

Lattice computation of QCD propagators and vertices in Landau gauge

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Outline

1 Two-point functions

- Gluon and ghost propagators
- High statistics and RGZ

2 Three point functions

- Three gluon vertex

3 Conclusions

Quantum Chromodynamics (QCD)

- current theory of the strong interaction
- quarks interact through gluon exchange
- nuclear forces binding together protons and neutrons in nuclei: residual strong force
 - cf. Van der Walls interaction binding together atoms in molecules
- Full understanding of QCD is beyond perturbative techniques
- Confinement: we are unable to observe free quarks in Nature

Lattice QCD

- Path integral formulation in Quantum Field Theories

$$\langle \mathcal{B} \rangle = \int \mathcal{D}A_\mu \mathcal{B}(A_\mu) \exp(iS(A_\mu))$$

- space-time lattice – natural regulator of the theory
- imaginary time formalism $t = -ix_4$
- path integrals computed via Monte Carlo integration
- Gauge fields on the lattice: links

$$U_\mu(x) = e^{iag_0 A_\mu(x + a\hat{e}_\mu/2)}$$

- $\{U^{(i)}\}_{i=1}^N$, $P(U) = \exp(-S_W[U])$

$$\langle \mathcal{B} \rangle = \frac{1}{N} \sum_{i=1}^N \mathcal{B}(U^{(i)}) + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right).$$

- statistical error $\sim \frac{1}{\sqrt{N}}$ due to Central Limit Theorem

Lattice simulations

- Configuration ensembles $\{U^{(i)}\}_{i=1}^N$ sampled from the probability distribution $P(U) = \exp(-S_W[U])$ generated using a Markov process
- typically we use a four dimensional lattice L^4 , with $L \sim 100$
 - quarks live on lattice sites
 - gluons live on the links connecting the sites
 - 4 links (SU(3) matrices) per site: 72 real numbers
- 128^4 lattice $\sim 1.9 \times 10^{10}$ real numbers ~ 73 GB in single precision
- 100 configurations: 7.3 TB in storage

QCD Green's functions

- Green's functions summarize the dynamics of the theory
 - QCD: information on confinement and chiral symmetry breaking
- generic n -point complete Green's function:

$$G^{(n)}(x_1, \dots, x_n) = \langle 0 | T(\phi(x_1) \cdots \phi(x_n)) | 0 \rangle,$$

- decomposition in terms of one particle irreducible (1PI) functions $\Gamma^{(n)}$
- access to form factors that define $\Gamma^{(n)}$
- lattice approach allows for first principles determination of the complete Green's functions of QCD

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Landau gauge $\partial_\mu A_\mu = 0$

- Landau gauge fixing on the lattice: numerical optimization of

$$F_U[g] = C_F \sum_{x,\mu} \text{Re}\{\text{Tr}[g(x)U_\mu(x)g^\dagger(x + \hat{\mu})]\}$$

(Fourier accelerated) Steepest Descent, Overrelaxation

- Second variation: defines a symmetric matrix

$$\begin{aligned} M_{x,y}^{ab} &= \sum_\mu \text{Re} \text{tr} \left[\left\{ t^a, t^b \right\} (U_\mu(x) + U_\mu(x - \hat{\mu})) \right] \delta_{xy} \\ &- 2 \text{Re} \text{tr} \left[t^b t^a U_\mu(x) \right] \delta_{x+\hat{\mu},y} - 2 \text{Re} \text{tr} \left[t^a t^b U_\mu(x - \hat{\mu}) \right] \delta_{x-\hat{\mu},y} \end{aligned}$$

- continuum limit $-\frac{1}{2} (\partial_\mu D_\mu^{ab} + D_\mu^{ab} \partial_\mu)$
- in Landau gauge: $= -\partial_\mu D_\mu^{ab}$ (continuum FP operator)

