; J. Nagle et. al. PRL 113, 112301; B. Schenke and R. Venugopalan, PRL 113, 102301].
- However, since these small systems are short-lived compared to those created in heavy ion collisions, one may look for some other, non-hydrodynamic mechanisms for generation of flow.

How to fake hydrodynamic signals [\[Paul Romatschke, NPA 956 \(2016\) 222\]](#)

- If not hydrodynamics, what else could it be?
- No single theoretical idea comes close to describing the multitude of experimental data as well as hydrodynamics.
- However, some theoretical models are able to describe certain types of signals.
- Some of these models are:
 - Hadronic Cascades. [\[Y. Zhou, X. Zhu, P. Li, H. Song, PRC 91, 064908\]](#).
 - AMPT. [\[J.D.OrjuelaKoop, A.Adare, D.McGlinchey, J.L.Nagle, PRC 92, 054903; P. Bozek, A. Bzdak, G.-L. Ma, PLB 748, 301\]](#).
 - The escape mechanism. [\[L. He, T. Edmonds, Z.-W. Lin, F. Liu, D. Molnar, F. Wang, PLB 753, 506; P. P. Bhaduri and AJ, PRC 97, 044909\]](#).
 - Free-streaming + hadronization. [\[Paul Romatschke, EPJC 75 \(9\) 429\]](#).
- Lessons learnt: transverse flow and femtoscopic measurements can easily be forged through non-hydrodynamic evolution, while large elliptic flow requires some non-vanishing interactions in the hot phase.

The escape mechanism [L. He, T. Edmonds, Z.-W. Lin, F. Liu, D. Molnar, F. Wang, PLB 753, 506.]

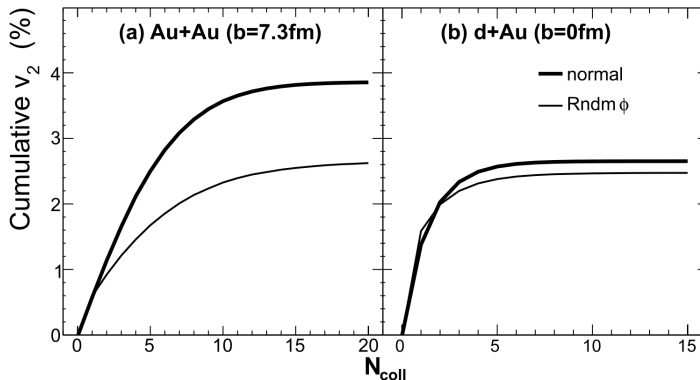
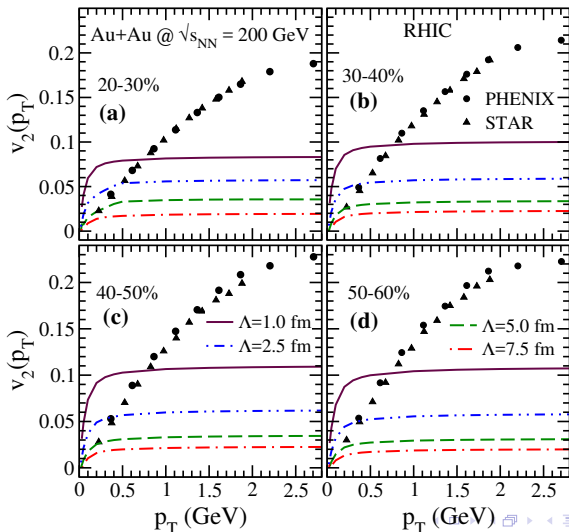


Fig. 7. Cumulative v_2 of all partons (see text) as a function of N_{coll} in (a) Au + Au and (b) d + Au collisions. Both normal (thick curves) and azimuth-randomized (thin curves) AMPT results are shown.

$$P_{\text{esc}} = \exp \left(- \int \rho \sigma dl \right)$$



- We study the role of anisotropic escape in generating the elliptic flow of bottomonia produced in ultrarelativistic heavy-ion collisions.
- We assume that the bottomonium does not thermalize with the medium due to its large mass.
- We implement temperature-dependent decay widths for the various bottomonium states and calculate their survival probability when traversing through the anisotropic medium formed in non-central collisions.
- We use initial conditions from a Glauber model and mimic the transverse expansion of the fireball in an effective way, assuming longitudinal boost invariance.
- We provide a quantitative prediction for bottomonium elliptic flow generated from an anisotropic escape mechanism.

