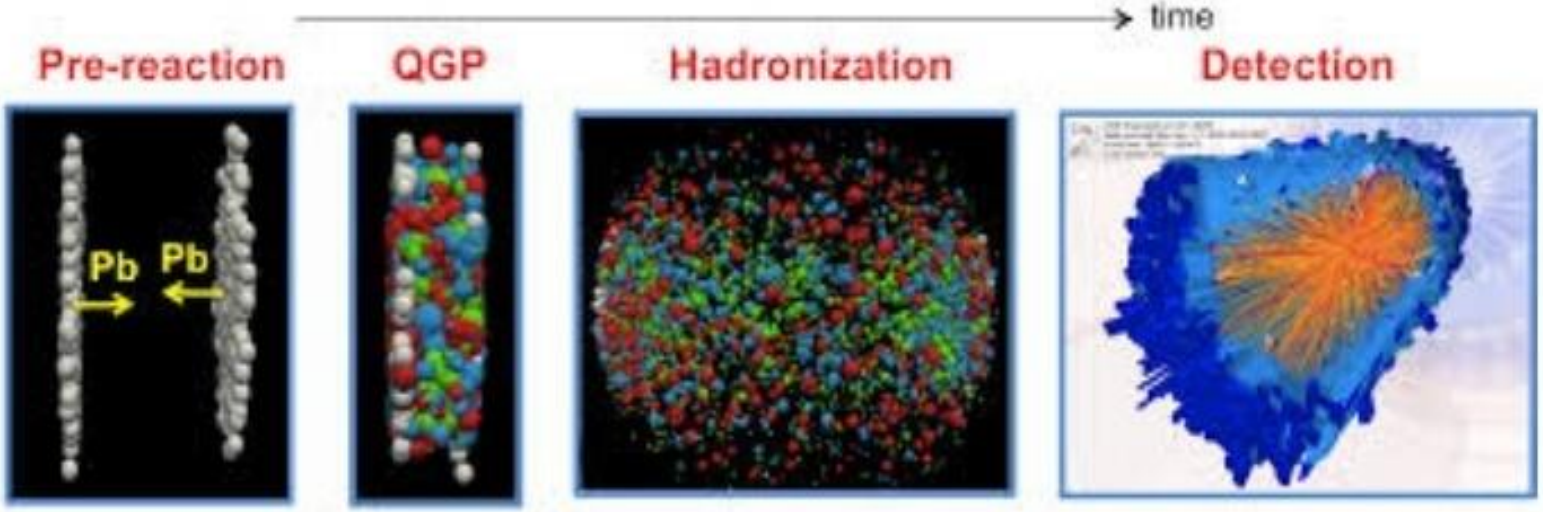
Heavy quarks dynamics in heavy ion collisions



Santosh Kumar Das

School of Physical Science
Indian Institute of Technology Goa
Goa, India

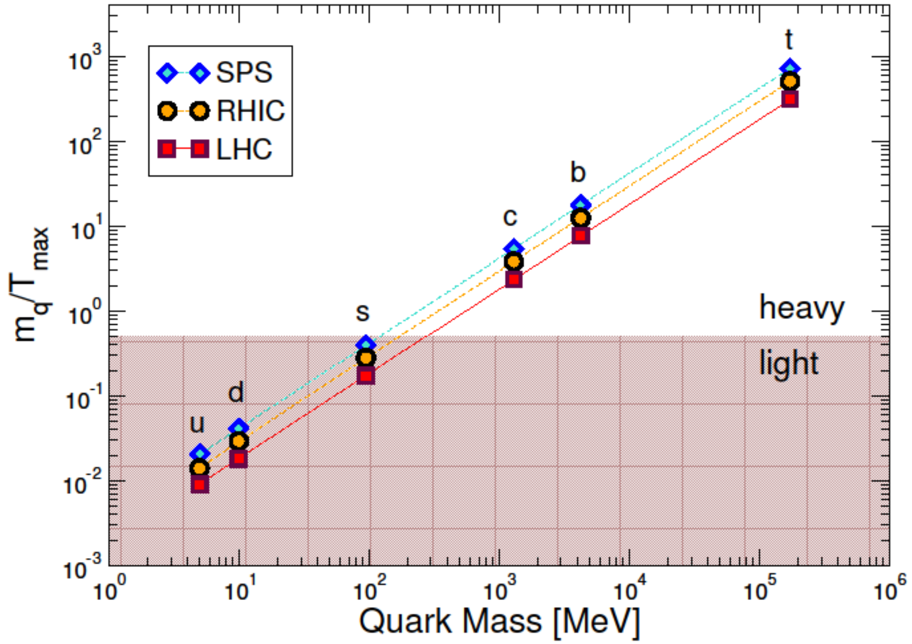
In Collaboration with:
V. Greco, M. Ruggieri, S. Plumari, F. Scardina

OUTLINE ……………

- Introduction
- Quark Gluon Plasma - the primordial fluid
- Heavy quark momentum evolution: Langevin vs Boltzmann
- Probing of initial electromagnetic field by heavy quarks
- Heavy quark dynamics in small system
- Summary and outlook

Heavy Quark & QGP

At very high density and temperature hadrons melt to a new phase of matter called Quark Gluon Plasma (QGP).



SPS to LHC

$\sqrt{s} = 17.3 GeV\ to\ 2.76 TeV$ **~100 times**

$T_i = 200\ MeV\ to\ 600\ MeV$ **~3 times**

$$M_{c,b} >> \Lambda_{QCD}$$

Produced by pQCD process (out of Equil.)

$$\tau_{c,b} >> \tau_{QGP}$$

They go through all the QGP life time

$$M_{c,b} >> T_0$$

No thermal production

Boltzmann Kinetic equation

$$\left(\frac{\partial}{\partial t}+\frac{P}{E}\frac{\partial}{\partial x}+\mathrm{F}.\frac{\partial}{\partial p}\right)f\left(x,\, p,\, t\right)=\left(\frac{\partial f}{\partial t}\right)_{col}$$

- The plasma is uniform ,i.e., the distribution function is independent of x.
- In the absence of any external force, F=0

$$\boxed{R\left(p,t\right)=\left(\frac{\partial f}{\partial t}\right)_{col}=\int d^3k\left[\omega\left(p+k,k\right)f\left(p+k\right)-\omega\left(p,k\right)f\left(p\right)\right]}$$

$$\omega(p,k)=g\int\frac{d^3q}{(2\pi)^3}f'(q)v_{q,p}\sigma_{p,q\to p-k,q+k}$$

→ is rate of collisions which change the momentum of the charmed quark from p to p-k

$$\omega\left(p+k,k\right)f\left(p+k\right)\approx\omega\left(p,k\right)f\left(p\right)+k.\frac{\partial}{\partial p}\left(\omega f\right)+\frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i\partial p_j}\left(\omega f\right)$$

$$\boxed{\frac{\partial \mathbf{f}}{\partial \mathbf{t}}=\frac{\partial}{\partial \mathbf{p}_\mathbf{i}}\left[\mathbf{A}_\mathbf{i}(\mathbf{p})\mathbf{f}+\frac{\partial}{\partial \mathbf{p}_\mathbf{j}}\left[\mathbf{B}_\mathbf{ij}(\mathbf{p})\mathbf{f}\right]\right]}$$

B. Svetitsky PRD 37(1987)2484

where we have defined the kernels

, $\mathbf{A}_\mathbf{i}=\int \mathbf{d}^3\mathbf{k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k}_\mathbf{i}$ → **Drag Coefficient**

$\mathbf{B}_\mathbf{ij}=\int \mathbf{d}^3\mathbf{k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k}_\mathbf{i}\mathbf{k}_\mathbf{j}$ → **Diffusion Coefficient**

Langevin Equation

$$dx_j = \frac{p_j}{E}dt$$

$$dp_j = -\Gamma p_j dt + \sqrt{dt}\,C_{jk}(t, p+\xi dp)\rho_k$$

where Γ **is the deterministic friction (drag) force**

C_{ij} **is stochastic force in terms of independent Gaussian-normal distributed random variable**

$$\rho = (\rho_x, \rho_y, \rho_z) \quad , \quad P(\rho) = \left(\frac{1}{2\pi}\right)^3 \exp\left(-\frac{\rho^2}{2}\right)$$

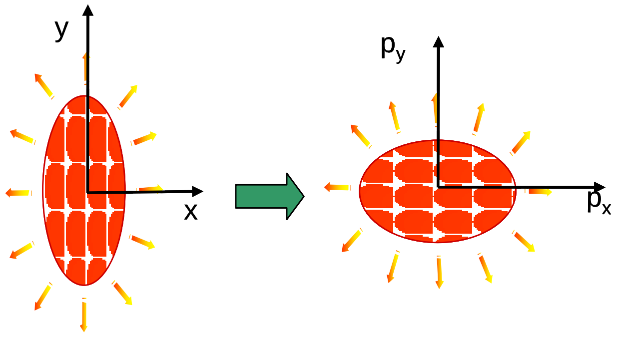
With $< \rho_i(t)\rho_k(t') > = \delta(t-t')\delta_{jk}$

H. v. Hees and R. Rapp
arXiv:0903.1096

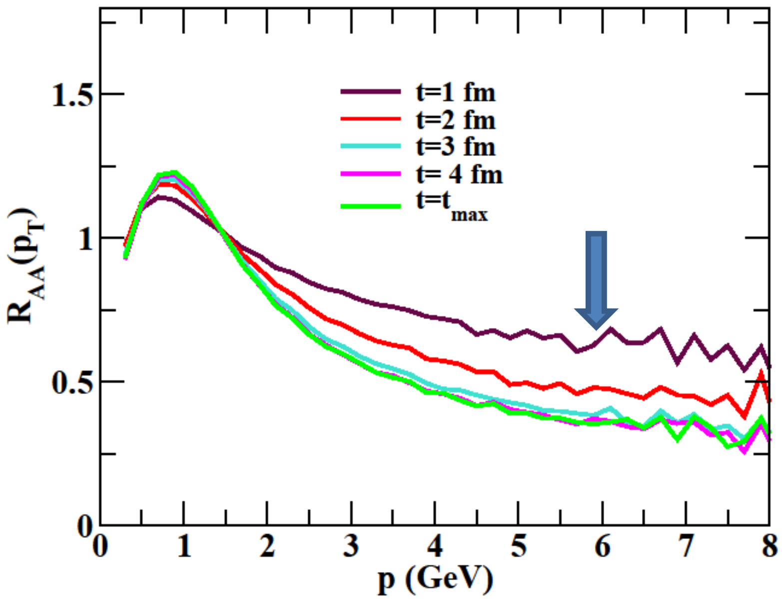
$\xi = 0$ **the pre-point Ito interpretation of the momentum argument of the covariance matrix.**

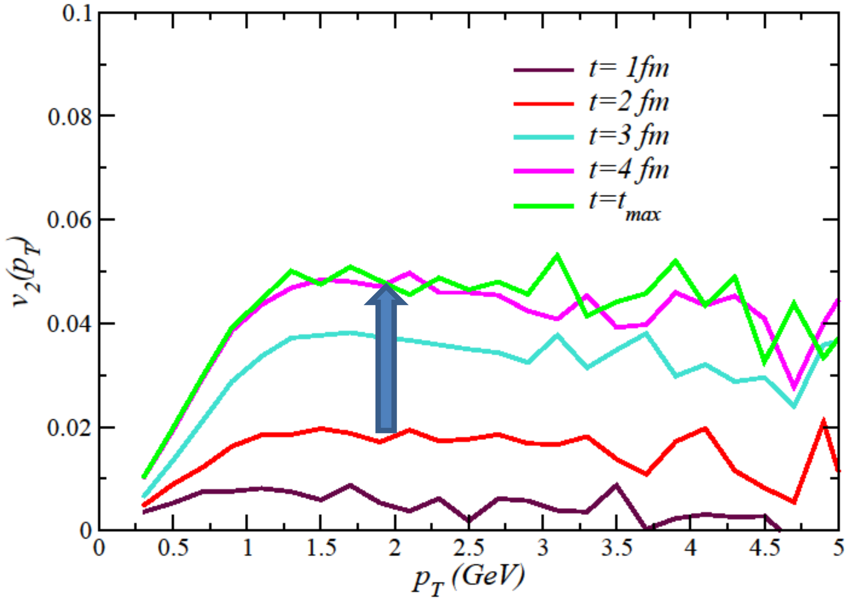
$$\boxed{R_{AA} = \frac{\left(\dfrac{dN}{d^2p_T dy}\right)^{Au+Au}}{N_{coll}\left(\dfrac{dN}{d^2p_T dy}\right)^{p+p}}}$$

$$\boxed{v_2(p_T) = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_x^2} \right\rangle}$$

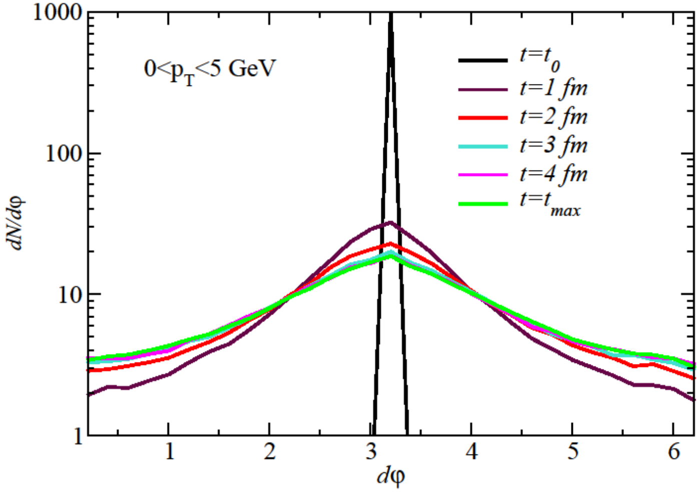


Time evolution of Heavy quarks observables





Das, Scardina, Plumari, Greco
J. Phys. Conf. Ser. 668 (2016)012051



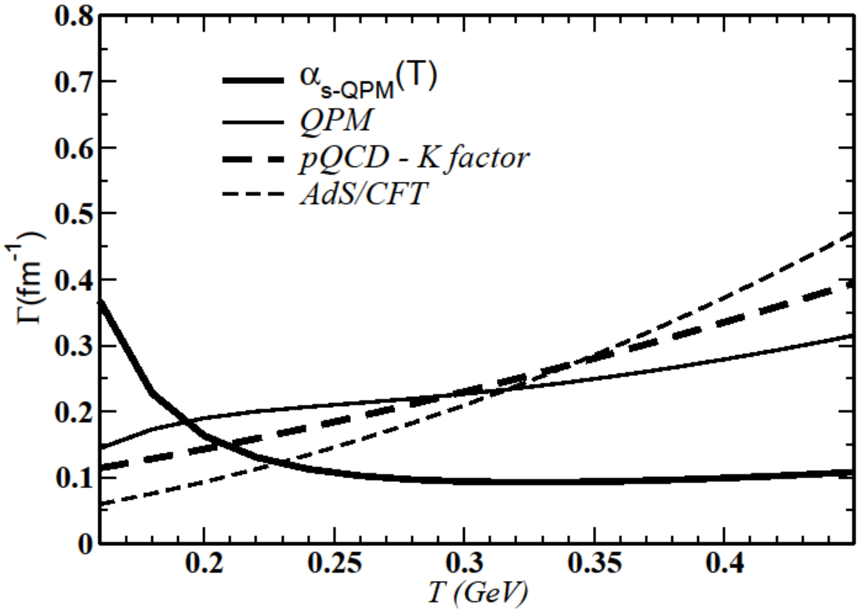
RAA and dN/dphi_ccbar developed during the early stage of the evolution → T_i

V2 developed duringing the later stage of the evolution → T_c

T dependence of the interaction i.e the transport coefficients are the essential ingredient for the simultaneous description of HQ observables.

