Generalisation of the Yang-Mills Theory and Physics Beyond the Standard Model

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Extension of Poincaré Group and of Yang-Mills Theory

Proton Structure, Tensorgluons, PDF, Grand Unification

1.Generalization of Yang-Mills theory
Phys. Lett. B 625 (2005) 341

2.Extension of Poincaré Group and Tensor Gauge Fields
Int.J.Mod.Phys. A25 (2010) 5765-5785

3.Supersymmetric Extensions of the Poincare group
Ignatios Antoniadis, Lars Brink, G.S.
J.Math.Phys. 52 (2011) 072303,

4.Asymptotic Freedom of non-Abelian Tensor Gauge Fields
Phys. Lett. B 732 (2014) 150

5. High Spin Parton Distribution Functions (PDF)
Roland Kirschner and G.S.
ArXiv:1701.06660 [hep-th]

The extension of the Poincaré algebra.
New generators $L_a^{\lambda_1 \ldots \lambda_s}$ carry *higher spins and charges*.

$$
\begin{aligned}
&[P^{\mu},\ P^{\nu}] = 0, \\
&[M^{\mu\nu},\ P^{\lambda}] = i(\eta^{\lambda\nu}\ P^{\mu} - \eta^{\lambda\mu}\ P^{\nu}), \\
&[M^{\mu\nu},\ M^{\lambda\rho}] = i(\eta^{\mu\rho}\ M^{\nu\lambda} - \eta^{\mu\lambda}\ M^{\nu\rho} + \eta^{\nu\lambda}\ M^{\mu\rho} - \eta^{\nu\rho}\ M^{\mu\lambda}),
\end{aligned}
$$

$$
\begin{aligned}
&[P^{\mu},\ L_a^{\lambda_1 \ldots \lambda_s}] = 0, \\
&[M^{\mu\nu},\ L_a^{\lambda_1 \ldots \lambda_s}] = i(\eta^{\lambda_1\nu} L_a^{\mu\lambda_2 \ldots \lambda_s} + \ldots + -\eta^{\lambda_s\mu} L_a^{\lambda_1 \ldots \lambda_{s-1}\nu}),
\end{aligned}
$$

$$
[L_a^{\lambda_1 \ldots \lambda_n}, L_b^{\lambda_{n+1} \ldots \lambda_s}] = i f_{abc} L_c^{\lambda_1 \ldots \lambda_s} \quad (s = 0, 1, 2, \ldots).
$$

Unification of the Poincaré and internal algebras.

The $L_a^{\mu_1 \cdots \mu_s}$ are high helicity generators

$$L_a^{\mu_1 \cdots \mu_s} = \prod_{n=1}^{s} (e^{i\varphi} e_+^{\mu_n} + e^{-i\varphi} e_-^{\mu_n}) \oplus L_a, \tag{1}$$

$e_\pm^\mu$ is the polarisation vector of helicity ± 1,

$$k_{\mu_1} L_a^{\mu_1, \ldots, \mu_s} = 0, \quad k^2 = 0, \; k^\mu e_\mu = 0 \tag{2}$$

The $L_a^{\mu_1 \cdots \mu_s}$ carry the helicities:

$$h = \pm s, \pm(s-2), \ldots\ldots, \tag{3}$$

The non-Abelian tensor gauge fields are:

$$\mathcal{A}_\mu(x,L) = \sum_{s=0}^{\infty} \frac{1}{s!}\; A^a_{\mu\lambda_1\ldots\lambda_s}(x)\; L_a^{\lambda_1\ldots\lambda_s}. \tag{4}$$

the field strength tensor is

$$\mathcal{G}_{\mu\nu}(x,e) = \partial_\mu \mathcal{A}_\nu(x,e) - \partial_\nu \mathcal{A}_\mu(x,e) - ig[\mathcal{A}_\mu(x,e)\; \mathcal{A}_\nu(x,e)] \tag{5}$$

The invariant Lagrangian

$$\mathcal{L}(x) = \langle\; \mathcal{G}^a_{\mu\nu}(x,L)\mathcal{G}^a_{\mu\nu}(x,L)\;\rangle \tag{6}$$

In terms of the field components the Lagrangian $\mathcal{L}$ is

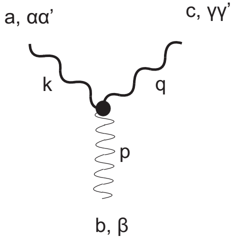
$$
\begin{aligned}
\mathcal{L} = \; & - \; \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} \\
& - \; \frac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\nu,\lambda} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu,\lambda\lambda} + \\
& + \; \frac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\lambda,\nu} + \frac{1}{4} G^a_{\mu\nu,\nu} G^a_{\mu\lambda,\lambda} + \frac{1}{2} G^a_{\mu\nu} G^a_{\mu\lambda,\nu\lambda} +
\end{aligned}
$$