Fireball expansion

- For the longitudinal expansion, we consider boost-invariant flow, i.e. $v^z = z/t$. Accordingly, we work in the Milne coordinate system $(\tau, r, \theta, \eta_s)$, where $\tau = \sqrt{t^2 - z^2}$ and $\eta_s = \tanh^{-1}(z/t)$.
- In the absence of transverse expansion, longitudinal boost invariance results in the well-known Bjorken scaling solution

$$\varepsilon(x, y, \tau) = \varepsilon(x, y; \tau_i) \left(\frac{\tau}{\tau_i} \right)^{-4/3}$$

- For purely radial expansion, the medium which is at $x_i = r_i \cos \theta$, $y_i = r_i \sin \theta$ at the initial time τ_i is at

$$x(\tau) = r(\tau) \cos \theta \quad , \quad y(\tau) = r(\tau) \sin \theta$$

- Putting this radial transverse motion into the Bjorken solution

$$\varepsilon(x(\tau), y(\tau); \tau) = \varepsilon(x_i, y_i; \tau_i) \left(\frac{\tau}{\tau_i} \right)^{-4/3}$$

Fireball expansion contd.

- We parametrize the (radial) transverse velocity profile of the medium:

$$\beta_T(\tau, r) = [\beta_0(\tau) + 2\beta_2(\tau) \cos 2\theta] \frac{r}{R_0}.$$

- For the functions encoding the time dependence of the transverse velocity profile, we parametrize

$$\beta_0(\tau) = b_0 \left(\frac{\tau - \tau_i}{\tau_i} \right), \quad \beta_2(\tau) = b_2 \left(\frac{\tau - \tau_i}{\tau_i} \right).$$

- The values of b_0 and b_2 are tuned to match the blast wave fitted values of $\beta_0^{\text{BW}} = \beta_0(\tau_f^{\text{BW}})$ and $\beta_2^{\text{BW}} = \beta_2(\tau_f^{\text{BW}})$

$$b_0 = \beta_0^{\text{BW}} \left(\frac{\tau_i}{\tau_f'} \right)^{n_0}, \quad b_2 = \beta_2^{\text{BW}} \left(\frac{\tau_i}{\tau_f'} \right)^{n_2}, \quad \tau_f' \equiv \tau_f^{\text{BW}} - \tau_i.$$

- Using the fact that $\beta_T(\tau', r) = dr/d\tau'$, where $\tau' = \tau - \tau_i$, we get

$$r_i = r(\tau') \exp \left[- (\beta_0(\tau') + 2\beta_2(\tau') \cos 2\theta) \frac{\tau'}{2R_0} \right].$$

Initial distribution of Bottomonium

- We use Glauber model for spatial initialization of the fireball energy density profile.
- The bottomonium states are produced in initial hard scattering processes during the very earliest stages of the heavy-ion collision.
- The spatial distribution of the bottomonium production points in the transverse plane is assumed to follow that of the number of binary collisions, $N_{\text{coll}}(x, y)$.
- We assume a power-law transverse momentum (p_T) distribution of the Υ 's obtained from PYTHIA simulations for $p + p$ collisions, scaled by the mass number of the colliding nuclei [K. Zhou, N. Xu and P. Zhuang, Nucl. Phys. A 931, 654.].
- This kind of scaling implicitly assumes that the bottomonia do not “flow” with the medium and any v_2 that we obtain in our model will be purely due to the anisotropic escape mechanism.

- The initial Υ distribution for $p + p$ collision is given by

$$\frac{d^2\sigma_{\Upsilon}^{pp}}{p_T dp_T dY} = \frac{4}{3\langle p_T^2 \rangle_{pp}} \left(1 + \frac{p_T^2}{\langle p_T^2 \rangle_{pp}} \right)^{-3} \frac{d\sigma_{\Upsilon}^{pp}}{dY},$$

with Y being the longitudinal rapidity in momentum space.

