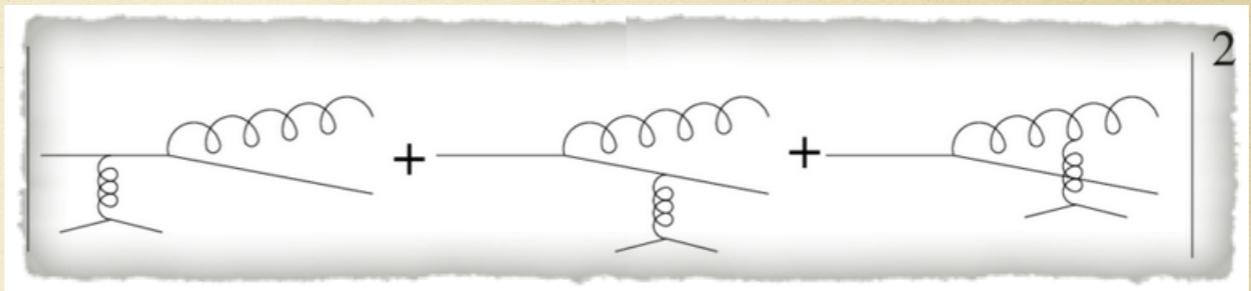


Chronicles of (heavy) quark (radiative) energy loss in nuclear medium (1993-2004)

**Raktim Abir
Aligarh Muslim University**

Gluon bremsstrahlung - Single scattering (1982)



- Asymptotic Freedom (1973)

- Light front perturbation

Gunion and Bertsch (1982)

- Gunion-Bertsch formula for gluon bremsstrahlung in vacuum

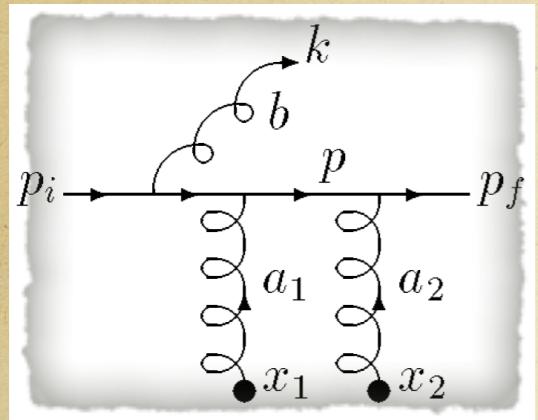
$$\omega \frac{dn_g}{d\omega dk_{\perp}^2} = \frac{C_A \alpha_s}{\pi} \frac{q_{\perp}^2}{k_{\perp}^2 (q_{\perp} - k_{\perp})^2}$$

Single scattering

- Matrix element $i\mathcal{M}_1(x_1) \propto ig \left(\frac{\epsilon \cdot p_0}{k \cdot p_0} - \frac{\epsilon \cdot p_1}{k \cdot p_1} \right) e^{ikx_1}$
- Matrix element squared $\omega \frac{dn_g}{d\omega dk_{\perp}^2} \propto |\mathcal{M}(x_1)|^2$

How this gluon radiation spectrum would modify in presence of medium?

Gluon bremsstrahlung – Multiple scatterings (1993–96)



Double scattering

- Matrix element

$$i\mathcal{M}_2(x_1, x_2) \propto ig \left(\frac{\epsilon \cdot p_0}{k \cdot p_0} - \frac{\epsilon \cdot p_1}{k \cdot p_1} \right) e^{ikx_1} + ig \left(\frac{\epsilon \cdot p_1}{k \cdot p_1} - \frac{\epsilon \cdot p_2}{k \cdot p_2} \right) e^{ikx_2}$$

- Matrix element squared

$$|\mathcal{M}(x_1, x_2)|^2 = |\mathcal{M}(x_1)|^2 + |\mathcal{M}(x_2)|^2 + \mathcal{M}(x_2)\mathcal{M}^*(x_1)e^{ik(x_1-x_2)} + \mathcal{M}(x_2)\mathcal{M}^*(x_1)e^{-ik(x_1-x_2)}$$