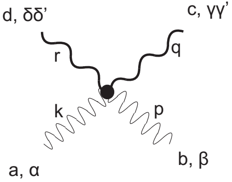
where the field strength tensors are:

$$
\begin{aligned}
G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}\; A^b_\mu \; A^c_\nu, \\
G^a_{\mu\nu,\lambda} &= \partial_\mu A^a_{\nu\lambda} - \partial_\nu A^a_{\mu\lambda} + g f^{abc}(\; A^b_\mu \; A^c_{\nu\lambda} + A^b_{\mu\lambda} \; A^c_\nu \;), \\
G^a_{\mu\nu,\lambda\rho} &= \partial_\mu A^a_{\nu\lambda\rho} - \partial_\nu A^a_{\mu\lambda\rho} + g f^{abc}(\; A^b_\mu \; A^c_{\nu\lambda\rho} + A^b_{\mu\lambda} \; A^c_{\nu\rho} + A^b_{\mu\rho} \; A^c_{\nu\lambda} \\
\ldots\ldots \; &. \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

Helicity spectrum of the tensor-gluons

$$
\begin{array}{ll}
A^a_{\mu} & \pm 1 \\
A^a_{\mu\lambda_1} & \pm 2, \\
A^a_{\mu\lambda_1\lambda_2} & \pm 3, \\
A^a_{\mu\lambda_1\lambda_2\lambda_3} & \pm 4, \\
\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots, \\
A^a_{\mu\lambda_1\ldots\lambda_s} & \pm s, \\
\ldots\ldots\ldots & \ldots\ldots\ldots\ldots,
\end{array}
$$

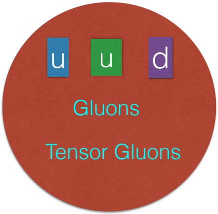




Splitting of gluons into tensorgluons,
the coupling constants are dimensionless g and g^2

We consider a possibility that inside the proton and, more generally, inside hadrons there are additional partons - tensorgluons, which can carry a part of the proton momentum.

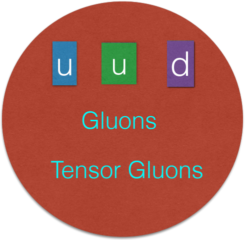
If there is an additional emission of tensorgluons in the proton, then one should introduce the corresponding density $T(x, t)$ of tensorgluons



PROTON

What we whant to know - are the Parton Distribution Functions - PDF

for the new partons $\rightarrow$ TensorGluons



PROTON

The generalisation of the parton evolution equations

$$\dot{q}^i_t(x) = \frac{\alpha(t)}{2\pi} \int_x^1 \frac{dy}{y}[q^j(y,t)\ P_{q^i q^j} + G(y,t)\ P_{q^i G}], \tag{8}$$

$$\dot{G}_t(x) = \frac{\alpha(t)}{2\pi} \int_x^1 \frac{dy}{y}[q^j(y,t) P_{G q^j} + G(y,t) P_{GG} + T_r(y,t) P_{G_r T}],$$

$$\dot{T}_r(x) = \frac{\alpha(t)}{2\pi} \int_x^1 \frac{dy}{y}[G(y,t)\ P_{T_r G} + \sum_s T_s(y,t)\ P_{T_r T_s}].$$

The $\alpha(t)$ is the running coupling constant ($\alpha = g^2/4\pi$)

$$\alpha(t) = \frac{\alpha}{1 + b_1\ \alpha\ t}, \tag{9}$$

where b_1 is the one-loop Callan-Symanzik beta coefficient,
it receives an additional contribution from the tensorgluons.

$$b_1 = b_{quarks} + b_{gluons} + b_{tensorgluons}$$

G.S. Phys.Lett.B732, 2014

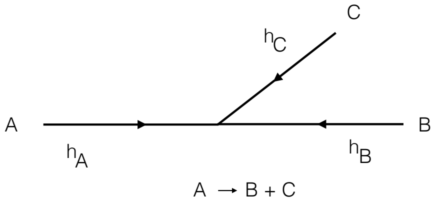
Asymptotic Freedom of Tensor-Gluons

$$\beta_1 = \frac{\sum_s (-1)^{2s}(12s^2-1)C_2(G) - 4n_f T(R)}{48\pi^2} g^3$$

where s= 1, 2,..... at s=1 it reproduces the Gross-Wilczek-Politzer result

Tensorgluons "accelerate" the asymptotic freedom !

