N=8 Supergravity at Five Loops

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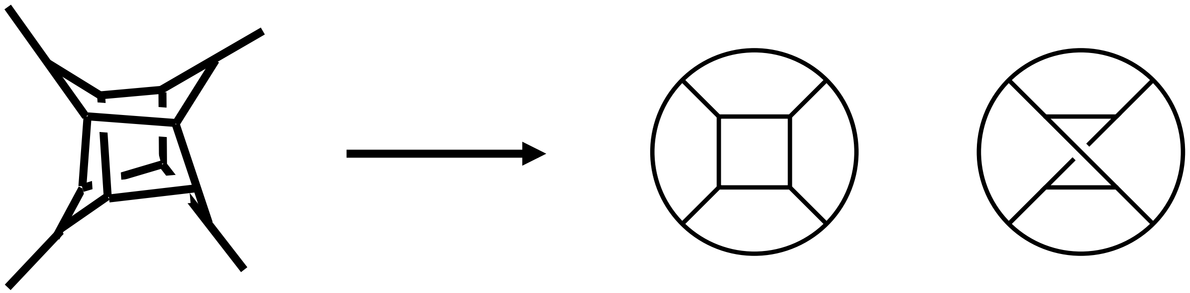
Amplitudes in the LHC era

GGI Florence, Oct 31, 2018



Based on recent work: 1701.02519, 1708.06807, 1804.09311
w/ Zvi Bern, John Joseph Carrasco, Wei-Ming Chen,
Alex Edison, Julio Parra-Martinez, Radu Roiban, Mao Zeng
and older work: 0702112, 0905.2326, 1008.3327, 1201.5366
w/ Zvi Bern, John Joseph Carrasco, Lance Dixon, David Kosower, Radu Roiban

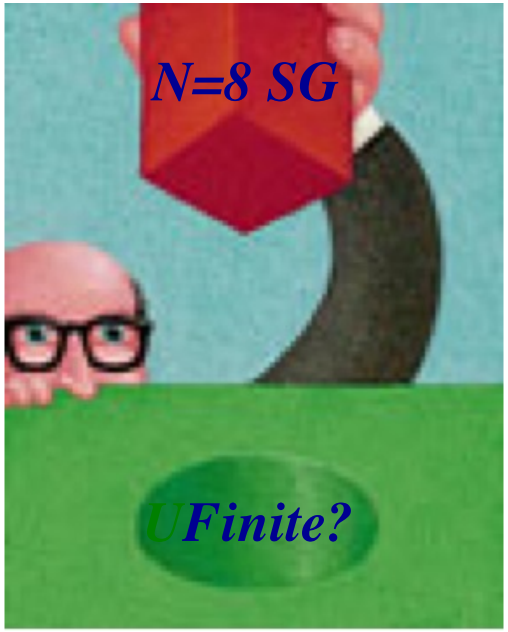
Outline

- Motivation & Review:
  - Status of N=8 SUGRA UV behavior
  - Previous 3,4 loop results
- Key steps in calculation
  - Generalized double copy for gravity ampl.
  - Controlling UV behavior of N=4 SYM
  - Improved UV integration, IBP & vacuum diag.
- Results at 5 loops
  - The critical UV behavior at 5 loops
  - Simplicity in pattern of diagrams
- Conclusion

SUGRA status on one page

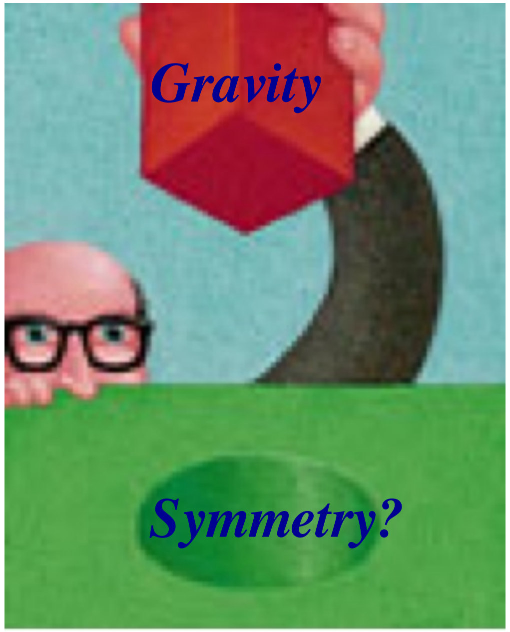
Known facts:

- Susy forbids 1,2 loop div. ~~R^2, R^3~~ Ferrara, Zumino, Deser, Kay, Stelle, Howe, Lindström, Green, Schwarz, Brink, Marcus, Sagnotti
- Pure gravity 1-loop finite, 2-loop divergent Goroff & Sagnotti
- With matter: 1-loop divergent 't Hooft & Veltman
- Naively susy allows 3-loop div. R^4
- $\mathcal{N}$=8 SG and $\mathcal{N}$=4 SG 3-loop finite!
  Bern, Carrasco, Dixon, HJ, Kosower, Roiban, Davies, Dennen, Huang
- $\mathcal{N}$=8 SG: no divergence before 7 loops
- *D*>4 divergences @ *L*=2,3,4 $D_c = 4 + \frac{6}{L}$
  Marcus, Sagnotti, Bern, Dixon, Dunbar, Perelstein, Rozowsky, Carrasco, HJ, Kosower, Roiban
- Only known *D*=4 *SG* divergence: Bern, Davies, Dennen, Smirnov[2]
  $\mathcal{N}$=4 @ 4 loops (→ more questions than answers)
- 7-loop *D*=4 calculation difficult
  instead work out 5 loops in *D*=24/5 → this talk



Why is it interesting ?

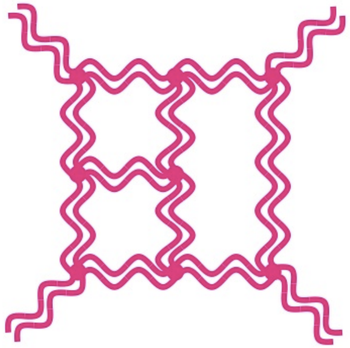
- If $\mathcal{N}$=8 SG is perturbatively finite, why is it interesting ?
- It might be finite for a good reason!
  - hidden new symmetry
  - Other mechanism or structure → open a host of possibilities
- Any indication of hidden structures yet?
  - Gravity is a double copy of gauge theories
  - Color-Kinematics: kinematics = Lie algebra
    Bern, Carrasco, HJ
  - Constraints from E-M duality ? Kallosh et al., Nicolai, Roiban, Freedman
  - Hidden superconformal symmetry ?
    Ferrara, Kallosh, Van Proeyen; Loebbert, Mojaza, Plefka; HJ, Mogull, Teng; Caron-Huot, Trinh, ...
  - Extended *N*=4 superspace ? Bossard, Howe, Stelle
  - Exceptional field theory Bossard, Kleinschmidt



UV problem = basic power counting

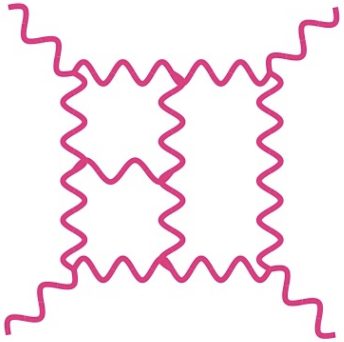
Naively expect gravity to behave worse than Yang-Mills