Gluon field and propagator

- gluon field

$$ag_0 A_\mu(x + a\hat{e}_\mu) = \frac{U_\mu(x) - U^\dagger(x)}{2ig_0} - \frac{\text{Tr} [U_\mu(x) - U^\dagger(x)]}{6ig_0}$$

in momentum space

$$A_\mu(\hat{p}) = \sum_x e^{-i\hat{p}(x+a\hat{e}_\mu)} A_\mu(x + a\hat{e}_\mu) \quad , \quad \hat{p}_\mu = \frac{2\pi n_\mu}{a L_\mu}$$

- tree level improved momentum

$$p_\mu = \frac{2}{a} \sin \left(\frac{a\hat{p}_\mu}{2} \right)$$

- Gluon propagator

$$D_{\mu\nu}^{ab}(\hat{q}) = \frac{1}{V} \langle A_\mu^a(\hat{q}) A_\nu^b(-\hat{q}) \rangle = \delta^{ab} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) D(q^2)$$

Ghost propagator and α_s

- Ghost propagator

$$G^{ab}(p) = \delta^{ab} G(p^2)$$

- inversion of lattice FP matrix $M_{x,y}^{ab}$ on a point source
- Renormalization group invariant strong coupling

$$\alpha_s(p^2) = \frac{g_0^2}{4\pi} d_D(p^2) d_G^2(p^2)$$

- dressing functions

$$d_D(p^2) = p^2 D(p^2) \quad \text{and} \quad d_G(p^2) = p^2 G(p^2)$$

Lattice setup

β	a (fm)	$1/a$ (GeV)	L	La (fm)	Conf	Sources
5.7	0.1838(11)	1.0734(63)	44	8.087	100	3
6.0	0.1016(25)	1.943(47)	64	6.502	100	2
			80	8.128	70	2
			128	13.005	35	1
6.3	0.0627(24)	3.149(46)	128	8.026	54	3

- Renormalization: MOM scheme

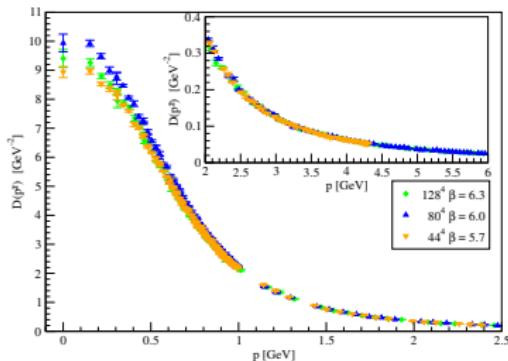
$$D, G_{Lat}(\mu^2) = \frac{1}{\mu^2}, \mu = 4 \text{ GeV}$$