T- dependence of the Drag Coefficient

Drag Coefficient



Das, Scardina, Plumari, Greco
Phys. Lett. B 747 (2015)260-264

Chandra and Das
Phys. Rev. D,93 (2016) , 094036

pQCD *(Combridge)*

$$\alpha_{pQCD} = \frac{4\pi}{11\ln\left(2\pi T\Lambda^{-1}\right)}, \quad m_D^2 = 4\pi\alpha_{pQCD}(\mathrm{T})T^2$$

AdS/CFT

$$\gamma_{AdS/CFT} = k\frac{T^2}{M}$$

Gubser
PRD,74,126005 (2006)
Akamatsu, Hatsuda, Hirano
PRC, 79, 054907 (2009)
Das and Davody
PRC, 89,054912 (2014)

Quasi-Particle-Model *(fit to lQCD ε,P)*

$$g^2_{QP}(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

λ=2.6
T_s=0.57

$$m_g^2 = \frac{1}{6}\left(N_c + \frac{1}{2}N_f\right)g^2T^2$$

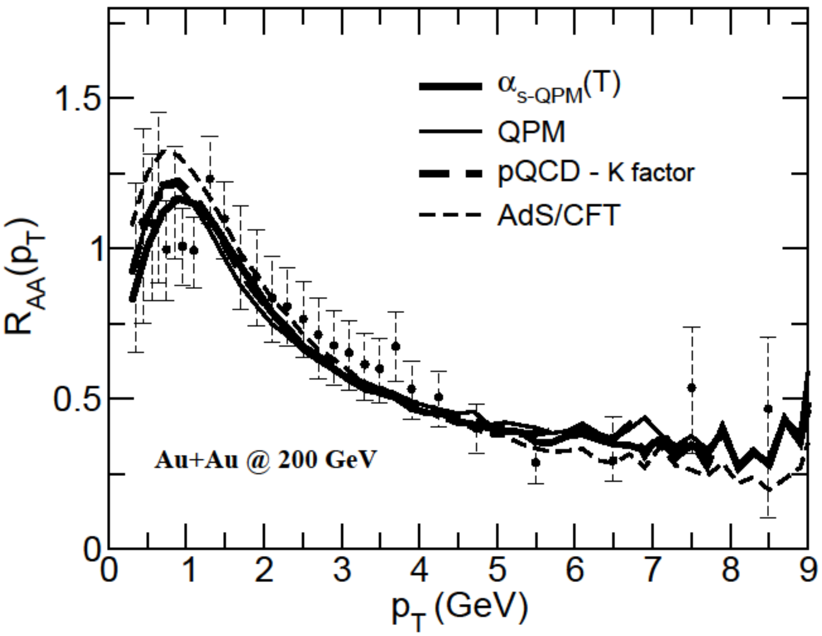
$$m_q^2 = \frac{N_c^2 - 1}{8N_c}g^2T^2($$

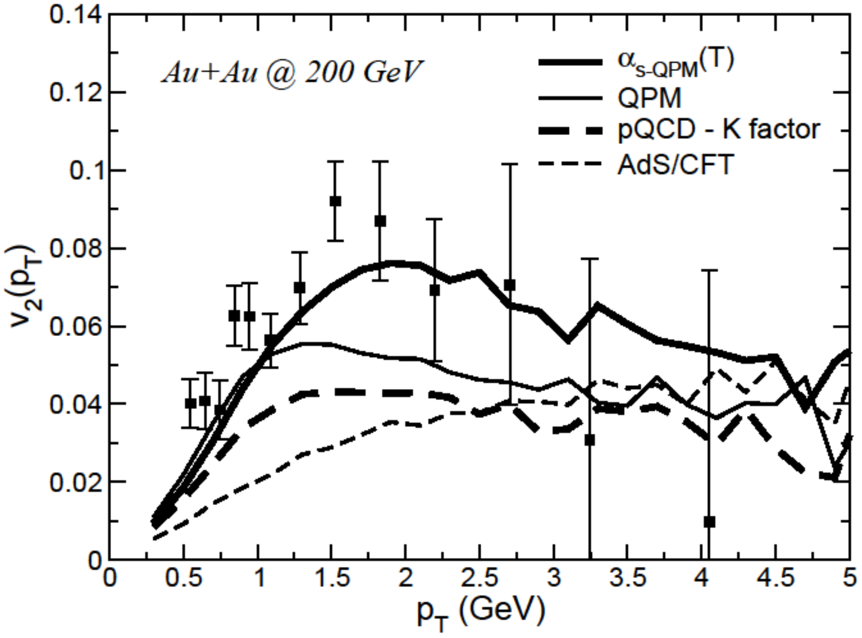
α_{QPM}(T) , $m_{q,g}$=0

we mean simply the coupling of the QPM,
but with a bulk of massless q and g .

RAA and v2 @ RHIC

(Au+Au@200AGeV, b=8 fm)



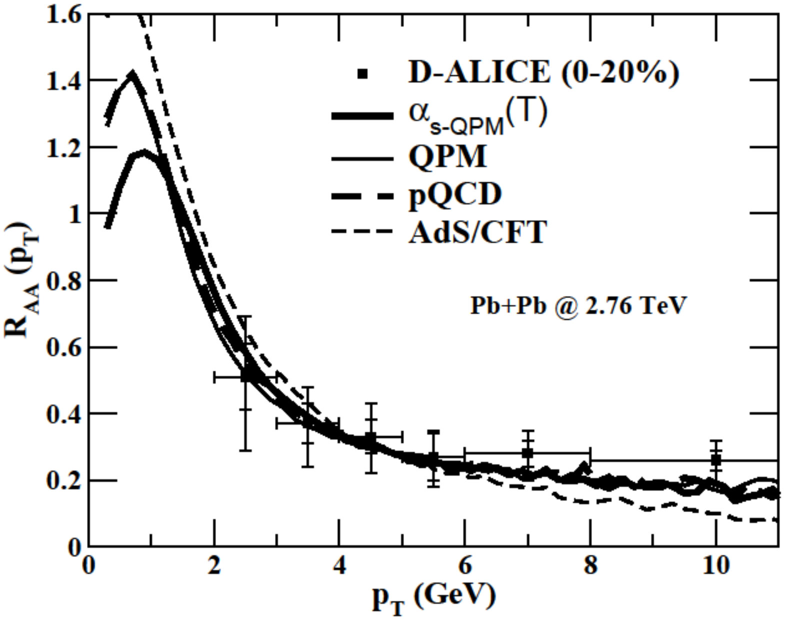


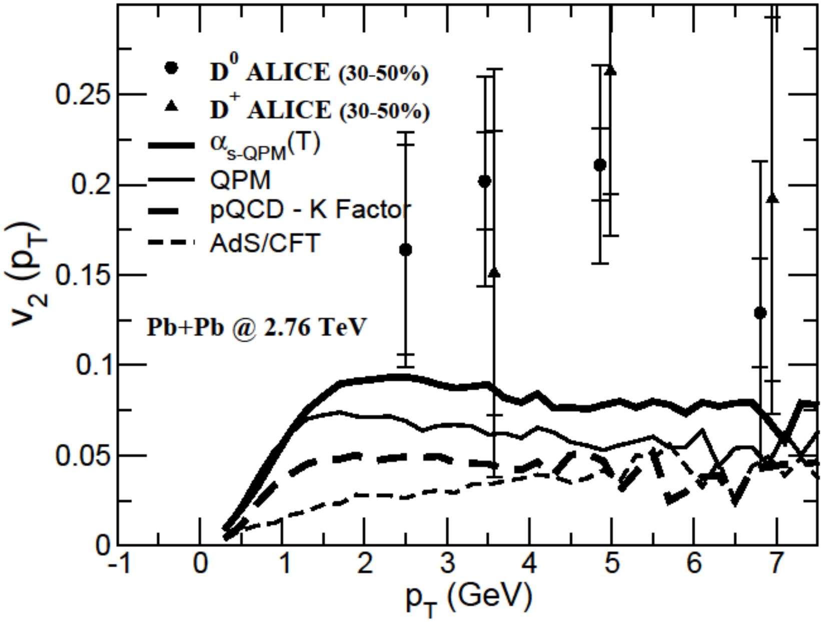
Das, Scardina, Plumari, Greco
Phys. Lett. B 747 (2015)260-264

The Larger the interaction /drag coefficients at Tc, the larger the v2 for the same RAA.

This indicates we need to go beyond pQCD.

RAA and v2 @ ALICE





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Phys. Lett. B 747 (2015) 260-264

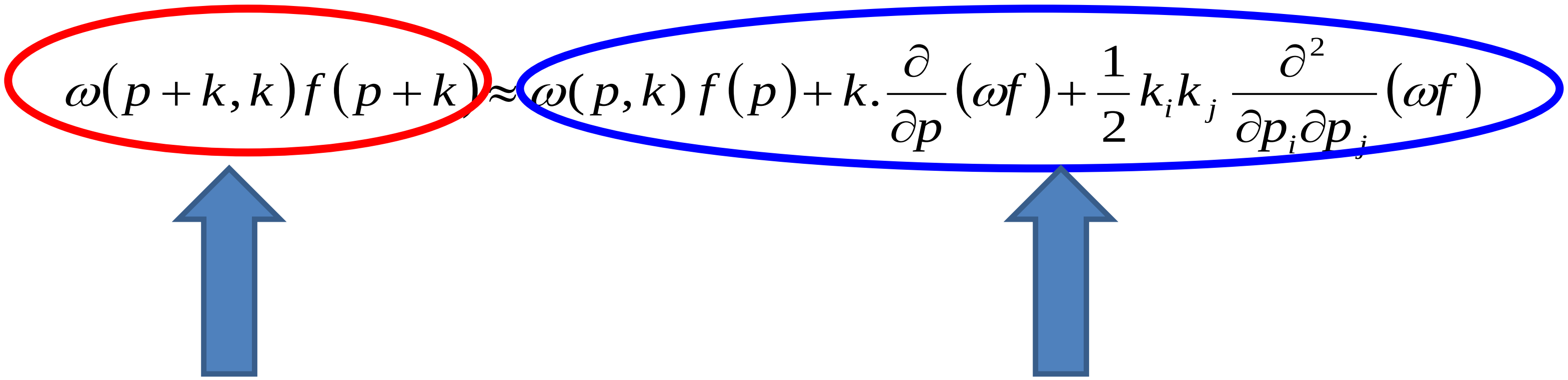
At LHC we underestimate the v2.

Unified attempts to understand heavy quark transport coefficients in QGP

Rapp , Gossiaux, Andronic, Averbeck, Masciocchi et. al. NPA 979, 2018 (HQ-RRTF)

Cao et. al arXiv:1809:07894 (JET-HQ)

Heavy quark momentum evolution: Langevin vs Boltzmann



$$\omega(p+k,k)f(p+k) \approx \omega(p,k)f(p)+k.\frac{\partial}{\partial p}(\omega f)+\frac{1}{2}k_i k_j\frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

Boltzmann Equation

Fokker Planck

It will be interesting to study both the equation in a identical environment to ensure the validity of this assumption at different momentum transfer and their subsequent effects on RAA and v2.

Langevin dynamics:

$$dx_j = \frac{p_j}{E}dt$$

$$dp_j = -\Gamma p_j dt + \sqrt{dt}\,C_{jk}(t, p+\xi dp)\rho_k$$

Das, Scardina, Plumari and Greco
Phys. Rev. C,90,044901(2014)

Γ **is the deterministic friction (drag) force**

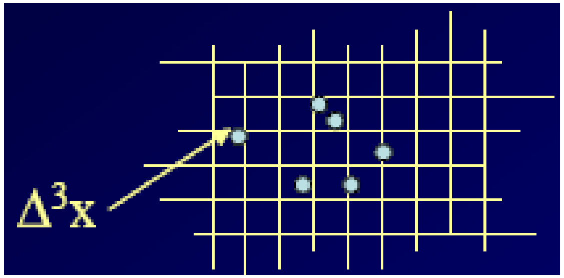
C_{ij} **is stochastic force in terms of independent Gaussian-normal distributed random variable.**

Transport theory

$$p^{\mu}\partial_{\mu} f(x,p) = C_{22}$$

We consider two body collisions

$$\begin{aligned} C_{22} = &\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'} f_1' f_2' |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p_1'+p_2'-p_1-p_2) \\ &-\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'} f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p_1'-p_2') \end{aligned}$$



$$\Delta t \to 0$$

$$\Delta^3 x \to 0$$

Exact solution

Collision integral is solved with a local stochastic sampling

Das, Scardina, Plumari and Greco
Phys. Rev. C,90,044901(2014)

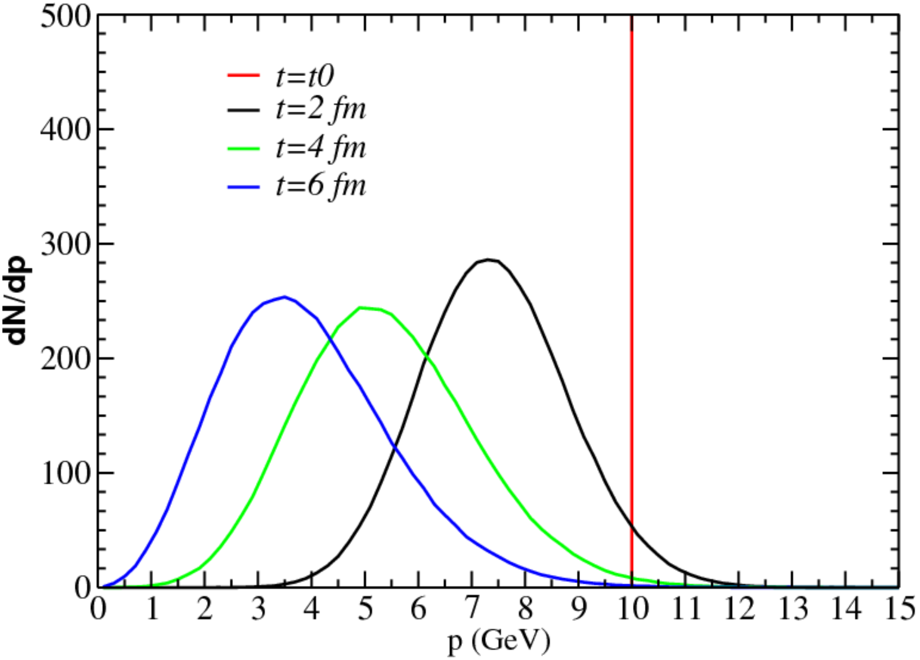
$$P_{22} = \frac{\Delta N_{\text{coll}}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{\text{rel}}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

Evolution: Boltzmann vs Langevin (Charm)

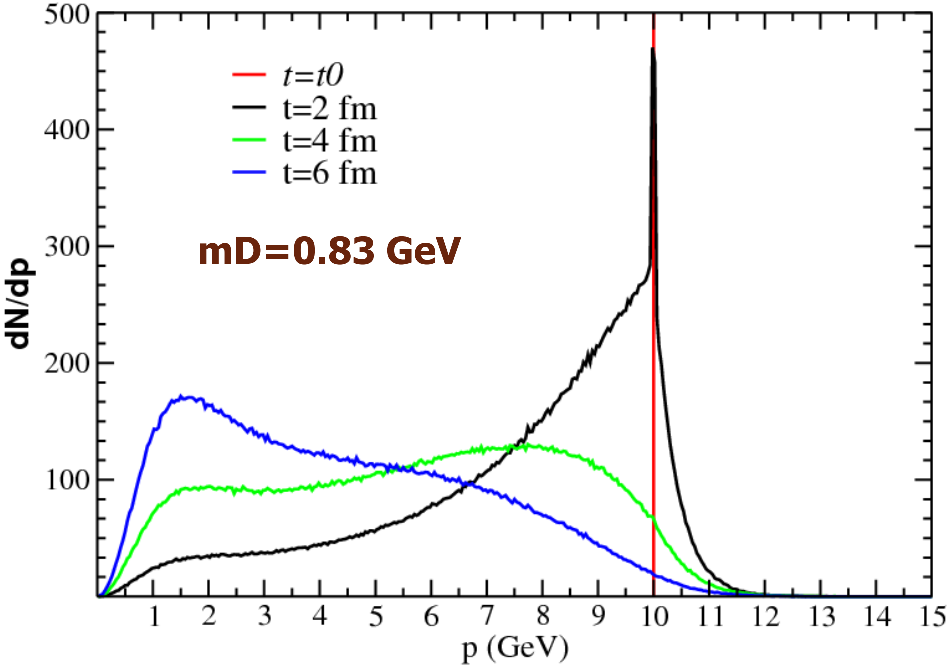
Momentum evolution starting from a δ (Charm) in a Box

$$\left.\frac{dN}{d^3p}\right|_{initial} = \delta\left(p - 10GeV\right)$$

Langevin



Boltzmann



In case of Langevin the distributions are Gaussian as expected by construction

In case of Boltzmann the charm quarks does not follow the Brownian motion

Das, Scardina, Plumari and Greco Phys. Rev. C,90,044901(2014)

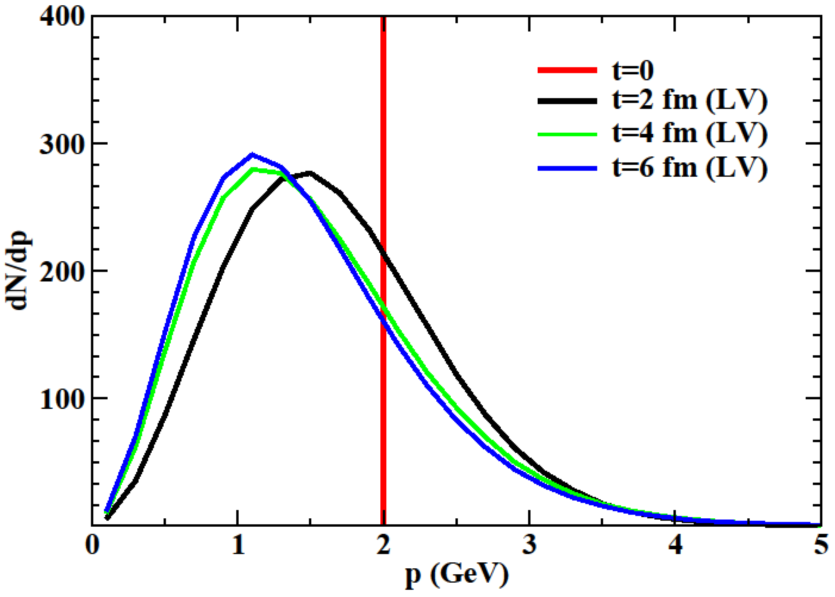
Evolution: Boltzmann vs Langevin (Charm)