Splitting probabilities $P^{C}_{BA}(z)$



The DGLAP quark and gluon Splitting Probabilities in QCD are:

$$
\begin{aligned}
P_{qq}(z) &= C_2(R)\left[\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(z-1)\right], \\
P_{Gq}(z) &= C_2(R)\frac{1+(1-z)^2}{z}, \qquad (10) \\
P_{qG}(z) &= T(R)[z^2+(1-z)^2], \\
P_{GG}(z) &= 2C_2(G)\left[\frac{z}{(1-z)_+}+\frac{1-z}{z}+z(1-z)\right]+ \\
&+ \frac{\sum_s(12s^2-1)C_2(G)-4n_f T(R)}{6}\,\delta(z-1), \qquad (11)
\end{aligned}
$$

where $C_2(G)=N, C_2(R)=\frac{N^2-1}{2N}, T(R)=\frac{1}{2}$ for the SU(N).

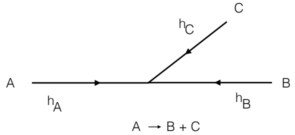


Figure: Splitting of the tensorgluons P^C_{BA}.

$$P^{h_C}_{h_B h_A} = \frac{1}{z^{2\eta h_B - 1}(1-z)^{2\eta h_C - 1}}, \quad h_C + h_B + h_A = \eta = \pm 1.$$

The formula describes all known splitting probabilities found earlier in Gauge theories and the generalised Yang-Mills theory.

The splitting probabilities of gluons into tensorgluons is

$$
\begin{aligned}
P_{T_s G}(z) &= C_2(G)\left[\frac{z^{2s+1}}{(1-z)^{2s-1}} + \frac{(1-z)^{2s+1}}{z^{2s-1}}\right],\\
P_{G T_s}(z) &= C_2(G)\left[\frac{1}{z(1-z)^{2s-1}} + \frac{(1-z)^{2s+1}}{z}\right],\\
P_{T_s T_s}(z) &= C_2(G)\left[\frac{z^{2s+1}}{(1-z)} + \frac{1}{(1-z)z^{2s-1}}\right].
\end{aligned}
$$

For spin 2 we shall have

$$
\begin{aligned}
P_{TG}(z) &= C_2(G)\left[\frac{z^5}{(1-z)^3}+\frac{(1-z)^5}{z^3}\right],\\
P_{GT}(z) &= C_2(G)\left[\frac{1}{z(1-z)^3}+\frac{(1-z)^5}{z}\right],\\
P_{TT}(z) &= C_2(G)\left[\frac{z^5}{(1-z)}+\frac{1}{(1-z)z^3}+\sum_{j=1}^{5}\frac{1}{j}\ \delta(1-z)\right]
\end{aligned}
\tag{12}
$$

Tree level scattering amplitudes of (n-2)-gluons and 2-tensorgluons calculated using BCFW formalism.G.Georgiou and G.S. IJMP 2011.

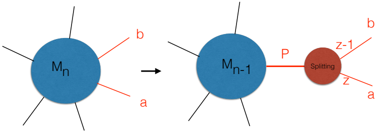
$$\hat{M}_n(1^+, ..i^-, ...k^{+s}, ..j^{-s}, ..n^+) =$$

$$= ig^{n-2}(2\pi)^4\delta^{(4)}(P^{a\dot{b}})\frac{< ij >^4}{\prod_{l=1}^{n} < ll+1 >}\Big(\frac{< ij >}{< ik >}\Big)^{2s-2},$$

They reduce to the Parke-Taylor formula when $s = 1$.

The collinear behavior:

$$M_n^{tree}(..., a^{\lambda_a}, b^{\lambda_b}, ...) \xrightarrow{a \parallel b} \sum_{\lambda=\pm 1} Split_{-\lambda}^{tree}(a^{\lambda_a}, b^{\lambda_b}) \times M_{n-1}^{tree}(..., P^{\lambda}, ..),$$



Antoniadis and Savvidy, Mod.Phys.Lett.(2012)

Spin sum rule of helicity weighted distributions:

$$\frac{1}{2}\Delta\Sigma + \Delta G + \sum_s s\ \Delta T_s + L_z = \frac{1}{2}\hbar.$$

The lowest moment of the spin-dependent structure function

$$\Gamma_1^p \ = \ \int_0^1 dx\ g_1^p(x, Q^2) = \ I_3 + I_8 + I_0$$

The singlet part of the proton spin structure function,
contribution of a tensorgluon of the helicity s

$$I_0 \ = \ \frac{1}{9}\ \left(\Delta\Sigma - n_f \frac{\alpha(Q^2)}{2\pi}\Delta G\right)\ \left(1 - \frac{\alpha(Q^2)}{2\pi}\frac{3(12s^2-1)-8n_f}{3(12s^2-1)-2n_f}\right) +$$
$$+ n_f \frac{\alpha(Q^2)}{2\pi}\Delta T_s\ \frac{\sum_{k=1}^{2s+1}\frac{1}{k}}{3(12s^2-1)-2n_f}\ .$$