A. G. Duarte, O. Oliveira, PJS, PRD94(2016)014502

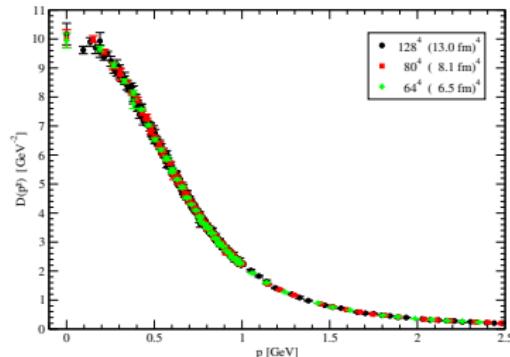


Gluon propagator on the lattice

Constant volume

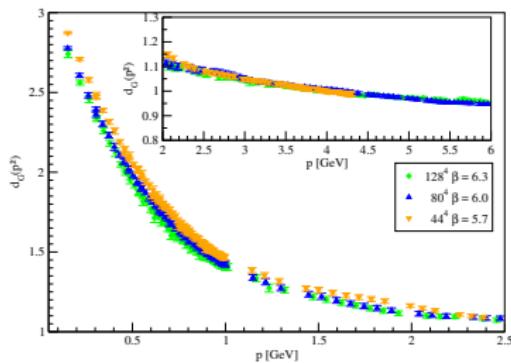


Constant lattice spacing

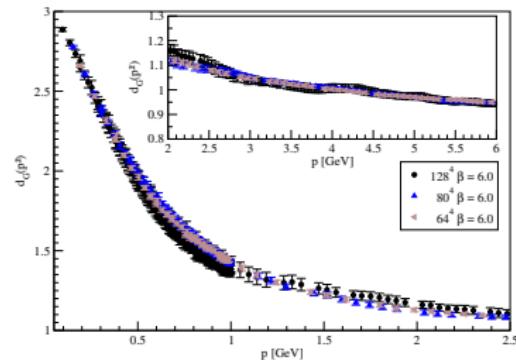


Ghost propagator on the lattice

Constant volume

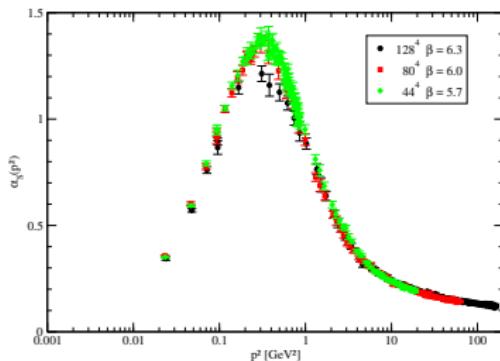


Constant lattice spacing

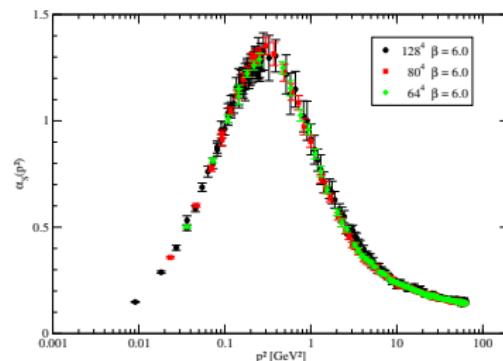


Running coupling on the lattice

Constant volume

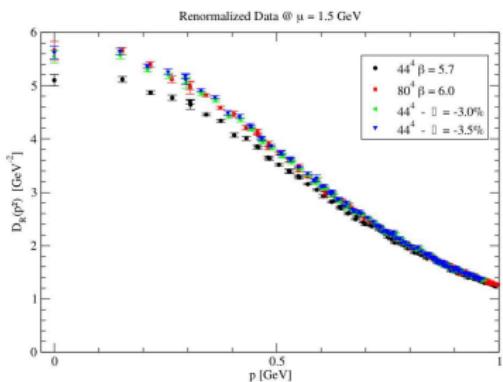


Constant lattice spacing

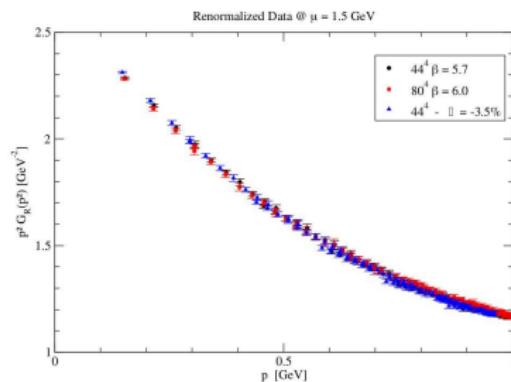


Lattice spacing recalibration

Gluon propagator



Ghost dressing function



Boucaud et al. PRD96(2017)098501
A. G. Duarte, O. Oliveira, PJS, PRD96(2017)098502



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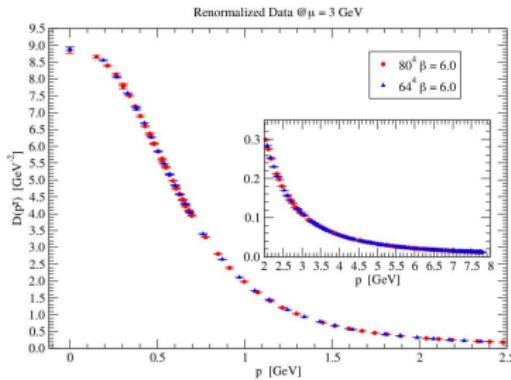
High Precision Statistical Gluon Propagator