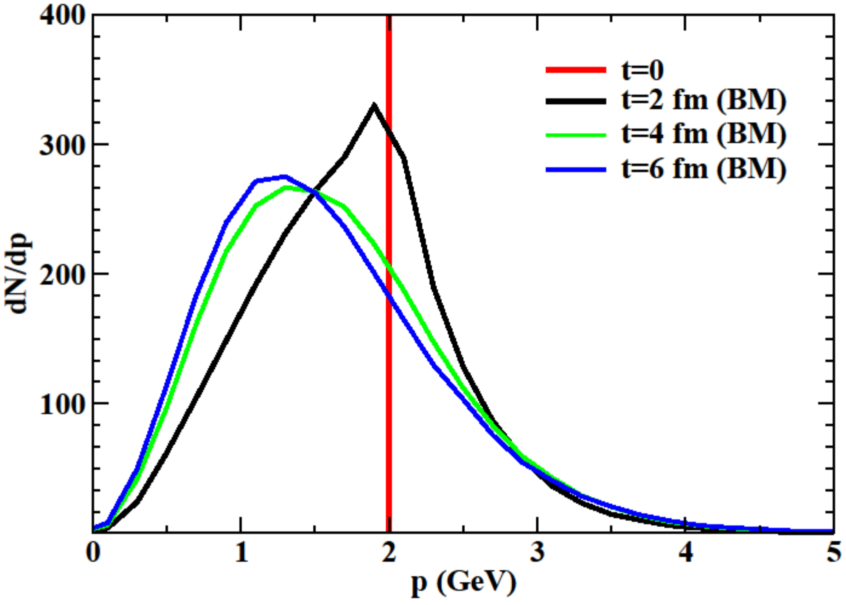
Momentum evolution starting from a δ (Charm) in a Box

$$\frac{dN}{d^3p}_{initial} = \delta(p - 2GeV)$$

Langevin

Boltzmann





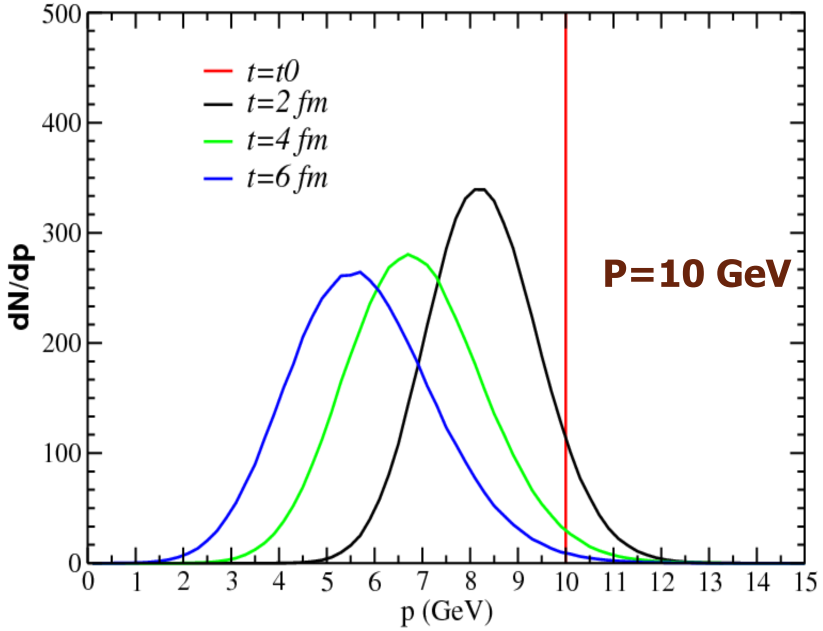
In case of Langevin the distributions are Gaussian as expected by construction

In case of Boltzmann the charm quarks follow the Brownian motion: At Low Momentum.

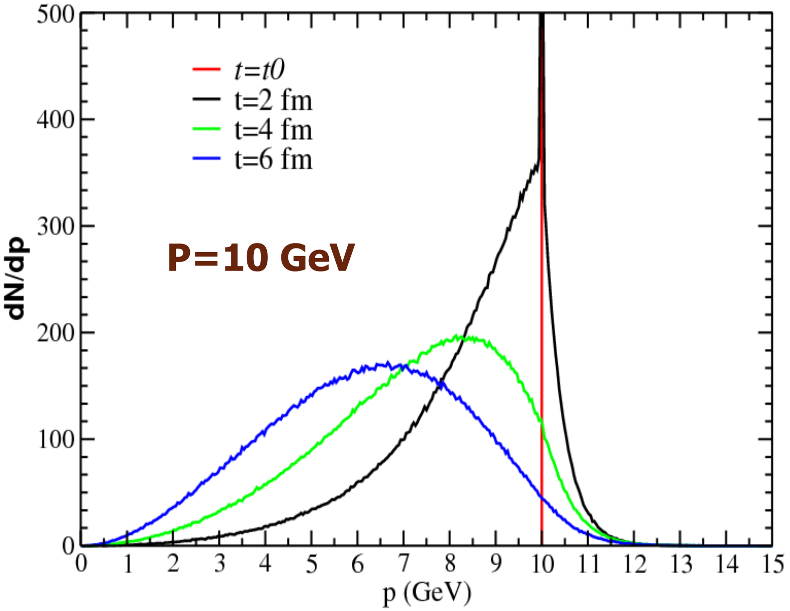
Momentum evolution starting from a δ (Bottom)

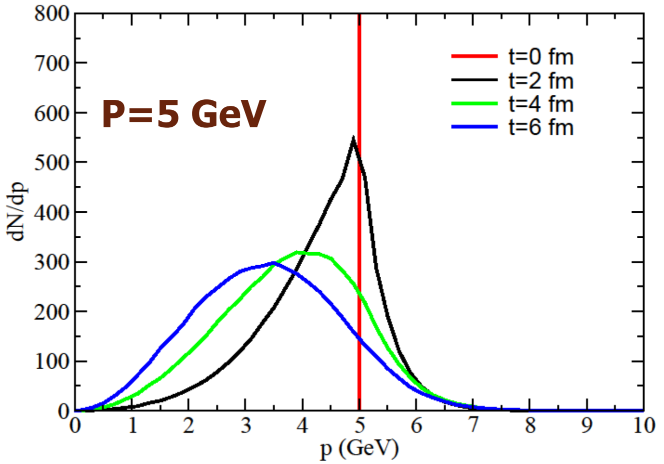
In a Box

Langevin



Boltzmann

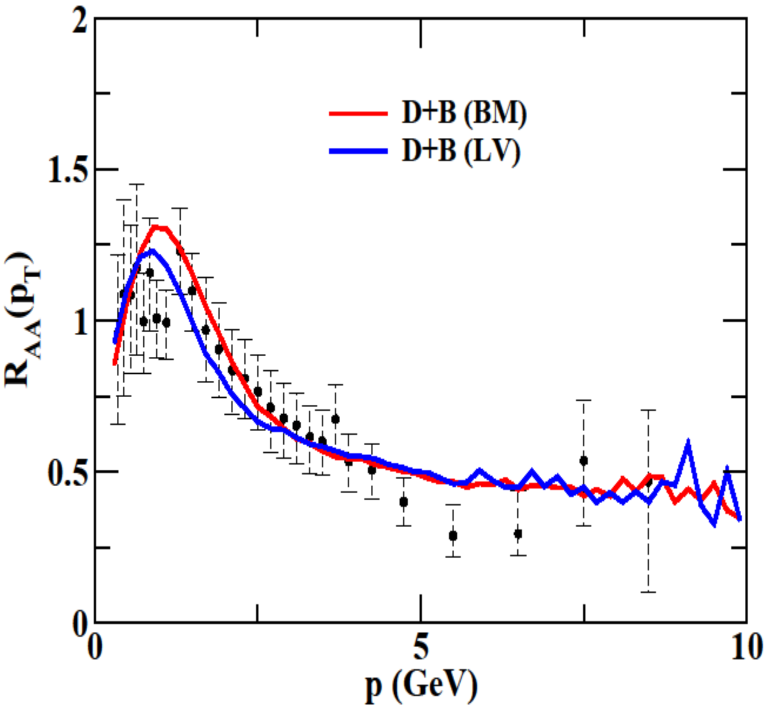


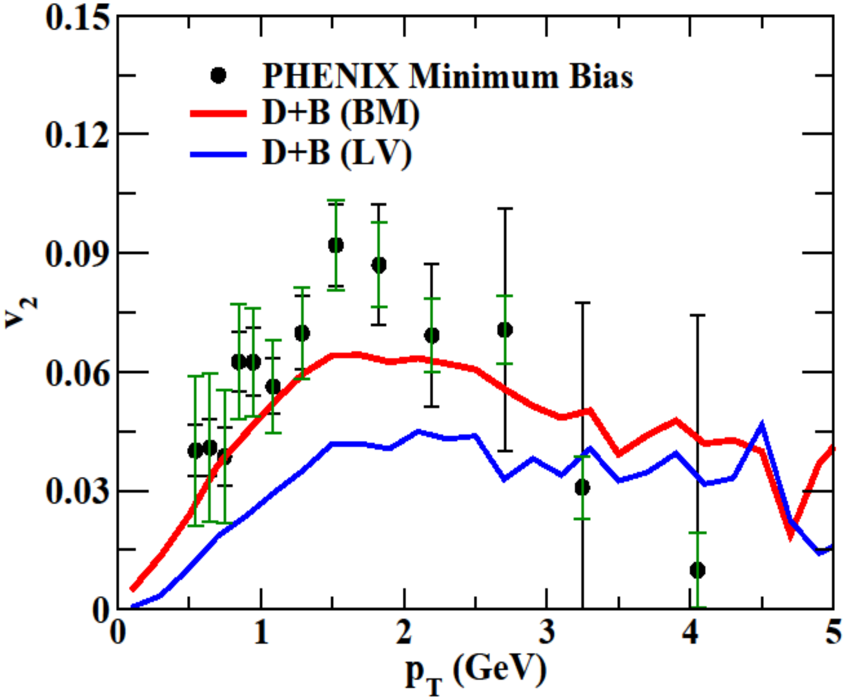


For bottom quarks it works better.

R_{AA} and v2 at RHIC

(With near isotropic cross-section)





Das, Scardina, Plumari and Greco
Phys. Rev. C,90,044901(2014)

At fixed RAA Boltzmann approach generate larger v2 .
(depending on mD and M/T)

With isotropic cross section one can describe both RAA and V2 simultaneously within the Boltzmann approach !

Hadronization: Coalescence plus Fragmentation

Fragmentation function gives the probability to get a hadron from a parton:

$$f_H(p_T) = \sum_p f_p(p_T / z) \otimes D_{p\to H}(z)$$

<z>~0.9 for charm quark and <z>~0.5 for light quark

Coalescence is the convolution of two /three parton distribution folded by a wave function:

$$\frac{dN_{Meson}}{d^2 p_T} = g_M \sum_{i,j} P_q(i) P_q(j) \delta^{(2)}(p_T - p_{iT} - p_{jT}) f_M(x_i, x_j; p_i, p_j)$$

Hadron wave function

$$\frac{dN_{Baryon}}{d^2 p_T} = g_B \sum_{i,j,k} P_q(i) P_q(j) P_q(k) \delta^{(2)}(p_T - p_{iT} - p_{jT} - p_{kT}) f_B(x_i, x_j, x_k; p_i, p_j, p_k)$$

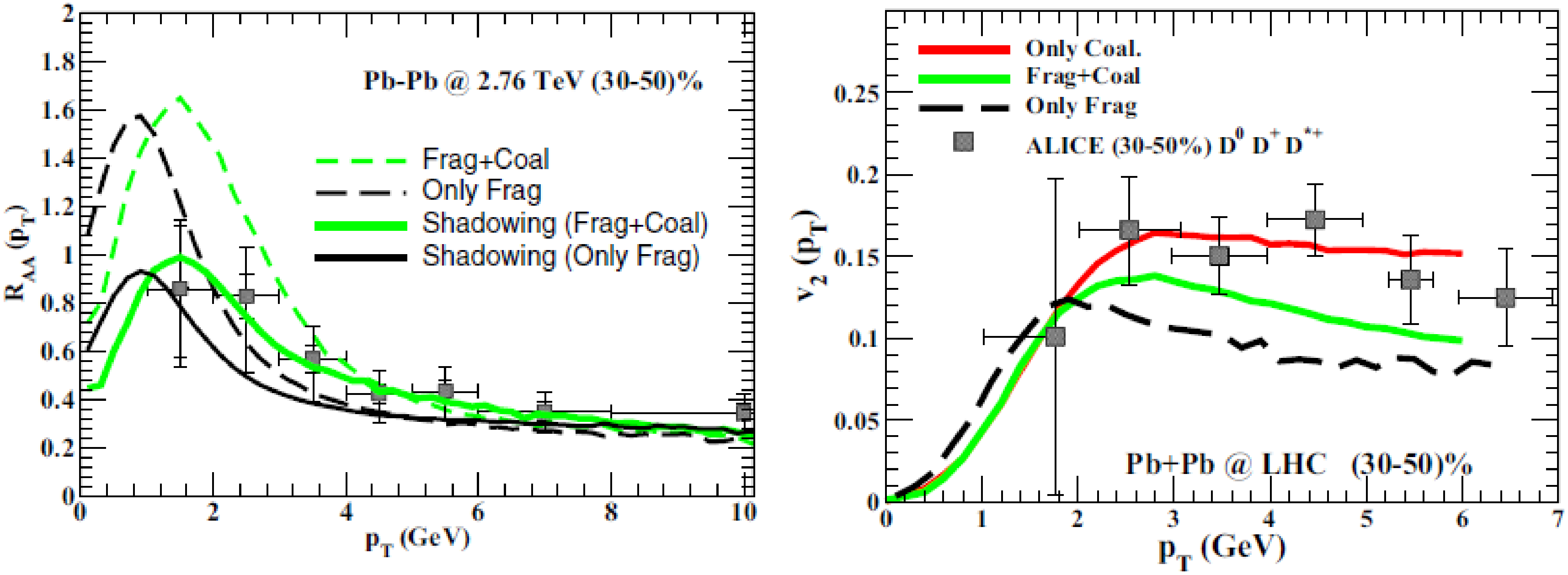
V. Greco, C.M. Ko, and P. L´evai
PRL 90 , 202302 (2003)

Das, Torres-Rincon, Tolos, Minissale, Scardina, Greco
PRD,94,114039,2016

Our Approach:

Charm quark production: NLO pQCD
Heavy quark momentum evolution: Boltzmann equation
Heavy quark –bulk interaction: Quasiparticle model
Bulk evolution: Boltzmann equation
Hadronization: Coalescence plus Fragmentation

LHC results: RAA vs v2

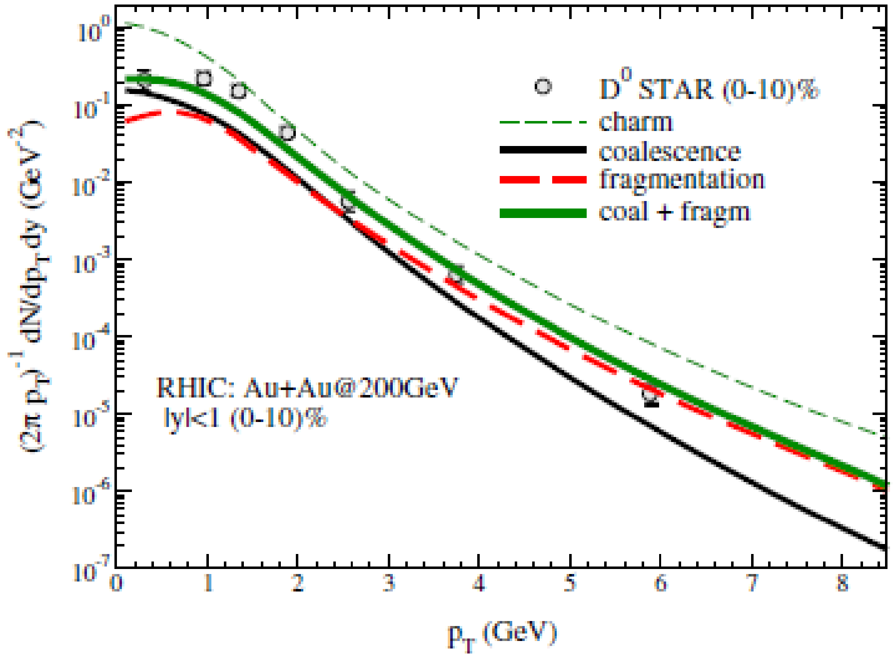


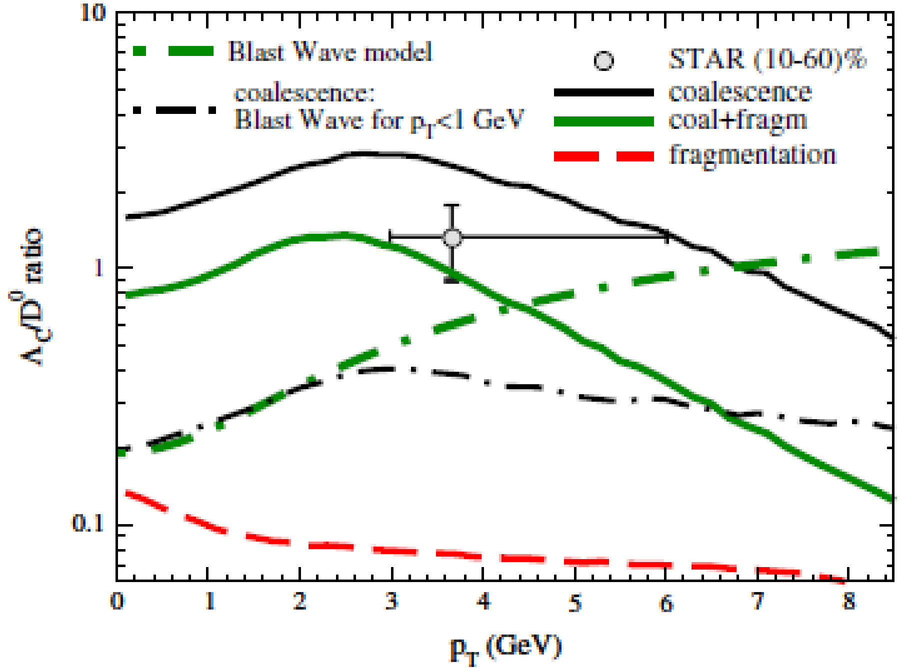
T-dependence of interaction: QPM
Boltzmann equation for HQ momentum evolution
Hadronization by coalescence plus fragmentation

