Chromoelectric Field Fluctuations and Heavy Quarks in Hot and Dense QCD Medium

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Content:

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- 🔴 **Heavy Quark Production**

- 🔴 **Heavy Quark Propagation**

  - ➥ **Collisional Energy Loss**

  - ➥ **Radiative Energy Loss**

  - ➥ **Heavy-flavoured hadron spectra (Jet Quenching)**

- 🔴 **Chromoelectric Field Fluctuations**

  - ➥ **Collisional Energy gain**

  - ➥ **Heavy-flavoured Nuclear Suppression factor**

  - ➥ J/ψ **Nuclear Suppression factor**
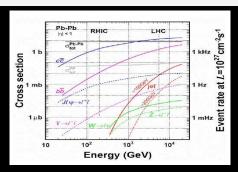
- 🔴 **Conclusion**

Why Heavy Quarks ?

- Heavy Quarks ➤ **Charm (c) & Bottom (b)**
  - ☛ **Charm:** $M_c = 1.27^{+0.07}_{-0.09}$ **GeV**
  - ☛ **Bottom:** $M_b = 4.19^{+0.18}_{-0.06}$ **GeV**
- Production time: $\tau_Q = 1/2M_Q \leq 0.1$ fm/c
- Thermalisation time (τ_{th}):
  - ☛ Successive equlibrium: $\tau_{th}^g < \tau_{th}^{u,d} < \tau_{th}^c < \tau_{th}^b < \tau_{th}^{life}$

    [J. Alam, S. Raha & B. Sinha, PRL 73 (1994) 1895]

  - ☛ Therma. time for HQs $\tau_{th}^Q \sim \frac{M_Q}{T} \times \tau_{th}^{u,d}$ [G. Moore & D. Teaney, PRC 2005]

  - ☛ $\tau_{th}^{u,d} < \tau_{th}^Q$:   no production at QGP and hadronic phase

  - ☛ Produced at very early time interactions in Hard Scattering of partons in Nucleons. Initial distribution of HQs ➤ frozen

  - ☛ Since g, u, d thermalise early and provide an expanding thermal background

Why Heavy Quarks ?

🔴 Heavy Quarks propagation:

- ☛ propagates in foreground with gluons & light quarks as an expanding thermal background for the non-equilibrated HQs ➤ expansion dynamics

- ☛ interacts with equilibrated degrees of freedom ➤ energy-loss of HQ

- ☛ requires dynamics ➤ final distr^n of HQs (Transport Eq.)

🔴 Uniqueness of Heavy Quarks:

- ☛ distinguishable down to lowest momenta ➤ medium coupling (Hydro)

- ☛ cleaner energy-loss probe ➤ reflected in leading particle $p_\perp$ spectra, flow

- ☛ tests understanding of mass dependence

- ☛ Quarkonia: QGP thermometer
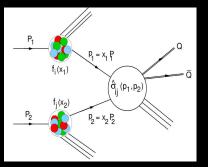
Heavy Quark Production in Heavy-Ion Collisions:

🔴 Heavy-Ion Coll.:   **ALICE: 0809.1062[nucl-ex]**

☞ **Charm: LHC $\sim 10\times$ RHIC**   ☞**Bottom: LHC $\sim 100\times$ RHIC**

🔴 Produced in pairs $(Q\bar{Q})$ at early times

🔴 QCD provides a framework to compute production x-section and distribution

HQs Production and Initial Distribution

🔴 Factorisations allows that an observable can be expressed as a convolution of short distance (hard scattering of partons) and the long distance contribution describing the initial hadrons
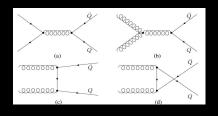
$$\frac{d\sigma}{dy_1\,dy_2\,dp_\perp} = x_1 x_2 p_\perp \sum_{ij} f_i^{(1)}(x_1, \mu_F^2) f_j^{(2)}(x_2, \mu_F^2) \hat{\sigma}_{ij}(x_i, x_j, \alpha_s(\mu_F^2), \mu_F^2)$$
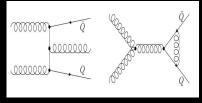