- two large physical volume lattice simulations
 - Wilson gauge action, $\beta = 6.0$
 $1/a = 1.943 \text{ GeV}$, $a = 0.1016(25) \text{ fm}$,
 - 64^4 and 80^4 lattices
 - physical volumes: $(6.57 \text{ fm})^4$, $(8.21 \text{ fm})^4$
 - number of configurations: 2000, 550
 - rotated to the Landau gauge
 - $p_{min} = 191 \text{ MeV}$, 153 MeV ; $p_{max} = 7.7 \text{ GeV}$
- renormalization: MOM scheme, scale $\mu = 3 \text{ GeV}$

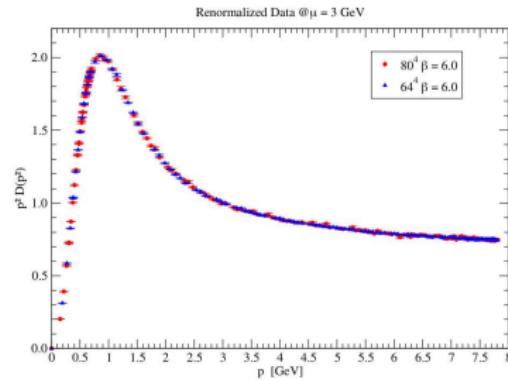
D. Dudal, O. Oliveira, PJS, Annals Phys. 397(2018)351

High Precision Statistical Gluon Propagator

Gluon propagator



Gluon dressing function



Refined Gribov-Zwanziger (RGZ) framework

- implements the functional restriction to the first Gribov horizon
- defines a local renormalizable quantum field theory

Dudal, Sorella, Vandersickel, Verschelde, PRD77(2008)071501

Dudal, Gracey, Sorella, Vandersickel, Verschelde, PRD78(2008)065047

Capri, Fiorentini, Pereira, Sorella, PRD96(2017)054022

Gracey, PRD82(2010)085032

Dudal, Sorella, Vandersickel, Verschelde, PRD84(2011)065039

- RGZ takes into account some $d = 2$ condensates
- (Very) Refined Gribov-Zwanziger *ansätze*

$$D_{RGZ}(p^2) = Z \frac{p^2 + M_1^2}{p^4 + M_2^2 p^2 + M_3^4}$$

$$D_{VRGZ}(p^2) = \frac{p^4 + M_2^2 p^2 + M_1^4}{p^6 + M_5^2 p^4 + M_4^4 p^2 + M_3^6}$$

- for $D_{VRGZ}(p^2)$ we do not include an overall normalization factor Z

Fitting $D_{RGZ}(p^2)$

- $L = 64$

$$\nu = \chi^2/\text{d.o.f.}$$

p_{max}	ν	Z	M_1^2	M_2^2	M_3^4
1.10	1.83	0.959(62)	2.67(32)	0.511(26)	0.285(15)
1.19	1.77	1	2.478(27)	0.496(9)	0.275(3)

- $L = 80$

p_{max}	ν	Z	M_1^2	M_2^2	M_3^4
1.10	1.16	0.957(66)	2.73(34)	0.527(29)	0.290(16)
3.00	1.31	0.730(4)	4.16(7)	0.592(10)	0.335(3)
1.25	1.60	1	2.454(35)	0.489(11)	0.273(3)

- lattice data compatible with RGZ up to $p \sim 1$ GeV
- agreement between ensembles for similar p_{max}

Fitting $D_{VRGZ}(p^2)$

- no significant difference between the RGZ and the VRGZ scenarios
- M_1^4, M_3^6 compatible with zero: VRGZ reduces to RGZ

$$D(p^2) = \frac{p^2 + M_2^2}{p^4 + M_5^2 p^2 + M_4^4}$$

$$M_2^2 \Big|_{VRGZ} \sim M_1^2 \Big|_{RGZ}, \quad M_5^2 \Big|_{VRGZ} \sim M_2^2 \Big|_{RGZ}, \quad M_4^4 \Big|_{VRGZ} \sim M_3^4 \Big|_{RGZ}$$

- IR lattice data ($p \lesssim 1$ GeV) well described by tree level Refined Gribov-Zwanziger prediction
- if the VRGZ reduces to the RGZ, the equality between the condensates $\langle \bar{\varphi} \bar{\varphi} \rangle = \langle \varphi \varphi \rangle$ should hold.