Scardina, Das, Minissale, Plumari, Greco
PRC, 96,044905 (2017)

Heavy Baryon to meson ratio

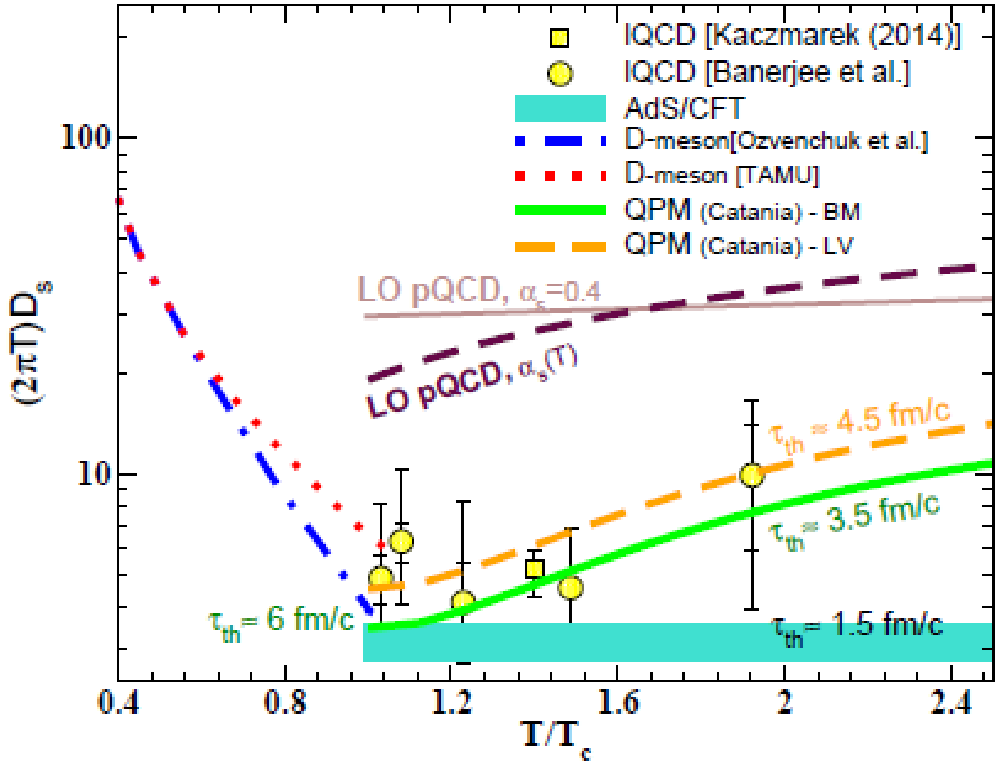
(Serve as a tool to disentangle different hadronization mechanisms)





Plumari, Minissale, Das, Coci, Greco
EPJC, 78, 2018

Connection to lattice QCD



$$D_s(p=0) = \frac{T}{m_Q \gamma} = T\, m_Q\, \tau_{th}$$

$$\tau_{th} = \frac{M}{2\pi T^2}(2\pi T D_s) \cong 1.8\, \frac{2\pi T D_s}{(T/T_c)^2}\ \mathrm{fm/c}$$

Open Heavy Flavour physics in heavy-ion collisions have the potential to link the phenomenology to lattice QCD and experiments.

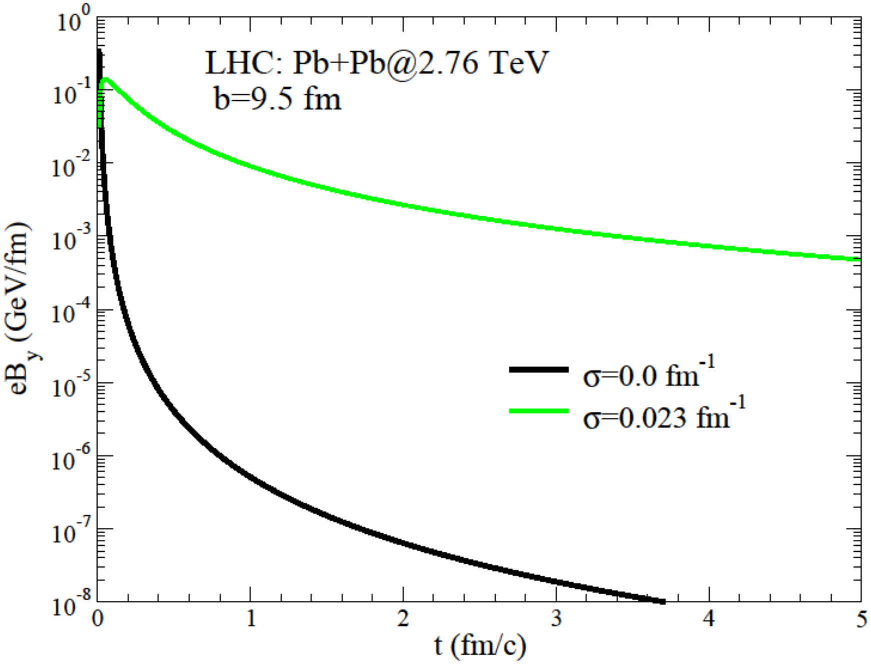
Scardina, Das, Minissale, Plumari, Greco
PRC, 96,044905 (2017)

Impact of EM field on heavy quark dynamics at LHC

$$dp_j = -\Gamma p_j dt + \sqrt{dt}\, C_{jk}(t, p + \xi dp)\rho_k + F_{ext} dt$$

$$F_{ext} = q(E' + v \times B')$$

$$E' = \gamma (E + v \times B) - (\gamma - 1)(E \cdot \hat{v})\hat{v}$$
$$B' = \gamma (B - v \times E) - (\gamma - 1)(B \cdot \hat{v})\hat{v}$$



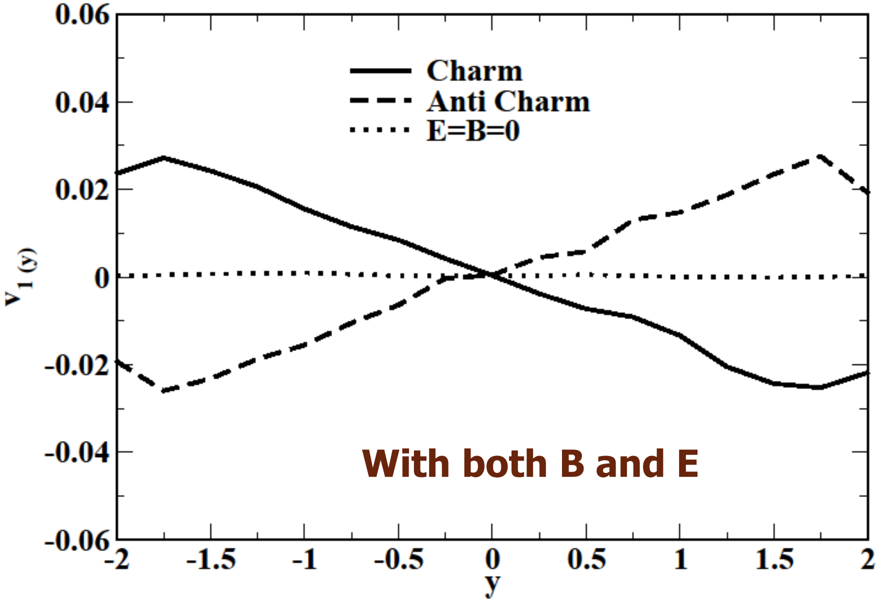
Electromagnetic field has been included in the Langevin equation as a external force.

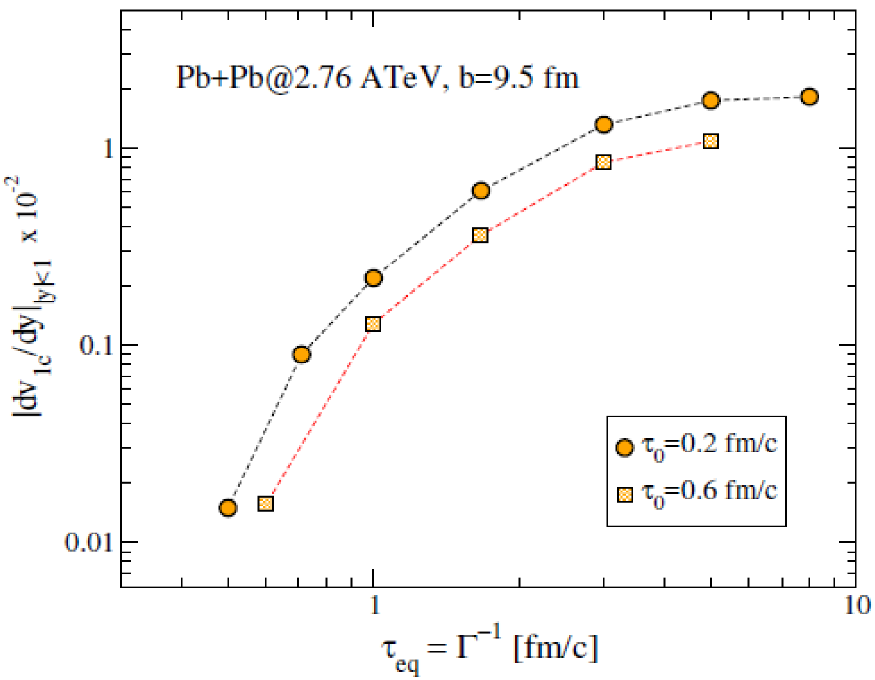
We consider both E and B.
Bx=Bz=0
And Ey=Ez=0

$$v_1 = < \frac{p_x}{p_T} >$$

Das, Plumari, Chartarjee, Scardina, Greco, Alam
Phys. Lett. B, 768 (2017) 260

Heavy quark v1@LHC



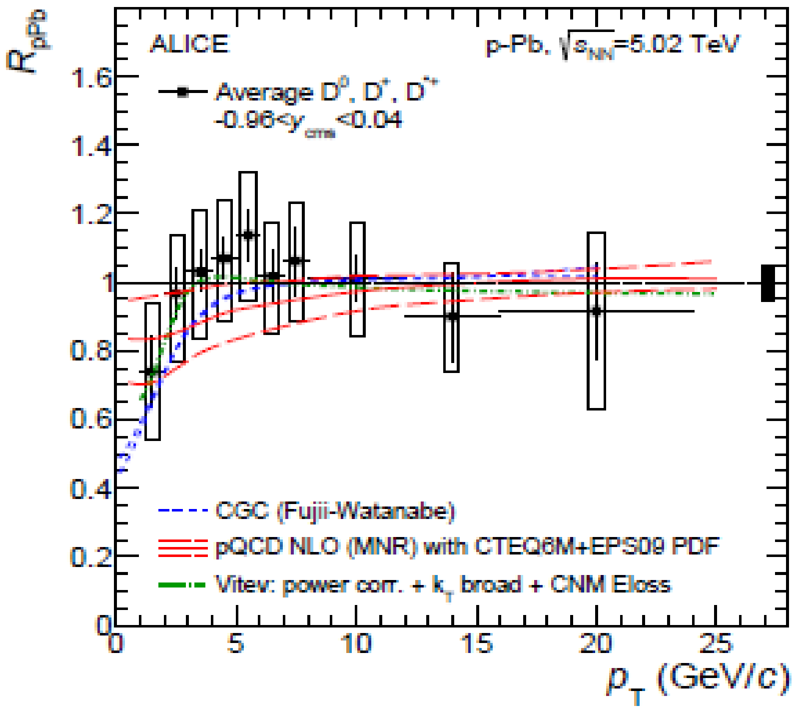


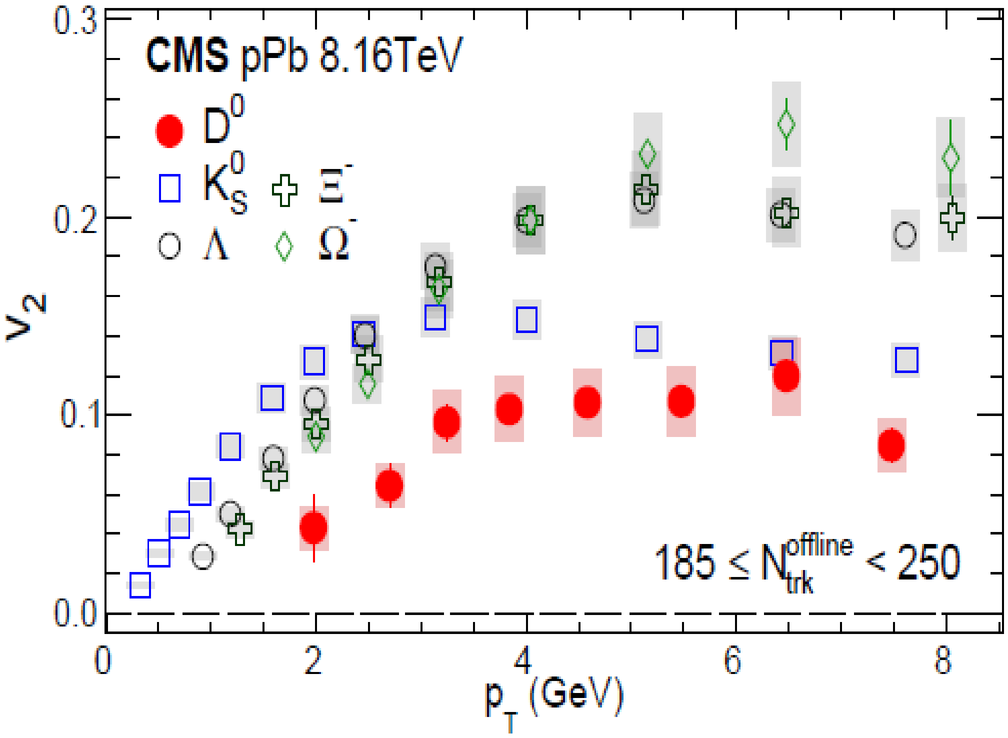
Das, Plumari, Chartarjee, Scardina, Greco, Alam
Phys. Lett. B, 768 (2017) 260

Heavy quark v1 is larger than light quark v1.

Recent data from ALICE indicates splitting in D and Dbar v1.
(Hard probes 2018)

Heavy quark in small system (p-nucleus)



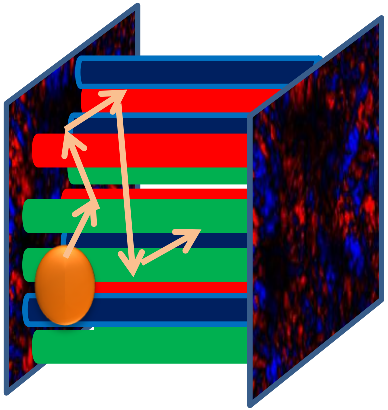


ALICE Collaboration
Phys. Rev. Lett. 113 (2014) 232301

CMS Collaboration
arXiv:1804.09767v2

What mechanism could build up v_2 without energy loss?