LPM Effect

Landau, Pomeranchuk (1953), Migdal (1956)

Bethe-Heitler limit

Large

(independent scatterings. No LPM effect)

$$k \cdot (x_2 - x_1) \approx \frac{L}{\tau(k)}$$

Small

(destructive interference by LPM effect)

Factorisation limit

Gyulassy, Wang, Plumer (1993–94)

Baier, Dokshitzer, Peigne and Schiff (1994)

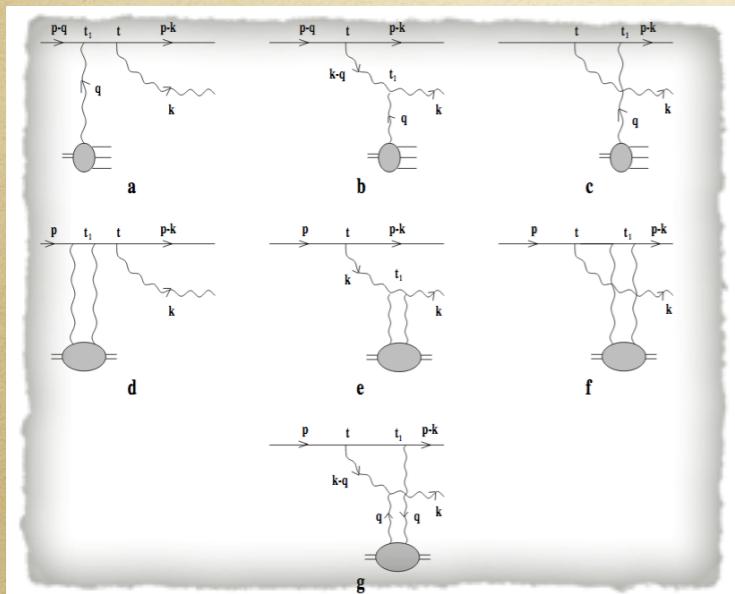
Zakharov (1996)

How to implement LPM effect for multiple scatterings in QCD medium?

Radiative Energy loss in finite sized QCD medium (1996-97)

Baier, Dokshitzer, Peigne, Muller and Schiff (1996)
(BDMPS)

- Soft gluon approximation, modelled the interaction by static scattering centres/Glauber approximation.
- Energy loss is related to transverse momentum broadening.
- Total energy loss is proportional to the square of the length of the traversed nuclear matter.



Zakharov (1997)

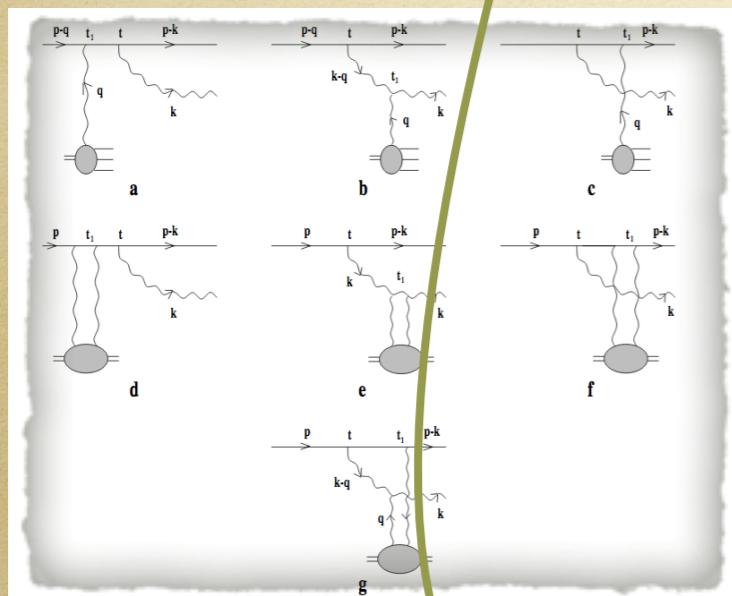
- Path integral formulation
- **No** soft gluon approximation
- Radiative energy loss $\Delta E_q \propto L^2$ (High energy quarks)
 $\propto L$ (Low energy quarks)

Radiative energy loss of the high energy quark is described by the interaction of a high energy colour-neutral quark-antiquark-gluon system with the QCD medium it passes through.

