

JIMWLK evolution and small- x asymptotics of 2n-tuple Wilson lines correlators

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3rd Heavy Flavour Meet-2019, IITI, Indore, India.

Outlines

Deep Inelastic Scattering

Map of high energy QCD

Color Glass Condensate

JIMWLK Equation

High Energy Evolution of Color 2n-tuple Correlator

Dipole Evolution

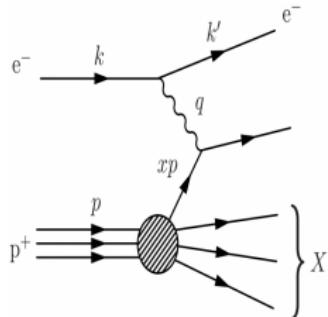
Quadrupole Evolution

2n-tuple correlator in the unitary limit

Deep Inelastic Scattering

DIS of a lepton on a hadron is the cleanest environment to probe the dynamics of strong interactions.

$$e^- + p \rightarrow e^- + X$$



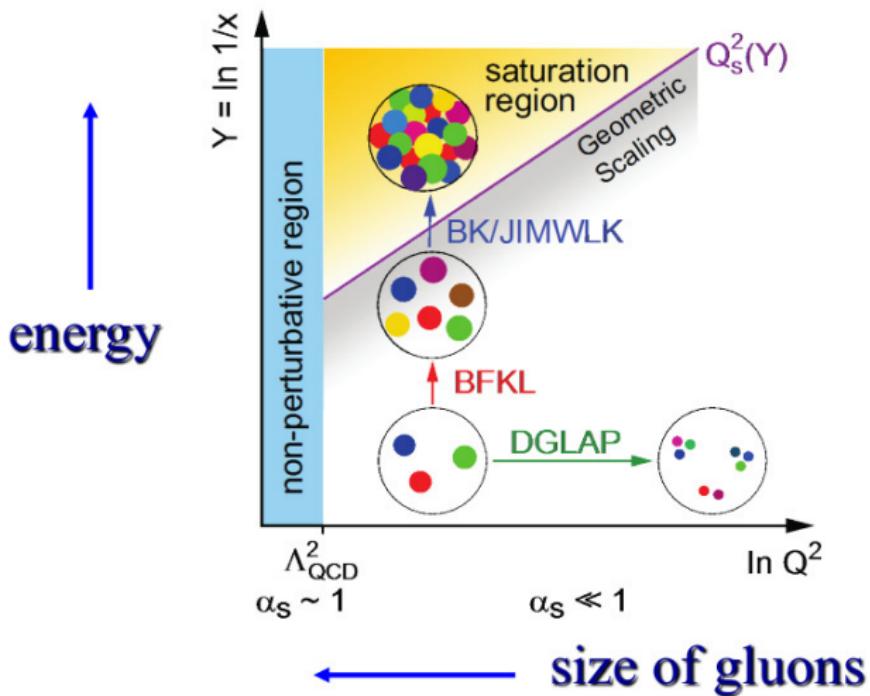
$$Q^2 = -q^2 \text{ (Virtuality of photon)}$$