Heavy quarks as probes of the evolving Glasma



(Adapted from M. Ruggieri)

$$t_{\text{formation}} \approx \frac{1}{2m_c} \approx 0.06\ \text{fm/c}$$

● ***HQs can probe the very early evolution of the Glasma fields***

Hamilton equations of motion of *c*-quarks:

$$\frac{dx_i}{dt} = \frac{p_i}{E} \qquad E = \sqrt{\boldsymbol{p}^2 + m^2}$$

$$\boldsymbol{v} \equiv \frac{\boldsymbol{p}}{E}$$

(Relativistic) Velocity

$$E\frac{dp_i}{dt} = gQ_a F^a_{i\nu} p^\nu \, .$$

$$\frac{d\boldsymbol{p}}{dt} = q\boldsymbol{E} + q\left(\boldsymbol{v} \times \boldsymbol{B}\right)$$

Lorentz force

$$E\frac{dQ_a}{dt} = -gQ_c \varepsilon^{cba} \boldsymbol{A}_b \cdot \boldsymbol{p}$$

Wong (1979)

$$D_\mu J^\mu_a = 0$$

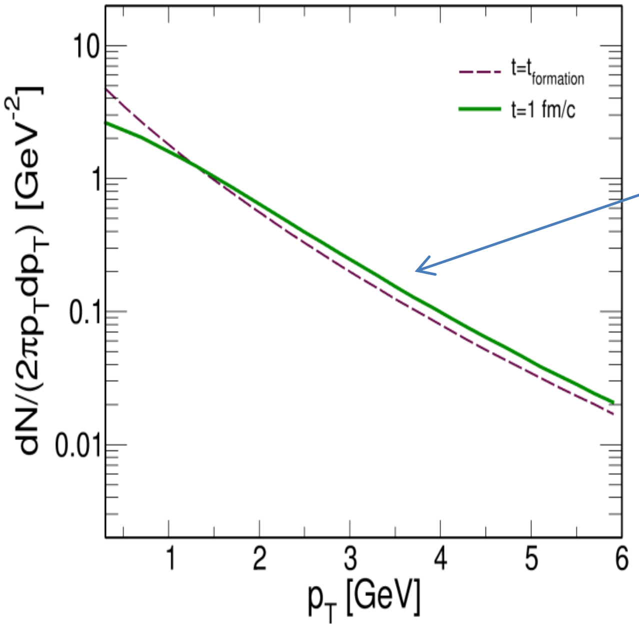
$$J^\mu_a = \bar{c}\gamma^\mu T_a c$$

Gauge-invariant conservation of the color Current carried by charm quaks + gluons

Equations of motion of heavy quarks are solved in the background given by the evolving Glasma fields

p-Pb @ 5.02 TeV

Nuclear modification factor (R_{pPb}) for p-Pb collisions

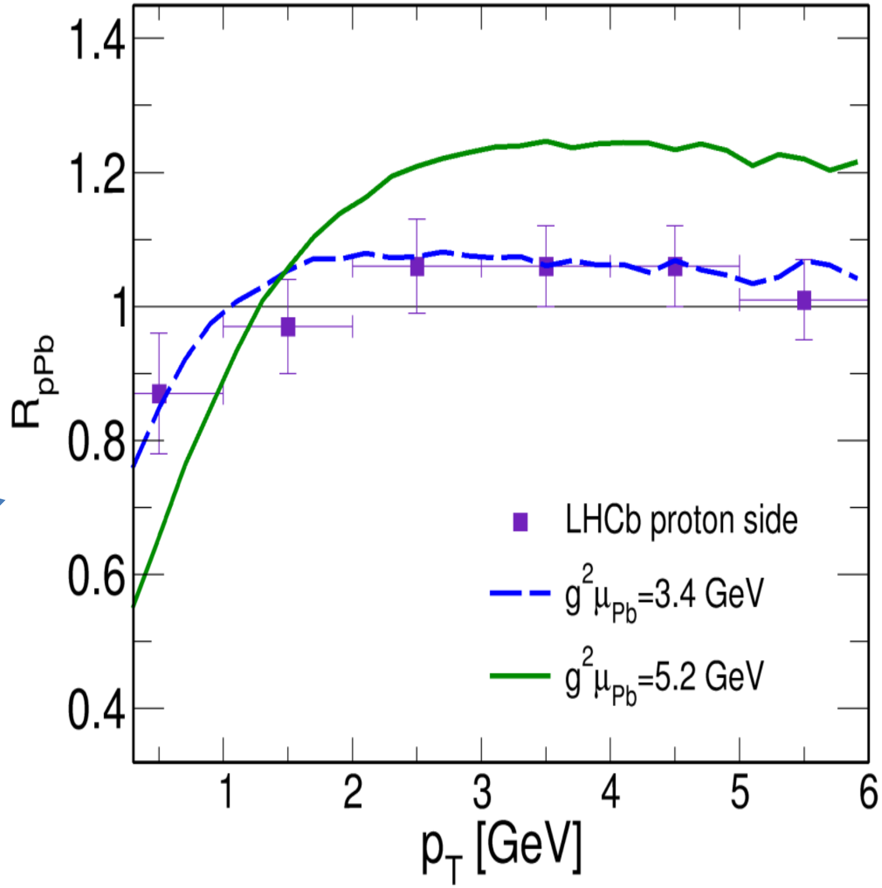


Initial distribution: from perturbative QCD

Evolution: interaction with the Glasma

D-mesons R_{pPb}

Standard fragmentation [Peterson *et al.*(1983)]



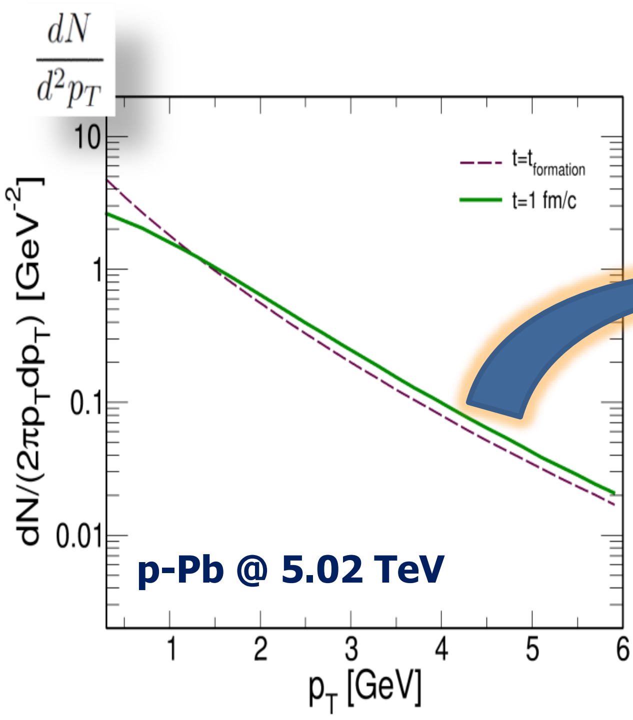
$$R_{\mathrm{pPb}} = \frac{(dN/d^2p_T)_{\mathrm{final}}}{(dN/d^2p_T)_{\mathrm{pQCD}}}$$

$R_{pPb} \neq 1$

Interaction with the fields created by the collision

Ruggieri and Das
PRD, 98, 094024(2018)

Diffusion results in acceleration



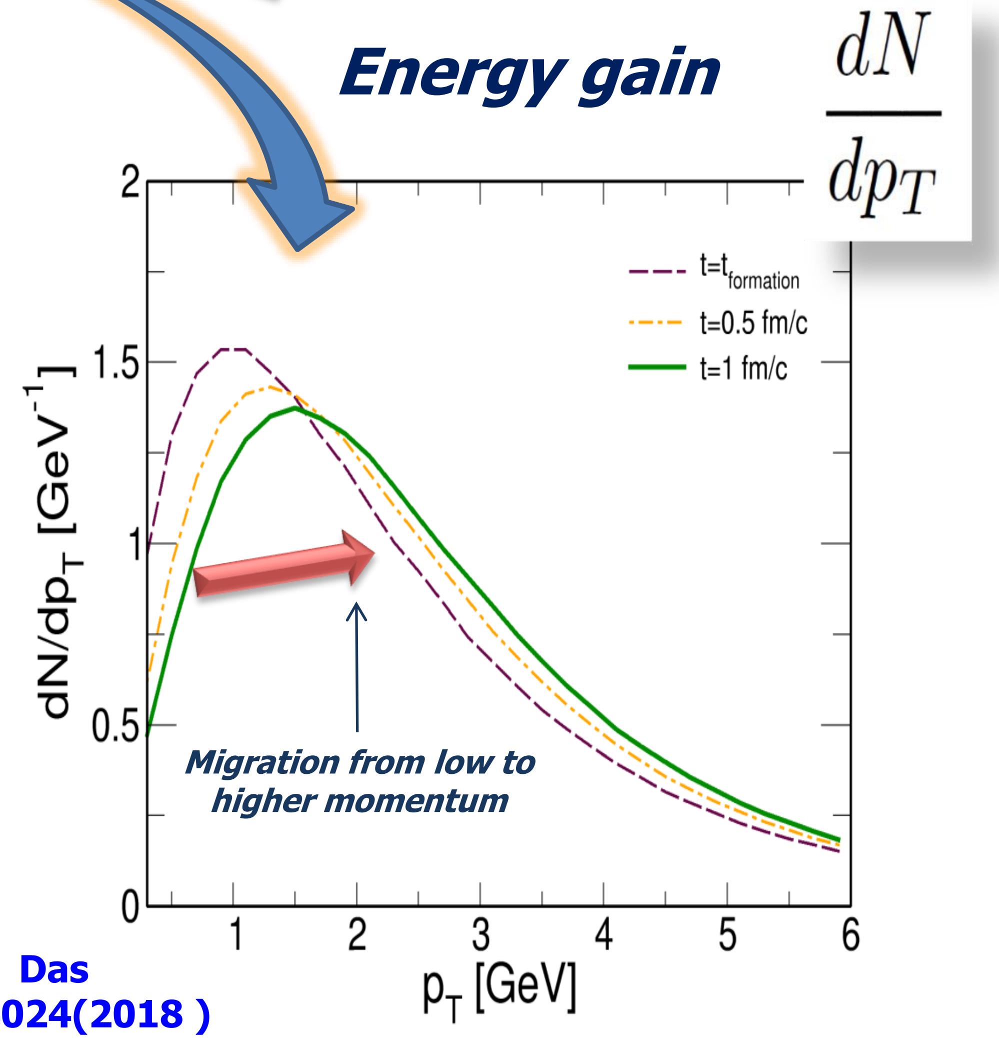
p-Pb @ 5.02 TeV

$$\frac{dN}{d^2p_T}$$

$$\times p_T(\times 2\pi)$$

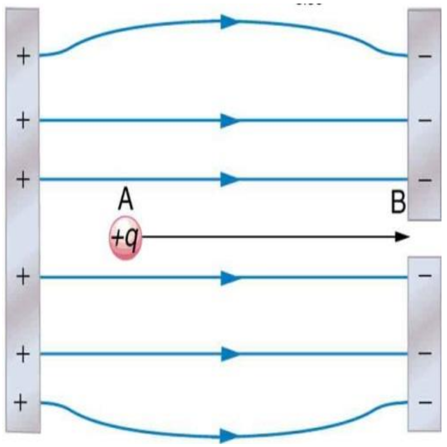
Energy gain

$$\frac{dN}{dp_T}$$



Migration from low to higher momentum

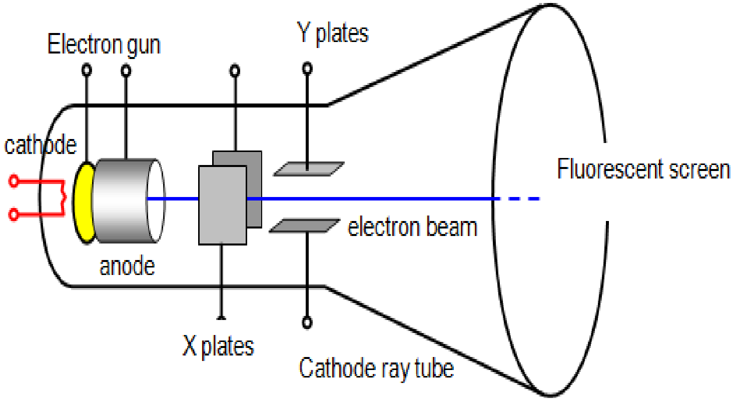
Heavy quarks seem to be accelerated by the transverse (color-)electric field



$$\frac{\Delta p}{\Delta t} = q\boldsymbol{E}$$

Ruggieri and Das
PRD, 98, 094024(2018)

The cathode tube effect

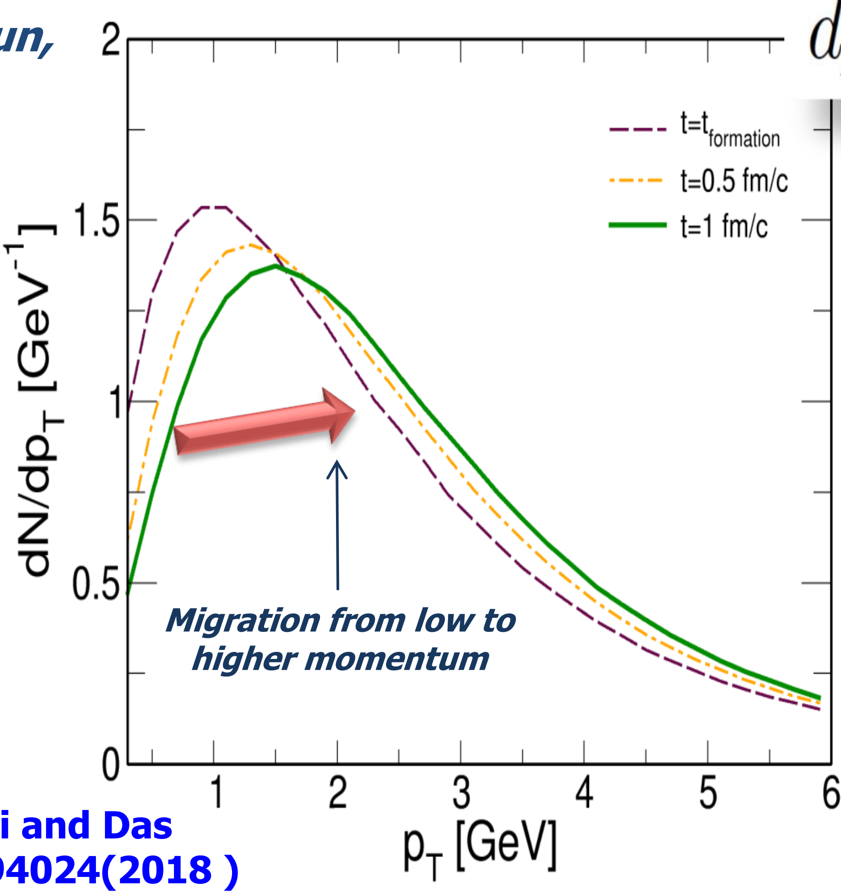


Why cathode tube?

Electrons are produced by the electron gun, then accelerated by the electric field

Heavy quarks seem to be accelerated by the (color-)electric field

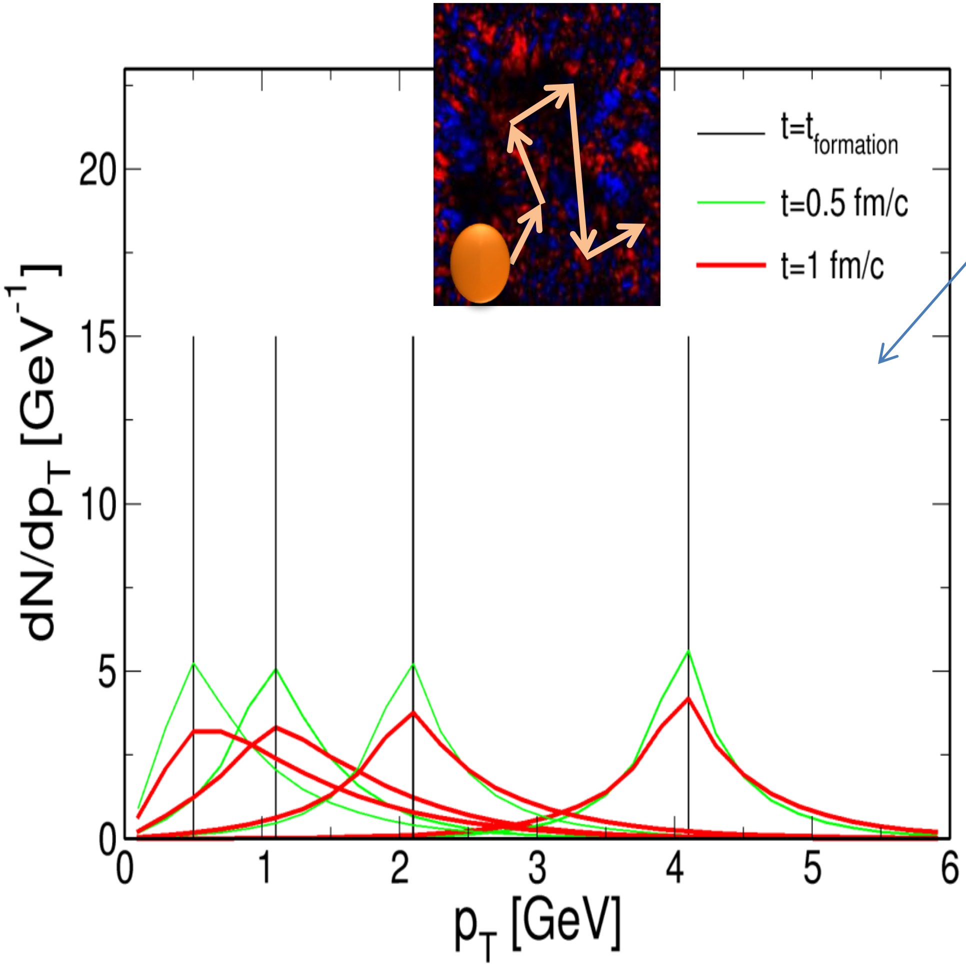
$$\frac{dN}{dp_T}$$



Migration from low to higher momentum

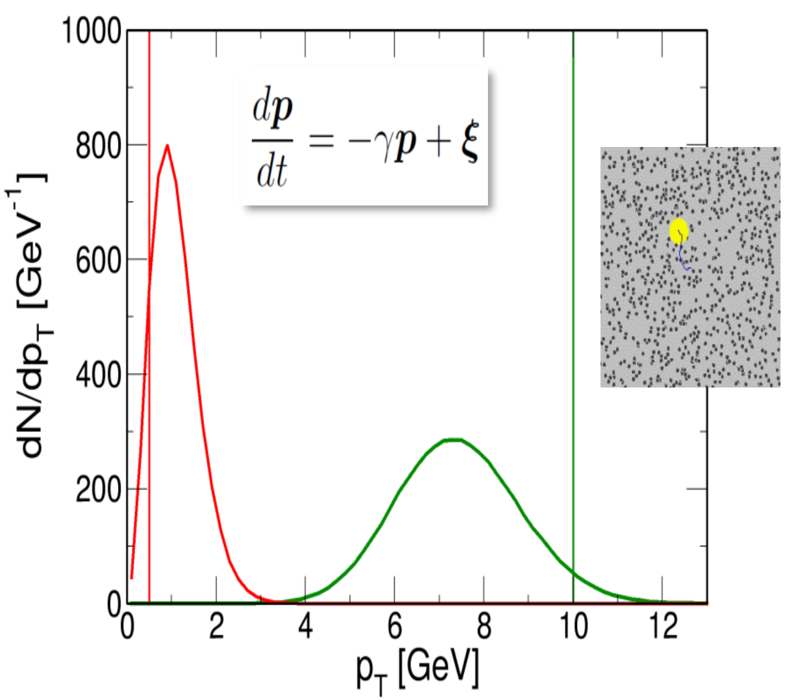
Ruggieri and Das
PRD, 98, 094024(2018)