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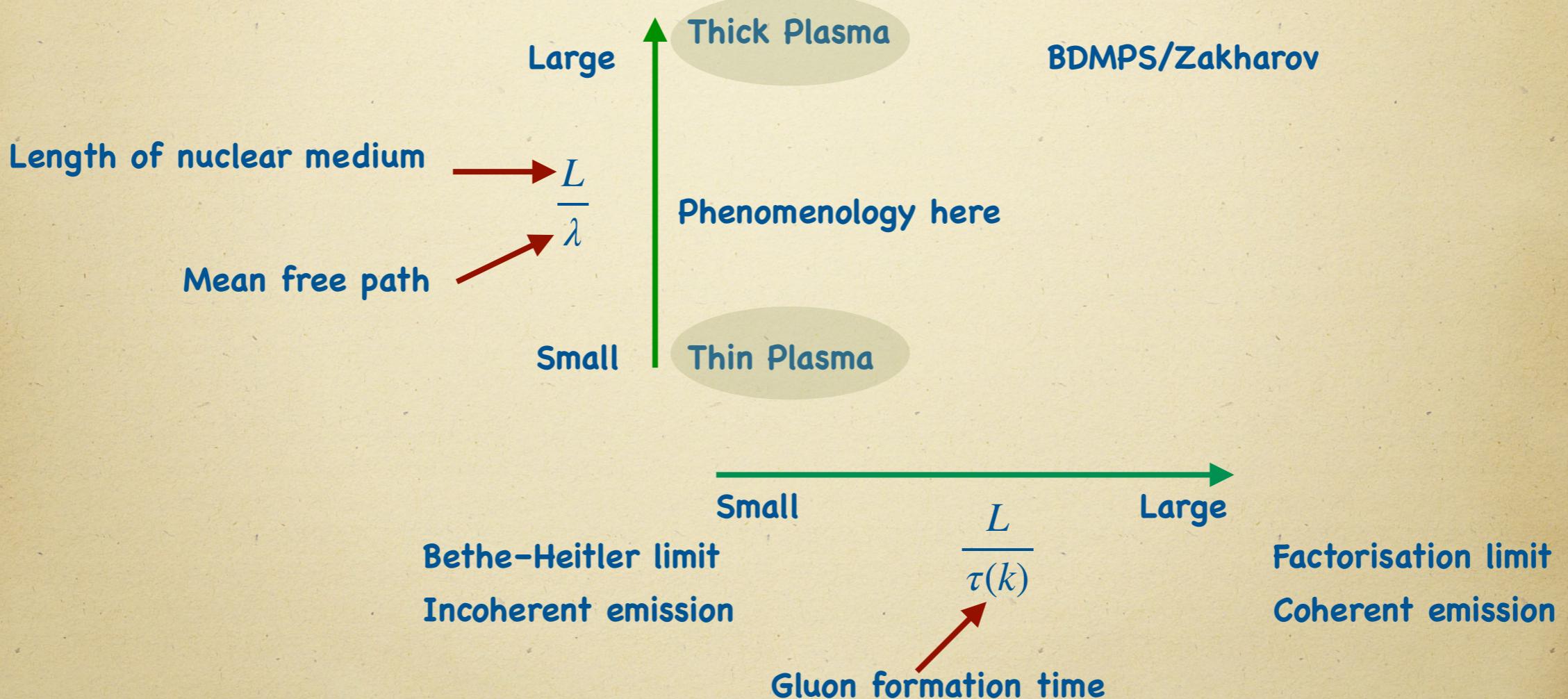
Radiative energy loss of the high energy quark is described by the interaction of a high energy colour-neutral quark-antiquark-gluon system with the QCD medium it passes through.