- ☞ i, j are partons in hadrons

- ☞ σ_{ij} are partonic x-sections ➤ pQCD

- ☞ f_i are parton distr^n fn. (PDFs) in hadron which are nonperturbative

- ☞ μ_F is the factorization scale between hard processes and the non-perturbative PDFs

- ☞ μ_F dependence should cancel in order by order in pQCD calculation

HQs Production and Initial Distribution

🔴 Factorisations:

☞ $gg \rightarrow Q\bar{Q}; \quad q\bar{q} \rightarrow Q\bar{Q}; \quad gg \rightarrow Q\bar{Q}g; \quad q\bar{q} \rightarrow Q\bar{Q}g$ [Leading Order (LO), NLO & FONLL]

☞ The short distance contribution describing the matrix elements of hard processes are free from mass singularities and calculable in pQCD

☞ PDFs obey DGLAP evolution equation ➤ knowing PDFS at one scale can be calculated at any other scale

☞ In elimentary [pp and $p\bar{p}$] collisions an observable is well controlled through factorization

Heavy-Ion Collisions & Uncertainties

🔴 Does factorisation work in A+A ? May or may not! But assumed!

🔴 PDFs in nuclei (nPDFs) are different from those in hadrons: geometry, wavefunction are different, nPDFs require appropriate modifications

🔴 Several nPDFS that prametrise the nuclear dependence of incoming parton distribution functions

👉 "uncertainity in intial state interactions"

🔴 The frgmentation function, used to obtain $p_\perp$ spectra, should also be afftected by the presence of hot and dense QCD medium!

👉 "uncertainty in final state interactions"

Heavy Quark Propagation and Final distribution

🔴 Produced at early time ($\tau_Q < \tau_{th}$); No production at later time

☛ **Total no. of HQ gets frozen very early in the history of collisions**

☛ **Immediately upon their production they will propagate through QGP**

☛ **One is left with the task of determining the HQ distribution**

☛ **Details of the distribution may reflect the characteristics and development of QGP**

Fokker-Planck Equation

🔴 Dynamics of a heavy quark propagation as a foreground particle in a QGP can be studied using the simplified Boltzmann approach when the background bulk medium is in thermal equilibrium
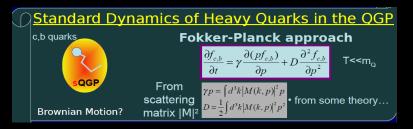
Extensive Body of Literatures

▸ Chakraborty & Syam, Lett. Nuvo Cim.'84
▸ Svetitsky, PRD'88
▸ Alam, Raha & Sinha, PRL'94
▸ Mustafa, Pal & Srivastava, PRC'97
▸ Mustafa PRC'05
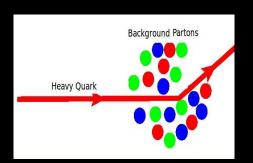▸ Moore & Teany PRC'05; Rapp &Hees'05

Fokker-Planck Equation

🔴 **Fokker-Planck Eq. ➤ Simplified Boltzmann Eq.**

☛ Boltzmann Eq ➤ No external force and isotropic in space

☛ Soft Scatt. ➤ Taylor expan. of Coll. Integral ➤ Landau Eq.

☛ Landau Eq. ➤ an integro-diff^l Eq. involving transport coeffs. ➤ describes collision processes of two particles

☛ Landau Eq. ➤ FP Eq. when distribution of the background particles (QGP) is thermal whereas foreground particle (HQs) is non-thermal
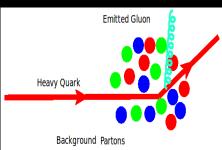

Standard Dynamics of Heavy Quarks in the QGP

c,b quarks

Fokker-Planck approach

$$\frac{\partial f_{c,b}}{\partial t} = \gamma \frac{\partial (p f_{c,b})}{\partial p} + D \frac{\partial^2 f_{c,b}}{\partial p^2}$$