Global Fits: from Infrared to Ultraviolet

- for large p^2 one expects to recover the usual perturbative behaviour

$$D(p^2) \propto \frac{1}{p^2} \left[\ln \left(\frac{p^2}{\Lambda_{QCD}^2} \right) \right]^{\gamma_{gl}}$$

- $\gamma_{gl} = -\frac{13}{22}$ → 1-loop gluon anomalous dimension
- Interpolation between
 - RGZ for low p
 - 1-loop RG-improved expression for high p

$$D(p^2) = Z \frac{p^2 + M_1^2}{p^4 + M_2^2 p^2 + M_3^4} \left[\omega \ln \left(\frac{p^2 + m_g^2(p^2)}{\Lambda_{QCD}^2} \right) + 1 \right]^{\gamma_{gl}}$$

- $\omega = 11N\alpha_s(\mu)/(12\pi)$
- $\Lambda_{QCD} = 0.425 \text{ GeV}, \alpha_s(3 \text{ GeV}) = 0.3837$

Global Fits: from Infrared to Ultraviolet

- regularisation of the leading log — introduction of a “log-regularisation mass” $m_g^2(p^2)$ which should become negligible at high momentum
- several regularizing mass definitions:

$$m_g^2(p^2) = \frac{M_3^4 + (M_2^2 - M_1^2) p^2}{M_1^2 + p^2} \quad (\text{RGZ inspired})$$

$$m_g^2(p^2) = m_0^2 \quad (\text{Constant})$$

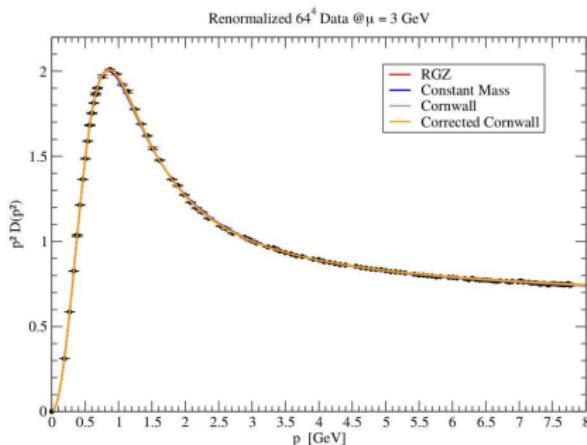
$$m_g^2(p^2) = \frac{m_0^4}{p^2 + \lambda^2} \quad (\text{Cornwall})$$

$$m_g^2(p^2) = \lambda_0^2 + \frac{m_0^4}{p^2 + \lambda^2} \quad (\text{Corrected Cornwall})$$

Global Fits: from Infrared to Ultraviolet

- best 64^4 fit:
corrected Cornwall
regularisation mass
($\chi^2/\text{d.o.f.} = 1.11$)

$$\begin{aligned} Z &= 1.36992 \pm 0.00072 \\ M_1^2 &= 2.333 \pm 0.042 \text{ GeV}^2 \\ M_2^2 &= 0.514 \pm 0.024 \text{ GeV}^2 \\ M_3^4 &= 0.2123 \pm 0.0032 \text{ GeV}^4 \\ m_0^2 &= 1.33 \pm 0.13 \text{ GeV}^4 \\ \lambda^2 &= 0.100 \pm 0.035 \text{ GeV}^2 \\ \lambda_0^2 &= -0.954 \pm 0.070 \text{ GeV}^2 \end{aligned}$$



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Three gluon vertex

- momentum space three point function
- allow to measure e.g. strong coupling constant
- fundamental role in the structure of Dyson-Schwinger equations
 - DSE studies predict that some form factors associated with the three gluon 1PI change sign in the infrared region — **zero crossing**
 - zero crossing observed in continuum approaches, SU(2) 3d lattice simulations, and in SU(3) 4d lattice simulations