Gravity: non-renormalizable
dimensionful coupling



$$\sim \int d^{4L}p \frac{\ldots(\kappa p^\mu p^\nu)\ldots}{p_1^2 p_2^2 p_3^2 \ldots p_n^2}$$

Yang-Mills: renormalizable
dimensionless coupling



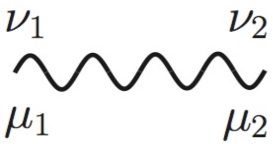
$$\sim \int d^{4L}p \frac{\ldots(g p^\mu)\ldots}{p_1^2 p_2^2 p_3^2 \ldots p_n^2}$$

For finite gravity → vast cancellations needed $\sim \quad (p^\mu)^{2L} \to (k^\mu)^{2L}$
seems implausible, but exists for *N*=8 SG in all known ampl's.

external momenta

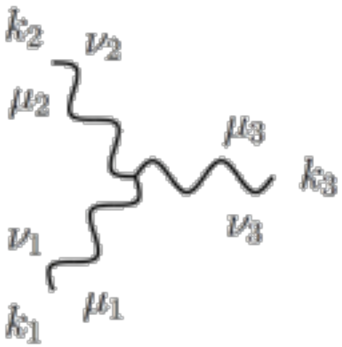
Textbook perturbative gravity is complicated !

$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{g}R, \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$



$$= \frac{1}{2}\left[\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \frac{2}{D-2}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\right]\frac{i}{p^2+i\epsilon}$$

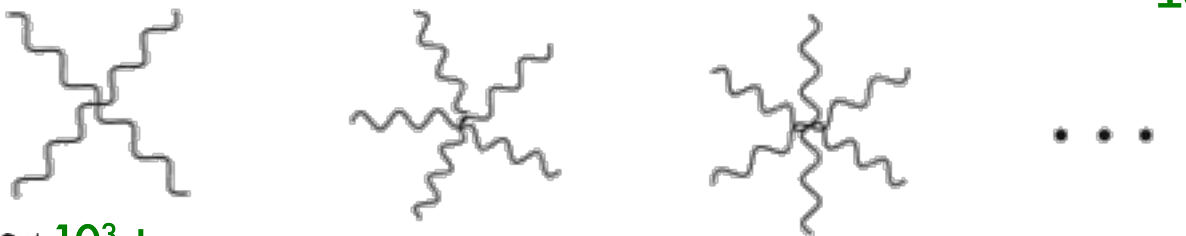
de Donder gauge



$$\begin{aligned} = \mathrm{sym}[&-\frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}) - \frac{1}{2}P_6(k_{1\mu_1}k_{1\nu_2}\eta_{\mu_1\nu_1}\eta_{\mu_3\nu_3}) + \frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\eta_{\mu_3\nu_3}) \\ &+P_6(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}) + 2P_3(k_{1\mu_2}k_{1\nu_3}\eta_{\mu_1\nu_1}\eta_{\nu_2\mu_3}) - P_3(k_{1\nu_2}k_{2\mu_1}\eta_{\nu_1\mu_1}\eta_{\mu_3\nu_3}) \\ &+P_3(k_{1\mu_3}k_{2\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + P_6(k_{1\mu_3}k_{1\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + 2P_6(k_{1\mu_2}k_{2\nu_3}\eta_{\nu_2\mu_1}\eta_{\nu_1\mu_3}) \\ &+2P_3(k_{1\mu_2}k_{2\mu_1}\eta_{\nu_2\mu_3}\eta_{\nu_3\nu_1}) - 2P_3(k_1\cdot k_2\eta_{\nu_1\mu_2}\eta_{\nu_2\mu_3}\eta_{\nu_3\mu_1})]\end{aligned}$$

After symmetrization $\sim$ 100 terms !

higher order vertices...



$\sim 10^3$ terms

complicated diagrams:

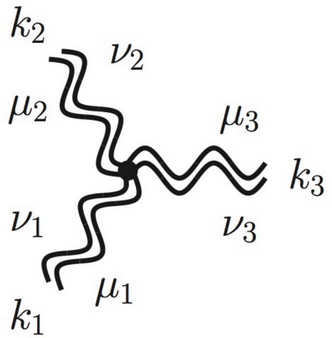
$\sim 10^4$ terms $\sim 10^7$ terms $\sim 10^{21}$ terms $\sim 10^{31}$ terms

On-shell simplifications

Graviton plane wave: $\varepsilon^{\mu}(p)\varepsilon^{\nu}(p)\, e^{ip\cdot x}$

Yang-Mills polarization

On-shell 3-graviton vertex:

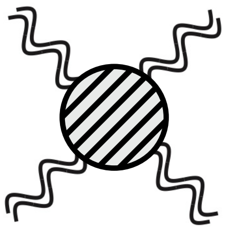


$$= \; i\kappa\Big(\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3}+\text{cyclic}\Big)\Big(\eta_{\nu_1\nu_2}(k_1-k_2)_{\nu_3}+\text{cyclic}\Big)$$

Yang-Mills vertex

Gravity scattering amplitude:

Yang-Mills amplitude



$$M^{\text{GR}}_{\text{tree}}(1,2,3,4)=\frac{st}{u}A^{\text{YM}}_{\text{tree}}(1,2,3,4)\otimes A^{\text{YM}}_{\text{tree}}(1,2,3,4)$$

Kawai, Lewellen, Tye

Gravity processes = “squares” of gauge theory ones - entire S-matrix

$$\text{gravity} = (\text{gauge th})\otimes(\text{gauge th})$$

Bern, Carrasco, HJ

Historical record – where is the $\mathcal{N} = 8$ div. ?

3 loops	Conventional superspace power counting	Green, Schwarz, Brink (1982) Howe and Stelle (1989) Marcus and Sagnotti (1985)
5 loops	Partial analysis of unitarity cuts; *If* $\mathcal{N} = 6$ harmonic superspace exists; algebraic renormalisation	Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998) Howe and Stelle (2003,2009)
6 loops	*If* $\mathcal{N} = 7$ harmonic superspace exists	Howe and Stelle (2003)
7 loops	*If* $\mathcal{N} = 8$ harmonic superspace exists; string theory U-duality analysis; lightcone gauge locality arguments; $E_{7(7)}$ analysis, unique 1/8 BPS candidate	Grisaru and Siegel (1982); Green, Russo, Vanhove; Kallosh; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Bossard, Howe, Stelle, Vanhove
8 loops	Explicit identification of potential susy invariant counterterm with full non-linear susy	Howe and Lindström; Kallosh (1981)
9 loops	Assume Berkovits' superstring non-renormalization theorems can be carried over to $\mathcal{N} = 8$ supergravity	Green, Russo, Vanhove (2006)
Finite	Identified cancellations in multiloop amplitudes; lightcone gauge locality and $E_{7(7)}$, inherited from hidden N=4 SC gravity	Bern, Dixon, Roiban (2006), Kallosh (2009–12), Ferrara, Kallosh, Van Proeyen (2012)

note: above arguments/proofs/speculation are only lower bounds

→ only an explicit calculation can prove the existence of a divergence!