T<<m$_Q$

From scattering matrix |M|²

$$\gamma p = \int d^3 k |M(k,p)|^2 p$$
$$D = \frac{1}{2} \int d^3 k |M(k,p)|^2 p^2$$

· from some theory…

Brownian Motion?

sQGP

Propagation of Heavy Quark in QGP

🔴 Energy-Loss

☞ **Collisional Loss**   ☞**Radiative Loss**
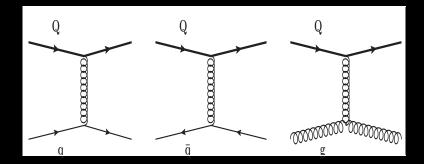
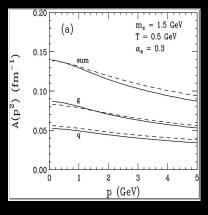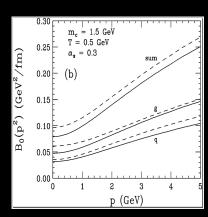# Collisional Processes

🔴 Feynman Diagram

Heavy Quark Transport Coefficients from FP Equation
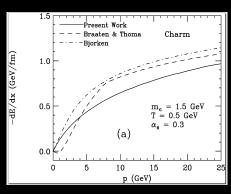
🔴 Drag $\gamma(p)$ and Diffusions $\mathcal{B}_0(p)$

🔴 **Mustafa, Pal & Srivastava, PRC'97**

🔴 **Mustafa PRC'05**
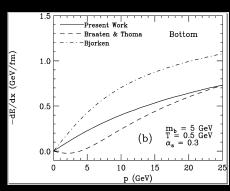
Differential Collisional Energy Loss

- Drag Coeff.: $\gamma(p) = -\frac{1}{p}\frac{dE}{dx}$
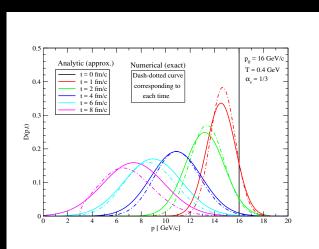
  - Differnetial Collisional Energy-Loss: $\frac{dE}{dx}$

    [Mustafa, Pal, Srivastava, PRC57 (1998) 889]

HQ Energy-Loss Distribution **[Mustafa, PRC72 (2005) 014905]**

🔴 Collisional Energy-Loss ▶ Transport-coeffs. ▶ Fokker-Planck Eq. ▶ Energy-Loss Distribution
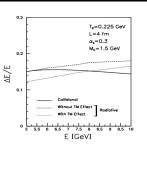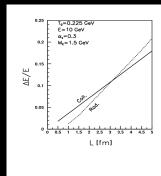

🔴 Mom. diffusion begins

☞ Peak Shifts to lower p ▶ **Energy Loss**
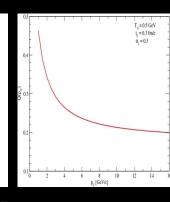
☞ Peak broadens ▶ **Momentum Dispersion**

Substantial Collisional E-Loss

🔴 **Mustafa, PRC72 (2005) 014905**
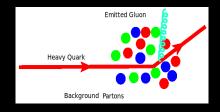

🔴 Coll. E-Loss is important   🔴 Predicted R_{AA} but no data then

🔴 For light quarks R_{AA} :

☛ **Mustafa et al., PRL100 (2008) 072301; Acta Phys.Hung. A22 (2005)**

Radiative Energy Loss

🔴 HQ encounters inelastic scattering as it traveses a medium

☞ Emits a gluon

🔴 Differential Loss: dE/dx

🔴 Very simple but first estimation:

☞ **Mustafa, Pal, Srivastava, Thoma, PLB 428 (1998) 234**

🔴 Improvements in formalism

Radiative Energy Loss

🔴 Singnificant Points:

☛ The nature of the medium through which the energetic parton propagates ➤ thermally equilibrated perturbative medium with a collection of static scatt. centres with specified density

☛ Kinematic approximations for interation between medium partons (thermal background) and the projectile parton that is propagating in the medium