HQs in Glasma



(Anomalous) Diffusion in momentum space

T=500 MeV ***HQs in hot plasma***

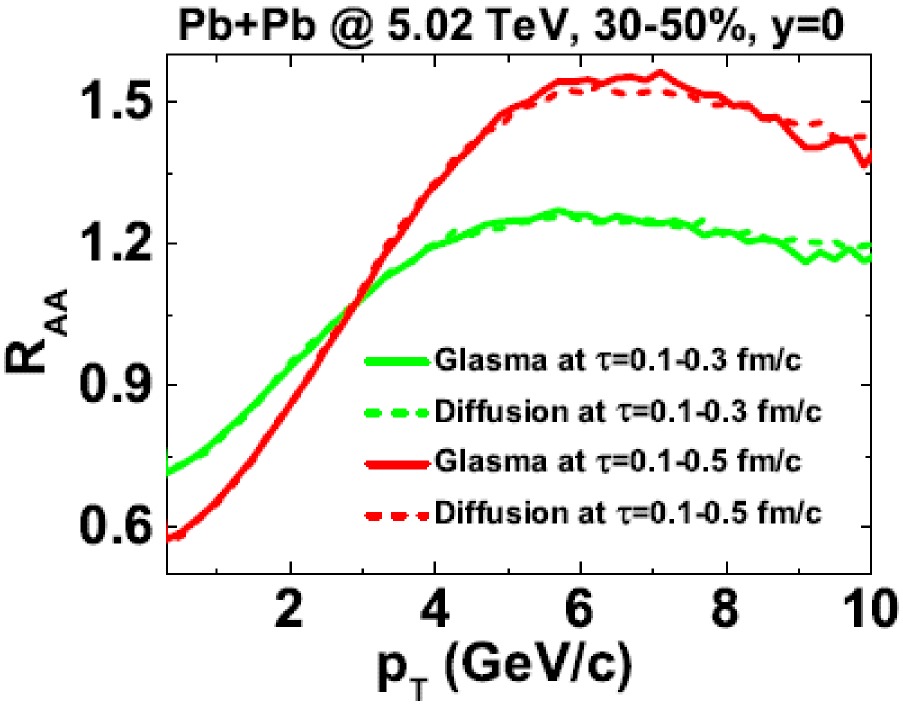


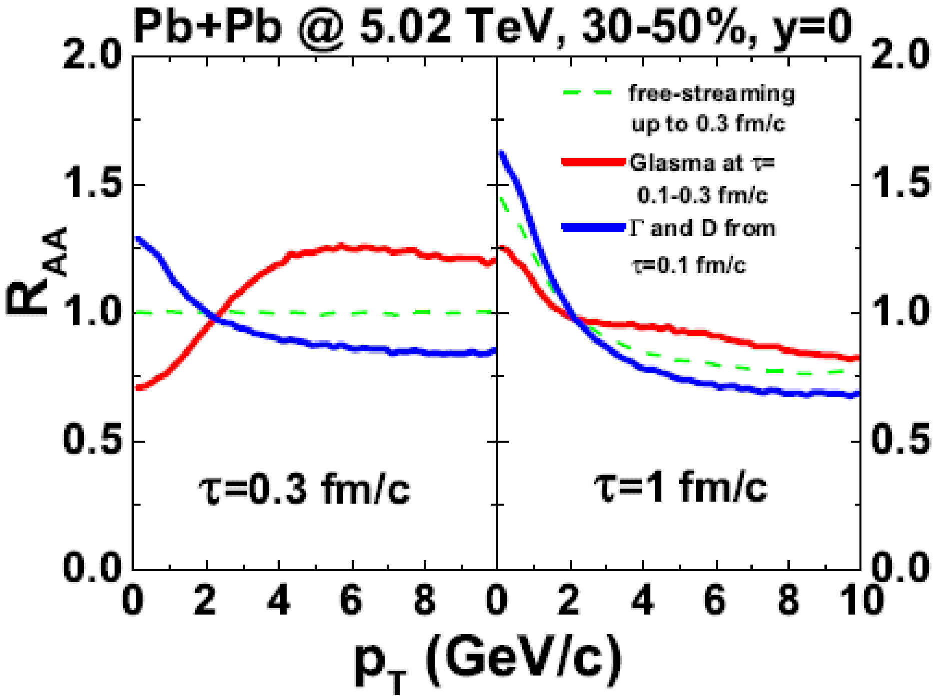
$$\frac{dp}{dt} = -\gamma p + \xi$$

For anomalous diffision see Havlin and Avraham (1987)

Liu , Coci, Das, Ruggieri
Under preparation

Impact of Glasma on a heavy quark observables at LHC

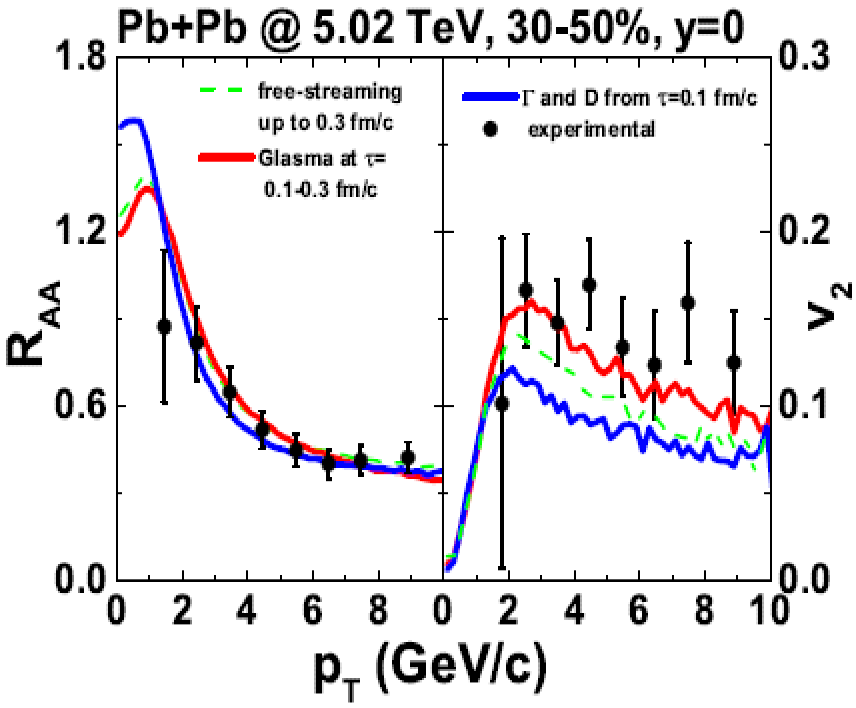


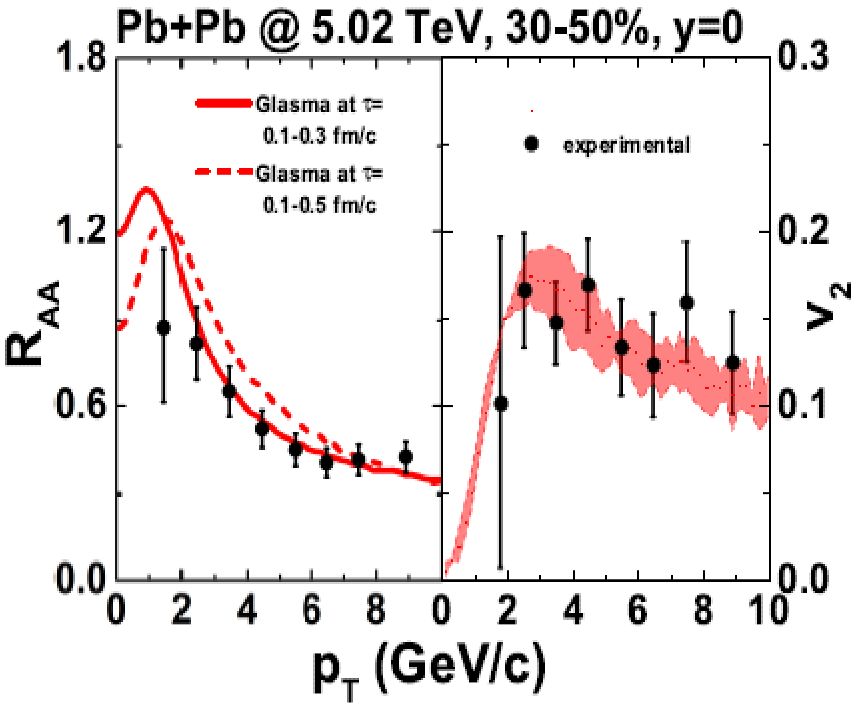


Glasma induce a diffusion of charm quarks in momentum space resulting in a tilt of their spectrum without a significant drag.

Sun, Coci, Das, Plumari, Ruggieri, Greco
arXiv: 1902.06254

Impact of Glasma on a heavy quark observables at LHC





This indicates an initial pre-thermal stage is unlikely to be described in terms of a standard drag and diffusion dynamics, because even if one tune such coefficients to reproduce the same $R_{AA}(p_T)$ this would imply a significantly smaller v_2.

Sun, Coci, Das, Plumari, Ruggieri, Greco
arXiv: 1902.06254

Summary & Outlook ……

- Heavy quarks are the novel probe to characterized QGP and to probe initial state.
- Our study indicates the temperature of the system produced at RHIC (T=340 MeV) and LHC (T=510 MeV) energies are much larger than the temperature needed to create the QGP.
- Several new experiments are coming up (FAIR, FCC) and we are looking for new observables which help us to understand several basic issues …

  - Heavy quark diffusion coefficient in QGP and in Glasma.
  - Hadronization.
  - Einstein relation will be studied.
  - Heavy quark thermalization.
  - QGP in small system (p-Au)
  - To probe the effect of initial magnetic field.

Thank You . . . .

Motivation for Transport approach

$$\left\{p^{*\mu}\partial_{\mu}+\left[p^{*}_{\nu}F^{\mu\nu}+m^{*}\partial^{\mu}m^{*}\right]\partial^{p^{*}}_{\mu}\right\}f(x,p^{*})=C[f]$$

Free streaming Field Interaction Collisions -> $\eta\neq0$

- Starting from 1-body distribution function f(x,p) and not from $T_{\mu\nu}$:
  - **f(x,p) out-of-equilibrium: CGC-Qs scale or high p_T**
  - **Extract viscous correction δf to f(x,p)**
  - **Relevant at LHC due to large amount of minijet production**
  - **Freeze-out self-consistently related to η/s(T)**
  - **HQ dynamics in the same framework**

DISADVANTAGES?!

- Relaxation times fixed by kinetic theory
- Hadronization needed: coal.+frag . under progress

Simulate at fixed shear viscosity

Usually input of a transport approach are *cross-sections and fields*, but here we reverse it and start from η/s with aim of creating a more direct link to viscous hydrodynamics

Chapmann-Enskog

$$\frac{\eta}{s} = \frac{1}{15}\langle p\rangle \cdot \tau_\eta = \frac{1}{15}\frac{\langle p\rangle}{g(\frac{m_D}{T})\sigma_{TOT}\rho}$$

$$g(a) = \frac{1}{50}\int dy y^6 \left[(y^2+\frac{1}{3})K_3(2y) - yK_2(2y)\right] h\left(\frac{a^2}{y^2}\right)$$

g(a=m_D/2T) correct function that fix the relaxation time for the shear motion

Transport code

$$\sigma_{tot}(n(\vec{r}),T) = \frac{1}{15}\frac{\langle p_\alpha\rangle}{g(a)n_\alpha}\frac{1}{\eta/s}$$

Space-Time dependent cross section evaluated locally

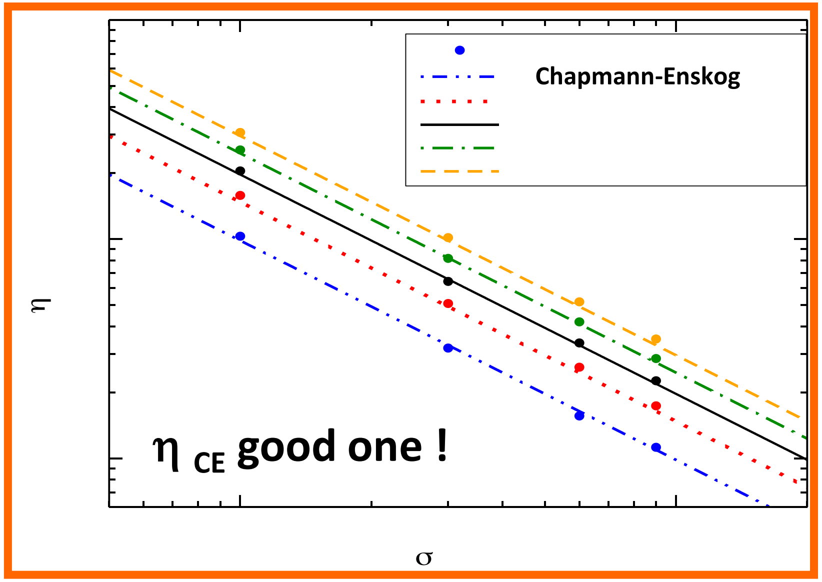
0 < g(m_D/2T) < 2/3

forward peaked

Isotropic $m_D \to \infty$

S.Plumari et al., PRC86(2012

Chapman-Enskog agrees with Green-Kubo



η CE good one !

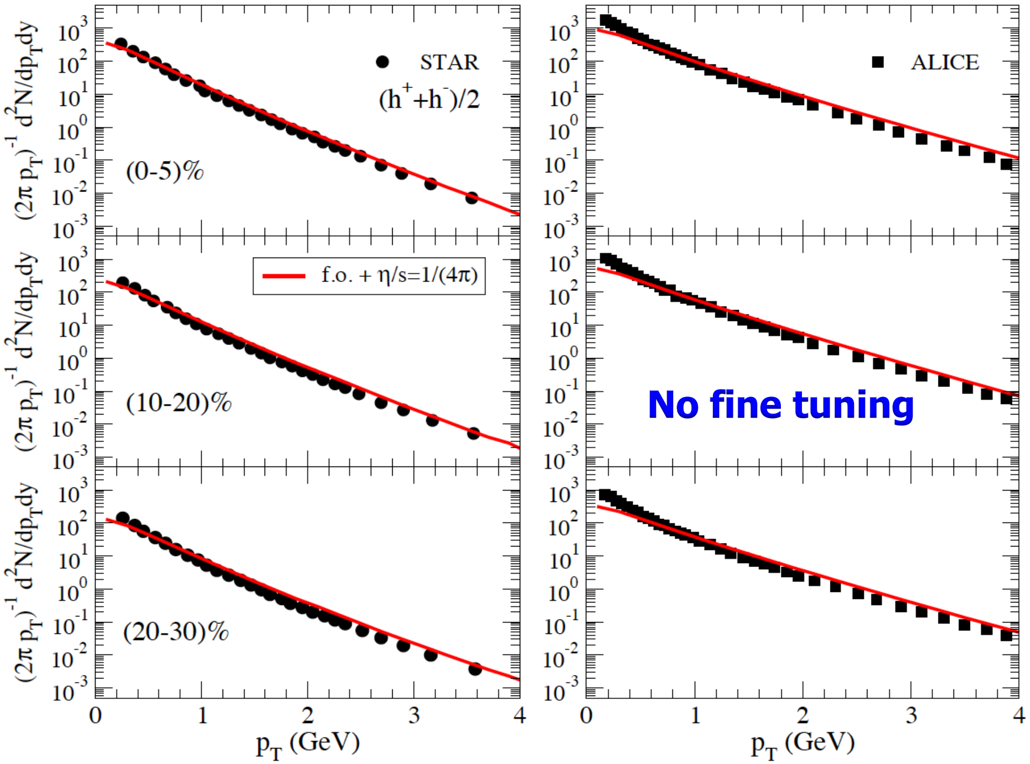
Bulk Initial Conditions

- ✧ r-space: standard Glauber model
- ✧ p-space: Boltzmann-Juttner at T+ minijet [p_T>3GeV]