- Here $\langle p_T^2 \rangle_{pp}(Y) = 20(1 - Y^2/Y_{\max}^2) (\text{GeV}/c)^2$, where $Y_{\max} = \cosh^{-1}(\sqrt{s_{NN}}/(2m_{\Upsilon}))$ is the most forward rapidity of the bottomonia.
- Eventually, the momentum rapidity density follows a Gaussian distribution:
$$\frac{d\sigma_{\Upsilon}^{pp}}{dY} = \frac{d\sigma_{\Upsilon}^{pp}}{dY} \Big|_{Y=0} e^{-Y^2/0.33Y_{\max}^2}.$$
- For our calculations, we integrate over Y and consider the resulting p_T distribution.

Formation time of bottomonium states

- The formation of each bound bottomonium state requires a finite formation time τ_{form} .
- The value τ_{form}^0 of the latter in the bottomonium rest frame is assumed to be proportional to the inverse of the vacuum binding energy for each state.
- For the $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\chi_b(1P)$ and $\chi_b(2P)$ states we use $\tau_{\text{form}}^0 = 0.2, 0.4, 0.6, 0.4, 0.6$ fm/c, respectively [B. Krouppa and M. Strickland, arXiv:1605.03561].
- In the laboratory frame, relative to which a bottomonium state with mass M has transverse momentum p_T , the formation time becomes $\tau_{\text{form}} = E_T \tau_{\text{form}}^0 / M$ with $E_T = \sqrt{p_T^2 + M^2}$.

Propagation through the medium

- Since the bottomonium states are color neutral heavy particles, we assume that they propagate quasi freely following nearly straight-line trajectories.
- If (x_0, y_0) denotes the position in the transverse plane where a bottomonium with momentum p_T is at time τ_i , it will at a later time $\tau = \tau_i + \tau'$ be at

$$x' = x_0 + v_T \tau' \cos \phi \quad , \quad y' = y_0 + v_T \tau' \sin \phi,$$

where $v_T = p_T/E_T$ is the bottomonium transverse velocity and ϕ the azimuthal angle of its transverse momentum.

- Introducing now the polar coordinates (r', θ) of (x', y') ,

$$r' \cos \theta = x_0 + v_T \tau' \cos \phi \quad , \quad r' \sin \theta = y_0 + v_T \tau' \sin \phi,$$

where the radial coordinate is

$$r' = \sqrt{(x_0 + v_T \tau' \cos \phi)^2 + (y_0 + v_T \tau' \sin \phi)^2} \equiv r(\tau'),$$

Dissociation and width of bottomonium

- Using $r_i = r(\tau') \exp \left[-(\beta_0(\tau') + 2\beta_2(\tau') \cos 2\theta) \frac{\tau'}{2R_0} \right]$, we can to determine the initial position at $\tau' = 0$ of the piece of medium which is at (x', y') at τ' , and thus the medium energy density (or temperature) experienced by the $b\bar{b}$ pair.
- For this temperature, we obtain the thermal decay widths $\Gamma(T(x', y', \tau'))$ of the bottomonium states, adopting the recent state-of-the-art results of in-medium dissociation of different bound $b\bar{b}$ states [M. Strickland and D. Bazow, Nucl. Phys. A 879, 25].
- The final transmittance for a $b\bar{b}$ bound state labelled by j is given by

$$\mathcal{T}_j(x, y, p_T, \phi) = \exp \left[-\Theta(\tau_f - \tau_j^{\text{form}}) \int_{\max(\tau_j^{\text{form}}, \tau_i)}^{\tau_f} d\tau' \Gamma_j(T(x', y'; \tau')) \right],$$

where Θ is the usual step function.