☛ The virtuality and branching/splitting of hard parton to reduce its off-shellness ➤ multiple splitting may occure in the medium ➤ multigluon final state should include interference of emitted gluons

Suppression due to Mass and Dead Cone

🔴 General notion: **heavy quark radiates less than light quark**

$$\boxed{qq' \to qq'} \quad |\mathcal{M}_{qq' \to qq'}|^2 = \frac{8}{9} g^4 \frac{s^2}{t^2}$$

$$\boxed{Qq \to Qq} \quad |\mathcal{M}_{Qq \to Qq}|^2 = \frac{8}{9} g^4 \frac{s^2}{t^2} \left(1 - \frac{M^2}{s}\right)^2$$

$$\boxed{qq' \to qq'g} \quad |\mathcal{M}_{qq' \to qq'g}|^2 = 12g^2 \frac{8}{9} g^4 \frac{s^2}{t^2} \frac{1}{k_\perp^2}$$

$$\boxed{Qq \to Qqg} \quad |\mathcal{M}_{Qq \to Qqg}|^2 = 12g^2 \frac{8}{9} g^4 \frac{s^2}{t^2} \frac{1}{k_\perp^2} \quad \boxed{?}$$
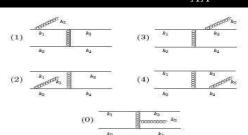
2001 Dokshitzer and Kharzeev proposed (Phys. Lett. B 519, 199 (2001)) "dead cone" effect => heavy quark small energy loss.

$$\left(1 + \frac{\theta_0^2}{\theta^2}\right)^{-2} \qquad \theta_0 = M/E$$

Generalised Dead Cone

🔴 RHIC data (PHENIX) ▶ $R_{AA}^{HQ} \sim R_{AA}^{LQ}$ ▶ **(Heavy quark puzzle)**

Hierarchy employed in this study

$$\sqrt{s}, E \gg \sqrt{|t|} \sim q_\perp \gg \omega > k_\perp \gg m_D$$

Mass Range | $0 < M/E < 1$

Emission angle Range | $-\pi < \theta < +\pi$

$$|\mathcal{M}_{Qq \to Qqg}|^2 = 12g^2 |\mathcal{M}_{Qq \to Qq}|^2 \frac{1}{k_\perp^2} \left(1 + \frac{M^2}{s \tan^2(\frac{\theta}{2})}\right)^{-2}$$

$$= 12g^2 |\mathcal{M}_{Qq \to Qq}|^2 \frac{1}{k_\perp^2} \left(1 + \frac{M^2}{s} e^{2\eta}\right)^{-2}$$

$$\mathcal{D} = \left(1 + \frac{M^2}{s \tan^2(\frac{\theta}{2})}\right)^{-2}$$

..... *Dead Cone Factor*

☛ **Abir, Greiner, Martinez, Mustafa, Uphoff, PRD85 (2012) 054012**

M G Mustafa (SINP)   3rd HF Meet · · ·   IIT, Indore: 19/3/19   21 / 41

Radiation Spectrum [Abir,Greiner,Martinez,Mustafa,Uphoff, PRD85 (2012)054012]

- $Qq \to Qqg$ and $Q\bar{q} \to Q\bar{q}g$;   Replace $Q \to \bar{Q}$

$$\left.\frac{dn_g}{d\eta \; dk_\perp^2}\right|_{Qq \to Qqg} = \frac{C_A \alpha_s}{\pi} \frac{1}{k_\perp^2} \mathcal{D}$$

- $Qg \to Qgg$ and   Replace $Q \to \bar{Q}$

$$\left.\frac{dn_g}{d\eta \; dk_\perp^2}\right|_{Qg \to Qgg} = \frac{C_A}{C_F} \left.\frac{dn_g}{d\eta \; dk_\perp^2}\right|_{Qq \to Qqg} ; \; \frac{C_A}{C_F} = \frac{9}{4}$$

