Non-perturbative study of heavy $Q\bar{Q}$ potential

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outline

1. Heavy quark potential definition
2. Method
3. Results

The correlator $D(r,t_m) =< M^\dagger(r,t_m)M(r,0) >$ in the heavy quark limit satisfy Schrodinger equation

$$[-\frac{\nabla^2}{m_Q} + V(r)]D(r,t_m) = i\frac{\partial D(r,t_m)}{\partial t_m} \quad (1)$$

where $M(r = |\vec{x}-\vec{y}|, t_m) = \bar{\psi}(\vec{x},t_m)U(\vec{x},\vec{y};t_m)\psi(\vec{y},t_m)$ for singlet channel and
$M(r = |\vec{x}-\vec{y}|, t_m) = \bar{\psi}(\vec{x},t_m)U(\vec{x},\vec{z};t_m)T^aU(\vec{z},\vec{y};t_m)\psi(\vec{y},t_m)$ for octet channel.
The heavy quark potential $V(r)$ is defined by the following equation

$$i\frac{\partial w_m(r,t_m)}{\partial t_m} = V(r)w_m(r,t_m), \quad (2)$$

as $t_m \to \infty$. Here $w_m(r,t_m)$ is the Wilson loop given by

$$w_m(r,t_m) =< Tr(exp(-ig\oint_C A^\mu(x)\,dx_\mu)) > \quad (3)$$

M. Laine et al JHEP03(2007)054

- On the lattice Wilson loop is calculated in Euclidean time .
- In lattice Wilson loop is defined as

$$w(r,t) = \frac{1}{3}\,\mathrm{Tr}\left(\prod_{x} U_{x,x+\mu}\right) \tag{4}$$

- $w(r,t) = \int e^{-\omega t}\rho(r,\omega)$ and $w_m(r,t_m) = \int e^{-i\omega t_m}\rho(r,\omega)$
  Y. Burnier,O. Kaczmarek & A. Rothkopf, Phys. Rev. Lett, 114, 082001 (2015)
- Lattice spacing
  $a_t = 0.024 fm$
  $a_s = 0.072 fm.$
  The spatial extension of our lattice is 1.7 fm.
  We change the temperature from 0.6Tc to 1.45Tc by varying the the temporal extent of the lattice.
  To remove the non-potential part of the Wilson loop we use smearing along the spatial direction.

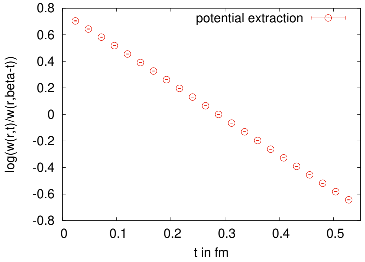
We write the Wilson loop as product a periodic and a non-periodic part in β (inverse temperature)

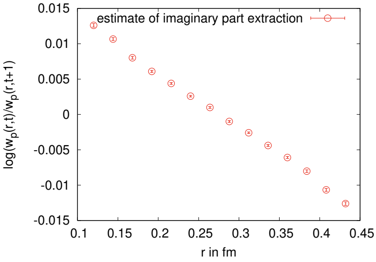
$$w(r,t) = w_{np}(r,t)w_p(r,t) \tag{5}$$

where $w_p(r,t) = w_p(r,\beta - t)$ and $w_{np}(r,t) = ce^{-tV_r}$
Y. Burnier & A. Rothkopf, Phys. Rev. D 87, 114019 (2013)
Then

$$V(r) = i\frac{\partial log(w(r,it_m))}{\partial t_m} = V_r(r) + i\sum_n p_n\omega_n sinh(\omega_n t_m) \tag{6}$$

Here we have calcuated the real part of the potential from the slope of $log(\frac{w(r,t)}{w(r,\beta-t)})$.
Currently we do not have any model independent result for the imaginary part of the potential, but we give a measure of the imaginary part through $\Gamma^2 = \frac{\partial^2 log(w_p(r,t))}{\partial t^2}$, which assume a Gaussian spectral function in small width limit.