Results

- We numerically calculate the elliptic flow of bottomonia for Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.
- The energy density in the transverse plane is generated from a Glauber model with Woods-Saxon nuclear density distribution, following a linear combination of the spatial profiles of the number of participant nucleons N_{part} and number of binary collisions N_{coll} as $\varepsilon_i(x, y) \propto 0.85N_{\text{part}}(x, y) + 0.15N_{\text{coll}}(x, y)$.
- The inelastic nucleon-nucleon interaction cross section is taken to be $\sigma_{NN} = 62$ mb and the initial energy density in the central cell of central Pb+Pb collisions at $\tau_i = 0.4$ fm is taken to be 85 GeV/fm³, which corresponds to an initial temperature of 480 MeV in the central region.
- In order to compare to the experimental results, we integrate over the entire temperature profile in the transverse plane.

Effect of hydrodynamic parameters

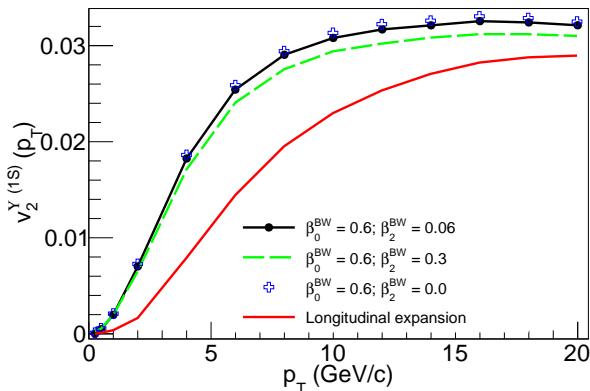


Figure: Transverse momentum dependence of v_2 of $\Upsilon(1S)$ for different combinations of parameters for the medium expansion for Pb + Pb collisions at $\sqrt{s_{\text{NN}}} = 2.76$ TeV in 40 – 50% centrality class. The curve labeled “longitudinal expansion” corresponds to $\beta_0^{\text{BW}} = \beta_2^{\text{BW}} = 0$.

Feed down

- To account for the post-QGP feed down of the excited states, we use a p_T -averaged feed down fraction obtained from a recent compilation of $p + p$ data at LHC [B. Krouppa and M. Strickland, arXiv:1605.03561].
- The inclusive spectra for $\Upsilon(1S)$ is then calculated from a linear superposition of the raw spectra for each state:

$$\frac{d^2 N_{\Upsilon(1S)}^{all}}{d^2 p_T} = \sum_i f_i \frac{d^2 N_i}{d^2 p_T}.$$

- The contribution from different states are as follows: $f_{2S \rightarrow 1S} = 8.6\%$, $f_{3S \rightarrow 1S} = 1\%$, $f_{1P \rightarrow 1S} = 17\%$, $f_{2P \rightarrow 1S} = 5.1\%$ and $f_{3P \rightarrow 1S} = 1.5\%$.
- Since contribution from $3S$ and $3P$ states are small, we include their percentage contributions in $2S$ and $2P$ states, respectively.
- While considering feed down, the transverse momentum of the mother and daughter bottomonium states are assumed to be the same.

Effect of feed down

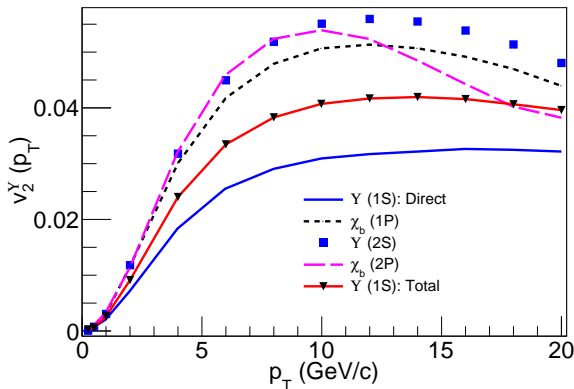


Figure: Transverse momentum dependence of elliptic flow parameter for different bottomonium states with $\beta_0^{\text{BW}} = 0.6$ and $\beta_2^{\text{BW}} = 0.06$, for Pb + Pb collisions at $\sqrt{s_{\text{NN}}} = 2.76$ TeV in 40 – 50% centrality class. For $\Upsilon(1S)$, directly produced states and the inclusive yield including feed down contributions are shown.