Radiative E-Loss [Abir,Jamil,Mustafa, Srivastava, PLB715 (2012) 183]
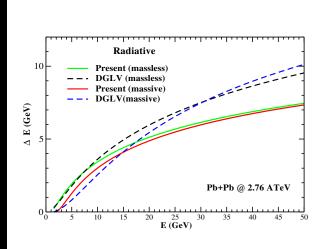
🔴 Differential Radiative E-Loss

$$-\frac{dE}{dx} = \frac{\langle\omega\rangle}{\langle\lambda\rangle}$$

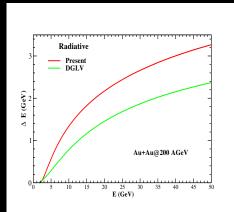$\langle\omega\rangle =$ **Mean energy of the emitted gluon**

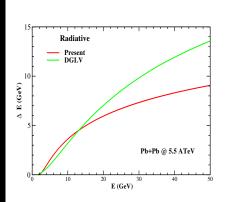$\langle\lambda\rangle =$ **Mean free path of the heavy quark**

Radiative E-Loss [Abir,Jamil,Mustafa,Sivastava, PLB715 (2012) 183]

DGLV: PRL85 (2000); NPA784 (2007); NPA783 (2000); NPA733 (2004)
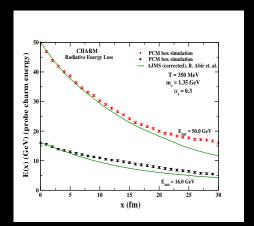
Radiative E-Loss [Abir,Jamil,Mustafa,Sivastava, PLB715 (2012) 183]
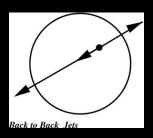
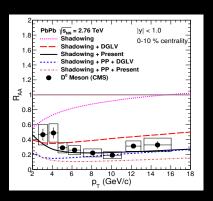# Comparison with Parton Cascade Model [Younus et al, Phys.Rev. C91 (2015) 024912]
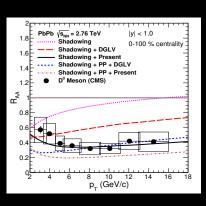
Jet Quenching

Back to Back Jets

- Jets are produced back to back
  - ☞ **Awayside will pass through medium**
  - ☞ **Interact with the medium**
  - ☞ **Lose energy and quenched**
  - ☞ **Results in suppression of hadronic yields in AA than NN**

- Nuclear Suppression Factor: $R_{AA} = \dfrac{(\text{Yield})^{AA}}{N_{\text{coll}} (\text{Yield})^{NN}}$

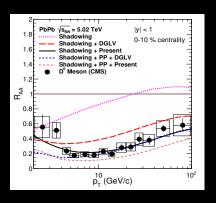- Quenching depends on amount of energy-loss suffered in the medium

D-Meson @ LHC 2.76ATeV

☛ DATA:**CMS Collaboration , CMS-PAS-HIN-16-011.**

☛ Theory: **Abir, Jamil, Mustafa, Srivastava, PLB715 (2012) 183**

☛ **Saraswat, Shukla, Singh, NPA961(2017)169**

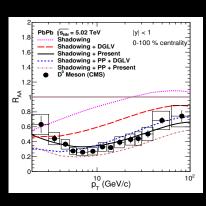# D-Meson @ LHC 5.02ATeV

- Data:**CMS Collaboration, CMS-PAS-HIN-16-001.**
- Theory: **Abir, Jamil, Mustafa, Srivastava, PLB715 (2012) 183**
- **Saraswat, Shukla, Singh, NPA961(2017)169**

Collisional Energy

- 🔴 Assumed that collisional energy lost by the particle per unit time is small compared to the energy of the particle itself

- ☛ the change in the velocity of the particle during the motion may be neglected, i.e, the particle moves in a straight line trajectory.