potential extraction
log(w(r,t)/w(r,beta-t))
t in fm
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.1
0.2
0.3
0.4
0.5


estimate of imaginary part extraction
log(w_p(r,t)/w_p(r,t+1))
r in fm
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45

For octet potential

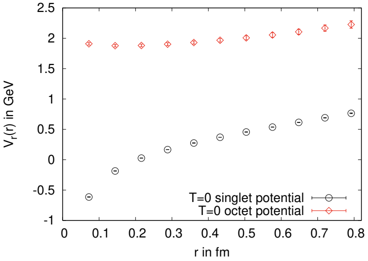
$M(r = |\vec{x} - \vec{y}|, t_m) = \bar{\psi}(\vec{x}, t_m)U(\vec{x}, \vec{z}; t_m)T^a U(\vec{z}, \vec{y}; t_m)\psi(\vec{y}, t_m)$.

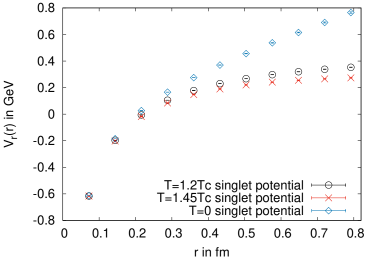
We can not implement this operator directly in lattice.

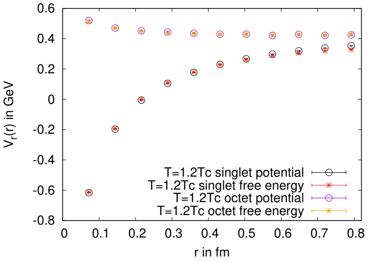
To make it gauge invariant we use the following operator

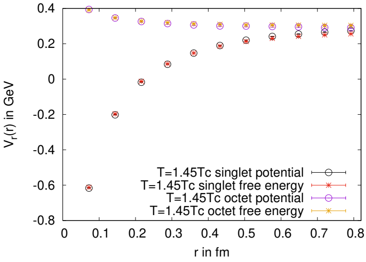
$M(r = |\vec{x} - \vec{y}|, t_m) =$

$\bar{\psi}(\vec{x}, t_m)U(\vec{x}, \vec{z}; t_m)T^a B_a(z)U(\vec{z}, \vec{y}; t_m)\psi(\vec{y}, t_m)$.

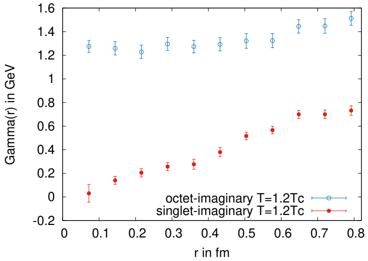


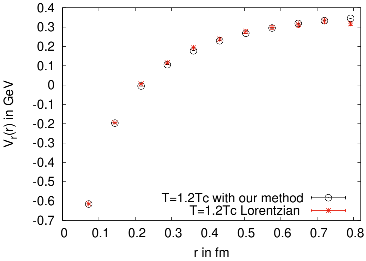



0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
V_r(r) in GeV
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
r in fm
T=1.2Tc singlet potential
T=1.2Tc singlet free energy
T=1.2Tc octet potential
T=1.2Tc octet free energy


0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
V_r(r) in GeV
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
r in fm
T=1.45Tc singlet potential
T=1.45Tc singlet free energy
T=1.45Tc octet potential
T=1.45Tc octet free energy

$$\rho(r,\omega,V_r(r),V_i(r),c)=\frac{c}{\pi}\frac{V_i(r)}{(\omega-V_r(r))^2+V_i(r)^2} \tag{7}$$





- A. Bazavov and P. Petreczky, 2013 J. Phys.: Conf. Ser. 458 012012
- A. Rothkopf, T. Hatsuda & S. Sasaki, Phys. Rev. Lett. 108, 162001 (2012)
- P. Petreczky, Eur.Phys.J.C43:51-57,2005.
- P. Petreczky & J. Weber, Nucl Phys A 00 (2018) 1-4.