$\mathcal{N}$=8 Amplitude and Counter Term Structure

Loop order	4pt amplitude form (any dimension)	divergence first occurs in	Counter term	$M_4^{\text{tree}} stu \sim R^4$
1	$R^4 \times (\text{intgrl.})$	$D_c = 8$	$\sim R^4$	Green, Schwarz, Brink
2	$\partial^4 R^4 \times (\text{intgrl.})$	$D_c = 7$	$\sim \partial^4 R^4$	Bern, Dixon, Dunbar, Perelstein, Rozowsky
3	$\partial^6 R^4 \times (\text{intgrl.})$	$D_c = 6$	$\sim \partial^6 R^4$	Bern, Carrasco, Dixon, HJ, Kosower, Roiban
4	$\partial^8 R^4 \times (\text{intgrl.})$	$D_c = 5.5$	$\sim \partial^8 R^4$	Bern, Carrasco, Dixon, HJ, Roiban
5	? $\partial^8 R^4 \times (\text{intgrl.})$	$D_c = 24/5$?	$\sim \partial^8 R^4$?	
	? $\partial^{10} R^4 \times (\text{intgrl.})$	$D_c = 26/5$?	$\sim \partial^{10} R^4$?	

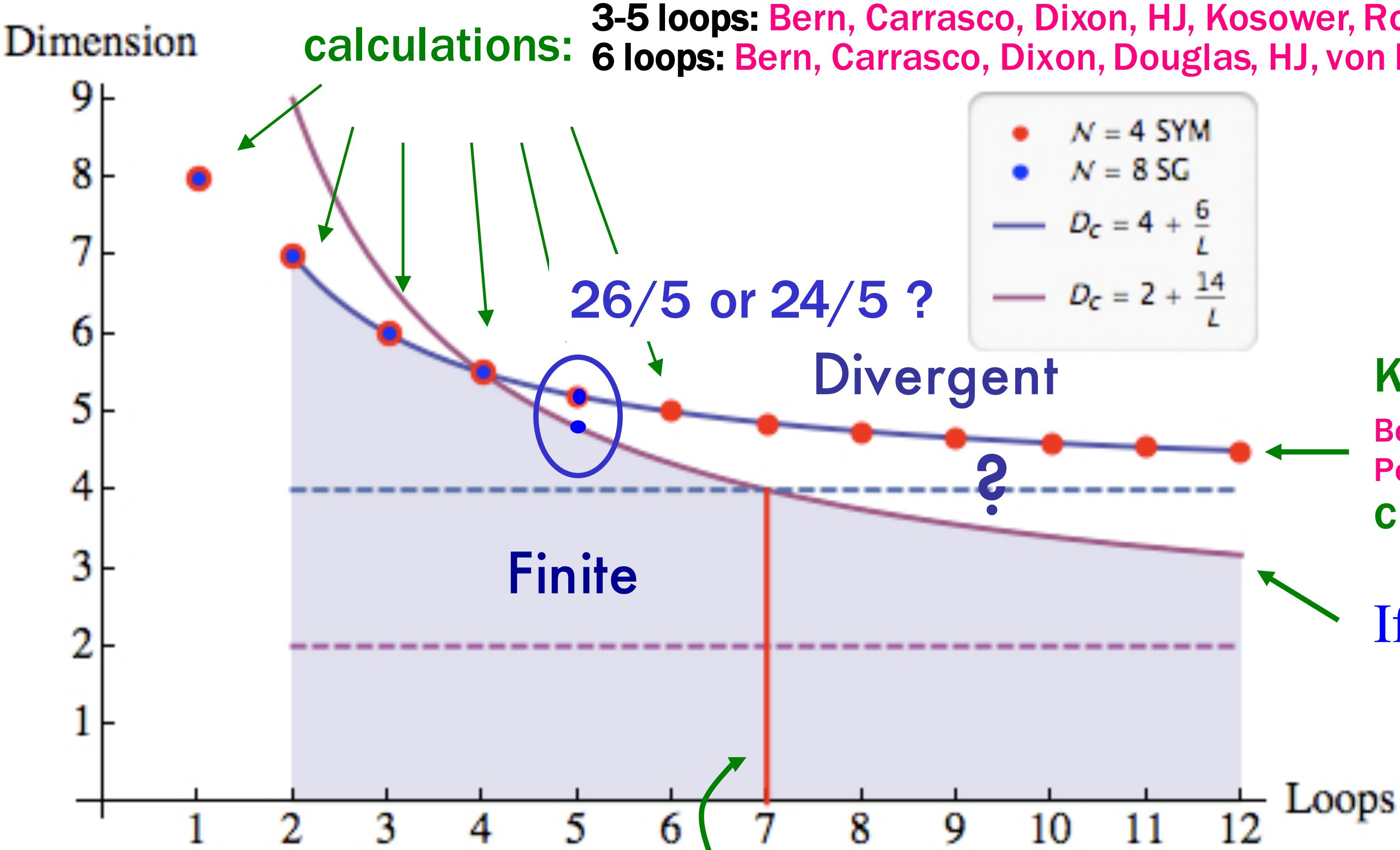
The critical dimension divergence tells us how many derivatives are pulled out of the integral → counter term structure

$$\partial^m R^4 \ @ \quad L \text{ loops} \quad \leftrightarrow \quad D_c = 2 + \frac{6}{L} + \frac{m}{L}$$

Known UV divergences in $D>4$

Plot of critical dimensions of $\mathcal{N}=8$ SUGRA and $\mathcal{N}=4$ SYM

calculations: **1-2 loops:** Green, Schwarz, Brink; Marcus and Sagnotti
3-5 loops: Bern, Carrasco, Dixon, HJ, Kosower, Roiban
6 loops: Bern, Carrasco, Dixon, Douglas, HJ, von Hippel



Known bound for $\mathcal{N}=4$

Bern, Dixon, Dunbar, Rozowsky, Perelstein; Howe, Stelle

current trend for $\mathcal{N}=8$

If $\mathcal{N}=8$ div. at $L=7$

$L=7$ lowest loop order for possible $D=4$ divergence

Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger;
Björnsson, Green, Bossard, Howe, Stelle, Vanhove Kallosh, Ramond, Lindström, Berkovits, Grisaru, Siegel, Russo, Cederwall, Karlsson, and more….

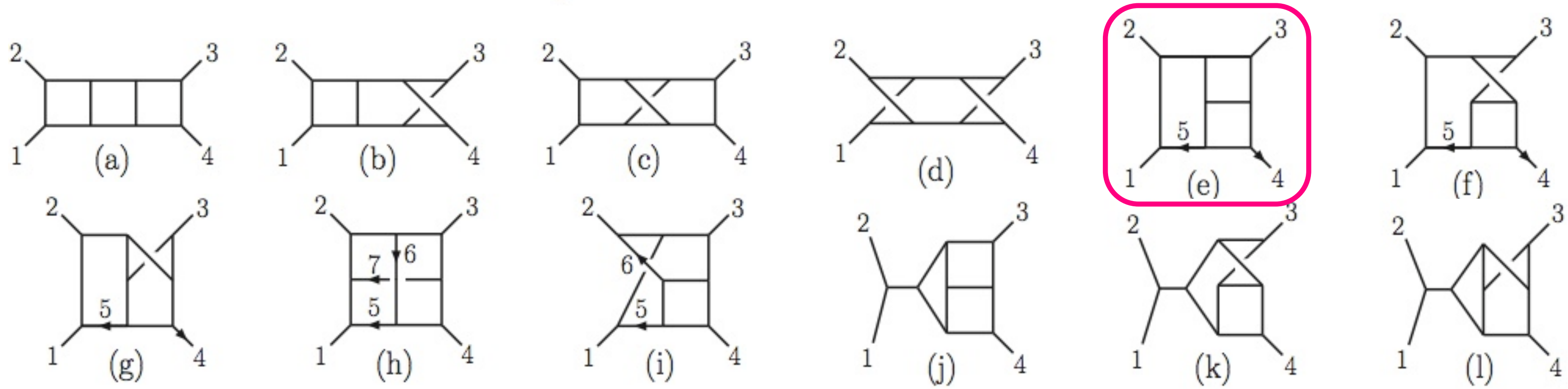
3,4,5-loop calculations

3-loop $\mathcal{N}$=8 SG & $\mathcal{N}$=4 SYM

Using color-kinematics duality: Bern, Carrasco, HJ

$$N^{(e)} = s(\tau_{45} + \tau_{15}) + \frac{1}{3}(t - s)(s + \tau_{15} - \tau_{25})$$