- 🔴 The energy loss of a particle is determined by the work done per unit time by the retarding chromo electric forces acting on the particle in the plasma while moving:

- ☛
$$\frac{dE}{dt}\Big|_{\text{coll}} = Q^a \vec{v} \cdot \vec{\mathcal{E}}^a \Big|_{\vec{r}=\vec{v}t}$$

- 🔴 The collisional energy loss does not take into account :

- ☛ the field fluctuation in the plasma

- ☛ the particle recoil in collisions

- 🔴 One needs to take in to account both : How??

Collisional Energy Gain [Chakraborty, Mustafa, Thoma, PRC75 (2007) 064908]

🔴 The work done per unit time reads as

☞
$$\frac{dE}{dt}\big|_{\text{fluc}} = \left\langle Q^a \vec{v}(t) \cdot \vec{\mathcal{E}}_t^a(\vec{r}(t), t) \right\rangle$$

☞ The electric field $\mathcal{E}_t^a$ consists of the induced field and a spontaneously generated microscopic field, the latter being a random function of position and time

☞ The stochastic time dependence comes because of explicit fluctuations of field in time due to change of the motion of the particle from one field point to another

☞ Both $\vec{r}(t)$ and $\vec{v}(t)$ can be obtained from EOM:

$$\frac{d\vec{p}}{dt} = Q^a \left[\mathcal{E}_t^a(\vec{r}(t), t) + \vec{v} \times \mathcal{B}_t^a(\vec{r}(t), t) \right]$$

☞ The magnetic field $\mathcal{B}_t^a$ consists of an induced and a microscopic part as in the case of $\mathcal{E}_t^a$

Collisional Energy Gain [Chakraborty, Mustafa, Thoma, PRC75 (2007) 064908]

☞ The spectrum of the chromo-electromagnetic fields follows from the fluctuation-dissipation theorem

$$\langle \mathcal{E}_i^a \mathcal{E}_j^b \rangle_{\beta;\omega,k} = \delta^{ab} \langle \mathcal{E}_i \mathcal{E}_j \rangle_{\beta;\omega,k}$$
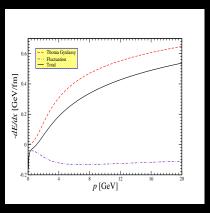
$$\langle \mathcal{E}_i \mathcal{E}_j \rangle = \frac{8\pi}{e^{\beta\omega} - 1} \left[\mathcal{P}_{ij}^L \frac{\mathsf{Im}\ \varepsilon_L}{|\varepsilon_L|^2} + \mathcal{P}_{ij}^T \frac{\mathsf{Im}\ \varepsilon_T}{\left| \varepsilon_T - \frac{\omega^2}{k^2} \right|^2} \right]$$
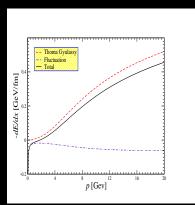
☞ This is completely determined by the dielectric functions of the medium

☞ This leads to energy gain as

$$\begin{aligned}
\frac{dE}{dx}\bigg|_{fluc} &= \frac{C_F \alpha_s}{16\pi^3 E} \int d^3 k \left[\frac{\partial}{\partial \omega} \langle w \vec{\mathcal{E}}_L^2 \rangle + \langle \vec{\mathcal{E}}_T^2 \rangle \right]_{\omega = \vec{k} \cdot \vec{v}} \\
&= 2\pi C_F \alpha_s^2 \left(1 + \frac{N_f}{6} \right) \frac{T^3}{E v^2} \ln \frac{1+v}{1-v} \ln \frac{k_{\mathsf{max}}}{k_{\mathsf{min}}}
\end{aligned}$$

Collisional Energy Gain

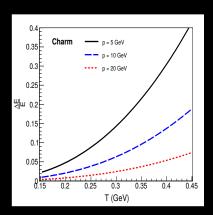🔴 The field fluctuations lead to energy gain:

☞ **Chakraborty, Mustafa, Thoma, PRC75 (2007) 064908**

Fractional Energy Gain of Charm Quark
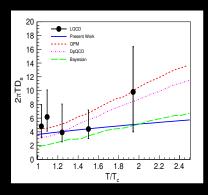
🔴 Coll. Gain/Energy$= \frac{\Delta E}{E}$