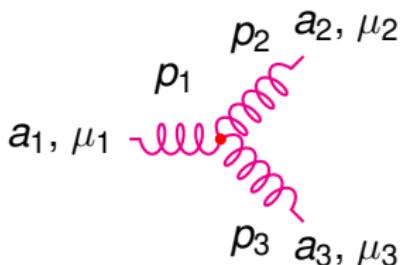
A. Athenodorou *et al*, PLB 761(2016)444

- DSE predicts a momentum scale $\sim 130 - 200$ MeV

Three point complete Green's function

$$\langle A_{\mu_1}^{a_1}(p_1) A_{\mu_2}^{a_2}(p_2) A_{\mu_3}^{a_3}(p_3) \rangle = V \delta(p_1 + p_2 + p_3) G_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3}(p_1, p_2, p_3)$$

$$G_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3}(p_1, p_2, p_3) = D_{\mu_1 \nu_1}^{a_1 b_1}(p_1) D_{\mu_2 \nu_2}^{a_2 b_2}(p_2) D_{\mu_3 \nu_3}^{a_3 b_3}(p_3) \Gamma_{\nu_1 \nu_2 \nu_3}^{b_1 b_2 b_3}(p_1, p_2, p_3)$$



- color structure:

$$\Gamma_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3}(p_1, p_2, p_3) = f_{a_1 a_2 a_3} \Gamma_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$$

Lattice setup

- Wilson gauge action, $\beta = 6.0$
 - 64^4 , 2000 configurations
 - 80^4 , 279 configurations

A. G. Duarte, O. Oliveira, PJS, PRD94(2016)074502

- rotation to the Landau gauge: FFT-SD method
- accessing the 1PI three gluon vertex from the lattice

$$\begin{aligned} G_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3) &= \text{Tr } \langle A_{\mu_1}(p_1) A_{\mu_2}(p_2) A_{\mu_3}(p_3) \rangle = \\ &= V \delta(p_1 + p_2 + p_3) \frac{N_c(N_c^2 - 1)}{4} D(p_1^2) D(p_2^2) D(p_3^2) \\ P_{\mu_1 \nu_1}(p_1) P_{\mu_2 \nu_2}(p_2) P_{\mu_3 \nu_3}(p_3) \Gamma_{\nu_1 \nu_2 \nu_3}(p_1, p_2, p_3) \end{aligned}$$



Case study: one vanishing momentum $p_2 = 0$

- used in the first lattice study

B. Allés et al, Nucl. Phys. B502, 325 (1997)

$$G_{\mu_1 \mu_2 \mu_3}(p, 0, -p) = V \frac{N_c(N_c^2 - 1)}{4} [D(p^2)]^2 D(0) \frac{\Gamma(p^2)}{3} p_{\mu_2} T_{\mu_1 \mu_3}(p)$$

$$\Gamma(p^2) = 2 \left[A(p^2, p^2; 0) + p^2 C(p^2, p^2; 0) \right]$$

$$G_{\mu \alpha \mu}(p, 0, -p) p_\alpha = V \frac{N_c(N_c^2 - 1)}{4} [D(p^2)]^2 D(0) \Gamma(p^2) p^2$$

A word about statistical errors on $\Gamma(p^2)$

- measurement of $\Gamma(p^2)$ requires to compute the ratio

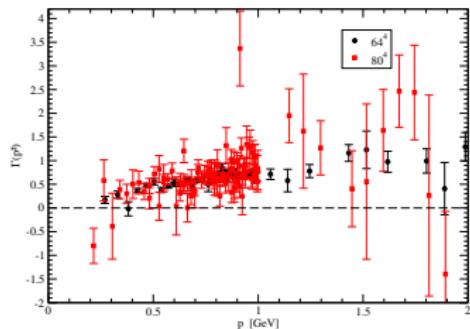
$$G_{\mu\alpha\mu}(p, 0, -p)p_\alpha / \left[D(p^2) \right]^2 D(0)$$