$\tau_{ij} = 2k_i \cdot l_j$

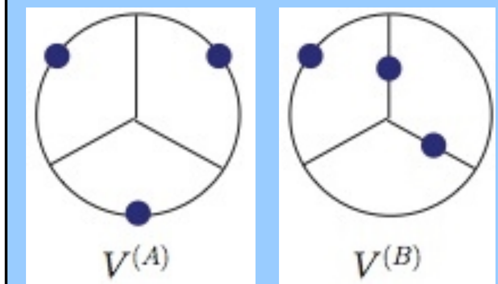


UV divergent in *D*=6: Bern, Carrasco, Dixon, HJ, Kosower, Roiban
Bern, Carrasco, Dixon, HJ, Roiban

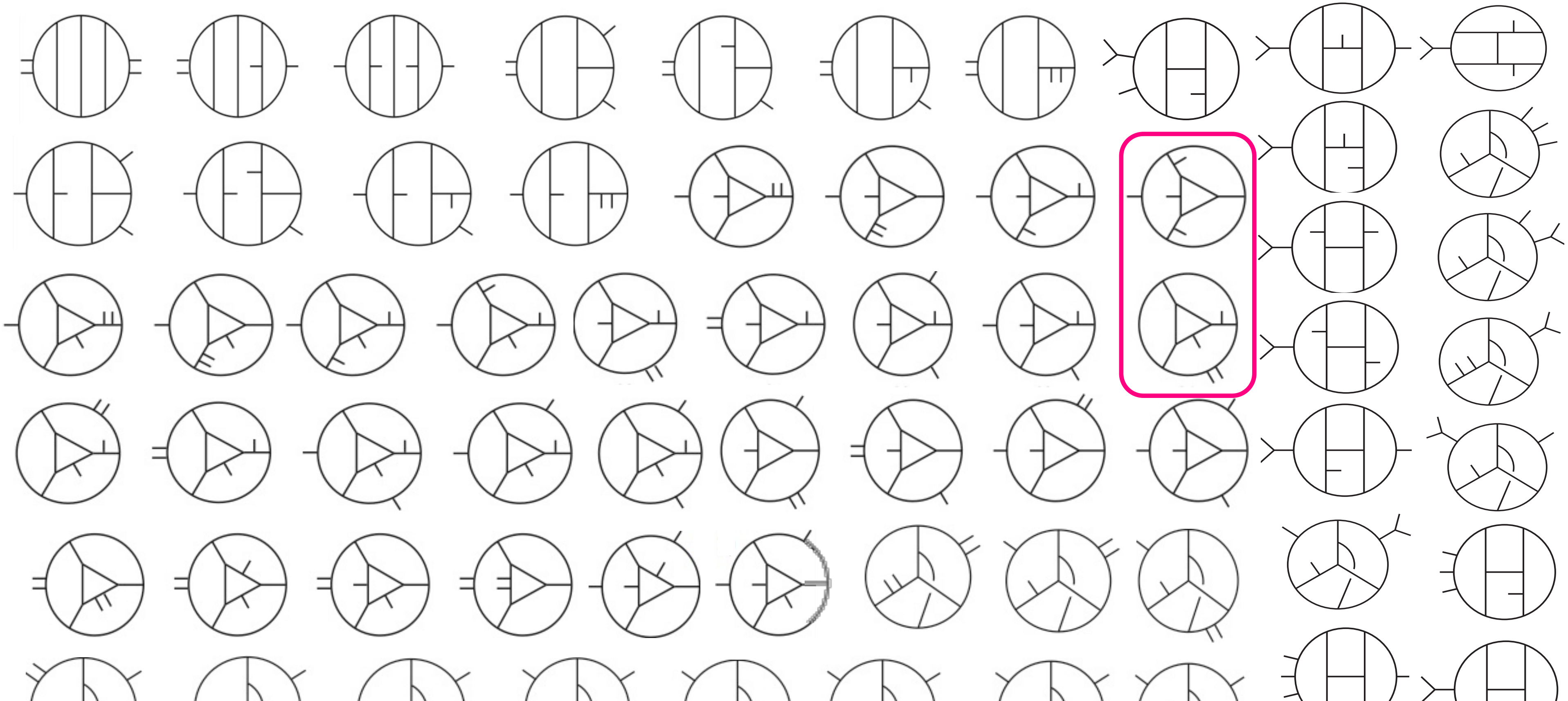
$$\mathcal{A}^{(3)}\Big|_{\rm pole} = 2g^8 st A^{\rm tree}(N_c^3 V^{(A)} + 12 N_c(V^{(A)} + 3V^{(B)})) \times (u{\rm Tr}[T^{a_1}T^{a_2}T^{a_3}T^{a_4}] + \text{ perms})$$

$$\mathcal{M}^{(3)}\Big|_{\rm pole} = 10\left(\frac{\kappa}{2}\right)^8 (stu)^2 M^{\rm tree}(V^{(A)} + 3V^{(B)})$$

$$V^{(A)} + 3V^{(B)} = \frac{\zeta_3}{6}$$



4-loops: 85 diagrams, 2 masters



$$
\begin{aligned}
N_{18} = \ & \tfrac{1}{4}(6u^2\tau_{25} + u(2s(5\tau_{25} + 2\tau_{26}) - \tau_{15}(7\tau_{16} + 6t)) \\
& + t(\tau_{15}\tau_{26} - \tau_{25}(\tau_{16} + 7\tau_{26})) + s(4\tau_{15}(t - \tau_{26}) + 6\tau_{36}(\tau_{35} - \tau_{45}) \\
& - \tau_{16}(4t + 5\tau_{25}) - \tau_{46}(5\tau_{35} + \tau_{45})) + 2s^2(t + \tau_{26} - \tau_{35} + \tau_{36} + \tau_{56}))\,, \\
N_{28} = \ & \tfrac{1}{4}(s(2\tau_{15}t + \tau_{16}(2t - 5\tau_{25} + \tau_{35}) + 5\tau_{35}(\tau_{26} + \tau_{36}) + 2t(2\tau_{46} - \tau_{56}) - 10u\tau_{25} \\
& - 4s^2\tau_{25} - 6u(\tau_{46}(t - \tau_{25} + \tau_{45}) + \tau_{25}\tau_{26}) - t(\tau_{15}(4\tau_{36} + 5\tau_{46}) + 5\tau_{25}\tau_{36}))\,.
\end{aligned}
$$

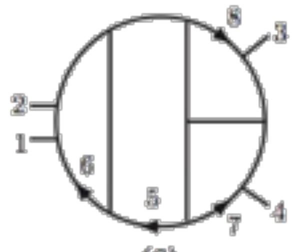
4-loop $\mathcal{N}$=8 SG and $\mathcal{N}$=4 SYM