Extension of Efremov-Altarelli-Ross formula

How the contribution of tensor-gluons changes the high energy behaviour of the coupling constants of the SM ?
The coupling constants evolve with scale as

$$\frac{1}{\alpha_i(M)} = \frac{1}{\alpha_i(\mu)} + 2b_i \ln \frac{M}{\mu}, \quad i = 1, 2, 3, \tag{13}$$

consider only the contribution of $s = 2$ tensor-bosons:
For the $SU(3)_c \times SU(2)_L \times U(1)$ group with its coupling constants α_3, α_2 and α_1 and six quarks $n_f = 6$ and $SU(5)$ unification group we will get

$$2b_3 = \frac{1}{2\pi} 54, \quad 2b_2 = \frac{1}{2\pi}\frac{10}{3}, \quad 2b_1 = -\frac{1}{2\pi} 4,$$

the solution of the system of equations (13) gives

$$\ln \frac{M}{\mu} = \frac{\pi}{58} \left(\frac{1}{\alpha_{el}(\mu)} - \frac{8}{3} \frac{1}{\alpha_s(\mu)} \right), \tag{14}$$

If one takes $\alpha_{el}(M_Z) = 1/128$ and $\alpha_s(M_Z) = 1/10$ one can get that coupling constants have equal strength at energies of order

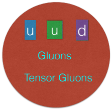
$$M \sim 4 \times 10^4 GeV = 40\ TeV,$$

it is much smaller than the previous GU scale $M \sim 10^{14} GeV$ the value of the weak angle remains intact :

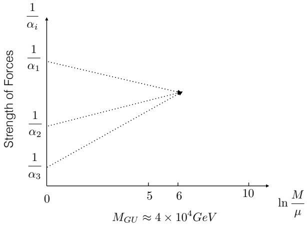
$$\sin^2 \theta_W = \frac{1}{6} + \frac{5}{9} \frac{\alpha_{el}(M_Z)}{\alpha_s(M_Z)}, \tag{15}$$

the coupling constant at the unification scale is of order $\bar{\alpha}(M) = 0,01$.

Summary



PROTON



$$\frac{1}{2}\Delta\Sigma + \Delta G + \sum_s s\ \Delta T_s + L_z = \frac{1}{2}\hbar$$

Thank You

We suggest an extension of the gauge principle which includes new tensor gauge bosons. The vector bosons of SM become the members of the bigger family of gauge bosons of larger spins. The extension is essentially based on the extension of the Poincaré algebra. We calculated the scattering amplitudes of tensor bosons at tree level, as well as their one-loop contribution into the Callan-Symanzik beta function. This contribution is negative and corresponds to the asymptotically free theory. This leads to a natural inclusion of the Standard theory of fundamental forces into theory in which vector bosons, leptons and quarks represent a low-spin subgroup. We consider a possibility that inside the proton and, more generally, inside hadrons there are additional partons - tensorgluons, which can carry a part of the proton momentum. The parton distribution functions (PDF) of the tensorgluons is calculated. The extension of QCD influences the unification scale at which the coupling constants of the Standard Model merge, shifting its value to lower energies.

The Killing forms are:

$$\begin{aligned}
\langle P^{\mu}; P^{\nu}\rangle \; &= 0\\
\langle M_{\mu\nu}; P_{\lambda}\rangle \; &= 0\\
\langle M^{\mu\nu}; M^{\lambda\rho}\rangle &= \eta^{\mu\lambda}\eta^{\nu\rho} - \eta^{\mu\rho}\eta^{\nu\lambda}
\end{aligned}$$

$L(\mathcal{P})$:

$$\begin{aligned}
\langle P^{\mu}; L_a^{\lambda_1 \ldots \lambda_s}\rangle &= 0,\\
\langle M^{\mu\nu}; L_a^{\lambda_1 \ldots \lambda_s}\rangle &= 0,
\end{aligned}$$

$L_G(\mathcal{P})$:

$$\begin{aligned}
\langle L_a^{\lambda_1 \ldots \lambda_n}; L_b^{\lambda_{n+1} \ldots . \lambda_{2s+1}}\rangle &= 0, \qquad\qquad s = 0, 1, 2, 3, \ldots\\
\langle L_a^{\lambda_1 \ldots \lambda_n}; L_b^{\lambda_{n+1} \ldots . \lambda_{2s}}\rangle &= \delta_{ab} s! (\eta^{\lambda_1\lambda_2}\eta^{\lambda_3\lambda_4} \ldots \eta^{\lambda_{2s-1}\lambda_{2s}} + \mathrm{per})
\end{aligned}$$