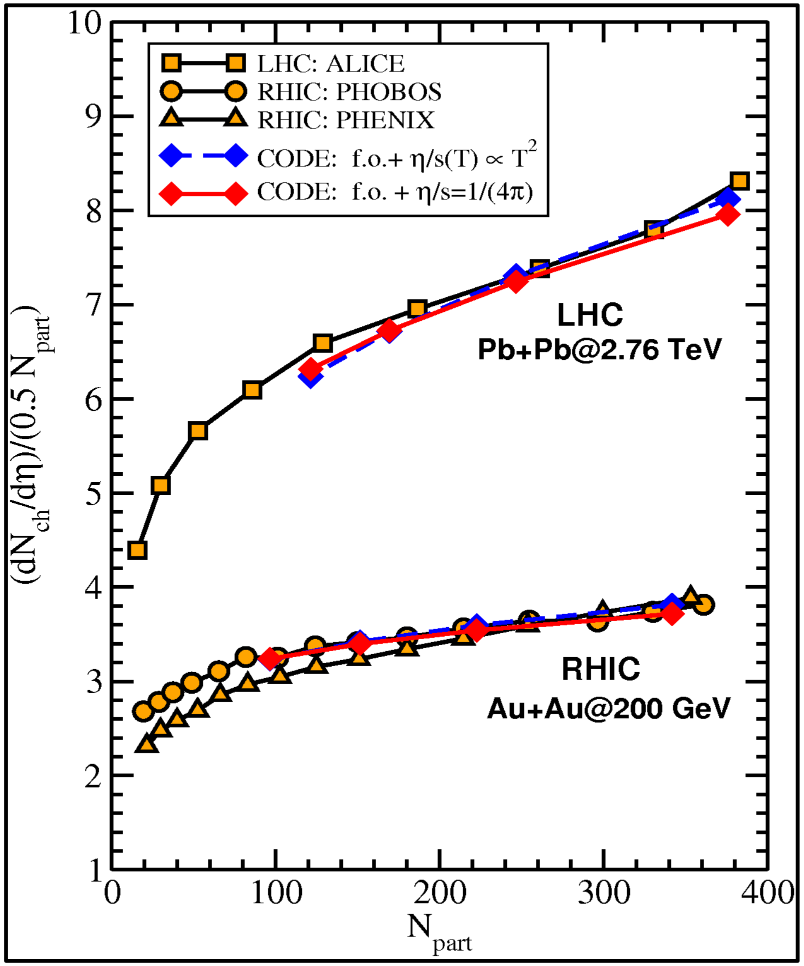
Typical hydro condition

	62 GeV	200 GeV	2.76 TeV
T_0	290 MeV	340 MeV	510 MeV
τ_0	0.7 fm/c	0.6 fm/c	0.3 fm/c

Spectra and multiplicity

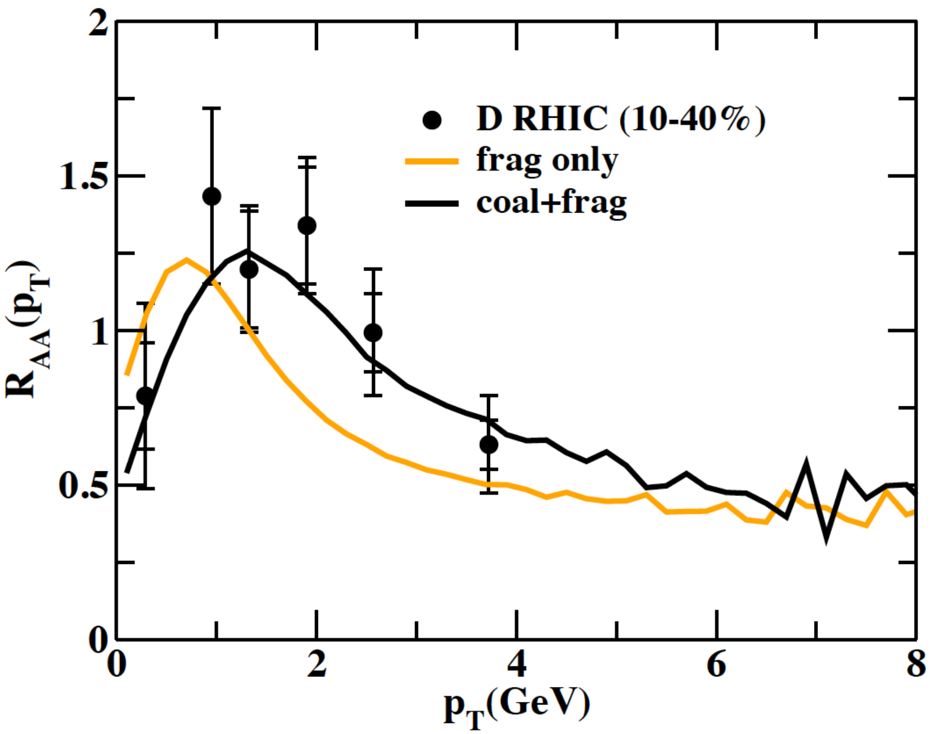


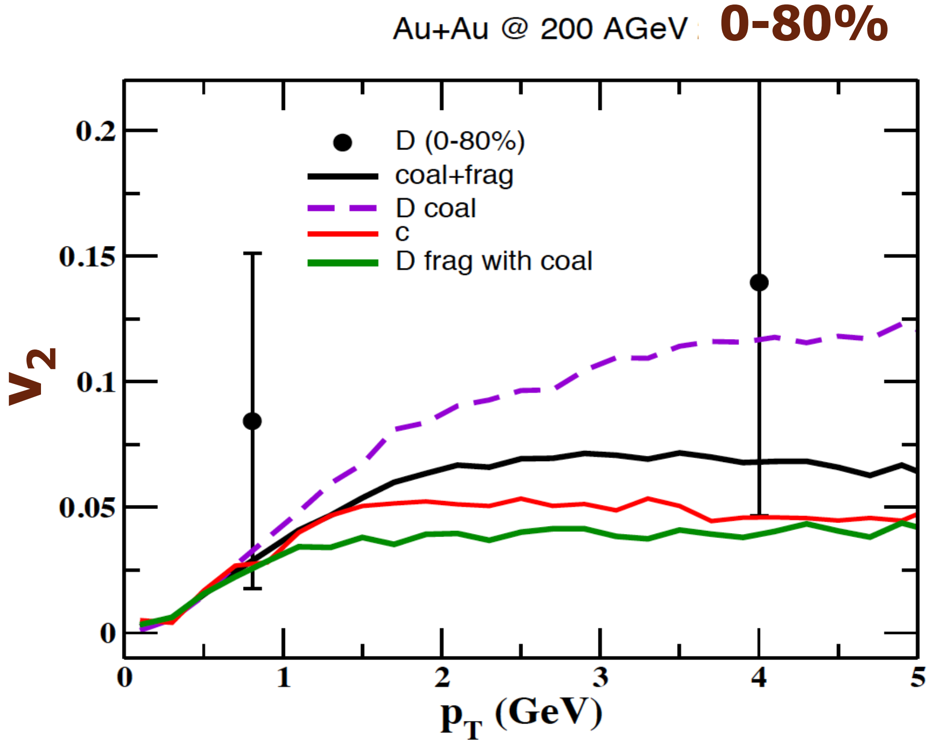
No fine tuning



RHIC – D meson

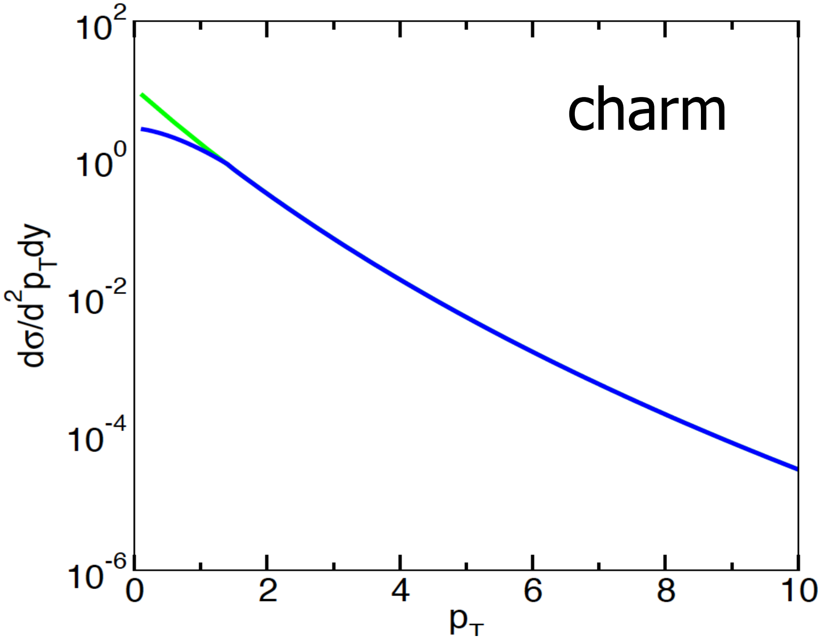
QPM - Boltzmann

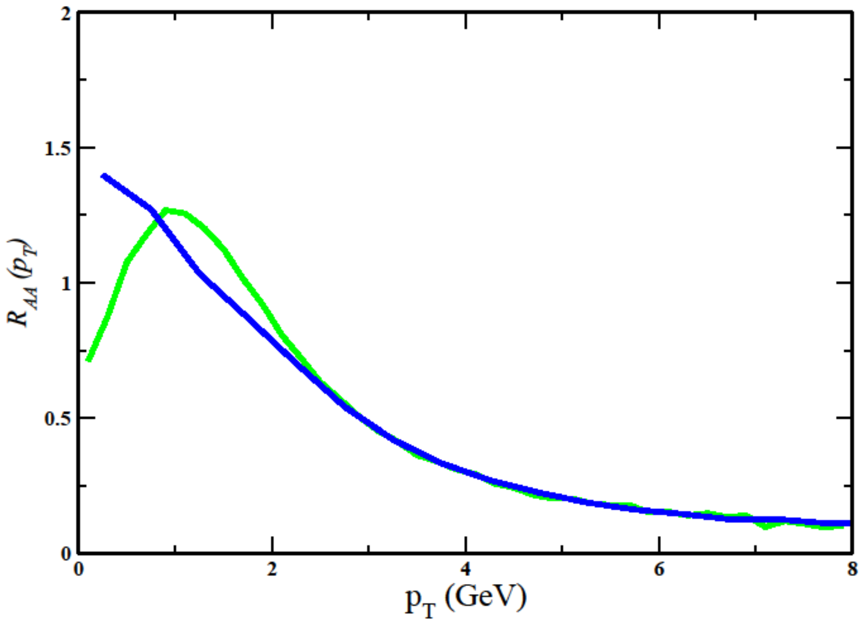


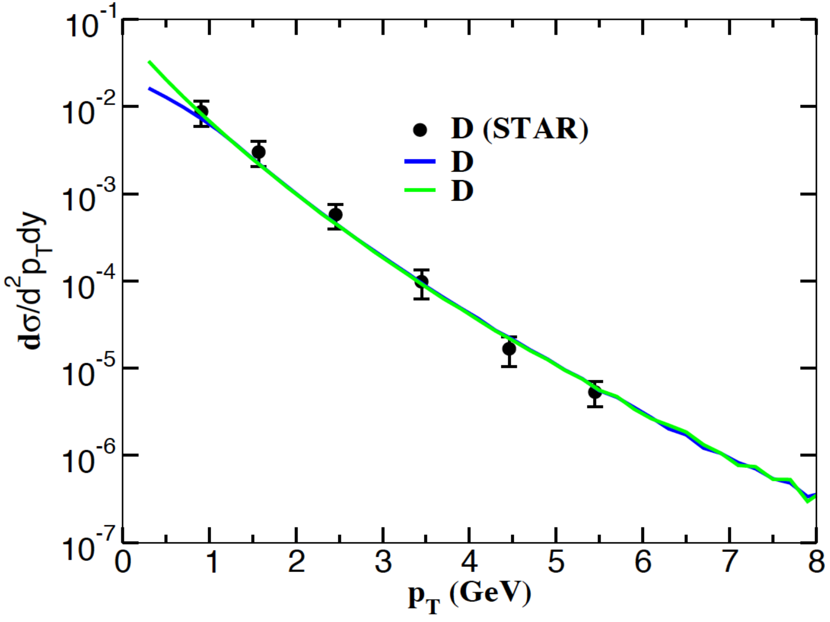


- No Hadronic Rescattering included
- Bump can be present also w/o coalescence
- Coalescence shift the bump

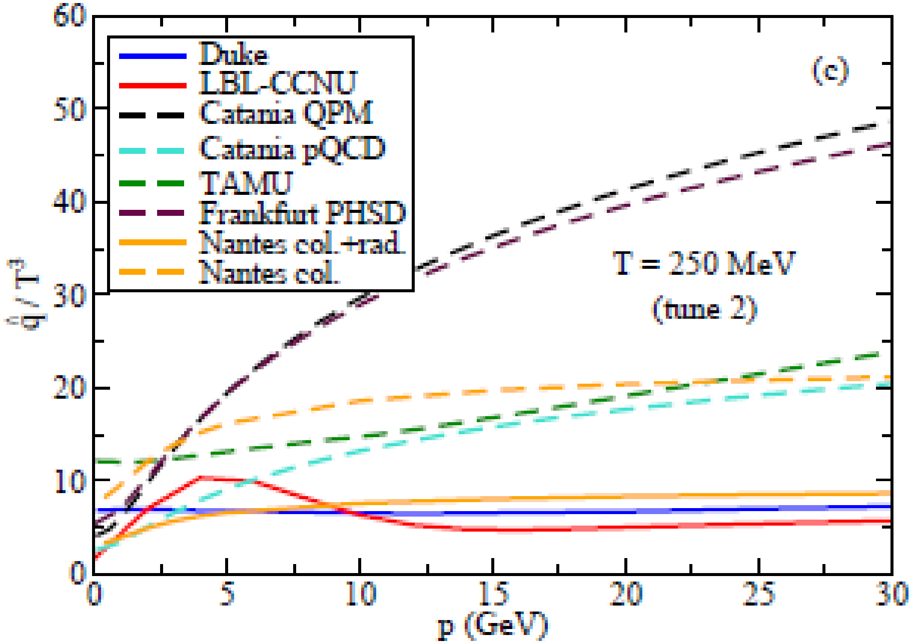
Impact of 2 dN/dp$_T$ well within FONLL



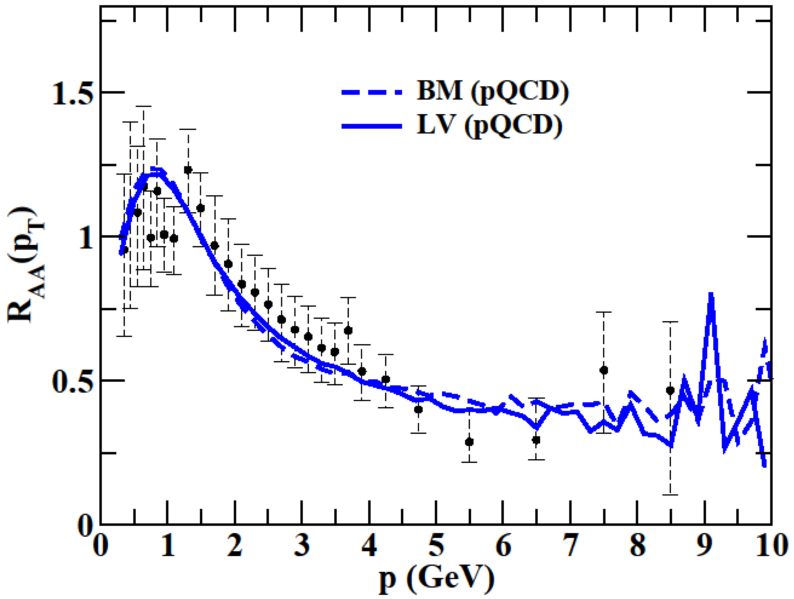


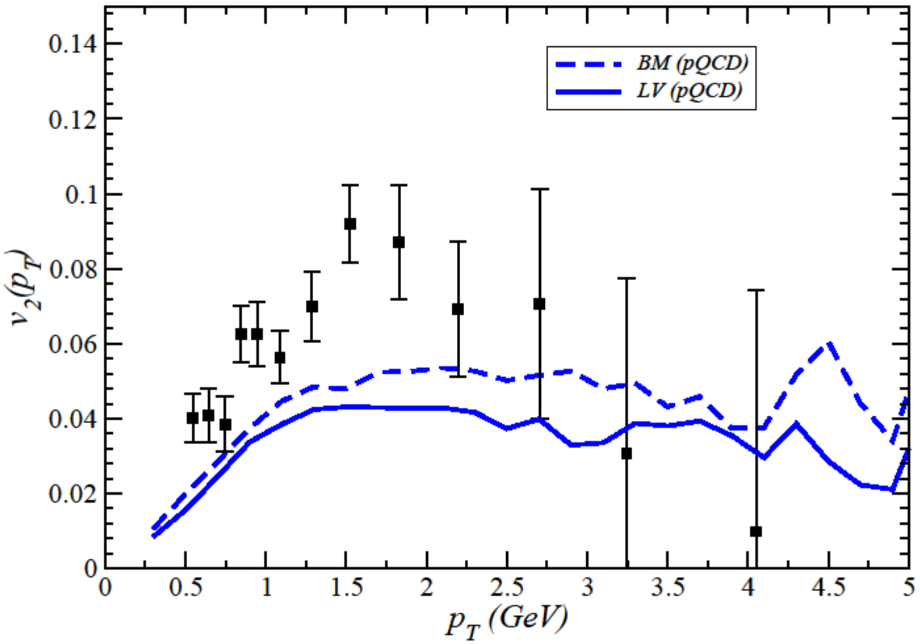


Especially for the bump
Look at distribution and data
down to 0 P$_T$ essential


60
50
40
30
20
10
0
$\hat{q} / T^3$
0
5
10
15
20
25
30
p (GeV)
Duke
LBL-CCNU
Catania QPM
Catania pQCD
TAMU
Frankfurt PHSD
Nantes col.+rad.
Nantes col.
(c)
T = 250 MeV
(tune 2)

R_{AA} and v2 at RHIC at mD=gT





Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

At fixed RAA Boltzmann approach generate larger v2 .
(depending on mD and M/T)

I) LPM effect : Suppression of bremsstrahlung and pair production.