A Ikbal, Z Ahamed, M G Mustafa, : arXiv:1902.06989

Ikbal, Ahamed, Shukla, Mustafa, Phys .Rev. C98 (2018) 034915

Diffusion Coefficient of Charm Quark

🔴 Langevin Dynamics
A Ikbal, Z Ahamed, M G Mustafa, : arXiv:1902.06989
Ikbal, Ahamed, Shukla, Mustafa, PRC98 (2018) 034915

☞ Drag : $\quad \gamma = -\frac{1}{p}\frac{dE}{dx}$
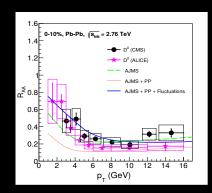
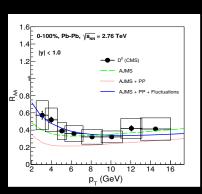☞ FD Theorem:

Mom. Diff.: $D = 2\gamma ET$

☞ Einstein Relation:

$$D_s \underset{p \to 0}{=} \frac{T}{\gamma M} = \frac{2T^2}{D}$$

☞ **Banerjee, Datta, Gavai and Majumdar, PRD 85 014510 (2012)**

Open Charm and Field Fluctuation

🔴 Nuclear Modification factor (R_{AA}) for D Meson at 2.76 TeV

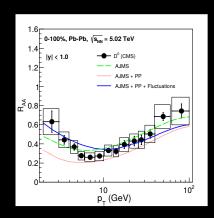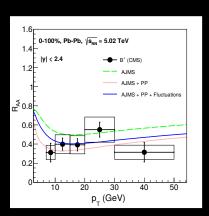**Ikbal, Ahamed, Shukla, Mustafa, Phys .Rev. C98 (2018) 034915**

☞ **DATA:CMS Collaboration , CMS-PAS-HIN-16-011**

☞ **ALICE Collaboration, JHEP 1603 (2016) 081, 2016**

Open Charm/Beauty and Field Fluctuation

🔴 Nuclear Modification factor (R_{AA}) for D & B-Meson at 5.02 TeV

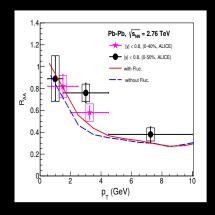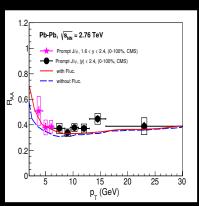**Ikbal, Ahamed, Shukla, Mustafa, Phys .Rev. C98 (2018) 034915**

☞ **DATA:CMS Collaboration , CMS-PAS-HIN-16-011**

Hidden Charm and Field Fluctuation

🔴 Nuclear Modification factor (R_{AA}) for J/ψ at 2.76 TeV

A Ikbal, Z Ahamed, M G Mustafa, : arXiv:1902.06989

☞ **ALICE Collaboration, J. High Energy Phys. 07, 051 (2015)**

☞ **CMS Collaboration, Eur. Phys. J. C 77, 252 (2017)**

Conclusion:

- 🔴 **Discussed why are heavy quarks important**
- 🔴 **Heavy quark production in high energy HIC: X-section in pQCD**
- 🔴 **Collisional E-Loss & E-Gain**
  - ☛ Relation to Transport Coeffs: Drag and Diffusion
  - ☛ Coll. E-Loss Distr. & **importance of Coll. E-Loss**
- 🔴 **Radiative E-Loss**
  - ☛ Generalised Dead Cone and radiative E-Loss
  - ☛ Light and heavy quark lose energy in a similar fashion
- 🔴 **Importance Chromoelectric Field Fluctuations & Energy Gain**
- 🔴 **Nuclear suppression R_{AA} for various heavy quarked meson**
  - ☛ Both D and B Meson R_{AA} @ LHC for 2.76ATeV and 5.02ATeV in Pb+Pb
  - ☛ J/Ψ R_{AA} @ LHC for 2.76ATeV in Pb+Pb without regeneration

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THANK YOU