- large statistical fluctuations at high momenta:

$$\begin{aligned} [\Delta\Gamma(p^2)]^2 &= \frac{1}{[D(p^2)]^4} \left\{ \left[\frac{\Delta G_{\mu\alpha\mu} p_\alpha}{D(0)} \right]^2 \right. \\ &+ \left[2 \Delta D(p^2) \frac{G_{\mu\alpha\mu} p_\alpha}{D(p^2) D(0)} \right]^2 \\ &\left. + \left[2 \Delta D(0) \frac{G_{\mu\alpha\mu} p_\alpha}{[D(0)]^2} \right]^2 \right\} \end{aligned}$$

- for large momenta:
 - $D(p^2) \sim 1/p^2$
 - $\Delta\Gamma(p^2) \sim p^4$

$\Gamma(p^2)$



- $\Gamma(p^2) = -0.80(37)$ at $p = 216$ MeV
- compatible with zero only within to 2.2σ
- $\Gamma(p = 270 \text{ MeV}) = 0.171(73)$ for 64^4
- $\Gamma(p = 264 \text{ MeV}) = 0.58(43)$ for 80^4
- zero crossing for $p \lesssim 250$ MeV
- compatible with earlier lattice results

$\Gamma(p^2)$ at high momenta

$$\Gamma_{UV}(p^2) = [D(p^2)]^2 D(0) \Gamma(p^2) p^2$$

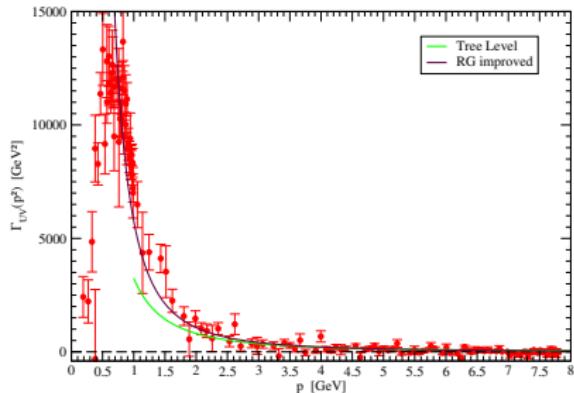
- D and Γ at high momenta:

$$Z \frac{\left[\ln \frac{p^2}{\mu^2} \right]^\gamma}{p^2}$$

- gluon ($\gamma_{2g} = -13/22$)
- 3g 1PI ($\gamma_{3g} = 17/44$)

$$\Gamma_{UV}(p^2) = \frac{Z}{p^2} \left[\ln \frac{p^2}{\mu^2} \right]^{\gamma_{3g} - 2\gamma}$$

- $\gamma' = \gamma_{3g} + 2\gamma_{2g} = -35/44$
- tree level: $\Gamma_{UV}(p^2) \sim \frac{1}{p^2}$



Conclusions

- good description of two-point functions
 - gluon lattice data compatible with RGZ
- Computation of the three gluon complete Green's function on the lattice
 - particular kinematical configuration $p_2 = 0$
 - two different lattice volumes: $(6.5 \text{ fm})^4$ and $(8.2 \text{ fm})^4$ ($a = 0.102 \text{ fm}$)
 - form factor $\Gamma(p^2)$ exhibits zero crossing for $p \sim 250 \text{ MeV}$
 - for high momenta: lattice data compatible with prediction of renormalisation group improved perturbation theory

Not covered in this talk

- finite temperature
 - gluon and quark propagators have a different behaviour below and above $T_c \sim 270$ MeV

PJS, O. Oliveira, P. Bicudo, N. Cardoso, PRD89(2014)074503

PJS, O. Oliveira, PRD93(2016)114509

PJS, O. Oliveira, arXiv:1812.00088; in preparation

- beyond Landau gauge
 - gluon and ghost propagators in Linear Covariant Gauges

P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira, PJS, PRD92(2015)114514

A. Cucchieri, D. Dudal, T. Mendes, O. Oliveira, M. Roelofs, PJS, PRD98(2018)091504