Formation length ($l_f = \frac{\hbar}{q_\perp}$) : The distance over which interaction is spread out

1) It is the distance required for the final state particles to separate enough that they act as separate particles.

2) It is also the distance over which the amplitude from several interactions can add coherently to the total cross section.

As $q_\perp$ increase → ℓ_f reduce → Radiation drops proportional

S. Klein, Rev. Mod. Phys **71** (1999)1501

(II) Dead cone Effect : Suppression of radiation due to mass

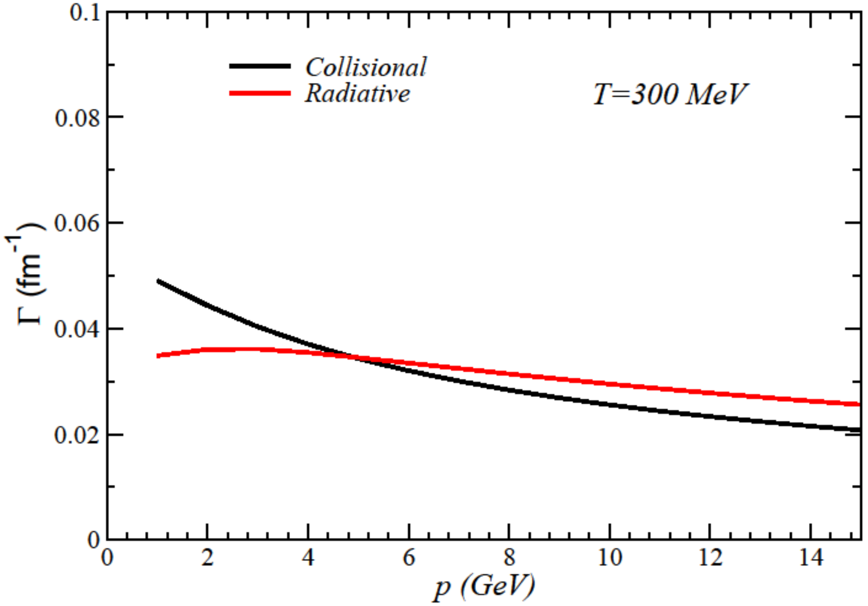
$$\boxed{\frac{1}{\sigma}\frac{d^2\sigma}{dz\,d\theta^2} \sim C_F \frac{\alpha_s}{\pi}\frac{1}{z}\frac{\theta^2}{\left(\theta^2+4\gamma\right)^2}}$$

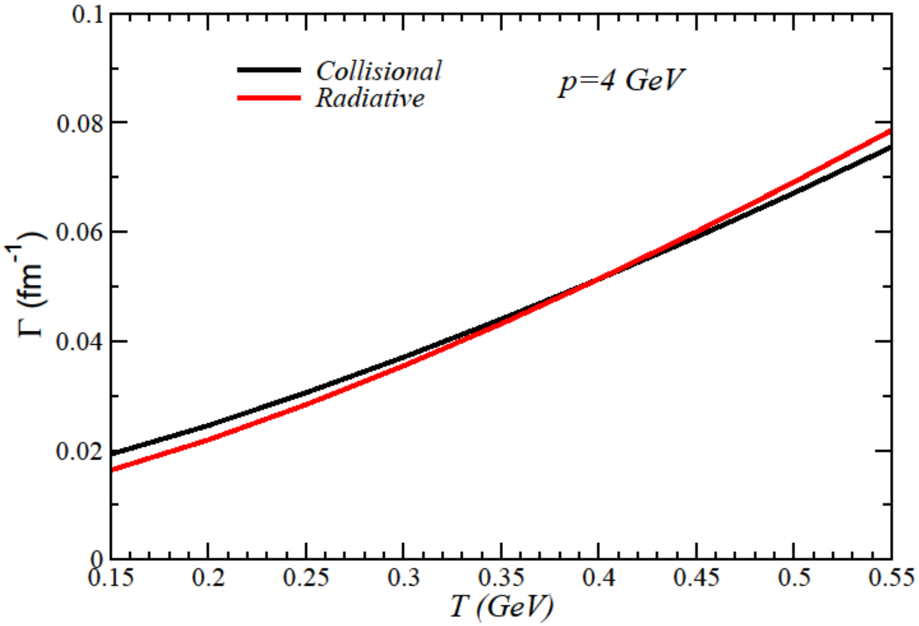
where $z = 2 - x_1 - x_2$ *and* $\gamma = \frac{m^2}{s}$

Where $x_1 = 2E_q/\sqrt{s}$ and $x_2 = 2E_{\bar{q}}/\sqrt{s}$ $\longrightarrow$ the energy fraction of the final state quark and anti-quark.

Radiation from heavy quarks suppress in the cone from θ =0 (minima) to θ=2 √γ (maxima)

Radiative vs Collisional

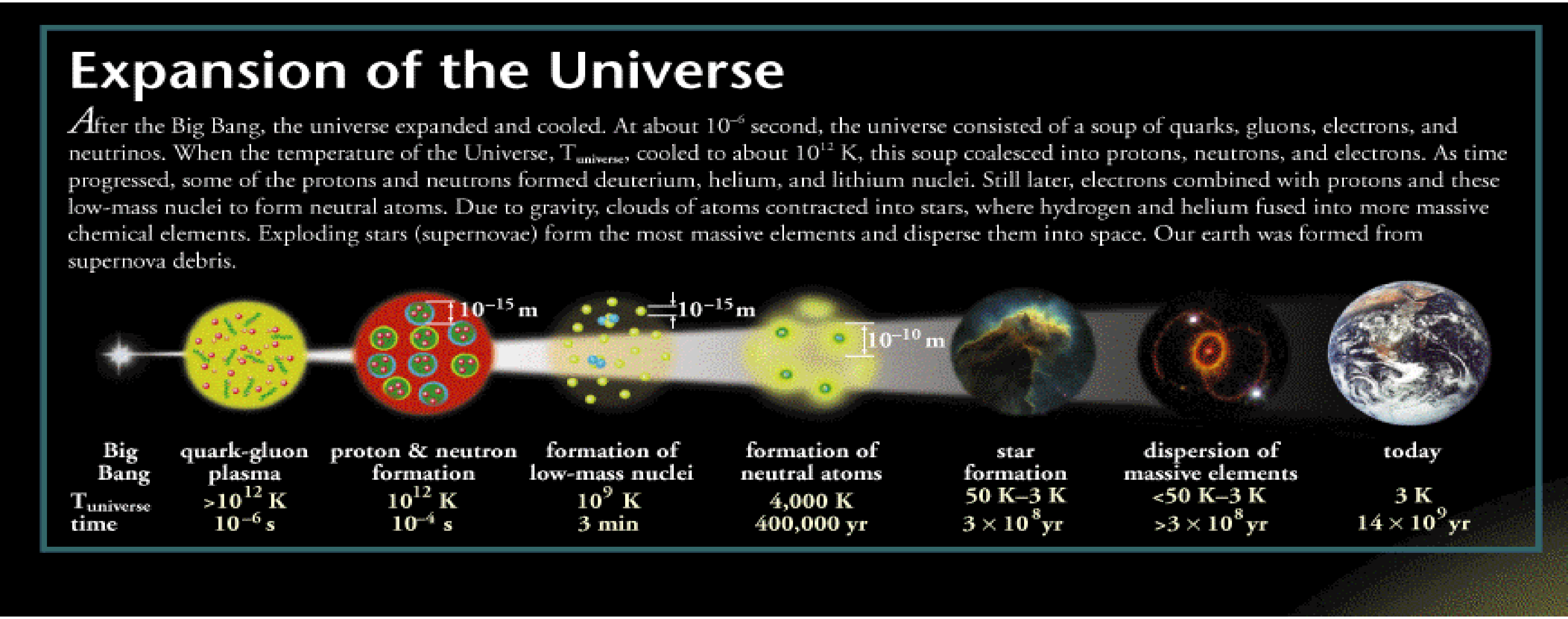




At High momentum radiative loss dominate over collisional loss

Das, Alam , Mohanty
PRC, 82,014908,2010

Introduction:



Expansion of the Universe

After the Big Bang, the universe expanded and cooled. At about 10^{-6} second, the universe consisted of a soup of quarks, gluons, electrons, and neutrinos. When the temperature of the Universe, $T_{universe}$, cooled to about 10^{12} K, this soup coalesced into protons, neutrons, and electrons. As time progressed, some of the protons and neutrons formed deuterium, helium, and lithium nuclei. Still later, electrons combined with protons and these low-mass nuclei to form neutral atoms. Due to gravity, clouds of atoms contracted into stars, where hydrogen and helium fused into more massive chemical elements. Exploding stars (supernovae) form the most massive elements and disperse them into space. Our earth was formed from supernova debris.

Big Bang	quark-gluon plasma	proton & neutron formation	formation of low-mass nuclei	formation of neutral atoms	star formation	dispersion of massive elements	today
$T_{universe}$	$>10^{12}$ K	10^{12} K	10^{9} K	4,000 K	50 K–3 K	<50 K–3 K	3 K
time	10^{-6} s	10^{-4} s	3 min	400,000 yr	3×10^{8} yr	$>3 \times 10^{8}$ yr	14×10^{9} yr

No direct way to look at the time before ~400,000 years

Nuclear collisions at high energy can create the micro second old universe.

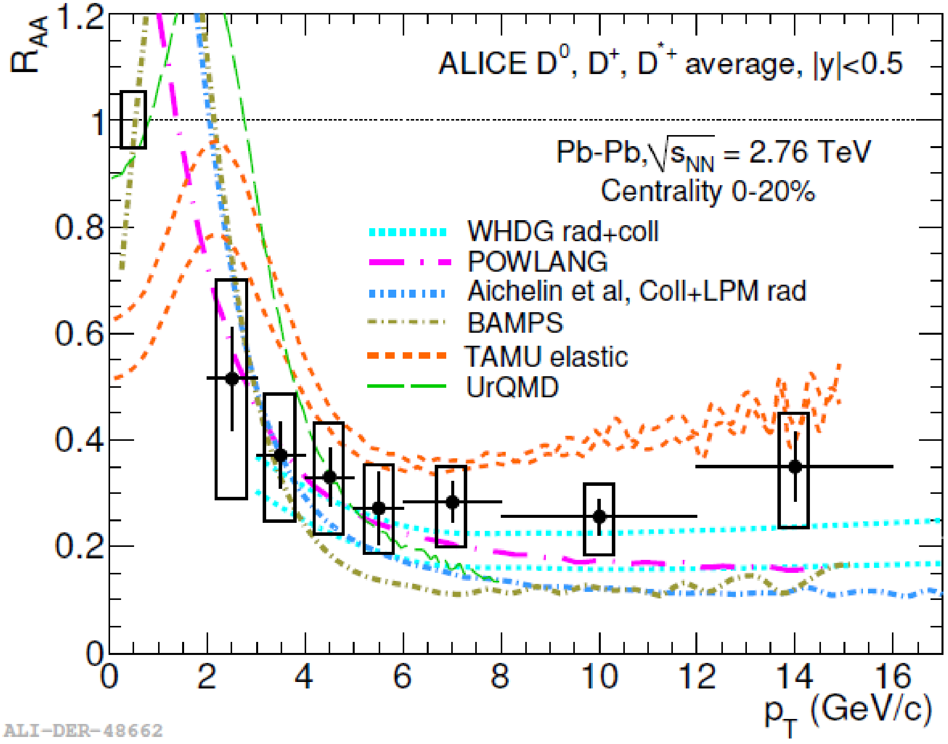
Quark-Hadrons transition occurred at T~170 MeV (Lattice-QCD)

Sun Temperature~KeV

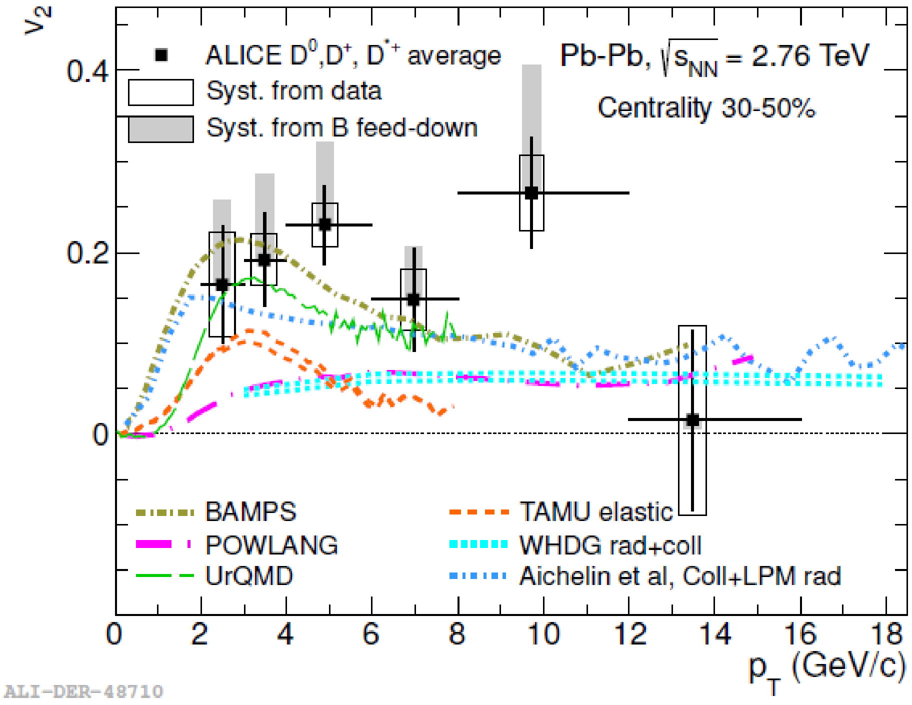
More than 10000 times of the temperature of sun !

Heavy Flavors at LHC

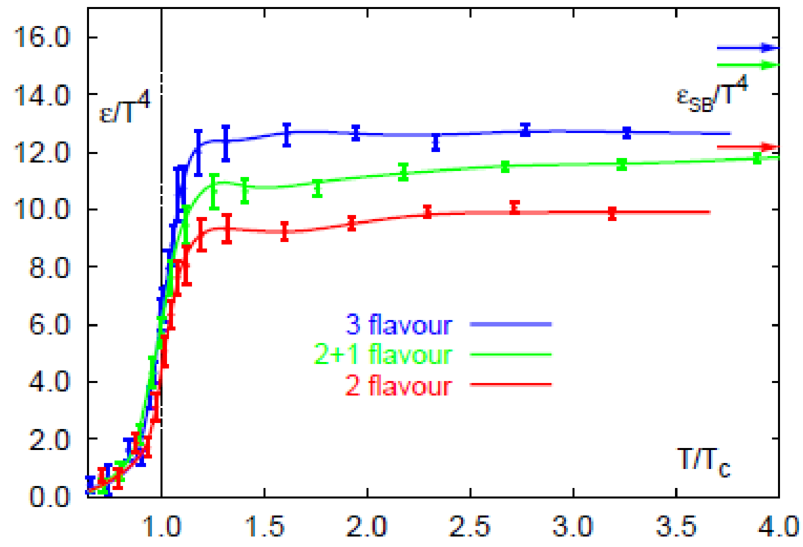
JHEP **1209** (2012) 112



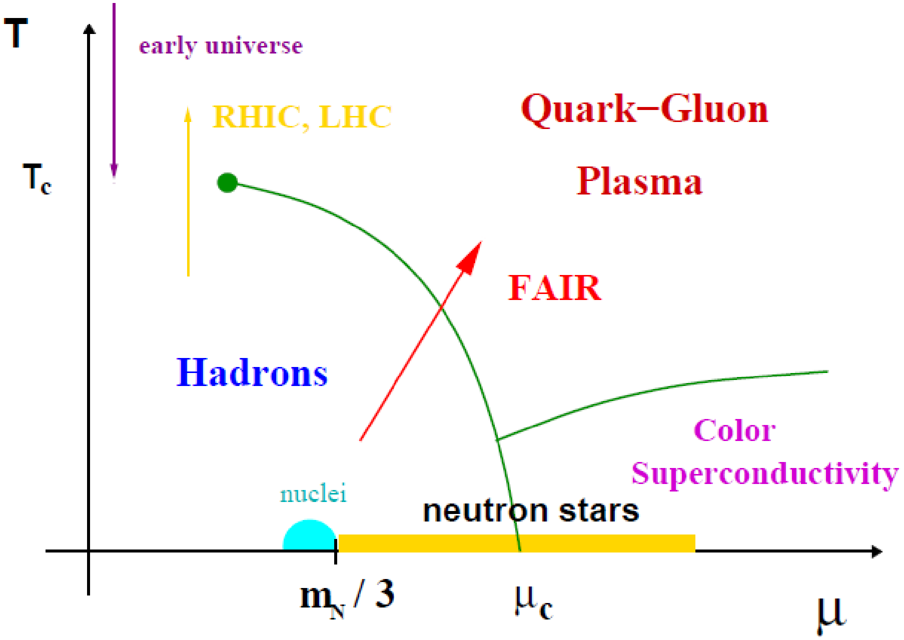
arXiv:1305.2707



How to probe it



Energy density as a function of temperature scaled by the critical temperature Tc.



Deconfined phases of QCD matter at two extreme conditions.

F. Karsch, Lect. Notes Phys.583 209 (2002)