Bern, Carrasco, Dixon, HJ, Roiban 1201.5366

- 85 diagrams
- Power counting manifest
- $\mathcal{N}$=4 & $\mathcal{N}$=8 diverge in D=11/2

$$N_6^{\rm SYM} = \tfrac{1}{2}s_{12}^2(\tau_{45} - \tau_{35} - s_{12})$$

$$N_6^{\rm SG} = \left[\tfrac{1}{2}s_{12}^2(\tau_{45} - \tau_{35} - s_{12})\right]^2$$



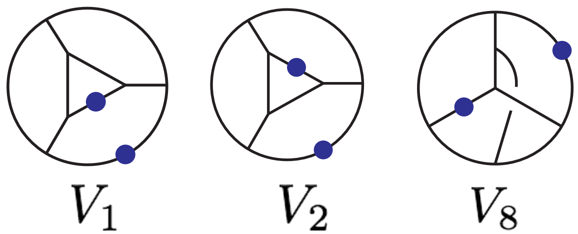
(6)

$\tau_{ij} = 2k_i \cdot l_j$

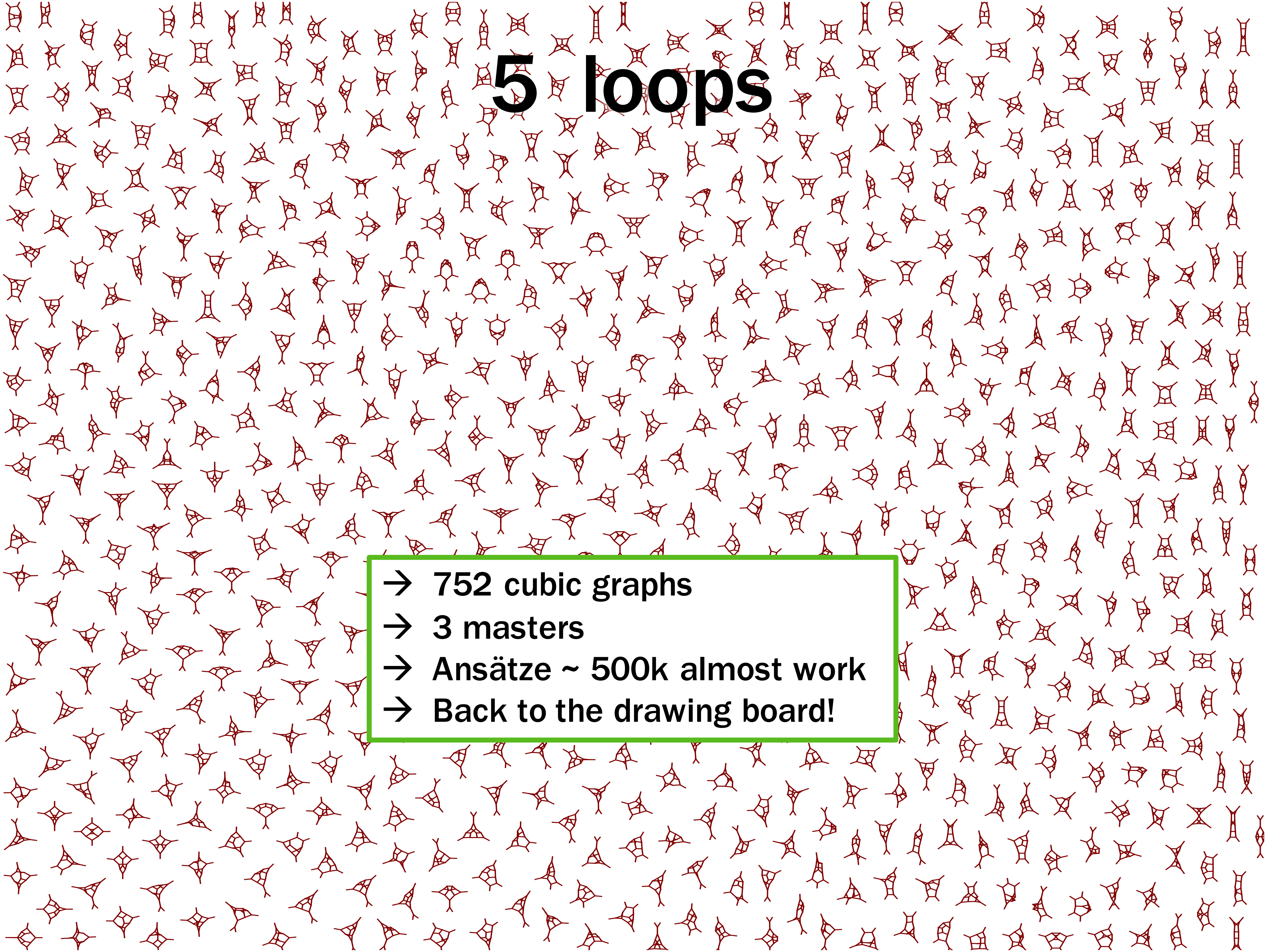
$$\mathcal{A}_4^{(4)}\Big|_{\rm pole} = -6g^{10} st A^{\rm tree} N_c^2 \Big(N_c^2 V_1 + 12(\underline{V_1 + 2V_2 + V_8})\Big) \times (u{\rm Tr}_{1234} + {\rm perms})$$

$$\mathcal{M}_4^{(4)}\Big|_{\rm pole} = -\frac{23}{8}\left(\frac{\kappa}{2}\right)^{10} stu(s^2+t^2+u^2)^2 M^{\rm tree}(\underline{V_1 + 2V_2 + V_8})$$

up to overall factor, divergence
same as for $\mathcal{N}$=4 SYM $1/N_c^2$ *part*



5 loops



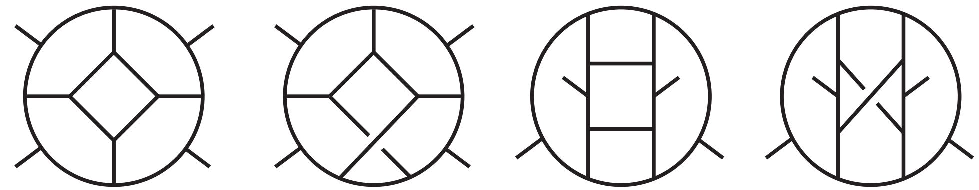
- → 752 cubic graphs
- → 3 masters
- → Ansätze ~ 500k almost work
- → Back to the drawing board!