Equivalence: BDMPS (1998), Wiedeman (2000)

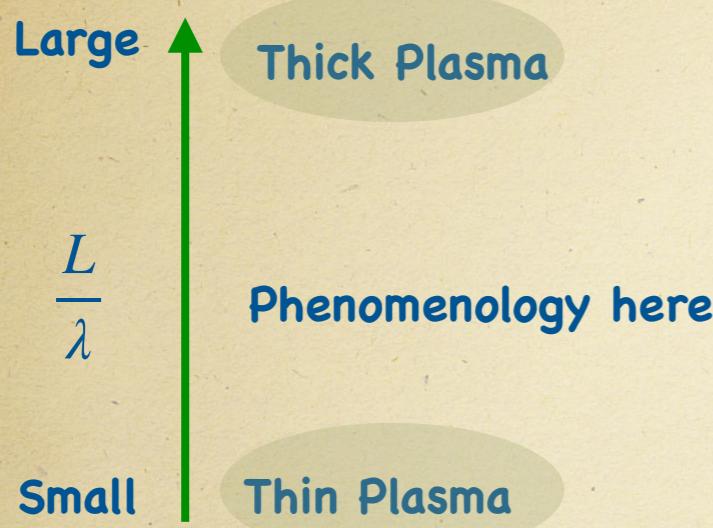
Phenomenology: Opacity expansion (2000)

M. Gyulassy, P. Levai and I. Vitev (2000)

Wiedeman (2000)



Opacity expansion: GLV formalism



$$\omega \frac{dn_g}{d\omega dk_{\perp}^2} = \frac{C_A \alpha_s}{\pi} \frac{q_{\perp}^2}{k_{\perp}^2 (q_{\perp} - k_{\perp})^2}$$

- Single emission kernel

$$x \frac{dN_g}{dx} \propto \frac{L}{\lambda} \int d^2 q d^2 k \frac{\mu^2}{(q^2 + \mu^2)^2} \frac{k \cdot q (k - q)^2 - \beta^2 q \cdot (k - q)}{[k^2 + x^2 M^2][(k - q^2) + x^2 M^2]} \int dz \left[1 - \cos \left(1 - \cos \left(\frac{(k - q^2) + x^2 M^2}{2xE} \right) \right) \right] \rho(z)$$

- Multiple gluon emission

Static scattering center, Poisson distributed independent emissions (Stochastic emission).

M. Gyulassy, P. Levai and I. Vitev (GLV)
(2000)

↓ + Heavy quark + adhoc gluon mass

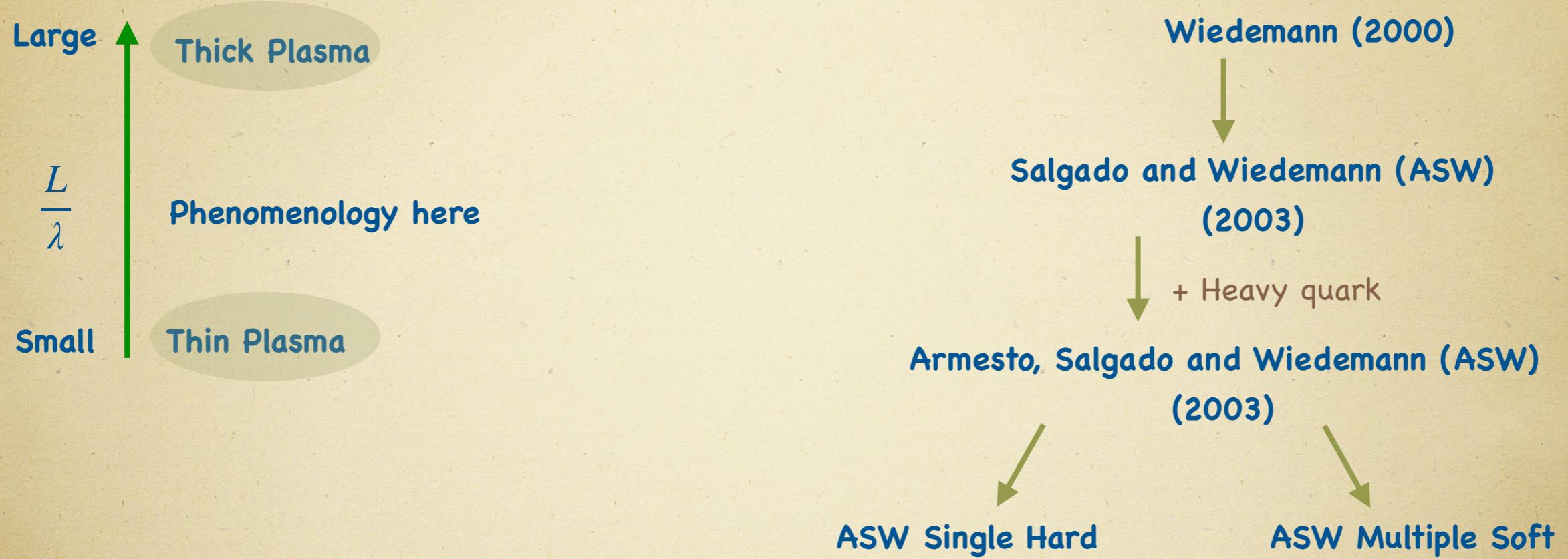
Djordjevic, Gyulassy (DGLV)
(2003)