x = Bjorken variable

$$s = \frac{Q^2}{x}$$

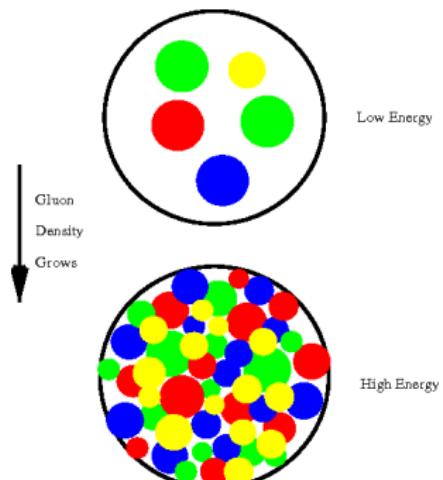
$$Y = \log(1/x)$$

Map of high energy QCD



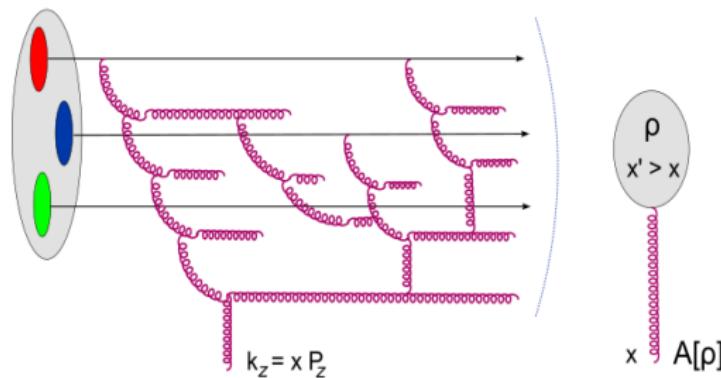
Color Glass Condensate(CGC)

- ▶ **Color:** The effective degrees of freedom in this framework are color sources ρ at large- x and gluon fields at small- x .
- ▶ **Glass:** Due to time dilation, the evolution of fields is smaller than natural time scale and are disordered like glass.
- ▶ **Condensate:** Due to large occupation number of saturated gluons $\mathcal{O}(1/\alpha_s)$.



Color Glass Condensate (CGC)

- ▶ Color Glass Condensate (CGC) is an effective theory which incorporates saturation dynamics at high energy.



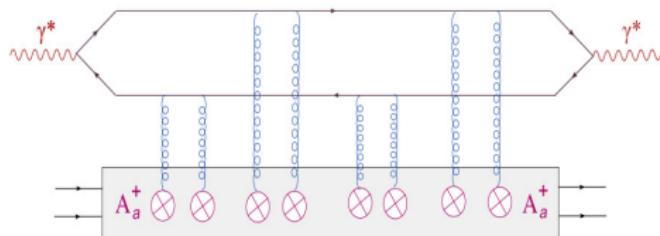
- ▶ It separates large- x partons from small- x partons in the hadrons wavefunction.
- ▶ Fast partons are considered as static color current sources ρ for the small- x partons.

JIMWLK (Jalilian-Iancu-McLerran-Weigert-Leonidov-Kovner) Equation

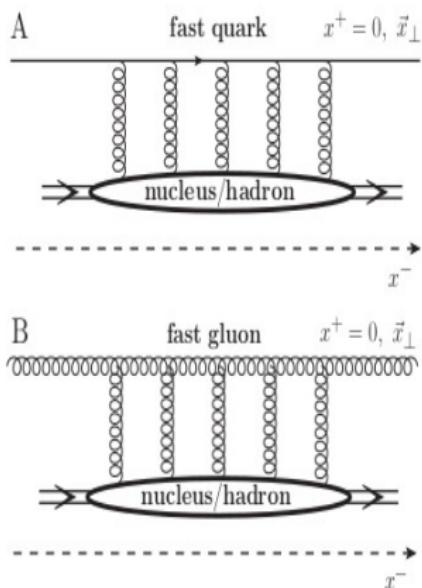
- ▶ Evolution equation for $\mathcal{W}[\rho]$ (JIMWLK)

$$\frac{\partial}{\partial Y} \mathcal{W}_Y[\rho] = \mathcal{H} \mathcal{W}_Y[\rho]$$

- ▶ $\mathcal{W}[\rho]$ (weight function) gives much more information about the saturated gluons and these are universal.



$$\langle \mathcal{O} \rangle \equiv \int \mathcal{D}\rho \mathcal{O}[\rho] \mathcal{W}_Y [\rho]$$



- $U_x = \mathcal{P} \exp \left(ig \int dx^- A^a(x^-, x) t^a \right)$
- $U_x = \mathcal{P} \exp \left(ig \int dx^- A^a(x^-, x) T^a \right)$

- Color dipole amplitude

$$\mathcal{O}^{(2)} \equiv \text{Tr} [U(x_1)U^\dagger(x_2)]$$

- Quadrupole amplitude

$$\mathcal{O}^{(4)} \equiv \text{Tr} [U(x_1)U^\dagger(x_2)U(x_3)U^\dagger(x_4)]$$