5-loop $\mathcal{N}$=4 SYM the traditional way

$\mathcal{N}$=4 SYM important stepping stone to $\mathcal{N}$=8 SG

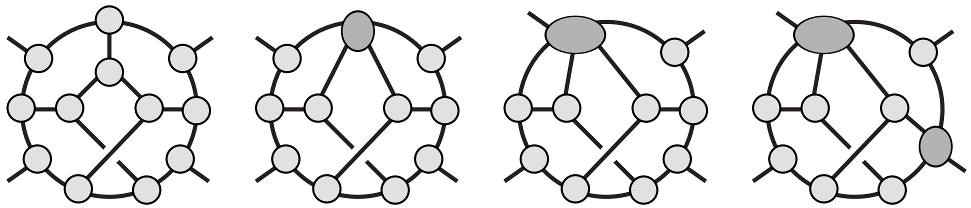
1207.6666 [hep-th]
Bern, Carrasco, HJ, Roiban

- 416 nonvanishing integral topologies:



(335) (370) (404) (410)

- Used maximal cut method
  Bern, Carrasco, HJ, Kosower
  - Maximal cuts: 410
  - Next-to-MC: 2473
  - N^2MC: 7917
  - N^3MC: 15156



MC NMC N^2MC N^3MC

Unitarity cuts done in *D* dimensions
integrated UV div. in *D*=26/5

Non-Planar UV divergence in D=26/5:

(a) (b) (c)

$$\mathcal{A}_4^{(5)}\Big|_{\rm div} = -\frac{144}{5} g^{12} s t A_4^{\rm tree} N_c^3 \left(N_c^2 V^{(\rm a)} + 12(V^{(\rm a)} + 2V^{(\rm b)} + V^{(\rm c)})\right) \times {\rm Tr}[T^{a_1}T^{a_2}T^{a_3}T^{a_4}]$$

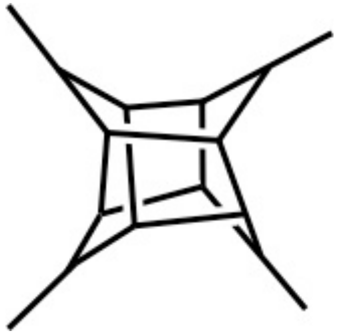
Key methods for 5 loops

Double copy is necessary

Unitarity & Ansätze possible way forward?

- Works for 5-loop *N*=4 SYM
- 5-loop SG seems too difficult

  (ansatz: billions of terms)



Pessimistic counting:

$$n^{\text{SYM}} \sim 8000 \text{ terms}$$

$$n^{\text{SG}} \sim (8000)^2/2$$

$$\sim 30\,000\,000 \text{ terms}$$

Only way: use some form of double copy

- On maximal cuts → naïve double copy works → square SYM numerators
- On non-maximal cuts → KLT works in principle, but not in practice
- KLT relations are non-local, non-crossing symmetric → bad for loops
- Need something better than KLT, and less constraining than BCJ

Generalized double copy --- when color-kinematics duality is non-manifest

Generalized Double copy

Bern, Carrasco, Chen, HJ, Roiban

Consider 4pt tree-level as warm-up:

YM $$A_4^{\mathrm{YM}} = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$ Assume: not BCJ numerators

Gravity $$M_4^{\mathrm{GR}} = \frac{n_s \tilde{n}_s}{s} + \frac{n_t \tilde{n}_t}{t} + \frac{n_u \tilde{n}_u}{u} + \text{contact}$$

Contact terms have to to vanish if numerator Jacobi relation holds

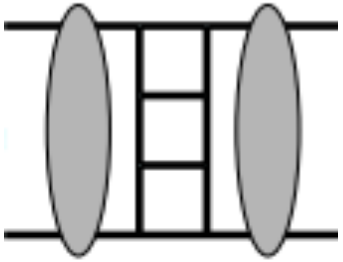
$$\left.\begin{aligned} \text{contact} &\propto J = n_s + n_t + n_u \\ \text{contact} &\propto \widetilde{J} = \tilde{n}_s + \tilde{n}_t + \tilde{n}_u \end{aligned}\right\} \Rightarrow \quad \text{contact} \sim J\widetilde{J}$$

Note: example too simple since all 4pt tree numerators obey BCJ

Generalized Double copy

Bern, Carrasco, Chen, HJ, Roiban

Consider two 4pt trees in a unitarity cut:



$$\text{YM} \quad \text{cut} = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{n_{i,j} c_{i,j}}{s_i^{\text{L}} s_j^{\text{R}}}$$

$$\text{GR} \quad \text{cut} = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{(n_{i,j})^2}{s_i^{\text{L}} s_j^{\text{R}}} + \text{contact}$$

sum rows or columns

$$\begin{bmatrix} n_{1,1} & n_{1,2} & n_{1,3} \\ n_{2,1} & n_{2,2} & n_{2,3} \\ n_{3,1} & n_{3,2} & n_{3,3} \end{bmatrix} \Rightarrow \begin{bmatrix} J_1^{\text{L}} \\ J_2^{\text{L}} \\ J_3^{\text{L}} \end{bmatrix}$$

Jacobi

$$\Downarrow$$

$$\begin{bmatrix} J_1^{\text{R}} & J_2^{\text{R}} & J_3^{\text{R}} \end{bmatrix}$$

Jacobi

In fact, the contact is given by

$$\text{contact} = \frac{J_i^{\text{L}} J_j^{\text{R}}}{s_j^{\text{L}} s_i^{\text{R}}}$$

independent of i and j

→ contact terms are bilinears in the Jacobi discrepancies

→ appears to work for general cuts

We can compute the integrand

Bern, Carrasco, Chen, Johansson, Roiban, Zeng

In 1708.06807 we compute the integrand

In addition to 410 cubic "top-level" diagrams we considered the contacts:

level	total # of diagrams	number of contact interactions						#vanishing
		1	2	3	4	5	6	
1	2,473	2,473	0	0	0	0	0	2473
2	7,917	1,597	6,320	0	0	0	0	6158
3	15,156	940	6,710	7,506	0	0	0	11894
4	19,567	434	5,232	9,510	4,391	0	0	14980
5	17,305	203	3,012	7,792	5,185	1,113	0	13239
6	10,745	83	1,567	4,407	3,694	896	98	7941

Total number of diagrams $\sim$ 17000

Superficial divergence in D=4 → Λ^4 power divergence in D=24/5

Too difficult to integrate! (it seemed)

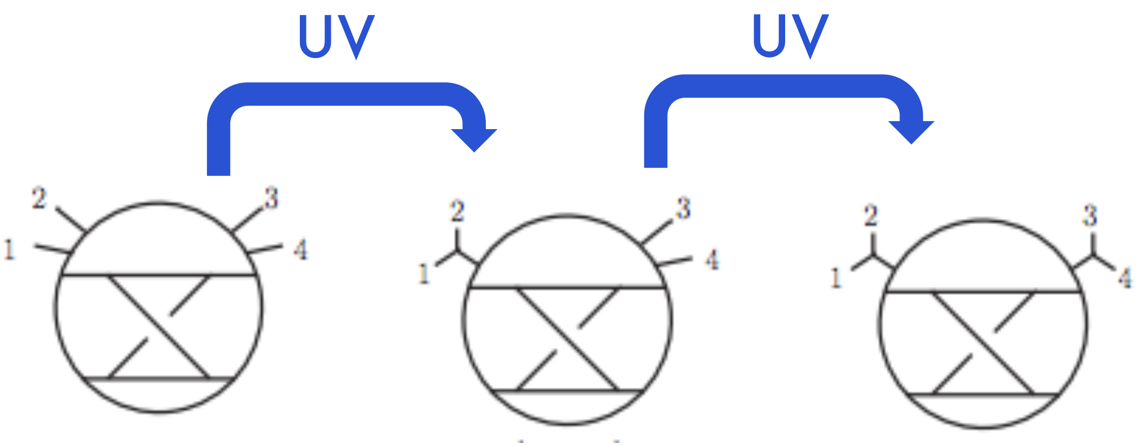
Controlling UV behavior of *N*=4 SYM

Bern, Carrasco, Chen, Edison, HJ, Parra-Martinez, Roiban, Zeng

In order to remove the power divergence (of most diagrams)

→ need to improve *N*=4 SYM numerators

→ Push the SYM divergence into propagator diagrams



$n_i \sim \ell^3$ $\qquad$ $n_i \sim \ell^2$ $\qquad$ $n_i \sim \ell$

→ Used Ansatz ~ 500k free parameters to move terms

→ Imposed that all unitarity cuts remained unchanged

Now: using generlized double copy gives few power divergent integrals

Integrating the N=8 amplitude

Bern, Carrasco, Chen, Edison, HJ, Parra-Martinez, Roiban, Zeng

Power-divergent contact diagrams are series expanded around soft external momenta (= infinite loop momenta).