+ Collisional energy loss
(advocated by M.G.M.)

Wicks, Horowitz, Djordjevic
and Gyulassy (WHDG)
(2007)

↓ + Opacity expansion upto 9th order
Buzzatti + Gyulassy (CUJET 1.0)
(2012)

Opacity expansion: ASW formalism



- **Single emission kernel (ASW-SH)**

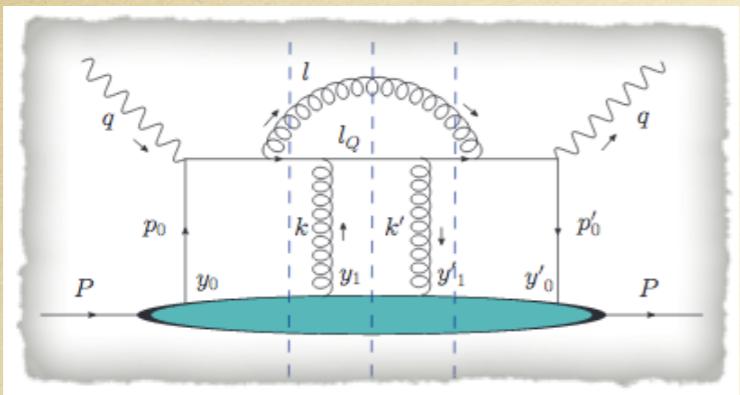
$$\omega \frac{dN_g}{d\omega} \propto \frac{L}{\lambda} \int d^2q d^2k \frac{\mu^2}{(q^2 + \mu^2)^2} \frac{k \cdot q (k - q)^2 - x^2 M^2 q \cdot (k - q)}{[(k - q^2) + x^2 M^2][k^2 + x^2 M^2]} \int dz \left[1 - \cos \left(1 - \cos \left(\frac{(k - q^2) + x^2 M^2}{2\omega} \right) \right) \right] \rho(z)$$

- **Multiple gluon emission**

Static scattering center, Poisson distributed independent emissions (Stochastic emission).

Higher twist formalism

HIGHER-TWIST-BERKELEY-WUHAN(HT-BW) MODEL



- Higher twist energy loss approach is based on applying factorisation, OPE to the problem of calculating the change in the fragmentation function of a quark produced by DIS of an electron on a nucleus.

- Single emission kernel:

$$\frac{dN_g}{dx dk_\perp^2} = \frac{2\alpha_s(k_\perp)}{\pi} \frac{P(x)}{x} \frac{1}{k_\perp^4} \left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2} \right)^4 \hat{q}$$

- Multiple gluon emission:

$$\hat{q} = \frac{4\pi C_F \alpha_s}{N_c^2 - 1} \int dy^- \langle F^{ai+}(0) F_i^{a+}(y^-) \rangle e^{i\xi p^+ y^-}$$

Through (medium-modified) DGLAP evolution.

Xiaofeng Guo and Xin-Nian Wang (2000-01)

Enke Wang and Xin-Nian Wang (2002)

+ Heavy quark

Zhang, Wang and Wang
(2004)

HT-BW

+ Light cone drag

Abir, Majumder (2014)

AMY formalism

- Based on Hard Thermal Loop Field perturbation theory.