- Multipole amplitude

$$\mathcal{O}^{(2n)} \equiv \text{Tr} [U(x_1)U^\dagger(x_2)U(x_3)U^\dagger(x_4)\dots U(x_{2n-1})U^\dagger(x_{2n})]$$

JIMWLK Equation for 2n-tuple

The energy evolution for gauge invariant operator \mathcal{O}

$$\frac{\partial}{\partial Y} \langle \hat{\mathcal{O}} \rangle_Y = \langle \mathcal{H} \hat{\mathcal{O}} \rangle_Y$$

$$\mathcal{H} \equiv -\frac{1}{16\pi^3} \int_z \mathcal{M}_{xzy} \left(1 + \tilde{U}_x^\dagger \tilde{U}_y - \tilde{U}_x^\dagger \tilde{U}_z - \tilde{U}_z^\dagger \tilde{U}_y \right)^{ab} \frac{\delta}{\delta \alpha_x^a} \frac{\delta}{\delta \alpha_y^b}$$

2n-tuple Correlator

$$\begin{aligned}
& \frac{\partial}{\partial Y} \text{Tr} \left[U(x_1) U^\dagger(x_2) U(x_3) U^\dagger(x_4) \dots U(x_{2n-1}) U^\dagger(x_{2n}) \right] = \frac{\bar{\alpha}_s}{4\pi} \left(\frac{1}{1 + \delta_{n,1}} \right) \times \\
& \int_z \sum_{k=0}^{\lfloor n/2 \rfloor - 1} \sum_{l=0}^{n-1} \mathcal{K}_{(2l+1; 2l+2k+1)}^{(2l; 2l+2k+2)} \text{Tr} \left[U(x_{2l+1}) U^\dagger(x_{2l+2}) \dots U(x_{2l+1+2k}) U^\dagger(z) \right] \text{Tr} \left[U(z) U^\dagger(x_{2l+2k+2}) \dots U(x_{2l-1}) U^\dagger(x_{2l}) \right] \\
& + \sum_{k=0}^{\lfloor n/2 \rfloor - 1} \sum_{l=0}^{n-1} \mathcal{K}_{(2l+2; 2l+2k+2)}^{(2l+1; 2l+2k+3)} \text{Tr} \left[U^\dagger(x_{2l+2}) U(x_{2l+3}) \dots U^\dagger(x_{2l+2+2k}) U(z) \right] \text{Tr} \left[U^\dagger(z) U(x_{2l+2k+3}) \dots U^\dagger(x_{2l}) U(x_{2l+1}) \right] \\
& + \sum_{k=0}^{\lfloor n/2 \rfloor - 2} \sum_{l=0}^{n-1} \mathcal{K}_{(2l+1; 2l+2k+2)}^{(2l; 2l+2k+3)} \text{Tr} \left[U(x_{2l+1}) U^\dagger(x_{2l+2}) \dots U^\dagger(x_{2l+1+2k+1}) \right] \text{Tr} \left[U(x_{2l+2k+3}) U^\dagger(x_{2l+2k+4}) \dots U^\dagger(x_{2l}) \right] \\
& + \sum_{k=0}^{\lfloor n/2 \rfloor - 2} \sum_{l=0}^{n-1} \mathcal{K}_{(2l+2; 2l+2k+3)}^{(2l+1; 2l+2k+4)} \text{Tr} \left[U^\dagger(x_{2l+2}) U(x_{2l+3}) \dots U(x_{2l+2k+3}) \right] \text{Tr} \left[U^\dagger(x_{2l+2k+4}) U(x_{2l+2k+5}) \dots U(x_{2l+1}) \right] \\
& + \delta_{1,n \bmod 2} \sum_{l=0}^{\lfloor n/2 \rfloor - 1} \mathcal{K}_{(2l+1; 2l+n)}^{(2l; 2l+n+1)} \text{Tr} \left[U(x_{2l+1}) U^\dagger(x_{2l+2}) \dots U(x_{2l+n}) U^\dagger(z) \right] \text{Tr} \left[U(z) U^\dagger(x_{2l+n+1}) \dots U(x_{2l-1}) U^\dagger(x_{2l}) \right] \\
& + \delta_{1,n \bmod 2} \sum_{l=0}^{\lfloor n/2 \rfloor - 2} \mathcal{K}_{(2l+2; 2l+n+1)}^{(2l+1; 2l+n+2)} \text{Tr} \left[U^\dagger(x_{2l+2}) U(x_{2l+3}) \dots U^\dagger(x_{2l+n+1}) U(z) \right] \text{Tr} \left[U^\dagger(z) U(x_{2l+n+2}) \dots U^\dagger(x_{2l}) U(x_{2l+1}) \right] \\
& + \delta_{0,n \bmod 2} \sum_{l=0}^{n/2-1} \mathcal{K}_{(2l+1; 2l+n)}^{(2l; 2l+n+1)} \text{Tr} \left[U(x_{2l+1}) U^\dagger(x_{2l+2}) \dots U^\dagger(x_{2l+n}) \right] \text{Tr} \left[U(x_{2l+n+1}) U^\dagger(x_{2l+n+2}) \dots U^\dagger(x_{2l}) \right] \\
& + \delta_{0,n \bmod 2} \sum_{l=0}^{n/2-1} \mathcal{K}_{(2l+2; 2l+n+1)}^{(2l+1; 2l+n+2)} \text{Tr} \left[U^\dagger(x_{2l+2}) U(x_{2l+3}) \dots U(x_{2l+n+1}) \right] \text{Tr} \left[U^\dagger(x_{2l+n+2}) U(x_{2l+n+3}) \dots U(x_{2l+1}) \right] \\
& - \mathcal{P}_{2n} \text{Tr} \left[U(x_1) U^\dagger(x_2) U(x_3) U^\dagger(x_4) \dots U(x_{2n-1}) U^\dagger(x_{2n}) \right]
\end{aligned}$$