→ Gives vacuum diagrams with dots (propagators to higher power)

$$I(p_i) \to I(\epsilon p_i) = \epsilon^{12} V_{\Lambda^4} + \epsilon^{14} V_{\Lambda^2} + \epsilon^{16} V_0 + \dots$$

The improved N = 8 supergravity integrand

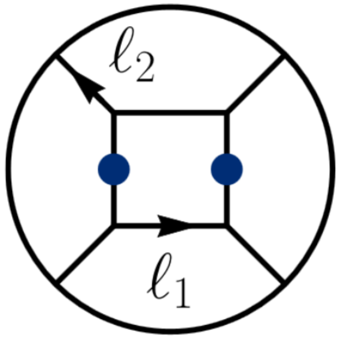
→ has 8473 distinct diagrams before integration

→ all cubic diagrams are manifestly log divergent in D=24/5.

→ vacuum diagrams with at most 4 dots are needed

→ ~140k distinct vacuum integrals

$(\ell_1 \cdot \ell_2)$



The old N = 8 supergravity integrand

→ has ~ 17000 distinct diagrams before integration

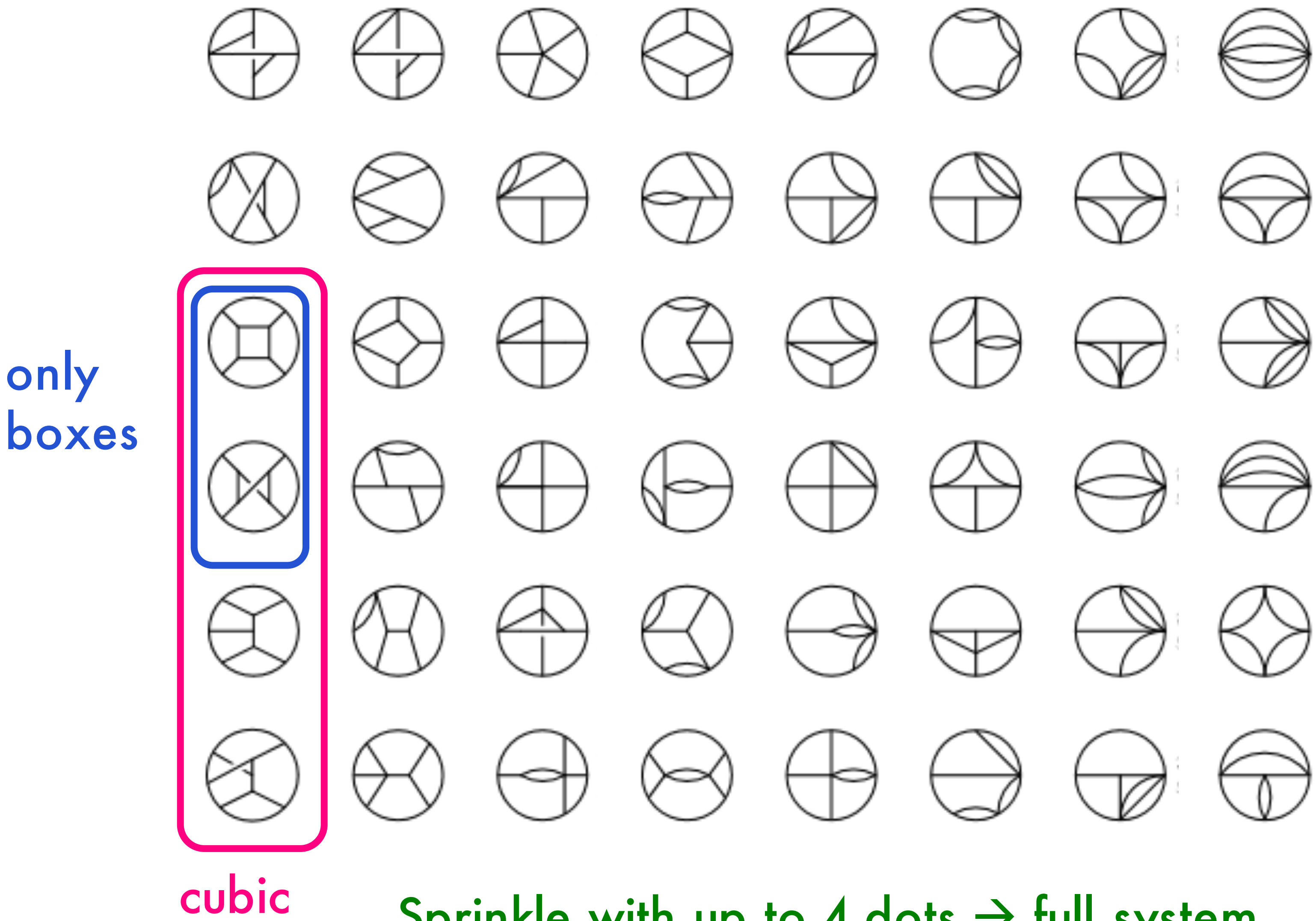
→ all cubic diagrams are quarticly divergent in D=24/5.

→ vacuum diagrams have up to 6 dots

→ ~ 17 million distinct vacuum integrals

The relevant vacuum topologies

Bern, Carrasco, Chen, Edison, HJ, Parra-Martinez, Roiban, Zeng



Sprinkle with up to 4 dots → full system

Summing up the result

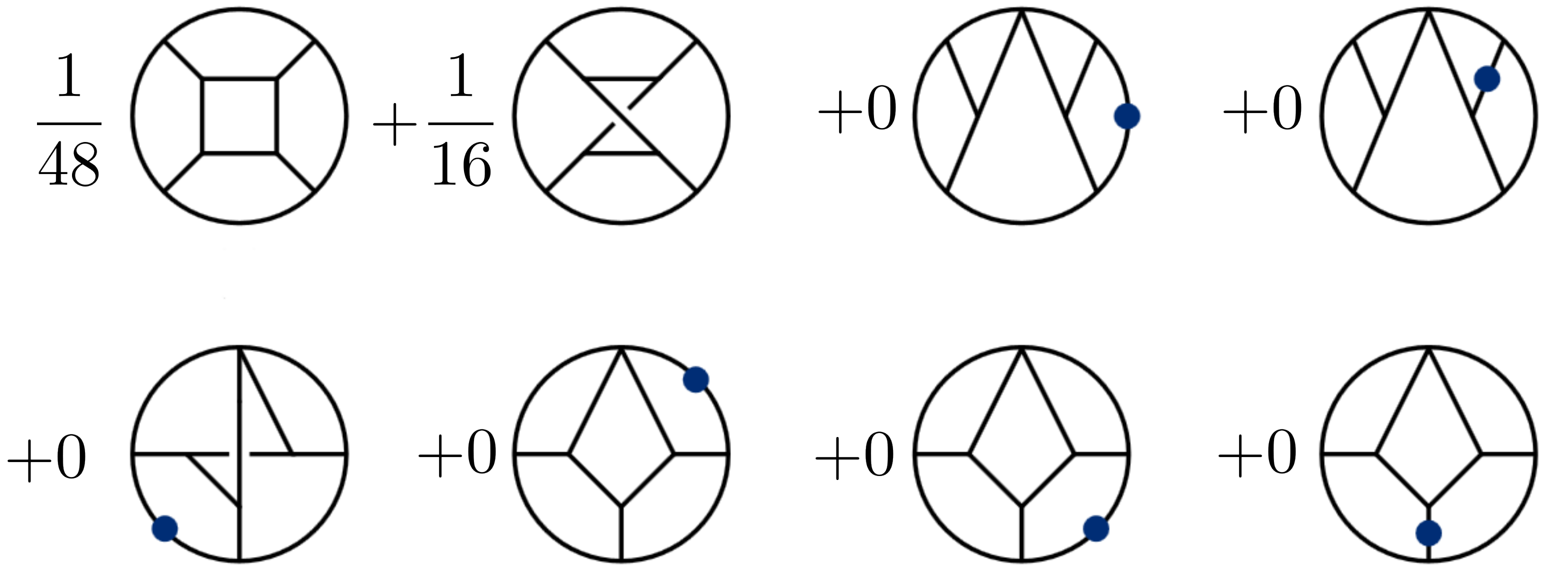
Bern, Carrasco, Chen, Edison, HJ, Parra-Martinez, Roiban, Zeng