P. Arnold, Guy D. Moore and L. G. Yaffe (AMY)
(2001-02)

- Infinite length thermally equilibrated medium at finite temperature, No static scattering centres.

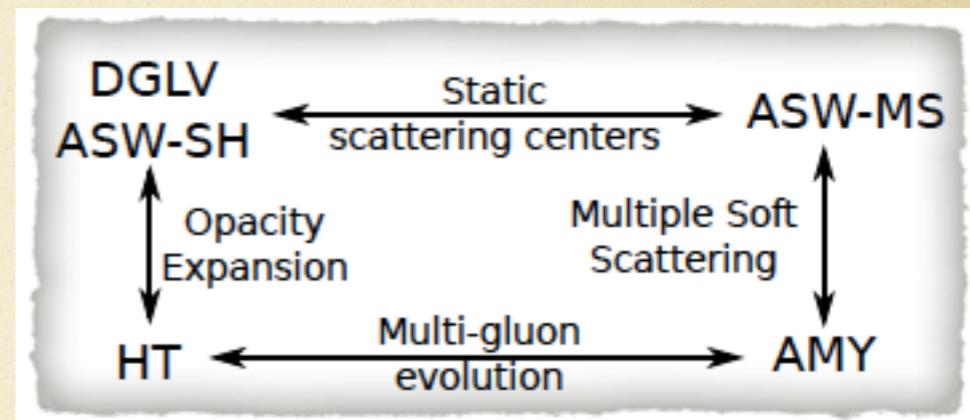
+ Collisional energy loss

↓
Qin, Ruppert, Gale, Jeon, Moore and Mustafa
(2007)

- Multiple gluon emission through rate equations (no vacuum radiation).

Landscape of four formalisms

- Medium modeling
- Kinematic limits
- Treatment of multi gluon emission



• arXiv:1106.1106 (Comparison of Energy loss models)

• arXiv:1312.5003 (JET collaboration) (Extractions of jet quenching parameter)

Cascades of models

Gyulassy and Wang (1993)

BDPS (1994)

Zakharov (1996)

LPM effects in QCD

BDMPS (1996)

Zakharov (1996)

Energy loss in finite sized QCD medium

Equivalence: BDMPS (1998), Wiedeman (2000)

GLV (2000)

Opacity expansion

SW(2003)

Quenching Weights

DGLV (2003)

+ Heavy Quark

WG (2000)

Twist four parton distributions

AMY (2001)

Thermal Field Theory

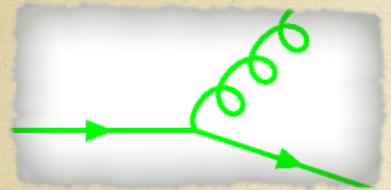
ZWW (2004)

+ Heavy Quark

WHDG (2007)

...+ Collisional Energy loss +
path length fluctuations

Light Cone Drag



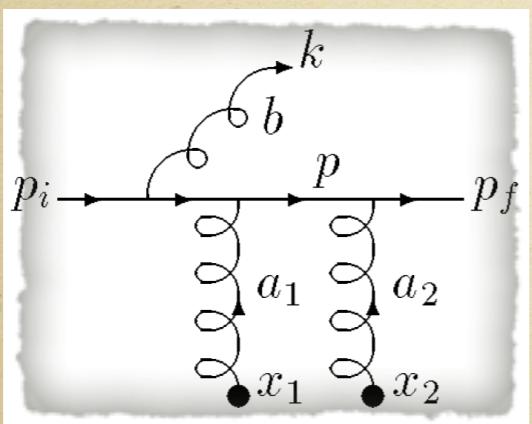
Two problem

$$k = (k^+, k^-, k_\perp)$$

Time Light Cone Drag

What is the role play of light cone drag?

Fixing the scale in coupling



Running Coupling

$$\alpha(\sim q_\perp^2)$$

Scale that enters in the coupling

How to fix this scale?

BLM prescription