Schematic representation of 2n-tuple correlators

$$\frac{d}{dY} \text{ (2n tuple)} = \int_Z \sum_{\text{all possible unique cuts}} \mathcal{K}_{(a,b)}^{(c,d)} + \mathcal{P}_{2n}$$

$$\equiv \text{ (2m tuple)} \otimes z \text{ (2n+2-2m) tuple} + \text{ (2m tuple)} \otimes z \text{ (2n-2m) tuple}$$

Dipole Evolution

- ▶ For dipole $n = 1$,

$$\frac{\partial}{\partial Y} \langle \text{Tr} [U(x_1)U^\dagger(x_2)] \rangle_Y = \frac{\bar{\alpha}_s}{4\pi} \frac{1}{2} \int_z \mathcal{K}_{(1;1)}^{(2;2)} \langle \text{Tr} [U(x_1)U^\dagger(z)] \text{Tr} [U(z)U^\dagger(x_2)] \rangle_Y - (\mathcal{P}_{(1,2)} + \mathcal{P}_{(2,1)}) \langle \text{Tr} [U(x_1)U^\dagger(x_2)] \rangle_Y$$

- ▶ In the large N_c limit for large nucleus

JIMWLK(dipole) Equation \Leftrightarrow **BK (Balitsky Kovchegov) Equation**

$$\frac{\partial}{\partial Y} S(x_1, x_2) = \frac{\bar{\alpha}_s}{4\pi} \int_z \frac{(x_1 - z)^2}{(x_1 - z)^2 (z - x_2)^2} [S(x_1, z)S(z, x_2) - S(x_1, x_2)]$$