In $D = 22/5$: UV finite, as expected.

In $D = 24/5$: considered 2.8 million relations between 850k integrals. System ~1 billion nonzero entries. Sparse Gaussian elimination over finite fields → 8 master integrals.

Schabinger, von Manteuffel, 2014; Peraro, 2016

After summing over all contribution, all but two master cancels out



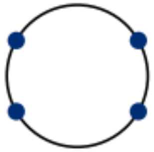
N=8 amplitude divergent in 24/5

Summary of *N*=8 SG divergences up to 5 loops

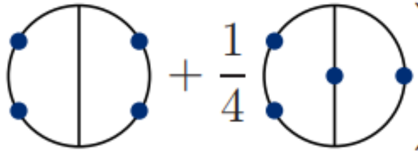
Bern, Carrasco, Chen, Edison, HJ, Parra-Martinez, Roiban, Zeng

In hindsight (after each calculation), the results are strikingly simple

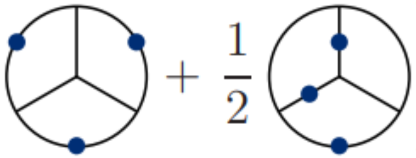
$$\mathcal{M}_4^{(1)}\Big|_{\text{leading}} = -3\,\mathcal{K}_{\text{G}} \left(\frac{\kappa}{2}\right)^4 \; [\text{diagram}],$$



$$\mathcal{M}_4^{(2)}\Big|_{\text{leading}} = -8\,\mathcal{K}_{\text{G}} \left(\frac{\kappa}{2}\right)^6 (s^2+t^2+u^2) \left(\frac{1}{4}\,[\text{diagram}] + \frac{1}{4}\,[\text{diagram}]\right),$$

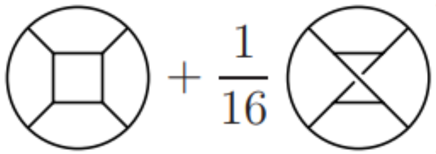


$$\mathcal{M}_4^{(3)}\Big|_{\text{leading}} = -60\,\mathcal{K}_{\text{G}} \left(\frac{\kappa}{2}\right)^8 stu \left(\frac{1}{6}\,[\text{diagram}] + \frac{1}{2}\,[\text{diagram}]\right),$$



$$\mathcal{M}_4^{(4)}\Big|_{\text{leading}} = -\frac{23}{2}\,\mathcal{K}_{\text{G}} \left(\frac{\kappa}{2}\right)^{10} (s^2+t^2+u^2)^2 \left(\frac{1}{4}\,[\text{diagram}] + \frac{1}{2}\,[\text{diagram}] + \frac{1}{4}\,[\text{diagram}]\right),$$

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16\times 629}{25}\,\mathcal{K}_{\text{G}} \left(\frac{\kappa}{2}\right)^{12} (s^2+t^2+u^2)^2 \left(\frac{1}{48}\,[\text{diagram}] + \frac{1}{16}\,[\text{diagram}]\right),$$



Can we use this pattern to predict behavior at $L = 6$ and $L = 7$?

Possible all-loop patterns?

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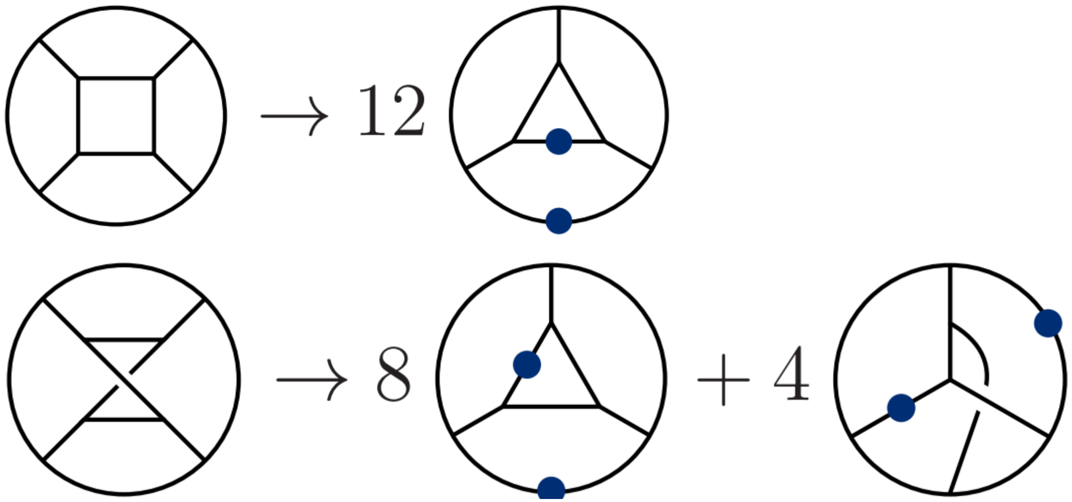
→ Cross-order relations from removing or cutting propagators



→ No triangle property for vacuum graphs

→ Color-kinematics duality for N=4 propagator diagrams predicts correct relative factors.

→ When integrating only vacuums diagrams with up to 4 dots needed allowed us to integrate old N=8 integrand → same answer.

Suggest that there is hope of obtaining 6 and 7 loop results !

Summary

- Explicit calculation in $\mathcal{N}=8$ SUGRA at five loops show that the theory is worse behaved in *D*>4 than $\mathcal{N}$=4 SYM.
- However, the implication for the *D*=4 theory is unclear.
- If good UV behavior of *N*=8 is tied to four-dimensional properties – as suggested by various proposed mechanisms – then *D*=24/5 might not mean much.
- 7 loop calculation in *D*=4 is thus more critical than ever
- Generalized double copy critical for 5-loop calculation, however color-kinematics duality has some glitch at 5 loops that is not yet understood
- Suggestive cross-order patterns in UV divergences of $\mathcal{N}$=8 SUGRA and as well as $\mathcal{N}$=4 SYM implies hidden simplicity for future calculations.
- Stay tuned for the 6- and 7-loop calculations!