Centrality dependence

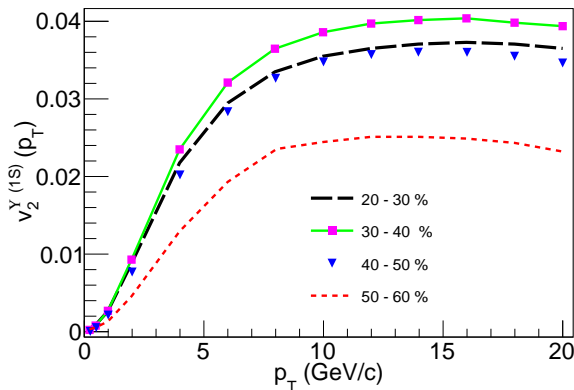


Figure: Transverse momentum dependence of v_2 of $\Upsilon(1S)$ including feed down contributions from higher excited states for different centrality for Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

Summary

- We have provided a quantitative prediction for the elliptic flow of bottomonia produced in mid-central collisions in $\sqrt{s_{\text{NN}}} = 2.76$ TeV Pb + Pb collisions at LHC via an anisotropic escape mechanism.
- We employed the Glauber model to generate initial distribution of energy density in the plane transverse to the beam axis.
- Using temperature-dependent decay widths for bottomonium states, we calculated their survival probability when traversing through the hot and dense anisotropic medium formed in non-central collisions.
- We have considered longitudinal Bjorken flow and a blast wave motivated effective expansion in the transverse plane.
- We also accounted for the feed down contribution to the bottomonium ground state from higher excited states.
- We found that the transverse momentum dependence of the elliptic flow of bottomonia is of the level of few percent.

Outlook

- One of the shortcomings of the present calculation is an effective expansion model.
- We plan to use the results from realistic hydrodynamic simulation to consider the expansion.
- It will be interesting to consider the effect of medium-induced transitions between bound states which is predicted from the open quantum system approach [N. Borghini and C. Gombeaud, EPJC 72, 2000; arXiv:1103.2945; Y. Akamatsu and A. Rothkopf, PRD 85, 105011].
- In this "state reshuffling" scenario, transitions between various bound states become possible which counteracts the usual suppression picture by allowing for the re-formation of otherwise suppressed states even above the hadronization temperature.
- Since the excited states of bottomonium acquire more elliptic flow due to anisotropic escape mechanism, one may expect to generate larger flow owing to the feed-down from excited states.

Backup: Imaginary part of the potential

2.2.7. Model for the imaginary part of the potential

The imaginary part of the potential $\Im[V]$ is obtained from a leading order perturbative calculation which was performed in the small anisotropy limit [44]. The resulting imaginary part of the potential is

$$\Im[V] = -\alpha_s C_F T \left[\phi(\hat{r}) - \xi (\psi_1(\hat{r}, \theta) + \psi_2(\hat{r}, \theta)) \right], \quad (35)$$

where $\hat{r} = m_D r$, $\alpha_s = g^2/(4\pi)$, $C_F = (N_c^2 - 1)/(2N_c)$, and

$$\phi(\hat{r}) = 2 \int_0^\infty dz \frac{z}{(z^2 + 1)^2} \left[1 - \frac{\sin(z \hat{r})}{z \hat{r}} \right], \quad (36)$$

$$\psi_1(\hat{r}, \theta) = \int_0^\infty dz \frac{z}{(z^2 + 1)^2} \left(1 - \frac{3}{2} \left[\sin^2 \theta \frac{\sin(z \hat{r})}{z \hat{r}} + (1 - 3 \cos^2 \theta) G(\hat{r}, z) \right] \right), \quad (37)$$

$$\psi_2(\hat{r}, \theta) = - \int_0^\infty dz \frac{\frac{4}{3} z}{(z^2 + 1)^3} \left(1 - 3 \left[\left(\frac{2}{3} - \cos^2 \theta \right) \frac{\sin(z \hat{r})}{z \hat{r}} + (1 - 3 \cos^2 \theta) G(\hat{r}, z) \right] \right), \quad (38)$$

with θ being the angle from the beam direction and

$$G(\hat{r}, z) = \frac{\hat{r} z \cos(\hat{r} z) - \sin(\hat{r} z)}{(\hat{r} z)^3}. \quad (39)$$