Quadrupole

- ▶ For quadrupole $n = 2$,

$$\begin{aligned}
 & \frac{\partial}{\partial Y} \langle \text{Tr} [U(x_1)U^\dagger(x_2)U(x_3)U^\dagger(x_4)] \rangle_Y \\
 = & \frac{\bar{\alpha}_s}{4\pi} \int_z \mathcal{K}_{(1;1)}^{(4;2)} \langle \text{Tr} [U(x_1)U^\dagger(z)] \text{Tr} [U(z)U^\dagger(x_2)U(x_3)U^\dagger(x_4)] \rangle_Y \\
 & + \mathcal{K}_{(2;2)}^{(1;3)} \langle \text{Tr} [U^\dagger(x_2)U(z)] \text{Tr} [U^\dagger(z)U(x_3)U^\dagger(x_4)U(x_1)] \rangle_Y \\
 & + \mathcal{K}_{(3;3)}^{(2;4)} \langle \text{Tr} [U(x_3)U^\dagger(z)] \text{Tr} [U(z)U^\dagger(x_4)U(x_1)U^\dagger(x_2)] \rangle_Y \\
 & + \mathcal{K}_{(4;4)}^{(3;1)} \langle \text{Tr} [U^\dagger(x_4)U(z)] \text{Tr} [U^\dagger(z)U(x_1)U^\dagger(x_2)U(x_3)] \rangle_Y \\
 & + \mathcal{K}_{(1;2)}^{(4;3)} \langle \text{Tr} [U(x_1)U^\dagger(x_2)] \text{Tr} [U(x_3)U^\dagger(x_4)] \rangle_Y \\
 & + \mathcal{K}_{(3;2)}^{(4;1)} \langle \text{Tr} [U(x_3)U^\dagger(x_2)] \text{Tr} [U(x_1)U^\dagger(x_4)] \rangle_Y \\
 & - \mathcal{P}_4 \langle \text{Tr} [U(x_1)U^\dagger(x_2)U(x_3)U^\dagger(x_4)] \rangle_Y
 \end{aligned}$$

2n-tuple correlator in the unitary limit

- ▶ In the strong scattering regime ($r_\perp \gg 1/Q_s(Y)$) transverse separation between the partons is very large.
- ▶ In the limit $Y \rightarrow \infty$

$$\frac{\partial}{\partial Y} \ln \mathcal{S}(x_1, x_2, \dots, x_{2n-1}, x_{2n}) = -\frac{\bar{\alpha}_s}{2} \left(\frac{1}{1 + \delta_{n,1}} \right) \ln \prod_{j=1}^{2n} |x_j - x_{j+1}|^2 Q_s^2(Y)$$

- ▶ Levin-Tuchin asymptotic solution for 2n-tuple Wilson line correlator

$$\mathcal{S}(x_1, x_2, \dots, x_{2n-1}, x_{2n}) = S_0^{(2n)} \exp \left[-\frac{1+2i\nu_0}{2(1+\delta_{n,1})\chi(0,\nu_0)} \ln^2 \left(\prod_{j=1}^{2n} |x_j - x_{j+1}|^2 Q_s^2(Y) \right) \right]$$

Conclusion

- ▶ JIMWLK equation for general 2n-tuple correlator is derived in their fundamental representation.
- ▶ Real terms(splitting) and Virtual terms(without splitting) are explicit in integro-differential equation.
- ▶ We study the solution of the general evolution equation in the unitarity limit.
- ▶ The solution exhibits complete geometric scaling as in the case of color dipole deep inside the saturation region.

Thank You

Backup slides

$$\mathcal{K}_{(a;b)}^{(c;d)} = \frac{(x_a - x_d)^2}{(x_a - z)^2(z - x_d)^2} + \frac{(x_b - x_c)^2}{(x_b - z)^2(z - x_c)^2} - \frac{(x_a - x_b)^2}{(x_a - z)^2(z - x_b)^2} - \frac{(x_c - x_d)^2}{(x_c - z)^2(z - x_d)^2}$$

$$\mathcal{P}_{2n} = \sum_{j=1}^{2n} \frac{(x_j - x_{j+1})^2}{(x_j - z)^2(z - x_{j+1})^2}$$

► *Real terms*

- ▶ Terms involving $U(z)$ or $U^\dagger(z)$
- ▶ $2N$ -tuple correlator to pair of $2m$ -tuple and $(2N + 2 - 2m)$ -tuple correlators.

► *Virtual terms*

- ▶ $2N$ -tuple correlator to pair of $2m$ -tuple and $(2N - 2m)$ -tuple correlators